Some Statistical Properties of Deregulated Electricity Prices in Alberta

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SOME STATISTICAL PROPERTIES OF DEREGULATED ELECTRICITY PRICES IN ALBERTA

by

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ABSTRACT

In this paper we investigate the stochastic properties of deregulated electricity prices in Alberta. These prices have a long memory, but not a unit root, and are subject to cluster volatility, much like other asset prices. We estimate mean reverting, time varying and jump diffusion models for these prices. Although the modeling strategy produces some interesting results, for instance a jump occurs on average every 3.6 hours, the short term forecasting performance of the models is poor. This suggests that recent advanced work volatility estimation (eg. Andersen et al. (2001)) may be helpful in characterizing the stochastic process. Alternatively, structural models of supply and demand may be estimated.
1. *Introduction*

Deregulation in the Alberta electricity market began on January 1, 1996. The framework for the new structure of Alberta’s electric industry is the Electric Utilities Act. As the first step towards a fully competitive electric market, Alberta Power Pool (the “pool” hereafter) was established on January 1, 1996, as a spot commodity exchange market through which all electric energy, whether generated in Alberta or imported, is traded. Anyone wishing to participate in the pool must become a member of the pool and all pool participants must have a valid contract with the pool before they can submit Bids or Offers. The pool price is determined from hour to hour, depending on the bids and offers to made to the pool.¹

Since deregulation removes price control, it becomes important for both electricity generators and purchasers to hedge price risk. Thus, a number of energy based financial products have become popular in recent energy portfolio analysis. However, the empirical studies on electricity prices are quite few and most of them were focused on UK and US electricity market.

In this paper, we investigate the behavior of Alberta electricity prices and apply several asset pricing models to the price series. This paper is organized as follows. Section 2 analyzes the characteristics of the Alberta electricity prices. Section 3 performs several unit root and long memory tests. Section 4 outlines and describes the econometric methodology we use, in addition to presenting our empirical results. Section 5 presents conclusions and further research.

2. *Data Analysis*

The data used in this paper consist of the Alberta Power Pool hourly electricity price over the period from January 1, 1998 to September 30, 2001, a total of 32,855 observations.² The plot of the entire hourly price series appears in Figure 1. Because electricity markets are characterized by distribution and transmission constraints, there

¹ The pool price is determined at every minute and the officially posted hourly pool price is calculated as the time weighted average of the 60 minutes System Marginal Prices at the end of every hour.

²
are dramatic clusters of swings representing strong peak prices. That is, electricity will be distributed and transferred along a network in which lines are designed to take the particles from the generation source to the demand source after it is generated. Since each line has a maximum capacity that can carry at a given moment, the marginal cost of transmission becomes infinite when the capacity is constrained. This makes the relevant market become isolated from the rest of the electricity market. In this situation, the generators in the isolated market may have market power to influence prices. Therefore, the spot price for electricity in Alberta can move from a price less than $160/MW H to $999.99/MWH within a day.

Electricity prices in Alberta had a dramatic rise from year 1998 to year 2000. In 1998, the average price for the year was $33.02/MW while in 2000 the average price was $133.21/MW. While average value could be distorted by extreme values, the median price may show a better picture of the price distribution. The median price in 1998 was $27.75/MW compared to $68.70/MW, which more than doubled in 2000. However, since the decrease in gas price from the beginning of 2001, the electricity prices also declined in 2001. A summary of the pool price statistics between January 1, 1998 — September 30, 2001 is presented in Table 1.

Figure 2 presents the average hourly electricity prices measured in dollars per megawatt hour ($/MWH) for the whole data set, weekdays and weekends. On average, electricity prices are higher in the weekday than the weekend. From this figure, we can see the daily average of the whole data set is between the average price of weekdays and that of weekends. Within one day, the price begins to increase roughly from 8:00 am, as the workday begins and to decrease roughly from 11:00 pm, as the decrease of residential usage because part of the populace has a rest by that time. The peak price happens between 6:00 pm and 7:00 pm most likely because of both the working usage and residential usage of the electricity.

Figure 3 plots a sample of hourly price data and demand data for the period May 1, 2001 to May 7, 2001. The left vertical axis units are $/MWH measuring price and the right vertical axis units are megawatt hours measuring demand. The horizontal axis is the corresponding hours during this period. This figure illustrates more clearly the daily

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2 This is available at the web site www.powerpool.ab.ca.
usage pattern and its persistence over time. However, the prices do not mimic the demand very well.

