Learning and Convergence to Nash Equilibria with Minimal Information

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February 2, 2005

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Abstract

We present evidence from a Cournot-Nash experiment in which participants were provided with no information regarding the structure of payoffs. Rather, participants had to discern how payoffs were determined based on the history of choices. In this minimal information environment, we find support for convergence to both stable and unstable Nash equilibria. Moreover, our results indicate that the behavior of many participants conforms to normative best-response dynamics.

JEL Codes: C72, C92, D83.

Keywords: Nash equilibrium, convergence, information, experiments.
1 Introduction

The concept of Nash equilibrium is central to much of the theoretical work in economics and game theory. While a large theoretical literature has explored the properties yielding convergence to stable and unstable Nash equilibria (e.g. Offerman et al., 2002; Nyarko, 1998), there exist significant questions regarding whether or not individuals actually play the type of strategies yielding convergence to Nash equilibria.

To this end, experimental research has explored the ways in which individuals learn to play strategies in both the context of stable and unstable Nash equilibria. For example, Cox and Walker (1998) find that individuals learn to play stable interior equilibria but do not play unstable or boundary equilibria in Cournot duopoly games. In five participant Cournot games, Rassenti et al. (2000) find only weak evidence that participants’ strategies converge to Nash equilibrium. Similarly, Huck et al. (2002) find no differences between experimental markets that should converge to a Nash equilibrium versus markets that should not. Thus, the experimental evidence suggests that the practical import of Nash equilibrium is suspect: The lack of strong evidence for Nash play in many experiments raises the issue of whether or not participants in experiments can learn to play a concept so central to economic analysis.

A potential explanation for these results may lie in the information participants use when “learning to play” in these experimental games. Namely, when participants have full information about the payoff structure of a game, their learning is focused on predicting the behavior of others in the game. As such, significant attention is concentrated on the strategic aspects of players’ interactions and learning is focused on better predicting others’ behaviors. On the other hand, when individuals must not only predict the behavior of others but also learn the structure of payoffs, the strategic aspects of players’ interactions may be muted in their decision calculus. As a result,
participants must learn everything about the game and may be more focused on identifying the relationship between choices and payoffs. This focus of attention stresses the use of best response dynamics (which are based on making optimal decisions given the behaviors of others) and thereby yields greater Nash-type play. The key here is that too much information may sway participants’ decision making away from the central idea underlying Nash best response dynamics (e.g. maximizing payoffs versus predicting the behavior of or regarding the payoffs of others).

As partial evidence of the import of this information, Gallego (1998) finds support for individuals playing Bertrand–Nash equilibria in a game where players have no information about market conditions. As a result, participants had to learn from experience over repeated rounds of decision making. It is in this environment that Nash equilibrium serves as an attractor for players’ strategies. This is offset by the results of Huck et al. (2002) in which participants had compete information about the game but choices yielded little evidence of convergence to Nash predictions.\(^1\)

In this paper, we provide an alternate test as to the strength and import of Nash equilibria in the ways individuals play experimental games. We report on a laboratory experiment in which individuals had minimal information about the decision environment. In our experiment, individuals participated in a two player, 11 strategy game in which symmetric payoffs were non-linear in each player’s strategy. This game (based on the common property game of Brooks et al., 1999) is characterized by stable, unstable and efficient (i.e. payoff dominant) equilibria. Moreover, participants in the game had no initial information regarding the structure of payoffs in the game. Thus, in addition to considering the strategic environment of the game, participants had to learn how payoffs are determined. It is in this minimal information design

\(^1\)In this experiment participants were explained the demand and cost functions of all firms and had profit calculators which provided individuals with their profit maximizing choices contingent upon hypothetical choices of others.
(i.e. no information regarding the structure of payoffs) that we find support for Nash behavior and best response dynamics. Not only do we find participants playing all of these equilibria, but we also find that traditional Nash best response dynamics characterize much of players’ behavior.

2 Experimental Design

In our experiment, participants were randomly paired and asked to choose integer valued decision numbers in $x \in [0,10]$. Payoffs to participants were based on a modified version of the common property game of Brooks et al. (1999).\(^2\) Letting $y \in [0,10]$ represent the choice of an individual’s partner, an individual’s single period payoff is given by

$$
\pi(x; y) = \frac{40x - (x + y)x}{x + y - 3.5}
$$

Restricting participants’ choices to integer values $x \in [0,10]$ yields the payoff table in figure 1.

Figure 1 about here.

The best response functions in a two player game with payoffs described by equation (1) are presented in figure 2, drawn with continuous strategies over the unit interval. Note that there are three “types” of equilibria in this game. The first equilibrium (A in figure 2; $(x, y) = (8, 8)$ in figure 1) corresponds to the standard, full rent dissipation equilibrium common to these types of games. Note that this equilibrium is stable in the sense that the best response dynamic in response to a minor deviation by one player will yield a return to this equilibrium. The second equilibrium (B in

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\(^2\)Brooks et al. present an infinite time, $n$ player, harvesting game with a stock dynamic. Interestingly, the authors identify equilibria which do not yield full rent dissipation under free-access. We refer the reader to their paper for full details on this model.
figure 2; \((x, y) = (4, 4)\) in figure 1) is unstable in that the best response to a deviation from this equilibrium yields movement towards equilibrium \(A\) \((x, y) = (8, 8)\) or towards the latter type of equilibria. This latter class is a continuum of equilibria (along the cord marked \(C\) in figure 2; \(x + y = 4\) in figure 1) which are efficient in that total payoffs are maximized.

