Effective Property Rights, Conflict and Growth

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Abstract

This paper shows how the interaction between conflict and growth can give rise to a nonmonotone relationship between property rights and social welfare. This interaction is illustrated in a model of endogenous growth in which equilibrium diversion of resources is the cost of securing effective property rights. A symmetric equilibrium allocation associated with more secure property rights and faster growth can be Pareto dominated by one associated with poorer property rights and slower growth. Faster growth can exacerbate the problem of diversion whenever property rights are sufficiently poor. These results call for caution before a society decides to pursue economic growth independently of the institutional structure of property rights. Furthermore, if this structure is inappropriate piecemeal reform might not be in the interest of society, and a substantial reform might be necessary if it is to be welfare-improving.

Keywords: property rights, conflict, diversion, investment, growth

JEL classification: D23, O10, O40

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1 Introduction

The efforts of men are utilized in two different ways: they are directed to the production or transformation of economic goods, or else to the appropriation of goods produced by others. ¹

Vilfredo Pareto

Lack of development or slow growth, insecure property rights and widespread rent-seeking activities are common features of many economies. Case studies suggest that insecure property often discourages investment and is accompanied by conflict over economic distribution. A consequence is that productive resources might be diverted towards appropriative activities (e.g., De Soto (1989)). Cross-country studies find that countries where property is more insecure suffer lower capital accumulation, productivity and growth (e.g., Keefer and Knack (1997), Hall and Jones (1999), Acemoglu et al. (2001)). The development of the rule of law and secure property rights is also commonly regarded as having played a critical role in the development of western societies (North (1990)). It is therefore tempting to view relatively more secure property rights as a more efficient institutional arrangement.

This paper analyzes the effect of property rights on the conflict-growth relationship and the efficiency of social allocations. The goal of the paper is to argue why the commonsense logic suggesting that more secure property rights are conducive to Pareto superior social allocations can be misleading. My focus is on individuals’ incentives to engage in appropriative activities when the government cannot perfectly enforce the law and protect property rights. This is a dimension receiving increasing attention in the context of the East European transition to capitalism (e.g., Johnson et al. (2002), Roland and Verdier (2003)) and the individualization of property rights in sub-Saharan Africa (e.g., Platteau (2000)).

¹As quoted by Hirshleifer (1988).
Decentralized conflict over economic distribution is modeled in the context of an AK growth model. A technology of conflict summarizes this process, accounting explicitly for the distinction between the defense and the challenge of claims to property. In this context, the security of property becomes the endogenous outcome of the conflict technology as a function of the resources allocated to the defense of property, the resources allocated to the challenge of property claims, and the exogenous institutional structure of property rights.

The model illustrates a surprisingly complex relationship between property rights and social welfare. Specifically, the interaction between conflict and growth can lead to an equilibrium allocation associated with more secure property and faster growth being Pareto dominated by one associated with less secure property and slower growth. To see why, consider the effect of an improvement in the institutional structure of property rights. In the present context, this corresponds to an exogenous increase in the ease of establishing effective property rights. Intuitively, this translates into endogenously more secure property. Now consider how this is achieved. On the one hand, more secure property raises the returns to productive activities relative to appropriative activities. Thus, it discourages diversion and promotes growth. This is the conventional tradeoff between growth-enhancing and appropriative activities. On the other hand, conflict rises along higher-growth paths, as individuals allocate resources across investment activities so their marginal returns are equated. Social welfare can decline because the diversion of resources that accompanies growth ultimately takes place at the expense of


consumption, and the welfare benefits from faster growth can be overwhelmed by the cost of increased conflict.

The analysis also illustrates how the poor enforcement of property rights can lead to situations where the dissipation of resources due to conflict is greater when growth is faster. In this context a tradeoff arises between the diversionary impact of conflict on growth and the increased diversion that accompanies faster output growth. Which effect dominates depends on the institutional structure of property rights. Specifically, growth that is not accompanied by sufficiently secure property can exacerbate the problem of diversion.

This possibility calls for caution before a society decides to pursue economic growth independently of the structure of property rights. Furthermore, the model implies that, if the institutional structure is inappropriate, piecemeal reform might not be in the interest of society, and substantial reform might be necessary if it is to be welfare-improving. Thus, the model supports the view that institutional reform, regarding the definition and enforcement of property rights, should, in certain cases, take the form of a major institutional change.

