Monetary Aggregation, Inflation, and Welfare

Apostolos Serletis
Department of Economics
University of Calgary
Calgary, Alberta T2N 1N4

and

Jagat Jit Virk
Department of Economics
Queens University
Kingston, Ontario K7L 3N6

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Apostolos Serletis†
Department of Economics
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Jagat Jit Virk
Department of Economics
Queens University
Kingston, Ontario

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Abstract

This paper investigates the welfare implications of alternative monetary aggregation procedures by providing a comparison among simple-sum, Divisia, and currency equivalent monetary aggregates at different levels of monetary aggregation. We find evidence that the choice of monetary aggregation procedure is crucial in evaluating the welfare cost of inflation.

Keywords: Divisia; Currency equivalent; Integration; Cointegration; Money demand; Interest elasticity.

JEL classification: E31, E41.

*Serletis acknowledges support from the Social Sciences and Humanities Research Council of Canada.
†Corresponding author. Tel.: +1-403-220-4092; fax: +1-403-282-5262; e-mail address: serletis@ucalgary.ca; web:http://econ.ucalgary.ca/serletis.htm
1 Introduction

Lucas (2000) provides estimates of the welfare cost of inflation based on U.S. time series for 1900-1994. In doing so, he defines the money supply as simple-sum M1, assumes that money pays no interest, and estimates the welfare cost of inflation using Bailey’s (1956) consumer surplus approach as well as the compensating variation approach. Lucas argues that money demand behavior at hyperinflation or at rates of interest close to zero is crucial for welfare cost calculations; in those cases the semi-log money demand function, used by Cagan (1956) and Bailey (1956), fits the data better and should be used for such calculations. However, the U.S. time series data includes only moderate inflation rates, and Lucas’ calculations, based on the double log demand schedule, indicate that reducing the interest rate from 3% to zero yields a benefit equivalent to an increase in real output of about 0.009 (or 0.9%).

More recently, Serletis and Yavari (2004) calculate the welfare cost of inflation for Canada and the United States, in the post-World War II period, from 1948 to 2001. In doing so, they use the same double log money demand specification used by Lucas (2000), but pay particular attention to the integration and cointegration properties of the money demand variables and use recent advances in the field of applied econometrics to estimate the interest elasticity of money demand. They conclude that the welfare cost of inflation is significantly lower than Lucas reported. In particular, for the United States, they find that reducing the interest rate from 3% to zero, would yield a benefit equivalent to 0.0018 (less than two tenths of one percent) of real income. This is much smaller than the 0.9% (nine tenths of one percent) figure obtained by Lucas under the assumption that the interest elasticity of money demand is $-0.5$. Similar welfare cost estimates are also reported by Serletis and Yavari (2005) for Italy, using the low frequency data from Muscatelli and Spinelli (2000) over the 1861 to 1996 period.

As Lucas (2000, p. 270) puts it in his conclusions, a direction for potentially productive research “is to replace M1 with an aggregate in which different monetary assets are given different weights.” In this paper, we take up Lucas on his suggestion and provide a comparison among the official simple-sum aggregates, Barnett’s (1980) Divisia aggregates, and Rotemberg’s (1991) currency equivalent (CE) aggregates, at four different levels of monetary aggregation, to investigate the welfare implications of alternative monetary aggregation procedures. We make the bold assumption that money is non-
interest bearing and also assume that the different monetary aggregates face
the same double log demand function, since our data does not include regions
of hyperinflation or rates of interest approaching zero. However, following
Serletis and Yavari (2004), we pay particular attention to the integration and
cointegration properties of the money demand variables and use the Fisher
and Seater (1993) long-horizon regression approach to obtain an estimate of
the interest rate elasticity of money demand.

The organization of the article is as follows. The next section provides a
brief summary of the theoretical issues regarding the estimation of the welfare
cost of inflation. Section 3 discusses the data and presents empirical evidence
regarding the interest elasticity of money demand, using recent advances in
the field of applied econometrics (such as integration theory, cointegration
theory, and long-horizon regression tests). Section 4 investigates the sensitiv-
ity of the welfare cost calculations to monetary aggregation procedures and
Section 5 closes with a brief summary and conclusion.