Figure 4 illustrates the hourly average price in each of the four seasons. Although the electricity price could reflect heating and cooling needs, we found that the seasonal pattern is not strong in Alberta electricity prices except the fall season. This is because the weather in Alberta is quite changeable and the seasons are not distinct. Most generators shut down and maintain their machine in fall, making short supply in this period. Thus, the highest price in a year mostly happened in fall. The reason for lower electricity prices in winter is that most heating needs are met by natural gas consumption.

A histogram of the price series is presented in Figure 5. This is consistent with all the statistics we obtained from skewness and kurtosis statistics demonstrated in Table 1. All skewness statistics are positive and all kurtosis statistics are greater than three, indicating that distribution of the price series does not resemble the normal distribution. It can be seen from Figure 5 that this series has a long, fat right hand side tail.

The QQ-plot of the price series is illustrated in Figure 6. The plot has the sample data displayed with the plot symbol ‘+’. Superimposed on the plot is a line joining the first and third quartiles of the data (a robust linear fit of the sample order statistics). This line is extrapolated out to the ends of the sample to help evaluate the linearity of the data. If the data does come from a normal distribution, the plot will appear linear. Other probability density functions will introduce curvature in the plot. Both Figure 5 and Figure 6 demonstrate that the distribution of Alberta electricity prices is not normal.

3. **Unit Root and Long Memory Tests for the Electricity Price Series**

Figure 7 is the plot of the autocorrelation function for the level of price series. The autocorrelations are statistically significant even beyond 1000 lags. Figure 7 is suggestive of an extremely long memory series. Given this, we initially investigated the low frequency properties of the data by performing standard augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatowski, Phillips, Schmidt and Shin (KPSS) unit root tests. All three tests check for integer orders of integration but they differ in terms of their null hypothesis. The ADF and Phillips-Perron test the null hypothesis of a unit root
against the alternative of stationarity while the KPSS tests the null hypothesis of stationary against the alternative of a unit root.

The test results obtained from applying these three tests to the electricity price series are shown in Table 2. The results show that the null hypotheses were all rejected at 1% significance level in the three tests. However, the ADF and the PP tests reject the unit root null, implying that the series are I(0), while the KPSS test rejects the null of I(0), implying that the series are I(1). Therefore, neither an I(1) nor an I(0) process appears to adequately describe the low frequency behavior of the price series.

The above results suggest that the electricity price series may be stationary with a long memory. Given this, we performed the Lo’s modified rescaled range (R/S) test and the Geweke and Porter-Hudak (GPH) spectrum analysis in which the null hypothesis of no long memory persistence is tested against the alternative of long memory process.

The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Since the R/S statistic is excessively sensitivity to the short-range dependence and will give biased results in the case of short-range dependence, heterogeneities and nonstationary, Lo (1991) suggested a modified R/S statistic to correct the shortcomings. In his test, the variance of the partial sum is not simply the variance of individual observations, but also includes the weighted autocovariances up to lag \( l \). The criterion used for choosing optimal lags is Andrews (1991) criterion in which is given by

\[
l = \text{int}[\{(3T / 2)^{1/3} \{2\delta / (1 - \delta^2)\}^{2/3}\}]
\]

where \( \text{int} \) denotes the integer portion of the elements in [] and \( \delta \) is the short-order autocorrelation coefficient of the data.
Geweke and Porter-Hudak (1983) suggested a semi-parametric procedure to estimate \( d \) using a simple spectral regression equation\(^3\)

\[
\ln \{ I(\omega_j) \} = \beta_0 - \beta_1 \ln \{ 4 \sin^2 (\omega_j / 2) \} + \varepsilon_j, \quad j = 1, ..., n
\]

A major issue in this application is the choice of \( n \), the number of Fourier frequencies included in the regression. If too few ordinates are included, the slope is calculated from a small sample so that it will increase the variability of the estimates. On the other hand, if too many ordinates are included, medium and high frequency components of the spectrum will bias the estimate. To balance these two factors of consideration, a range of \( \mu \) values, usually \( \mu \in [0.5, 0.8] \) are used for the sample size function, \( n = g(T) = T^\mu \).