The present paper is close in spirit to the work of Grossman and Kim (1996) although both the model and its implications are very different. Unlike the present paper, they consider the consequences of strategic appropriative interaction in an asymmetric context and they focus on the deterrent role of defensive activities. They formalize the idea that for secure property rights to arise as an equilibrium outcome, resources must be sacrificed, and the corresponding diversion of resources away from growth-enhancing activities can lead to lower growth. In the present context this avenue is closed down; more secure property always leads to faster growth; instead, the focus here is on the possibility that increases in the security of property and growth might not be in the interest of society.
The following section presents the basic model to be analyzed, and Section 3 discusses the paper’s main result. Section 4 discusses further implications and Section 5 concludes. Proofs of all propositions are provided in an appendix.

2 A Model of Growth, Conflict and the Security of Property

Consider a society consisting of a unit measure of individuals. Each agent seeks to maximize

$$\sum_{t=0}^{\infty} \beta^t \log c_i(t), \quad \beta \in (0, 1),$$

(1)

where $c_i(t)$ is agent $i$’s consumption at date $t \geq 0$. Every period agent $i$ can produce $Ak_i(t)$ ($A > 0$) units of output using an amount $k_i(t) \geq 0$ of resources. Agent $i$’s output becomes available the following period. However, his claims over next-period output are insecure. Instead, initial claims must be converted into effective property rights. Agent $i$ can influence this process by allocating resources to appropriative activities. Specifically, agent $i$ may allocate an amount $x_i(t) \geq 0$ of resources to the defense of his own claims to property against all other agents and an amount $z_i(t) \geq 0$ to the challenge of the claims of others.

To formalize the consequences of decentralized conflict over economic distribution as simply as possible I suppose that each agent competes against the economy’s average. Agent $i$ appropriates the share $p_i(t)$ of his date-$t$ output and the share $q_i(t)$ of the average output $Ak(t)$; at each date $t$ agent $i$ enjoys effective property rights over $p_i(t) Ak_i(t) + q_i(t) A k(t)$. At date 0 each agent is endowed with $Ak(0) > 0$ secured resources. Subsequently, agent $i$ allocates his secured resources across consumption and investment activities, facing the resources constraint

$$p_i(t) Ak_i(t) + q_i(t) A k(t) = c_i(t) + k_i(t + 1) + x_i(t + 1) + z_i(t + 1),$$

(2)

where I have assumed, for simplicity, that all capital stocks depreciate fully every period.
Letting $x(t)$ and $z(t)$ be the economy-wide average of each type of appropriative activity, agent $i$’s shares are determined according to

$$p_i(t) = \frac{\pi x_i(t)^m}{\pi x_i(t)^m + z(t)^m},$$

(3)

and

$$q_i(t) = \frac{z_i(t)^m}{\pi x(t)^m + z(t)^m},$$

(4)

for $x(t) + z(t) > 0$, where $0 < m \leq 1$ and $\pi \geq 1$. In order to ensure positive growth whenever $\pi \geq 1$ it is assumed that $\beta A > 2$. The share $p_i(t)$ provides a natural measure of the security of agent $i$’s claims to property. I view the pair $\{\pi, m\}$ as reflecting the (exogenous) ‘institutional structure of property rights’, which depends on the quality of the legal system of property rights as well as the set of social norms that influence the creation of effective property rights. The parameter $m$ determines the strength of the diminishing returns to both appropriative activities. Assuming that $m \leq 1$ ensures that each agent faces decreasing returns to each appropriative activity throughout. The parameter $\pi$ determines the effectiveness of the defense of property claims relative to the challenge of claims. Assuming that $\pi \geq 1$ simply rules out the cases where the challenge of claims is more effective than the defense of claims. As $\pi$ approaches infinity each agent’s private returns to productive activities become perfectly secure at a negligible cost.

The special features of the specification (3)–(4) are its symmetry across agents and the fact that $p_i(t)$ and $q_i(t)$ are homogeneous of degree zero. 6

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4To complete the specification of the conflict technology it is assumed that an arbitrary distribution $\{p_i(t), q_i(t)\} = \{\tilde{p}, \tilde{q}\}$, with $\tilde{p} + \tilde{q} = 1$, results whenever $x(t) + z(t) = 0$.