2 Theoretical Foundations

Consider the following money demand function

\[
\frac{M}{P} = L(i, y)
\]

where \(M\) denotes nominal money balances, \(P\) the price level, \(y\) real income,
and \(i\) the nominal rate of interest, all at time \(t\). Assuming that the \(L(i, y)\)
function takes the form \(L(i, y) = \Phi(i)y\), the money demand function can
be written as \(m = \Phi(i)y\), where \(m\) denotes real money balances, \(M/P\).
Equivalently, we can write

\[
z = \frac{m}{y} = \Phi(i)
\]

which gives the demand for real money balances per unit of income as a
function of the nominal interest rate \(i\).

The specification of the money demand function is crucial in the estima-
tion of the welfare cost of inflation. Bailey (1956) and Friedman (1969) use a
semi-log demand schedule whereas Lucas (2000) uses a double log (constant
elasticity) schedule on the grounds that the double log performs better on the
U.S. data that does not include regions of hyperinflation or rates of interest
approaching zero. We, like Lucas, use the double log functional form,

\[ z = \Phi(i) = Ai^\eta, \]  

where \( \eta \) is the interest elasticity.

### 2.1 The Consumer Surplus Approach

The traditional approach to estimating the welfare cost of inflation is the one developed by Bailey (1956). It uses tools from public finance and applied microeconomics and defines the welfare cost of inflation as the area under the inverse money demand schedule — the ‘consumer surplus’ that can be gained by reducing the nominal interest rate from a positive level of \( i \) to the lowest possible level (perhaps zero). In particular, based on Bailey’s consumer surplus approach, we estimate the money demand function \( z = \Phi(i) \), calculate its inverse \( i = \Psi(z) \), and define

\[ w(i) = \int_0^{\Phi(0)} \Psi(x)dx = \int_0^i [\Phi(x) - i\Phi(i)] dx, \]  

where \( w(i) \) is the welfare cost of inflation, expressed as a fraction of income. With (1), equation (2) takes the form

\[ w(i) = \left[ \frac{A}{\eta + 1} i^{\eta+1} \right]_0^i - iA i^{\eta} = -\frac{\eta}{\eta + 1} i^{\eta+1}. \]  

### 2.2 The Compensating Variation Approach

Lucas (2000) takes a ‘compensating variation’ approach to the problem of estimating the welfare cost of inflation. In doing so, he also provides theoretical, general equilibrium justifications for Bailey’s consumer surplus approach.

Lucas starts with Brock’s (1974) perfect foresight version of the Sidrauski (1967) model, and defines the welfare cost of a nominal interest rate \( i \), \( w(i) \), to be the income compensation needed to leave the household indifferent between living in a steady state with an interest rate constant at \( i \) and an otherwise identical steady state with an interest rate of zero. Thus, \( w(i) \) is the solution to the following equality

\footnote{In this regard, Bali (2000) performs tests based on the Box-Cox transformation and confirms that the double log fits the U.S. data better than the semi-log function.}
\[ u(c, m) = \frac{1}{1 - \sigma} \left[ cf \left( \frac{m}{c} \right) \right]^{1 - \sigma}, \quad \sigma \neq 1, \]  

(5)

where \( c \) and \( m \) are real consumption and money balances, and setting up the dynamic programming problem [see Lucas (2000) for details], Lucas obtains the differential equation

\[ w'(i) = -\Psi \left( \frac{\Phi(i)}{1 + w(i)} \right) \Phi'(i), \]  

(6)

in the welfare cost function \( w(i) \). For any given money demand function, (6) can be solved numerically for an exact welfare cost function \( w(i) \). In fact, with (1), equation (6) can be written as

\[ w'(i) = -\eta Ai^n (1 + w(i))^{-1/\eta}, \]

with solution

\[ w(i) = \exp \left[ -\frac{\eta \ln \left( \frac{-A(i \exp(\eta \ln i) - \frac{1}{\Lambda(i + 1)} - \frac{1}{\Lambda(i + 1)}}{\eta + 1} \right)}{\eta + 1} \right] - 1. \]

(7)

Thus the welfare cost of inflation is easily obtained using equation (7).