The results of long memory test on Table 3 are supportive of the hypothesis that the price series has long memory. From Lo’s modified R/S test, we found that the price series reject the null hypothesis of no long-range dependence at 99% confidence level. From GPH spectrum regression analysis, all estimates of the fractional integrated parameter \( d \) are significantly positive and less than 0.5, meaning that the series is stationary and invertible. The \( t \)-statistics are used to perform formal tests of the null hypothesis of a non-fractional process (\( d = 0 \)) against the alternative hypothesis of a fractional process (\( d \neq 0 \)). The results indicated that there is statistically significant evidence that Alberta electricity price series are well described by the long memory fractionally differenced process.

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\(^3\) \( \varepsilon_j \) is asymptotically i.i.d. across harmonic frequencies, with zero mean and variance known to be equal to \( \pi^2 / 6 \); \( n = g(T) = T^\mu \) with \( 0 < \mu < 1 \) is the number of Fourier frequencies included in the spectral regression and is an increasing function of \( T \). Assuming that \( \lim_{T \to \infty} g(T) = \infty \), \( \lim_{T \to \infty} \{ g(T) / T \} = 0 \) and \( \lim_{T \to \infty} \{ \ln(T)^2 / g(T) \} = 0 \), GPH showed that the least squares estimate of \( \beta_1 \) provides a consistent and asymptotically normally distributed estimator of the slope coefficient \( d \) for \( d < 0 \) while Robinson (1990) provided a proof of consistency and asymptotic normality for \( d \in (0, 0.5) \).
Figure 8 plots the autocorrelation function for the square of prices, which bears a striking resemblance to Figure 7. This plot confirms the volatility clustering which we found in Figure 1. The second moment of the price series exhibits a high degree of persistence even after several hundred lags.

4. Estimated Statistical Models

In this section we estimate several different statistical models often used to represent asset price processes. All models are estimated by conditional maximum likelihood. The data used in estimation is from January 1, 1998 to September 23, 2001. We withhold the last week of data, which is from September 24, 2001 to September 30, 2001 to measure the out-of-sample forecasting ability. A one-week forecast horizon was chosen is because the frequency of our data indicates that only short-term forecast is feasible and most electricity contracts are short-term contracts.

4.1 Mean-Reverting Model

The first model imposes the Ornstein-Uhlenbeck process in which the price series is defined as a continuous time series:

\[ dp(t) = \nu [\mu - p(t)] \, dt + \sigma \, dz(t), \quad p(0) = p_0 \]

where \( p(t) \) is the price of electricity at time \( t \), \( \nu, \mu \) and \( \sigma \) are unknown parameters and \( dz(t) \) is a standard Wiener process where the change in a variable during each short period of time of length \( \Delta t \) has a normal distribution with a mean equal to zero and a variance equal to \( \Delta t \). The first term on the right-hand side represents that the deviations of the price from the equilibrium level are corrected at rate \( \nu \). The second term is a random disturbance. By integration equation (3), becomes:

\[ p(t) = e^{-\nu t} p_0 + \mu (1 - e^{-\nu t}) + e^{-\nu (s-t)} \sigma dz(s) \]
The discrete time version of equation (4) is simply a first order autoregressive model

(5) \[ p_t = \alpha_0 + \beta_1 p_{t-1} + u_t \]

where \( \alpha_0 = \mu (1 - e^{-\nu}) \), \( \beta_1 = e^{-\nu} \), and \( u_t = e^{(s-t)} \sigma dz(s) \). Since the error term is Gaussian white noise, the prices are Markov process with a Gaussian transition density with conditional mean of \( \alpha + \beta p_{t-1} \) and conditional variance of \( \sigma_u^2 \).

The estimated parameters of equation (5) are presented in Table 4. \( \beta_1 \) is highly significant and consistent with what we obtained from the ADF test: \( p_t \) is not an I(1) process. The forecasted (dashed line) and actual (solid line) prices are plotted on Figure 9. Clearly, this model does not accurately capture many of the features of the data. As pointed out by Knittle and Roberts (2001), this type of model fails to capture all cyclical variability present in the series, and does not account for the extreme prices. Clearly, it is inappropriate to assume constant volatility and independence across time in the error structure.