5It is easily verified that $\partial^2 p_i(t)/\partial x_i(t)^2 < 0$ if and only if $p_i(t) > (m - 1)/(2m)$ whereas $\partial^2 q_i(t)/\partial z_i(t)^2 < 0$ if and only if $q_i(t) > (m - 1)/(2m)$.

6Clark and Riis (1998) axiomatize a related conflict technology. Alternatively, modeling the asymmetry between the defense and the challenge of property claims by assuming differences in the parameter $m$ requires either that $m$ differs across agents, in which case the conflict technology will not be symmetric, or that $m$ differs across appropriative activities, in which case the shares will no longer be homogeneous of degree zero.
I shall restrict attention to symmetric equilibria and focus directly on equilibrium behavior, taking advantage of the fact that, with a continuum of agents, individual deviations are undetectable and consequently, within an equilibrium, each individual does not need to know the full strategy of others. Accordingly, an individual allocation \( \{c_i(t), k_i(t), x_i(t), z_i(t)\}_{\forall t} \) and an average allocation \( \{c(t), k(t), x(t), z(t)\}_{\forall t} \) constitute an equilibrium if (a) the allocation \( \{c_i(t), k_i(t), x_i(t), z_i(t)\}_{\forall t} \) maximizes (1) subject to (2)–(4), and (b) \( \{c_i(t), k_i(t), x_i(t), z_i(t)\}_{\forall t} = \{c(t), k(t), x(t), z(t)\}_{\forall t} \).

3 Equilibrium Analysis

The analysis aims to show why symmetric equilibria of the model exhibit the following property:

If \( \beta \) and \( m \) are sufficiently high, there exists an interval \( (\bar{\pi}, \bar{\pi}) \) such that social welfare \( \left( \sum_{t=0}^{\infty} \beta^t \log c(t) \right) \) declines with \( \pi \) for \( \pi \in (\bar{\pi}, \bar{\pi}) \). This is so despite the fact that both the security of property \( (p_i(t)) \) and growth in production \( \left( \frac{k(t+1)}{k(t)} \right) \) increase with \( \pi \).

3.1 Symmetric Equilibria

Intuitively, symmetric equilibrium allocations must be interior. Accordingly, they satisfy the Euler equations

\[
\frac{c_i(t + 1)}{c_i(t)} = \beta p_i(t + 1) A, \\
\frac{c_i(t + 1)}{c_i(t)} = \beta \frac{\partial p_i(t + 1)}{\partial x_i(t + 1)} A k_i(t + 1), \\
\frac{c_i(t + 1)}{c_i(t)} = \beta \frac{\partial q_i(t + 1)}{\partial z_i(t + 1)} A k(t + 1),
\]

for all \( t \) and the transversality conditions

\[
\lim_{t \to \infty} \beta^t \frac{k_i(t)}{c_i(t)} = \lim_{t \to \infty} \beta^t \frac{x_i(t)}{c_i(t)} = \lim_{t \to \infty} \beta^t \frac{z_i(t)}{c_i(t)} = 0.
\]
together with agent $i$'s accumulation constraint. In the interest of clarity, I will characterize the solution to this problem in two steps:

**Step 1:** A key feature of interior equilibria is that agents will choose investment activities so marginal returns are equated for each use every period:

$$p_i(t + 1)A = \frac{\partial p_i(t + 1)}{\partial x_i(t + 1)}A k_i(t + 1) = \frac{\partial q_i(t + 1)}{\partial z_i(t + 1)}A k(t + 1).$$ (9)

In particular, any symmetric allocation satisfying (9), for all $t$, implies that (see Appendix)

$$p_i(t) = 1 - q_i(t) = \frac{\pi}{\pi + 1} \equiv p(\pi),$$ (10)

$$x(t) = z(t) = m (1 - p(\pi)) k(t).$$ (11)

Symmetric equilibria have the property that the security of property, $p(\pi)$, is determined solely by the property rights parameter $\pi$. In turn, the term $m (1 - p(\pi))$ determines the returns to appropriation relative to production. The somewhat unintuitive observation that $x(t) = z(t)$ relies upon the homogeneity and the symmetry of the conflict technology, the symmetry of the interior equilibrium, and the fact that $x(t)$ and $z(t)$ depreciate at the same rate. 7 This ensures that the incentives to engage in the defense and the challenge of claims respond symmetrically to changes in the parameters of the model and it thus simplifies the analysis, without obscuring the intuition behind the main results.