Lucas also investigates the robustness of his results to the non-existence of lump sum taxes and inelastic labor supply, by introducing theoretical modifications to the Sidrauski model. In particular, (5) is modified to include the consumption of leisure \( l \)

\[ u(c, m, l) = \frac{1}{1 - \sigma} \left[ cf \left( \frac{m}{c} \right) \phi(l) \right]^{1 - \sigma}, \quad \sigma \neq 1, \]  

(8)

the consumer’s constraint to reflect income taxation, and the resource constraint to include government consumption. In this case the welfare cost function \( w(i) \) is defined as the solution to the following equality

\[ U \left[ (1 + w(i)) c(i), \Phi(i), l(i) \right] = U \left[ c(\delta), \Phi(\delta), l(\delta) \right], \]  

(9)
where \( i = \delta \) is used as a benchmark rather than \( i = 0 \), because depending on the assumed functions \( f \) and \( \phi \), the system may not have a solution at \( i = 0 \). With (8), (9) is equivalent to

\[
(1 + w(i)) c(i) f \left( \frac{\Phi(i)}{(1 + w(i)) \omega(i)} \right) \phi(l(i)) = c(\delta) f \left( \frac{\Phi(\delta)}{\omega(\delta)} \right) \phi(l(\delta)),
\]

where \( \omega = c/y \). Moreover, with (1), equation (10) becomes [see Lucas (2000) for details]

\[
(1 + w(i)) \omega(i) \left[ \left( \frac{\Phi(i)}{(1 + w(i)) \omega(i)} \right)^{(b-1)/b} A^{1/b} + 1 \right]^{b/(b-1)} \phi(l(i)) = F(\delta),
\]

with solution\(^2\)

\[
w(i) = \frac{\left[ \left( \frac{F(\delta)}{\phi(l(i))} \right)^{(b-1)/b} (b-1)^{b-1} A^{1/b} \right]^{b/(b-1)} - 1. \]

Equation (11) is used to calculate the welfare cost of inflation in the case of income taxation and elastic supply of labor.

Lucas (2000) also uses a version of the McCallum and Goodfriend (1987) variation of the Sidrauski model to provide another general equilibrium rationale for Bailey’s consumer surplus approach. However, the relevant differential equation [equation (5.8) in Lucas (2000)] cannot be solved for interest rate elasticities different than -0.50 that Lucas assumes. As a result we are not exploring that approach in this paper.

### 3 The Demand for Money

To investigate the welfare implications of alternative monetary aggregation procedures, we use the official simple-sum monetary aggregates, Barnett’s

\(^2\)In equation (11),

\[
F(\delta) = \omega(\delta) \phi(l(\delta)) \left[ \left( \frac{\Phi(\delta)}{\omega(\delta)} \right)^{(b-1)/b} A^{1/b} + 1 \right]^{b/(b-1)}.
\]
(1980) Divisia aggregates (also known as “monetary services indices”), and
Rotemberg’s (1991) currency equivalent (CE) indices. The use of monetary
aggregates (in various forms and at different levels of aggregation) is subject
to a comment by Prescott (1996, p.114) that (in the case of M1)

“[t]he theory has households holding non-interest bearing money,
while the monetary aggregate used in the demand for money func-
tion is M1. Most of M1 is not non-interest bearing debt held by
households. Only a third is currency and half of that is probably
held abroad. Another third is demand deposits held by busi-
nesses, which often earn interest de facto. Households do not use
these demand deposits to economize on shopping time. The …
nal third is demand deposits held by households that, at least in
recent years, can pay interest.”