Figure 2 about here.

Our experiment used the payoffs in figure 1 to test the degree to which individuals choices conformed with Nash play given the above payoff structure. We conducted two treatments. In the partner treatment, 28 participants were randomly assigned into dyads which were maintained throughout 20 decision making rounds. In the random treatment, 14 subjects were randomly assigned to dyads in 30 decision making rounds. Thus a participant could not be certain that she was playing with the same partner across rounds.

In the experiment, participants were provided with no information regarding the structure of the game.\(^3\) That is, individuals were told only that their payoff was dependent upon an unknown relationship between their decision number and that of their partner. As the decision making rounds progressed, participants were provided with a history of their choices, that of their partners, and their corresponding payoff. The experiment was conducted in our university’s experimental economics laboratory with participants recruited from the population of undergraduate students. The experiment was programmed in z-Tree (Fischbacher, 1999).

\(^3\)See the instructions in appendix A.
3 Results

We analyze the data in two ways. First, we consider the extent to which participants played one of the aforementioned equilibria (A, B, or C). Our attention here is on whether or not participants converged to any of the equilibria given the lack of information regarding the structure of payoffs. Secondly, we analyze the data with respect to the types of response rules used in play. Specifically, we test whether participants’ choices conformed with either best response dynamics, fictitious play, imitation or inertia.

Convergence to Nash Equilibrium

To test for convergence to one of the Nash equilibria, we calculate the Euclidian distance between a participant’s choice and the proposed equilibrium, testing if this distance shrinks over rounds of play. That is, for each participant and equilibrium A (stable at \( x^*_A = 8 \)), B (unstable at \( x^*_B = 4 \)), and C (stable at \( x^*_C = 2 \)) we regress

\[
|x_{it} - x^*_j| = \beta_0 + \beta_j t + \varepsilon_{it},
\]

where \( x_{it} \) is participant \( i \)’s choice in period \( t \), \( x^*_j \) is the equilibrium choice for equilibria \( j \in \{A, B, C\} \), and \( \varepsilon_{it} \) is an error term. We test the null hypothesis \( \beta_1 \geq 0 \) against the alternative hypothesis \( \beta_1 < 0 \). In table 1 we report the number of times the null hypothesis is rejected as well as the percentage of times was rejected.

Table 1 about here.

In the partner treatment, 80% of participants’ choices converged to one of the equilibria, with the largest share converging to the Pareto dominant equilibrium C. In the random treatment, 50% of participants converged to one of the equilibria,
with the largest percentage converging to the unstable equilibrium $B$. What makes these results so striking is that participants had no information regarding the payoff structure. Thus, the only information participants used was the history of choices and payoffs emerging as the game was played.

We take two key findings from this result. First, participants can (and do) play strategies yielding each of the Nash equilibria. Thus, while in many experimental games we fail to observe convergence to traditional Nash play, in our game (where individuals did not ostensibly know the payoffs of others) Nash play emerges. Indeed, it is quite striking the extent to which those in the random treatment converged to the unstable equilibrium (21%) and those in the partner treatment converged to the Pareto dominant equilibrium (32%) given they initially had no information regarding how payoffs were determined.

**Choice Strategies**

We next analyze the strategies players used in the game. That is, given the lack of information we are interested in how participants played the game. To do so, we calculated each individual’s choice based on the previous choice of her partner for each of the following decision rules: a traditional best response dynamic using equation (1), fictitious play, imitation, and inertia. We then regressed the following:

\[
x_{it} = \beta_0 + \beta_1 BR_i(y_{it-1}) + \beta_2 BR_i(\bar{y}_{it-1}) + \\
\beta_3 IM_i(y_{it-1}) + \beta_4 IN_i(y_{it-1}) + \varepsilon_{it}. \quad (3)
\]

The variable $BR_i(y_{it-1})$ is the best-response by participant $i$ to $y_{it-1}$ (the previous choice of the person with whom $i$ was matched). Similarly $BR_i(\bar{y}_{it-1})$ is the best-response to the observed vector $\bar{y}_{it-1}$ of past choices by the person with whom $i$ was
(or had been) matched. This permits us to identify if the participant was responding to fictitious play. Finally, $IM_i(y_{it-1})$ and $IN_i(y_{it-1})$ are the simplest form of imitation and inertia by participant $i$.