4.2 Time-Varying Mean Model

In this model, we consider peak hour effect, weekend effect and seasonal effect in electricity prices. Thus, equation (3) becomes

(6) \[ dp(t) = \nu [\mu(t) - p(t)] dt + \sigma dz(t) \]

where

(7) \[ \mu(t) = \alpha_{11} (t \in \text{peak}) + \alpha_{21} (t \in \text{Off Peak}) + \alpha_{31} (t \in \text{Weekend}) + \alpha_{41} (t \in \text{Spring}) + \alpha_{51} (t \in \text{Summer}) + \alpha_{61} (t \in \text{Fall}) \]

The discrete time version of equation (6) can be viewed as \( \alpha_0 \) in equation (5) consisting of six dummy variables. That is, two of the dummy variables indicate whether the data falls in peak or off-peak hours. The standard definition for peak hours is from Monday to
Saturday 8:00 am – 23 pm. One of the dummy variables indicates whether the data falls on a weekend and other three dummy variables indicate the seasonal effect.

The estimated parameters are reported in Table 4. The coefficients for on peak and off-peak are significant, reflecting the usage pattern in a day and consistent with our plot represented in Figure 2. The coefficient on weekend has a designed negative sign, but is not significant. This is because the peak hours include most hours on Saturday and the peak is so strong that the weekend effect cannot offset the peak hour effect. Referring to the seasonal effect, only the coefficient on the fall period is significant and has positive sign, suggesting that, on average, electricity prices are higher during fall than other months. This is not a surprising result given the preliminary data analysis.

Next, we considered different dummy variables for equation (7). The discrete time version has the following form:

\[
p_t = \alpha_1 \text{Peak}_t + \alpha_2 \text{Off Peak}_t + \alpha_3 \text{Weekend}_t + \alpha_4 \text{PS}_t + \alpha_5 \text{SS}_t + \beta_1 p_{t-1} + u_t
\]

Where PS stands for peak demand months and SS stands for shoulder demand months. Table 4 shows the estimation result of equation (8). There is very little improvement in this model over the previous specification. Prices appear to be even higher in shoulder demand months. Thus, we can infer that the supply side of electricity is also playing a very important role in Alberta electricity prices.

Figure 11 and Figure 12 showed the out of sample forecasts for time-varying model indicated in equation (6) and (8) respectively. Although the up and down tendency of the forecasted data generally match the actual data, the difference between them is quite big, making the RMS even greater than the previous model. The forecasted price series still cannot capture the extreme prices.
4.3 Jump-Diffusion Model

Jump-Diffusion model was first suggested by Merton (1976). The key assumption made by Merton is that the jump component of the asset’s return represents nonsystematic risk. That is, the risk is not priced in the economy and can be diversified away.

By defining $\mu$ as the expected return from the asset, $\lambda$ as the rate at which jumps happen, $k$ as the average jump size measured as a proportional increase in the asset price, posted stock price returns can be written as a stochastic differential equation, in the following manner:

$$dS/S = (\mu - \lambda k) dt + \sigma dz + dq$$

where $dz$ is a Wiener process, $dq$ is a Poisson process generating the jumps, $\sigma$ is the volatility of the geometric Brownian motion, and $\lambda k$ is the average growth rate from the jumps, meaning that the expected growth rate provided by the geometric Brownian motion is $\mu - \lambda k$. The processes $dz$ and $dq$ are assumed to be independent. The proportional jump size is assumed to be drawn from a Poisson process that can be characterized as follows. The probability of an event occurring during a minor time interval of length $\Delta t$ can be written as

$$\text{Prob}[\text{no events occur in the time interval } (t, t + \Delta t)] = 1 - \lambda \Delta t + O (\Delta t)$$

$$\text{Prob}[\text{the event occurs once in the time interval } (t, t + \Delta t)] = \lambda \Delta t + O (\Delta t)$$

$$\text{Prob}[\text{the event occurs more than once in the time interval } (t, t + \Delta t)] = O (\Delta t)$$

Where $O (\Delta t)$ is the asymptotic order symbol defined by $\psi (\Delta t) = O (\Delta t)$ if $\lim_{h \to 0} [\psi (\Delta t)/\Delta t] = 0$, and $\lambda$ is the mean number of arrivals per unit time, the intensity of the process.
Given this, the price process can be written as

\[ dp(t) = \nu [\mu(t) - p(t)] \, dt + \sigma \, dz(t) + jdq(t) \]  \hspace{1cm} (11)

where \( dq(t) \) is a Poisson process with intensity of \( \lambda \), and \( j \) is a draw from a normal distribution with mean \( \mu_j \) and standard deviation \( \sigma_j \).