**Step 2:** Using (10), intertemporal optimality of each individual’s behavior, as reflected in (5), requires that consumption grows at a constant rate according to

$$c_i(t) = c_i(0) (\beta p(\pi) A)^t,$$ (12)

---

7One can verify that, if $\delta_k$, $\delta_x$ and $\delta_z$ are the depreciation rates of $k(t)$, $x(t)$ and $z(t)$, respectively, interior, symmetric equilibria satisfy $x(t) = \phi_1 z(t) = \phi_2 k(t)$ and $p_i(t) = p_1$, where $\phi_1$, $\phi_2$ and $p_1$ are all constant. Furthermore, $p_1 = p(\pi)$ and $\phi_1 = 1$ if and only if $\delta_x = \delta_z = \delta$; and $\phi_2 = m (1 - p(\pi))$ if $\delta_k = \delta$. 
where \( c_i(0) \) is agent \( i \)'s consumption at the initial date. Using (10)–(12) and (8), it is now a simple matter to solve the accumulation equation (2) for \( k(t) \), and show that symmetric equilibrium allocations are uniquely characterized by

\[
x(t + 1) = z(t + 1) = m \left( 1 - p(\pi) \right) k(t + 1) = m \left( 1 - p(\pi) \right) \beta p(\pi) A k(t),
\]

(13)

\[
c(t) = \left[ 1 - \left( 1 + 2m \left( 1 - p(\pi) \right) \right) \beta p(\pi) \right] A k(t).
\]

(14)

Symmetric equilibria exhibit inefficiently low investment and growth, because agents internalize the fact that a fraction \( 1 - p(\pi) \) of each agent’s own output accrues to others. However, agents do not internalize the fact that the social cost of future consumption in terms of current consumption is \( 1 + 2m \left( 1 - p(\pi) \right) \). This conflict externality has nontrivial implications for the existence of symmetric equilibria. In particular, a symmetric equilibrium exists if and only if the bracketed term in (14) is greater than zero. This condition does have bite, even though appropriative activities are subject to diminishing returns \( (m \leq 1) \). This is illustrated in Figure 1, which plots the bracketed term in (14) as a function of \( p(\pi) \). Panels (A) and (B) illustrate how this relationship changes with \( m \) and \( \beta \), respectively.

The bracketed term in (14) becomes negative for some values of \( \pi \) whenever \( \beta \) and \( m \) are sufficiently large. This could never happen if \( m = 0 \) or \( \pi = \infty \). Note that, for each \( \pi \), if a symmetric equilibrium exists for some \( (m, \beta) \), then it also exists for lower values of \( m \) and \( \beta \).

### 3.2 Property Rights and Social Welfare

Figure 2 plots social welfare, \( U = \sum_{t=0}^{\infty} \beta^t \log c(t) \), as a function of the security of property, \( p(\pi) \), for the case where \( \beta = 0.9 \) and \( A = 2.3 \), with \( k(0) \) normalized to \( k(0) = 1 \). Each of the
four panels illustrates this relationship for a different value of \( m \).

[Figure 2]

For sufficiently low values of \( m \), \( U \) is everywhere increasing in \( p(\pi) \), as in panel (A). As \( m \) rises, \( U \) exhibits an inflexion point, as in panel (B), beyond which higher values of \( m \) induce an interval over which \( U \) declines with \( p(\pi) \). Panel (C) illustrates this possibility for a case where the symmetric equilibrium exists for all \( \pi \geq 1 \). Panel (D) illustrates a different case where \( m \) is sufficiently high (relative to \( \beta \)) that a symmetric equilibrium only exists at sufficiently low and sufficiently high values of \( \pi \).