With Percott’s comment in mind, we use quarterly data over the period
from 1960:1 to 2001:4, obtained from the St. Louis MSI database, maintained
by the Federal Reserve Bank of St. Louis as a part of the bank’s Federal Re-
serve Economic Database (FRED) — see Anderson et al. (1997a, 1997b) for
details regarding the construction of the Divisia and CE monetary aggregates
and related data issues, and Serletis and Koustas (2001) for a brief discussion
of the problem of the definition (aggregation) of money. Moreover, we use
real GDP as the real output series, the GDP deflator as the price level series,
and the 90-day Treasury bill rate as the relevant interest rate series.

Welfare cost calculations are also sensitive to the specification of the
money demand function. As already noted, we follow Lucas (2000) and
assume a double log schedule. However, to obtain an estimate of the in-
terest elasticity, we first investigate the time series properties of the money
demand variables to avoid what Granger and Newbold (1974) refer to as
“spurious regression.” In doing so, we first test for stochastic trends (unit
roots) in the autoregressive representation of each individual logged series
$z_t$ and the logged interest rate series. In Table 1 we report $p$-values for
the weighted symmetric (WS) unit root test [see Pantula, Gonzalez-Farias,
and Fuller (1994)], the augmented Dickey-Fuller (ADF) test [see Dickey
and Fuller (1981) for more details], and the nonparametric $Z(t_{ah})$ test of Phillips
and Perron (1987). As discussed in Pantula et al. (1994), the WS test domi-
nates the ADF test in terms of power. Also the $Z(t_{ah})$ test is robust to a wide
variety of serial correlation and time-dependent heteroskedasticity. For the
WS and ADF tests, the optimal lag length is taken to be the order selected by the Akaike Information Criterion (AIC) plus 2 - see Pantula et al. (1994) for details regarding the advantages of this rule for choosing the number of augmenting lags. The $Z(t_{\hat{\alpha}})$ is done with the same Dickey-Fuller regression variables, using no augmenting lags.

According to the $p$-values [based on the response surface estimates given by MacKinnon (1994)] for the WS, ADF, and $Z(t_{\hat{\alpha}})$ unit root tests reported in the first three columns of each panel of Table 1, the null hypothesis of a unit root in levels cannot in general be rejected, except for the CE M2 and CE M3 monetary aggregates.\(^3\) Hence, we conclude that both the $z_t$ and $i_t$ series are integrated of order 1 [or I(1) in the terminology of Engle and Granger (1987)].\(^1\) We also tested the null hypothesis of no cointegration (against the alternative of cointegration) between each I(1) money measure and $i_t$ using the Engle and Granger (1987) two-step procedure, which is well suited for the bivariate case which can have at most one cointegrating vector.\(^5\) The tests were first done with $z_t$ as the dependent variable in the cointegrating regression and then repeated with the nominal interest rate $i_t$ as the dependent variable. The results, under the “Cointegration” columns of Table 1, suggest that the null hypothesis of no cointegration between $z_t$ and $i_t$ cannot be rejected (at the 5% level) for all I(1) money measures.

Since we are not able to find evidence of cointegration, to avoid the spurious regression problem we use the long-horizon regression approach developed by Fisher and Seater (1993) to obtain an estimate of the interest rate elasticity of money demand. One important advantage to working with the long-horizon regression approach is that cointegration is neither necessary nor sufficient for tests on the interest rate elasticity of money demand. Long-horizon regressions have received a lot of attention in the recent economics and finance literature, because studies based on long-horizon variables seem to find significant results where short-horizon regressions commonly used in

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\(^3\) See Serletis (2001, Chapter 10) for similar results over a different sample period.

\(^1\) It should also be noticed that time series data on nominal interest rates reflects time-series data on inflation, and the persistence of inflation episodes makes it difficult to reject I(1) behavior in interest rates.