Since empirical implementation of (18) is difficult, we follow Ball and Torous’s (1983) approach used by Knittel and Roberts (2001). Equation (11) can result an hourly electricity price series whose density \( f \) is a mixture of Gaussian densities:

\[ f(p) = (1 - \lambda)\phi(\alpha + \beta_1 p_{t-1}, \sigma_u^2) + \lambda\phi(\alpha + \beta_1 p_{t-1} + \mu, \sigma_u^2 + \sigma_j^2) \]  \hspace{1cm} (12)

Where \( \phi(\cdot) \) is a normal distribution.

The conditional likelihood function is

\[
L = \prod_{t=2}^{T} \left(1 - \lambda \right)\phi(p_t - (\alpha_i + \beta_1 p_{t-1}) / \sigma_\mu) \sigma_\mu^{-1} \\
+ \lambda \phi(p_t - (\alpha_i + \beta_1 p_{t-1} + \mu) / (\sigma_\mu^2 + \sigma_j^2)^{1/2}) (\sigma_\mu^2 + \sigma_j^2)^{-1/2}
\]  \hspace{1cm} (13)

where \( \phi(\cdot) \) is the standard normal density.

We use \( \xi \) as a volatility multiplier representing a proportional increase in volatility during the jump period. Thus, the conditional likelihood function becomes

\[
L = \prod_{t=2}^{T} \left(1 - \lambda \right)\phi(p_t - (\alpha_i + \beta_1 p_{t-1}) / \sigma_\mu) \sigma_\mu^{-1} \\
+ \lambda \phi(p_t - (\alpha_i + \beta_1 p_{t-1} + \mu) / (\xi \sigma_\mu)(\xi \sigma_\mu)^{-1})
\]  \hspace{1cm} (14)

The estimation results are presented in Table 4. The results on Table 4 indicate that the probability of a jump occurring at any hour are either 0.277 (column 4), or 0.2765 (column 5), meaning that, on average, a jump occurs every 3.6 hours. This result
is consistent with the dramatic cluster of swings seen on Figure 1. The average jump size $\mu$ is not significantly different from zero in both models, suggesting that the frequency of prices jumping up and down is almost the same. The price volatility is also quite close in both models. During the hours when jump occurs, it is 6 times larger than when jump does not occur. Although there is an increase in the AR coefficient $\beta_1$, it is still statistically stationary and is consistent with the previous model’s estimates. The parameter estimates of $\alpha_t$ verified our previous findings that Alberta electricity prices are affect largely by neither natural season nor demand season.

Figure 13 and 14 illustrate the forecasted results. As pointed out by Knittel and Roberts (2001), “generating useful forecasts from this model is difficult. One must simulate a forecasted path because of the model’s dependency on random jumps”. Given this, we created 10,000 Monte Carlo simulations of forecasted price, drawing from the appropriate conditional distribution. We then took the average price falling based on the mean of the two conditional distributions which make up the mixture in each time period, to obtain a final forecasted path. In spite of this effort, the forecasts still fail to capture the actual movement of the price series.

5. Conclusion and Further Research

Deregulation in Alberta electricity market has lead to the increased use of electricity derivatives to hedge price risk. Clearly this requires as understanding of the statistical properties of deregulated electricity prices. The objective of this study has been to present an empirical analysis of deregulated Alberta electricity prices by applying several asset pricing models. Alberta electricity prices appear to be characterized by a high level of persistence and cluster volatility, statistical properties common amongst asset prices. However, the three financial asset pricing models used in this paper fail to capture the extremely erratic nature of electricity prices. In addition the short term forecasting properties of these models is very poor.

In future research, a number of important issues ought to be considered. From a purely statistical perspective, more advanced stochastic processes could be considered, such as the recent work on volatility by Andersen et al (2001). Alternatively, structural
models of supply and demand may be better for explaining the erratic nature of the electricity prices.
References

Alberta Department of Energy, 1996, “Moving to Competition – A guide to Alberta’s new electric industry structure”.