To gain some intuition, consider the agents’ utility

\[
U = \frac{1}{(1 - \beta)^2} \left[ \beta \log (1 + \gamma) + (1 - \beta) \log c(0) \right],
\]

where \( \gamma \equiv \beta p(\pi)A - 1 \) denotes the common growth rate of consumption and investment activities, and the agents’ resources constraint links current consumption and growth,

\[
c(0) = Ak(0) - (1 + 2m(1 - p(\pi)))(1 + \gamma)k(0).
\]

As \( \pi \) approaches infinity, \((c(0), \gamma)\) approaches the no-externality optimum \((c^*(0), \gamma^*) = ((1 - \beta)Ak(0), \beta A - 1)\). For a comparison, first suppose that \( \pi = \infty \) and consider any (nonoptimal) pair \((c'(0), \gamma')\) such that \( c'(0) > c^*(0) \) and \( \gamma' < \gamma^* \) and linked by the feasibility conditions. Note that if we move towards the optimal growth policy by raising \( \gamma' \), \( c'(0) \) falls. With perfect property rights there is no need for concern; welfare necessarily increases, despite the fall in current consumption.

Now start at \( \pi = 1 \) and consider the effect of improvements in \( \pi \) towards the no-externality optimum. For sufficiently high values of \( \beta \) and \( m \) the monotonic utility path in the previous
paragraph — starting from a nonoptimal policy — cannot be mimicked. The route that \( \pi \) takes towards the no-externality optimum creates problems along the way, because conflict rises along a higher-growth path, and the increased misallocation of resources can outweigh the positive effect of growth. To see why write the resources constraint as

\[
Ak(t) = \left(1 + \frac{x(t+1) + z(t+1)}{c(t)}\right)c(t) + k(t+1),
\tag{17}
\]

and note that conflict imposes a tax on consumption at the rate

\[
\frac{x(t+1) + z(t+1)}{c(t)} = \frac{2m(1-p(\pi))(1+\gamma)}{A - \left(1 + 2m(1-p(\pi))\right)(1+\gamma)}.
\tag{18}
\]

It is immediate that this consumption tax is an increasing function of the growth rate \( \gamma \), everything else being equal. A sufficient increase in this tax rate as growth increases will cause welfare to fall. However, while \( \gamma \) increases with \( \pi \) (since \( \gamma \equiv \beta p(\pi)A - 1 \)), the opportunity cost of productive activities, as given by \( 2m(1-p(\pi)) \), is a decreasing function of \( \pi \). The balance of these effects is such that the tax rate given by (18) will increase with \( \pi \) whenever \( \pi \) is sufficiently low. \(^8\) Increased conflict then \textit{necessarily} creates a zone of non-monotonicity when the values of \( m \) and \( \beta \) are sufficiently high (as illustrated in Figure 2). The influence of \( \pi \) on welfare can be summarized as

**Proposition 1** There exists a nonempty interval \( (\bar{\pi}, \pi) \subset (1, \infty) \) such that a symmetric equilibrium exists and social welfare is strictly decreasing in \( \pi \) for all \( \pi \in (\bar{\pi}, \pi) \) if and only if \( \beta > 7/8 \) and \( 2m > 3 - 2\beta + \sqrt{(3 - 2\beta)^2 - 1} \).

The parameter \( \pi \) can be thought of as a policy variable, summarizing the government’s support — legal framework, police, court system — which leverages the investments of individuals.

\(^8\)In particular, the tax rate in (18) increases with \( \pi \) whenever \( p(\pi) < (1/\beta)(1 - \sqrt{1-\beta}) \).
to defend their property. Under this view Proposition 1 characterizes situations where piecemeal reform might not be in the interest of society.

In the present context the equilibrium share of resources that are dissipated, $\frac{x(t+1)+z(t+1)}{A k(t)}$, does fall with $\pi$. This can be understood in terms of the consumption tax discussed above: $c(0)$ falls by so much as $\pi$ rises that the total tax also falls, despite the fact that the tax rate rises. In turn, this may suggest that improvements in $\pi$ necessarily induce a negative relationship between growth and diversion. However, as will become clear below, this is not the case in general, and one will rather expect a nonmonotone relationship between growth and diversion.

4 Further Remarks about Conflict and Growth

There is a common perception that relatively more property security and relatively less diversion are simply two sides of the same coin. Related to this, conventional wisdom emphasizes the tradeoff between production and diversion and, therefore, suggests that improvements in productivity which promote growth will simultaneously mitigate the problem of diversion. In this section I re-examine these views in the context of a simple extension of the previous model.

Consider the previous model, but suppose that $k(t)$, $x(t)$ and $z(t)$ depreciate at the common rate $\delta \in [0, 1]$. This formulation introduces the possibility that a stock of appropriative capital (some of which may be intangible) is built over time. For simplicity, assume that agents retain control of their past capital stocks. The foregoing analysis refers to the case where $\delta = 1$, but it clearly goes through with the obvious modifications.