\(^5\) In cointegration tests using three or more variables the Engle and Granger (1987) approach does not distinguish between the existence of one or more cointegrating vectors. In those cases, the Johansen (1988) maximum likelihood extension of the Engle and Granger (1987) cointegration approach or the Pesaran et al. (2001) bounds testing approach should be used.
economics and finance have failed.

Following Fisher and Seater (1993), we consider the following bivariate autoregressive representation

$$z_t = z_t(L) + \varepsilon_t$$

where $z_t(L) = \alpha_{zz}(L)\Delta^{(\varepsilon)} z_t + \varepsilon_t$ and $i_t(L) = \alpha_{ii}(L)\Delta^{(i)} i_t + \varepsilon_t$

where $\alpha_{zz} = \alpha_{ii}^0 = 1$, $\Delta = 1 - L$, where $L$ is the lag operator, $z$ is the money-income ratio, $i$ is the nominal interest rate, and $\langle x \rangle$ represents the order of integration of $x$, so that if $x$ is integrated of order $\gamma$, then $\langle x \rangle = \gamma$ and $\langle \Delta x \rangle = \langle x \rangle - 1$. The vector $(\varepsilon_t^z, \varepsilon_t^i)'$ is assumed to be independently and identically distributed normal with zero mean and covariance $\Sigma_\varepsilon$, the elements of which are $\text{var}(\varepsilon_t^z), \text{var}(\varepsilon_t^i), \text{cov}(\varepsilon_t^z, \varepsilon_t^i)$.

According to this approach, the null hypothesis of $\eta = 0$ can be tested in terms of the long-run derivative of $z$ with respect to a permanent change in $i$, $LRD_{z,i}$, which is defined as follows. If $\lim_{k \to \infty} \partial z_{t+k}/\partial \varepsilon_t^i \neq 0$, then

$$LRD_{z,i} = \lim_{k \to \infty} \frac{\partial z_{t+k}/\partial \varepsilon_t^i}{\partial i_{t+k}/\partial \varepsilon_t^i}$$

and expresses the ultimate effect of an exogenous interest rate disturbance on $z$, relative to that disturbance’s ultimate effect on $i$. When $\lim_{k \to \infty} \partial i_{t+k}/\partial \varepsilon_t^i = 0$, there are no permanent changes in $i$ and thus $LRD_{z,i}$ is undefined.

The above bivariate autoregressive system can be inverted to yield the following vector moving average representation

$$\Delta^{(\varepsilon)} z_t = \theta_{zi}(L)\varepsilon_t^i + \theta_{zz}(L)\varepsilon_t^z$$

$$\Delta^{(i)} i_t = \theta_{ii}(L)\varepsilon_t^i + \theta_{iz}(L)\varepsilon_t^i$$

In terms of this moving average representation, Fisher and Seater (1993) show that $LRD_{z,i}$ depends on $\langle i \rangle - \langle z \rangle$, as follows

$$LRD_{z,i} = \frac{(1 - L)^{(i)-(\varepsilon)} \theta_{zi}(L)|_{L=1}}{\theta_{ii}(1)}$$

Hence, meaningful long-horizon regression tests can be conducted if both $z_t$ and $i_t$ satisfy certain nonstationarity conditions. In particular, long-horizon regression tests require that both $z_t$ and $i_t$ are at least $I(1)$ and of the same
order of integration. In fact, when \( \langle z \rangle = \langle i \rangle = 1 \), the long-run derivative becomes

\[
LRD_{z,i} = \frac{\theta_{zi}(1)}{\theta_{ii}(1)}
\]

where \( \theta_{zi}(1) = \sum_{j=1}^{\infty} \theta_{zij}^j \) and \( \theta_{ii}(1) = \sum_{j=1}^{\infty} \theta_{iij}^j \). The coefficient \( \theta_{zi}(1)/\theta_{ii}(1) \) is the long-run value of the impulse-response of \( z \) with respect to \( i \), suggesting that \( LRD_{z,i} \) can be interpreted as the long-run elasticity of \( z \) with respect to \( i \).