Figure 1:
Hourly Electricity Prices for January 1, 1998 to September 30, 2001
Figure 2: Average Hourly Electricity Prices

Figure 3: A Sample of Hourly Electricity Prices and Demand for the Period May 1, 2001 to May 7, 2001
**Figure 4:** Average Hourly Electricity Prices by Season

![Average Hourly Electricity Prices by Season](image)

**Figure 5:** Empirical Histogram of Electricity Prices with Normal Density Superimposed

![Empirical Histogram of Electricity Prices with Normal Density Superimposed](image)
Figure 6: Normal Probability Plot of Electricity Prices

![Normal Probability Plot](image)

Figure 7: Autocorrelation Function for Electricity Prices

![Autocorrelation Function](image)
Figure 8: Autocorrelation Function for the Squared Electricity Prices

Figure 9: Week-Ahead Forecasts for the Mean-Reverting Model
Figure 10: Week-Ahead Forecasts for the Time-Varying Model (Equation 6)

Figure 11: Week-Ahead Forecasts for the Time-Varying Model (Equation 8)
**Figure 12:** Week-Ahead Forecasts 1 for the Jump-Diffusion Model

**Figure 13:** Week-Ahead Forecasts 2 for the Jump-Diffusion Model
<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>72.34</td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
</tr>
<tr>
<td>Maximum</td>
<td>999.99</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>101.09</td>
</tr>
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<td>Skewness</td>
<td>4.03</td>
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<tr>
<td>Kurtosis</td>
<td>19.73</td>
</tr>
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</table>

**Table 1**  
**Power Pool Price Statistics**

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
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<tbody>
<tr>
<td>Average</td>
<td>33.02</td>
<td>42.74</td>
<td>133.21</td>
<td>82.86</td>
</tr>
<tr>
<td>Median</td>
<td>27.75</td>
<td>32.26</td>
<td>68.7</td>
<td>78.39</td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6.22</td>
</tr>
<tr>
<td>Maximum</td>
<td>999.50</td>
<td>998.00</td>
<td>999.99</td>
<td>879.2</td>
</tr>
</tbody>
</table>

**Note:**
If skewness is negative, the data are spread out more to the left of the mean than to the right. If skewness is positive, the data are spread out more to the right. The skewness of the normal distribution (or any perfectly symmetric distribution) is zero.

The kurtosis of the normal distribution is 3. Distributions that are more outlier-prone than the normal distribution have kurtosis greater than 3; distributions that are less outlier-prone have kurtosis less than 3.
Table 2  
Unit Root Test for Alberta Electricity Prices

<table>
<thead>
<tr>
<th>Series</th>
<th>Test with Trend and Drift</th>
<th></th>
<th>Test with Drift Only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lags (( l ))</td>
<td>t</td>
<td>Lags (( l ))</td>
</tr>
<tr>
<td>ADF Test</td>
<td>14</td>
<td>-27.523</td>
<td>14</td>
</tr>
<tr>
<td>Phillips-Perron Test</td>
<td>14</td>
<td>-55.218</td>
<td>14</td>
</tr>
<tr>
<td>KPSS Test</td>
<td>17</td>
<td>6.033</td>
<td>17</td>
</tr>
</tbody>
</table>

Note: For ADF test, \( l_{\text{max}} = 14 \) chosen by Schwartz criterion. If the last included lag is significant, we choose \( l = l_{\text{max}} \). If not, reduce the order by 1 until the last included lag is significant.
Critical values for trend stationary: 10%: -3.12  5% : -3.41  1% : -3.96
Critical values for stationarity with drift: 10%: -2.57  5% : -2.86  1% : -3.43

For Phillips-Perron test, \( l_{\text{max}} = 14 \) chosen by Schwertz criterion.
Critical values for trend stationary: 10%: -3.12  5% : -3.41  1% : -3.96
Critical values for stationarity with drift: 10%: -2.57  5% : -2.86  1% : -3.43

For KPSS test, \( l_{\text{max}} = 17 \) chosen by Schwertz criterion.
Autocovariances weighted by Bartlett kernel.
Critical values for trend stationary: 10%: 0.119  5% : 0.146  1% : 0.216
Critical values for stationarity with drift: 10%: 0.347  5% : 0.463  1% : 0.739
Table 3  Long Memory Test for Alberta Electricity Prices