Consider the equilibrium fraction of aggregate resources that are dissipated (see Appendix),

$$\frac{x(t + 1) - (1 - \delta)x(t) + z(t + 1) - (1 - \delta)z(t)}{A k(t)} = 2 m(1 - p(\pi)) \left(\frac{\gamma + \delta}{A}\right),$$

(19)
where $\gamma \equiv \beta \left( p(\pi)A + 1 - \delta \right) - 1$. Intuitively, an increase in $\pi$ leads to higher $p(\pi)$. This lowers the opportunity cost of production, which tends to lower diversion. But it also promotes growth, which tends to raise diversion. The net influence of $\pi$ on equilibrium diversion is recorded as

**Proposition 2** Diversion, as measured by (19), increases with $\pi$ if and only if $\pi < \pi^*$, where

$$p(\pi^*) = \frac{1}{2} + \frac{(1 - \beta)(1 - \delta)}{2 \beta A}.$$  

Diversion peaks exactly at $\pi = 1$ in the special case where $\delta = 1$. 9 Otherwise, the model predicts an inverted U-shaped relationship between equilibrium diversion and the security of property. 10 An empirical implication is that a presumption that relatively more property security and less diversion are flip sides of the same coin, without separately identifying the two outcomes, can lead to misleading inferences. A further corollary is that differences in $\pi$ can induce a positive relationship between growth and diversion.

Now consider the effect of productivity on diversion. In the special case where $\delta = 1$, such an influence is absent. However, provided that $\delta < 1$, it is obvious from (19) that diversion increases with $A$. 11 It should be noted that this is not the only possible mechanism. 12 The main insight is that higher productivity levels can exacerbate the problem of diversion, despite also increasing output growth and, furthermore, that this effect of productivity on diversion can ultimately lead to a welfare loss. Proposition 3 makes this precise.

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9 Diversion peaks at $\pi = 1$ whenever both $x(t)$ and $z(t)$ depreciate fully, even if $k(t)$ does not.

10 Assuming that $\beta \left( \frac{1}{2}A + 1 - \delta \right) - 1 > 0$ to ensure positive growth for all $\pi \geq 1$ also implies that $p(\pi^*) < 1$.

11 It is also evident from (19) that higher values of $\beta$ induce more diversion. Therefore, this model does not support cultural stereotypes that suggest that conflict is a problem for those who do not adequately take account of the future. This idea has been formalized by Skaperdas and Syropoulos (1996).

12 A similar effect would arise if one departed from logarithmic utility, allowing for stronger intertemporal substitution effects of changes in productivity. Alternatively, one could consider asymmetries in the depreciation rates of productive and appropriative activities, in which case increases in $A$ can also lead to increases in the relative return to appropriation and, through this channel, in diversion.
Proposition 3 Let $\delta < 1$, $\beta > 8/9$ and $m > (2 - \beta + 2\sqrt{1-\beta})/(2\beta)$. There exists a nonempty interval $(\pi', \pi'') \subset (1, \infty)$ such that for each $\pi \in (\pi', \pi'')$: (1) social welfare is strictly decreasing in $A$ if and only if $A > A'$, and (2) a symmetric equilibrium exists if and only if $A < A''$, where

$$A' = \frac{(1 - \beta)(1 - \delta)}{p(\pi) \left[ \left(1 + 2m(1 - p(\pi)) \right) \beta p(\pi) - 1 \right]} < A'' = \left(1 + 2m(1 - p(\pi)) \right) p(\pi) A'. $$

Part (1) indicates that when $m$ and $\beta$ are high enough there is a limit to how high $A$ can be without harming social welfare, for some interval of values of $\pi$. Part (2) implies that the negative effect of productivity on welfare can arise in equilibrium. It also implies that further increases in $A$ will eventually drive initial consumption to zero, at which point a symmetric equilibrium fails to exist.

5 Conclusion

The analysis has shown how the interaction between conflict and growth can lead to a symmetric equilibrium allocation associated with relatively more property security and faster growth being Pareto dominated by one associated with less property security and slower growth. The analysis has also illustrated the related possibility that growth which is not accompanied by sufficiently secure property can exacerbate the problem of diversion. These results call for caution before a society decides to pursue economic growth independently of the structure of property rights. Furthermore, if the institutional structure is inappropriate, piecemeal reform might not be in the interest of society, and substantial reform might be necessary if it is to be welfare-improving.