Under the assumptions that \( \text{cov}(\varepsilon_t^z, \varepsilon_t^i) = 0 \) and that \( i \) is exogenous in the long-run, the coefficient \( \theta_{zi}(1)/\theta_{ii}(1) \) equals the zero-frequency regression coefficient in the regression of \( \Delta(z)z \) on \( \Delta(i)i \) — see Fisher and Seater (1993, note 11). This estimator is given by \( \lim_{k \to 0} \eta_k \), where \( \eta_k \) is the coefficient from the regression

\[
\left[ \sum_{j=0}^{k} \Delta(z)z_{t-j} \right] = a_k + \eta_k \left[ \sum_{j=0}^{k} \Delta(i)i_{t-j} \right] + e_{kt}
\]

In fact, when \( \langle z \rangle = \langle i \rangle = 1 \), consistent estimates of \( \eta_k \) can be derived by applying ordinary least squares to the regression

\[
z_t - z_{t-k-1} = a_k + \eta_k [i_t - i_{t-k-1}] + e_{kt}, \quad k = 1, \ldots, K
\]  

We estimate equation (12) and in Table 2 report ordinary least squares estimates of \( \eta_k \), tests of the null hypothesis that \( \eta_k = 0 \), and Andrews (1989) IPFs at low type II error, \( \eta_{k,0.05} \), and high type II error, \( \eta_{k,0.50} \), at forecast horizons \( k = 10, 15, 20, 25, \) and \( 30 \), for each money measure that is I(1). An asterisk next to an \( \eta_k \) value indicates rejection of the null hypothesis that \( \eta_k = 0 \); the test statistic is the \( t \)-ratio of \( \hat{\eta}_k \) with \( T/k \) degrees of freedom, where \( T \) is the sample size and \( \hat{\sigma}_{\eta_k} \) is computed following Newey and West (1994). Clearly, the null hypothesis that \( \eta_k = 0 \) is rejected (at the 5% level and for all values of \( k \)) only with the M1 and MZM monetary aggregates (irrespective of how they are measured).

As noted by Andrews (1989), when the null hypothesis is not rejected the IPF allows us to gauge which alternatives are consistent with the data. In particular, when \( \eta_k = 0 \) is not rejected at the 5% level, there are alternatives to the null that can be ruled out at the same level of significance. In this case, the IPF at the 95% level of power provides a lower
bound, \( \eta_{k,0.05} \), that is significantly different than the true value of \( \eta_k \) at the 0.05 level, \( \{ \eta_k : |\eta_k| > \eta_{k,0.05} \} \). Following Andrews (1989), we compute the asymptotic approximation to the IPF at the 0.95 level of power as \( \eta_{k,0.05} = \lambda_{1,0.05}(0.95)\sigma_{\eta_k} \), where \( \sigma_{\eta_k} \) is the standard error of \( \hat{\eta}_k \) and the subscripts of \( \lambda_{1,0.05}(0.95) \) indicate a test of one restriction at a 0.05 level. From Andrews (1989, Table 1), \( \lambda_{1,0.05}(0.95) = 3.605 \). Similarly, the IPF allows us to examine alternatives that possess a high type II error. At a 0.50 error probability that is suggested by Andrews (1989), the asymptotic approximation to the IPF is \( \eta_{k,0.50} = \lambda_{1,0.05}(0.50)\sigma_{\eta_k} \) for those alternatives to \( \eta_k = 0 \) that are consistent with the data, \( \{ \eta_k : 0 < |\eta_k| \leq \eta_{k,0.50} \} \). From Andrews (1989, Table 1) \( \lambda_{1,0.05}(0.50) = 1.960 \).

Clearly, the IPFs at low and high power provide little evidence to doubt \( \eta_k = 0 \) for the simple-sum M2 and M3 and Divisia M2 and M3 monetary aggregates. This is consistent with Prescott’s (1996) argument that, except for currency, few if any components of monetary aggregates are non-interest bearing. Currency is a much smaller fraction of these broad aggregates than of M1, so small a component that any welfare cost is difficult to detect.