Lo Modified R/S test for price

Critical values for H0: price is not long-range dependent

90%: [ 0.861, 1.747 ]
95%: [ 0.809, 1.862 ]
99%: [ 0.721, 2.098 ]

Test statistic: 6.14702 (99 lags via Andrews criterion) N = 32588

GPH Spectrum Regression

<table>
<thead>
<tr>
<th>Series</th>
<th>d (0.50)</th>
<th>d (0.55)</th>
<th>d (0.60)</th>
<th>d (0.65)</th>
<th>d (0.70)</th>
<th>d (0.75)</th>
<th>d (0.80)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>0.4513</td>
<td>0.3726</td>
<td>0.4157</td>
<td>0.4482</td>
<td>0.3013</td>
<td>0.348</td>
<td>0.4167</td>
</tr>
<tr>
<td>Price</td>
<td>(0.051)</td>
<td>(0.04)</td>
<td>(0.031)</td>
<td>(0.023)</td>
<td>(0.019)</td>
<td>(0.014)</td>
<td>(0.011)</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.038)</td>
<td>(0.029)</td>
<td>(0.022)</td>
<td>(0.017)</td>
<td>(0.013)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

Notes: d (0.50), d (0.55) and d (0.60) give the d estimates corresponding to the spectral regression of sample size $n = T^{0.50}$, $n = T^{0.55}$ and $n = T^{0.60}$.

The t - statistics are given in parentheses and are constructed imposing both the known theoretical error variance of $\pi^2 / 6$ and the empirical error variance estimates.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean-Reverting Model</th>
<th>Time-Varying Model</th>
<th>Jump-Diffusion Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$ (Mean)</td>
<td>10.9635 (0.3643)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_1$ (Peak Hour Mean)</td>
<td>15.9112 (0.6067)</td>
<td>15.6206 (0.565)</td>
<td>0.1768 (0.032)</td>
</tr>
<tr>
<td>$\alpha_2$ (Off-Peak Hour Mean)</td>
<td>5.4476 (0.6661)</td>
<td>5.2235 (0.629)</td>
<td>-0.3949 (0.0352)</td>
</tr>
<tr>
<td>$\alpha_3$ (Weekend Effect)</td>
<td>-0.366 (0.6848)</td>
<td>-0.3238 (0.685)</td>
<td>0.1691 (0.0351)</td>
</tr>
<tr>
<td>$\alpha_4$ (Spring Effect)</td>
<td>-1.1249 (0.8383)</td>
<td></td>
<td>0.0073 (0.0438)</td>
</tr>
<tr>
<td>$\alpha_5$ (Summer Effect)</td>
<td>0.4782 (0.7341)</td>
<td></td>
<td>-0.547 (0.039)</td>
</tr>
<tr>
<td>$\alpha_6$ (Fall Effect)</td>
<td>4.712 (0.8939)</td>
<td></td>
<td>0.1265 (0.0497)</td>
</tr>
<tr>
<td>$\alpha_7$ (Peak Demand Effect)</td>
<td></td>
<td>1.2511 (0.8519)</td>
<td>-0.766 (0.0463)</td>
</tr>
<tr>
<td>$\alpha_8$ (Shoulder Demand Effect)</td>
<td></td>
<td>1.7506 (0.6584)</td>
<td>0.1095 (0.035)</td>
</tr>
<tr>
<td>$\beta_1$ (AR 1)</td>
<td>0.8489 (0.0029)</td>
<td>0.8351 (0.003)</td>
<td>0.8367 (0.003)</td>
</tr>
<tr>
<td>$\lambda$ (Jump Probability)</td>
<td></td>
<td></td>
<td>0.277 (0.0196)</td>
</tr>
<tr>
<td>$\mu$ (Jump Mean)</td>
<td></td>
<td></td>
<td>0.3882 (1.111)</td>
</tr>
<tr>
<td>$\xi$ (Volatility Multiplier)</td>
<td></td>
<td></td>
<td>6.1239 (1.508)</td>
</tr>
<tr>
<td>$\sigma$ (Volatility)</td>
<td>53.5458 (0.2094)</td>
<td>53.2776 (0.2084)</td>
<td>53.3014 (0.0039)</td>
</tr>
<tr>
<td>Forecast RMS</td>
<td>43.37</td>
<td>71.83</td>
<td>51.79</td>
</tr>
</tbody>
</table>

* Standard error are in parentheses.