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The proof of Proposition 3 makes clear that its assumptions are also necessary for welfare to decline with $A$.

Proposition 3 is similar in spirit to Grossman and Mendoza’s (2003) argument that anticipated resource increases can temporarily induce individuals to devote more time to appropriative activities to the point of lowering welfare, a “paradox of anticipated abundance”. Their argument relies on a sufficiently strong intertemporal effect of an individual’s current consumption on his probability of survival.
A Appendix

Lemma 1 Symmetric equilibrium allocations satisfy (10)–(11) and (13)–(14). A symmetric allocation can be supported in equilibrium if and only if

\[ 1 - \left( 1 + 2m (1 - p(\pi)) \right) \beta p(\pi) > 0. \]

Proof: Consider a symmetric equilibrium allocation, with \( k(t), x(t), z(t) > 0 \), for all \( t \). Then, it must satisfy (2) and (5)–(8). Hence the equality of marginal returns (9) must hold. The second equality in (9) can be written as

\[ \frac{x_i(t+1)}{z_i(t+1)} = p_i(t+1) \left( 1 - p_i(t+1) \right) k_i(t+1) \frac{k_i(t+1)}{q_i(t+1) \left( 1 - q_i(t+1) \right) k(t+1)}. \] (20)

A symmetric allocation must satisfy \( p_i(t+1) = 1 - q_i(t+1) \) since \( x_i(t+1) = x(t+1) \) and \( z_i(t+1) = z(t+1) \). Together with the fact that \( k_i(t+1) = k(t+1) \), this implies both that \( x(t+1) = x(t+1) \) and that \( p_i(t+1) = \frac{\pi}{\pi+1} \). Equation (11) follows from the latter result, together with the first equality in (9), which can be written as

\[ \frac{x_i(t+1)}{k_i(t+1)} = m \left( 1 - p_i(t+1) \right). \] (21)

Equation (12) then follows from (5). To derive (14), write agent \( i \)'s resources constraint (2) as \( A k(t) = c_i(0) \left( \beta p(\pi) A \right)^t + \left( 1 + 2m (1 - p(\pi)) \right) k(t+1) \). The solution to this difference equation, consistent with the transversality conditions given in (8) and with \( c_i(0) > 0 \), is

\[ k(t) = \frac{(\beta p(\pi) A)^t c_i(0)}{A - \left( 1 + 2m (1 - p(\pi)) \right) \beta p(\pi) A}. \] (22)

Hence (14) is satisfied. The resources constraint (2) implies (13). The first part of the lemma follows from noting that symmetric equilibria must be interior.

By construction, the interior allocation described in (13)–(14) solves (2) and (5)–(8). Hence, it must solve the problem of each agent \( i \) (Theorem 4.15 in Stokey and Lucas (1989)). Clearly,
it can be supported by a symmetric equilibrium if and only if
\[
1 - \left(1 + 2m (1 - p(\pi)) \right) \beta p(\pi) > 0
\]  
holds — so \(c(t) > 0\) for all \(t \geq 0\) — and the agents’ utility is bounded. That this latter condition holds when \(\beta < 1\) can be seen directly in equation (15).

Proof of Proposition 1

Note that \(\partial U/\partial \pi = \left(\partial U/\partial p(\pi)\right) \left(\partial p(\pi)/\partial \pi\right)\), where \(\partial p(\pi)/\partial \pi > 0\) for \(\pi \in [1, \infty)\). There are two cases to be considered. (1) Suppose that (23) is satisfied for all \(\pi \in [1, \infty)\). Simple calculation shows that \(\partial U/\partial p(\pi) \geq 0\) if and only if
\[
1 \geq p(\pi) \left[1 + 2m (1 - (2 - \beta)p(\pi)) \right].
\]  
The right side of (24) is quadratic in \(p(\pi)\). It fails to hold at some \(p(\pi) < 1\) if and only if
\[
m > \frac{1}{2} \quad \text{and} \quad 2 - \beta < \frac{(1 + 2m)^2}{8m}.
\]  
Noting that the right side of the second inequality is increasing in \(m\), for \(m > 1/2\), it is easy to verify that the two conditions \(\beta > 7/8\) and \(2m > 3 - 2\beta + \sqrt{(3 - 2\beta)^2 - 1}\) are equivalent to (25). To conclude the proof, note that (24) defines an interval \((\pi, \pi) \subset (1, \infty)\) such that (24) fails to hold if and only if \(\pi \in (\pi, \pi)\). (2) Suppose that (23) is not satisfied for some \(\pi \in [1, \infty)\). Simply note that (23) fails to hold if and only if \(\pi \in [\pi', \pi']\), where \([\pi', \pi'] \subset (1, \infty)\) is defined by (23) in the obvious manner. One can verify that the previous argument continues to hold, with \(\pi = \pi\).