For this reason, in the next section we use only the M1 and MZM money measures to calculate the welfare cost of inflation and provide a comparison among simple-sum, Divisia, and currency equivalent monetary aggregation procedures.

## 4 The Empirical Evidence

Figures 1 and 2 plot the welfare cost function \( w(i) \), based on equation (3), for each of the M1 and MZM money measures at each of the three levels of monetary aggregation. In applying (3), we use the values of \( \eta \) given in Table 2 (under \( k = 30 \)) and set \( A \) so that the welfare curve passes through the geometric means of \( i \) and the money-income ratio \( z \).\(^6\) Strong evidence exists of a positive relation between the welfare cost of inflation and the nominal rate of interest. Moreover, the results are substantially more favorable for the Divisia than for the simple-sum and currency equivalent monetary aggregates. In particular, the Divisia aggregates perform better than the other aggregates (in terms of producing a lower welfare cost) at both the

\(^6\)For example, if \( \bar{z} \) is the geometric mean of \( z \) and \( \bar{i} \) that of \( i \), then \( A \) is set equal to \( \bar{z}/\bar{i} \).
lowest (M1) and highest (MZM) levels of aggregation, with that degree of superiority tending to increase at higher interest rates.

Next, we calculate the welfare cost of inflation using the Lucas (2000) compensating variation approach based on the simplified version of the Sidrauski (1967) model, discussed in Section 2. That is, we use equation (7) for our calculations. As in Lucas, the welfare cost estimates (not reported here) are very close to those in Figures 1 and 2; in fact, they are identical to four decimal places, so that the choice between these two approaches is of no importance.

In Figures 3-10 we plot welfare cost functions using equation (11). Here, we follow Lucas (2000) and investigate the welfare cost of inflation at very low interest rates. In particular, we let the nominal interest rate \( i \) range from zero to 2% (instead of using the actual interest rate values as we do in Figures 1 and 2), set the benchmark interest rate \( \delta \) equal to 2% (thereby calculating the welfare cost relative to a 2% interest rate), set \( g = 0.35 \) (the approximate share of government expenditure in total income in the United States today), and assume that \( \phi(l) = l^\vartheta \). We plot the welfare cost function for each of the M1 and MZM money measures, at each of the three levels of monetary aggregation, and for four different values of \( \vartheta = 0.0001, 0.3, 0.6, \) and 0.9.

With almost inelastic labor supply (\( \vartheta = 0.0001 \)), economic agents receive little utility from leisure and the welfare cost functions in Figures 3 and 7 indicate a positive relationship between the welfare cost of inflation and the nominal rate of interest, for nominal interest rates in the range between zero and 2%. Moreover, the Divisia aggregates show a smaller welfare cost than the simple-sum and currency equivalent aggregates, at both the M1 and MZM levels of aggregation. For \( \vartheta = 0.3, \vartheta = 0.6, \) and \( \vartheta = 0.9 \), the welfare cost functions in Figures 4-6 (for M1) and Figures 8-10 (for MZM) also indicate that the Divisia series show a lower welfare cost for each \( \vartheta \) value and for each interest rate value.

5 Conclusion

We have investigated the welfare cost of inflation using tools from public finance and applied microeconomics and quarterly U.S. data from 1960:1 to 2001:4. We have used simple-sum, Divisia, and currency equivalent monetary aggregates to investigate the welfare implications of alternative monetary
aggregation procedures. Our results indicate that the choice of monetary aggregation procedure is crucial in evaluating the welfare cost of inflation. In particular, the Divisia monetary aggregates, which embody differentials in opportunity costs and correctly measure the monetary services furnished by the non-currency components (valued by households), suggest a smaller welfare cost than the simple-sum and currency equivalent aggregates. This result is robust to whether we use the traditional approach developed by Bailey (1956) or the compensating variation approach used by Lucas (2000).