To test for the presence of each strategy, we use the following. If the subject responds only by best-response, then $\beta_1 = 1$ and $\beta_2 = \beta_3 = \beta_4 = 0$ is expected. If the subject responds only to fictitious play, then $\beta_2 = 1$ and $\beta_1 = \beta_3 = \beta_4 = 0$ is expected. If the subject only plays the imitation strategy, then $\beta_3 = 1$ and $\beta_1 = \beta_2 = \beta_4 = 0$ is expected. Finally, if the subject only plays with inertia, then $\beta_4 = 1$ and $\beta_1 = \beta_2 = \beta_3 = 0$ is expected. More generally, we expect subjects to play some combination of these, in which case the hypothesis is that $\sum_{k=1}^{4} \beta_k = 1$ where $\beta_k \geq 0$ for all $k \in \{1, 2, 3, 4\}$.

Table 2 about here.

We estimate equation (3) for each participant across all periods of play (excluding the first period). For each treatment, we report in table 2 the number of participants for whom we cannot reject the hypothesis that $\beta_k = 1$ for each $k \in \{1, 2, 3, 4\}$ based on an $F$-test ($F(1, 24)$ in random treatment; $F(1, 15)$ in partner treatment). In the partner treatment, it is striking to note how many players’ choices conformed to a traditional Nash best response (64%) given the lack of information on the payoff function.\footnote{Given the equilibria are symmetric, the evidence of imitation may not be true imitation but rather a best response to one’s partner at an equilibria.} This suggests that participants were able to discern the optimal direction of play in this game. In the random treatment a significant proportion of participants’ behaviors conform to best-response dynamics (43%). These results are supportive of a decision maker utilizing the available information (the history of choices and corresponding payoffs) to respond optimally. It is interesting to note that relatively few participants appear to have followed a fictitious play rule (11% and 29% in the
partner and random treatments), perhaps owing to the complexity of the decision environment and the complexity of such a decision rule (cf. Erev and Roth, 1998).

4 Conclusion

In our experiment, individuals were faced with a complex decision environment. Not only were payoffs a non-linear function of the decision numbers chosen by each individual, but participants were provided with no information regarding the structure of the payoff relationship. Strikingly, we observe significant incidences of Nash behavior. Participants were not only able to converge to the various equilibria, but also appear to have used best-response dynamics in playing the game. We take this as evidence of the presence of best response dynamics in experimental games and the import of these dynamics in yielding convergence to Nash equilibria.
References


9
A Instructions

In the following experiment you will be asked to make a series of decisions. As these decisions are strategic in nature, we ask that you refrain from talking with one another during the experiment. If you have any questions, please ask one of the experimenters. You may receive a monetary payment for your participation in the experiment. This payment is both compensation for your time as well as the effort you put into your decisions. To receive your payment, you will have to sign a receipt. This information is confidential and there will be no information tying you to the data.

The experiment will proceed in rounds of decision making. There will be at least 20 rounds of decision making.

1. **Appearing in partner treatment**: At the start of the experiment you will be randomly matched with another participant. You will remain paired with this participant for all decision making rounds.

   **Appearing in the random treatment**: In each round you will be randomly paired with another individual in the room.

2. You will be asked to choose a decision number between zero (0) and ten (10), inclusive. This decision number along with that chosen by the person with whom your are paired will determine your payoff. Your payoffs (denominated in lab dollars) represent the money you will receive for your participation in the experiment. At the end of the experiment your payoff in lab dollars will be converted to dollars at a rate of 1 lab dollar = $0.01. Since you may incur losses as a result of your decisions, you will be initially allocated 500 lab dollars ($5.00).
You do not currently know the relationship between decision numbers and payoffs. In each round, your choices, your partner’s choices, and your payoff in each period will be displayed on the left-hand side of your computer screen. You may use this information to identify the relationship between your decision number and that of your partner in determining your payoffs.

You will have a maximum of two minutes to make a decision. Once you have chosen a number, enter that number in the space provided and click on the button marked “Make Decision.”

3. Once everyone has made their decision, a screen will appear indicating your decision number, that chosen by the person with whom you were paired, your payoff for that period, and your accumulated payoff. After viewing this information, please click on the button marked “Continue.” Once everyone has clicked continue, a new decision round will begin.

At the end of the experiment, there will be a series questions. Please complete these questions, clicking on the “OK” button as you finish each page.

This will conclude the experiment. Please remain seated as we discuss the experiment and prepare your payoffs.

Thank you for your participation.
Figure 1: Payoffs utilized in experiment. In each cell, the top number is the payoff of the individual choosing $x$ while the bottom number is the payoff to the individual choosing $y$. 

<table>
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<tr>
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<th>$X^1$</th>
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<th>$X^5$</th>
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Figure 2: Best response functions implied by payoff function. Figure is drawn using continuous strategies over the unit interval.
Table 1: For each participant in each treatment, number of times $\beta_1 \geq 0$ (from equation 2) is rejected for each equilibrium.

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<th>$B$</th>
<th>$C$</th>
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<td>9</td>
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<td></td>
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<tr>
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<td>3</td>
<td>2</td>
</tr>
<tr>
<td>percent</td>
<td>14.3</td>
<td>21.4</td>
<td>14.3</td>
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</table>

Table 2: Number of participants for whom we cannot reject the null hypothesis that one of the above strategies was used by the participant ($\beta_k = 1$ for $k \in \{1, 2, 3, 4\}$ in equation 3).

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