Proof of Proposition 2

Let \(\delta \in [0, 1]\). One can easily verify that (10)–(11) continue to hold, whereas (13)–(14) become
\[
x(t + 1) - (1 - \delta) x(t) = z(t + 1) - (1 - \delta) z(t) = m (1 - p(\pi)) \left(k(t + 1) - (1 - \delta) k(t)\right)
\]
\[ c(t) = \left( A - \left( 1 + 2m (1 - p(\pi)) \right) (\gamma + \delta) \right) k(t), \]  
\[ \text{where } \gamma \equiv \beta \left( p(\pi) A + 1 - \delta \right) - 1. \]  

Equation (19) in the main text follows from (26). Its right side is a strictly concave function of \( p(\pi) \), which achieves its maximum at \( p(\pi^*) \). Since \( \partial p(\pi) / \partial \pi > 0 \), for all \( \pi \geq 1 \), it follows that the right side of (19) is increasing in \( \pi \) if and only if \( \pi < \pi^* \). \[ \blacksquare \]

**Proof of Proposition 3**

Consider a symmetric equilibrium allocation and write social welfare as

\[ U \equiv \sum_{t=0}^{\infty} \beta^t \log (c(t)) = \frac{1}{(1 - \beta)^2} \left[ \beta \log (1 + \gamma) + (1 - \beta) \log (c(0)) \right], \]  
\[ \text{where } \gamma \equiv \beta \left( p(\pi) A + 1 - \delta \right) - 1 \text{ and } c(0) \text{ is given by (27). Then, } \partial U / \partial A \geq 0 \text{ if and only if } \]  
\[ \frac{\beta}{1 + \gamma} \frac{\partial \gamma}{\partial A} + \frac{1 - \beta}{c(0)} \frac{\partial c(0)}{\partial A} \geq 0. \]  

Since \( \partial \gamma / \partial A > 0 \), it follows that \( \partial U / \partial A < 0 \) only if \( \partial c(0) / \partial A < 0 \). Assume that \( \delta < 1 \).

Noting that \( \partial c(0) / \partial A \) is a convex function of \( p(\pi) \) and it reaches its minimum at \( p(\pi_0) = 1/2 + 1/(4m) \), one can verify that \( \partial c(0) / \partial A < 0 \), when evaluated at \( p(\pi_0) \), if and only if \( \beta > 8/9 \) and \( m > (2 - \beta + 2\sqrt{1 - \beta}) / (2\beta) \). Noting that \( \partial^2 U / \partial A^2 < 0 \), it is straightforward to verify that if \( \partial c(0) / \partial A < 0 \), then \( U \) reaches its maximum at \( A = A' \), where \( A' \) is given in the proposition.

A symmetric allocation can be supported in equilibrium if and only if \( c(0) > 0 \) or, equivalently, if and only if \( A < A'' \), where \( A'' \) is stated in the proposition. Hence the previous argument applies, for any \( A < A'' \), since \( U \) is continuous and twice differentiable. Clearly, \( A'' > A' \) whenever \( \partial c(0) / \partial A < 0 \). \[ \blacksquare \]
References


Figure 1: Consumption Rates

\[ 1 - (1 + 2m(1 - p(\pi)))\beta p(\pi) \quad (A) \, \beta = 0.95 \]

\[ 1 - (1 + 2m(1 - p(\pi)))\beta p(\pi) \quad (B) \, m = 0.9 \]
Figure 2: Social Welfare and Property Rights

Parameter values: $\beta=0.9$, $A=2.3$, $k(0)=1$. 