We have used the long-horizon regression approach of Fisher and Seater (1993) to obtain an estimate of the interest elasticity of money demand and also investigated the power of the long-horizon regression tests using Andrews’ (1989) inverse power functions. In doing so, however, we made the bold assumption (mentioned in the introduction) that money is non-interesting bearing and used the 90-day T-bill rate to capture the opportunity cost of holding money. Investigating how much this matters, and also dealing with the issues raised in the last section of Marty (1999), is an area for potentially productive future research.
References


TABLE 1

MARGINAL SIGNIFICANCE LEVELS OF UNIT ROOT AND COINTEGRATION TESTS

<table>
<thead>
<tr>
<th>Level of aggregation</th>
<th>Simple sum</th>
<th>Divisia</th>
<th>Currency equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit root</td>
<td>Cointegration</td>
<td>Unit root</td>
</tr>
<tr>
<td></td>
<td>WS ADF $Z(t_{\alpha})$ $z$ $i$</td>
<td>WS ADF $Z(t_{\alpha})$ $z$ $i$</td>
<td>WS ADF $Z(t_{\alpha})$ $z$ $i$</td>
</tr>
<tr>
<td>M1</td>
<td>.751 .395 .837 .525 .357</td>
<td>.810 .448 .835 .400 .223</td>
<td>.668 .550 .844 .124 .051</td>
</tr>
<tr>
<td>M2</td>
<td>.769 .407 .389 .420 .226</td>
<td>.937 .560 .452 .530 .249</td>
<td>.005 .013 .019 n/a n/a</td>
</tr>
<tr>
<td>M3</td>
<td>.814 .568 .693 .582 .157</td>
<td>.965 .750 .613 .776 .323</td>
<td>.002 .005 .017 n/a n/a</td>
</tr>
<tr>
<td>MZM</td>
<td>.923 .912 .964 .383 .064</td>
<td>.965 .946 .983 .466 .055</td>
<td>.689 .609 .482 .284 .129</td>
</tr>
<tr>
<td>$i$</td>
<td>.796 .673 .536</td>
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</table>

NOTE: Sample period, logged quarterly U.S. data: 1960:1-2001:4. Numbers are tail areas of tests. All tests use a constant and trend. n/a = not applicable.
<table>
<thead>
<tr>
<th>Level of aggregation</th>
<th>Simple sum</th>
<th>Divisia</th>
<th>Currency equivalent</th>
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<td>$k=10$</td>
<td>$k=15$</td>
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<tr>
<td>M1</td>
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<td>-0.14*</td>
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<td>$\eta_{k,0.05}$</td>
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<tr>
<td>M2</td>
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<td>-0.03</td>
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<tr>
<td>M3</td>
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<tr>
<td>MZM</td>
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<td>-0.21*</td>
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</tbody>
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**NOTE:** Sample period, logged quarterly U.S. data: 1960:1-2001:4. An asterisk indicates rejection (of the null that $\eta_k = 0$) at the 5% asymptotic level.
Figure 1. Welfare Cost Functions for M1 Money Measures
Figure 2. Welfare Cost Functions for MZM Money Measures
Figure 3. Welfare Cost Functions for M1, Based on Equation (11) with $\theta = 0.0001$
Figure 4. Welfare Cost Functions for M1, Based on Equation (11) with $\theta = 0.3$
Figure 5. Welfare Cost Functions for M1, Based on Equation (11) with $\theta = 0.6$
Figure 6. Welfare Cost Functions for M1, Based on Equation (11) with $\theta = 0.9$
Figure 7. Welfare Cost Functions for MZM, Based on Equation (11) with $\theta = 0.0001$
Figure 8. Welfare Cost Functions for MZM, Based on Equation (11) with $\theta = 0.3$
Figure 9. Welfare Cost Functions for MZM, Based on Equation (11) with $\theta = 0.6$
Figure 10. Welfare Cost Functions for MZM, Based on Equation (11) with $\theta = 0.9$