Rehabilitated or Not?: To Release (?) is the Question*

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Abstract

We first demonstrate why a parole board that cares only about whether inmates are rehabilitated, would release one inmate, while continuing to incarcerate another one about whom they have identical beliefs, but has a longer sentence. The state learns about whether an inmate is rehabilitated by observing his behavior in prison. The longer it has discretion over whether to release the inmate, the more valuable is additional information. In turn, an increase in sentence length can lead to an even greater increase in expected time served. We also consider the effect of increased sentences on inmates’ incentives to undertake rehabilitative effort. Sentences that encourage effort cannot be too short, but maximal sentences may also not be desirable when inmates discount the future heavily. We also look at the possibility for multiple equilibrium in rehabilitation effort, and at the effect of parole eligibility.

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1 Introduction

In this paper, we consider a simple problem of a public authority who must decide whether to release prisoners before the end of their terms. The state does not value punishment for punishment’s sake, and in contrast to many economic models of crime,\textsuperscript{1} it ignores deterrence as a motive for incarceration. The sole reason for incarceration in this model is crime prevention through the incapacitation of criminals, and hopefully through their rehabilitation. The state gains from releasing those who have been rehabilitated and will become productive contributors to society, and loses from releasing recidivists who will return to a life of crime.

Within this environment, we consider the incentive for an inmate to undertake costly effort to rehabilitate themselves. We show that in order to encourage such effort sentences must not be too short, but excessively long sentences may also discourage rehabilitation. If sentences are short the gain in terms of early release is too small relative to the cost. As the sentence becomes increasingly long however, agents may also not wish to undertake this effort since increased sentences imply not only an increased benefit to being released at time $t$, but also a reduced probability of release at time $t$ due to an option value argument. However, only when inmates discount sufficiently the future that maximal sentences in fact reduce rehabilitation effort.

This paper also suggests that sentencing discretion is important in eliciting rehabilitative effort. However, legislators concerned with deterrence may have an incentive to restrict judicial discretion in order to guarantee punishment severity; restrictions have been widely imposed in the United States in recent years in the form of both sentencing guidelines and mandatory minimum sentences. Although an increase in sentences through discretion restrictions will reduce crime, it may also imply less rehabilitation and therefore higher rates of recidivism. In spite of the reductions in crime, increased sentences can imply increased prison populations since agents will have longer to serve prior to release. Moreover, parole officials may have more pessimistic views about inmates’ prospects for rehabilitation.

The choice of whether to grant early release is not blind. Common sense dictates that public authorities should release inmates early only if they believe that inmates will be successful on parole – in other words, they believe inmates are rehabilitated

\textsuperscript{1}For example, Becker (1968) and much of the following literature consider deterrence the primary objective of incarceration.
and will not commit more crimes. However, a formal model describing such a process, and its implications for prisoners’ behaviour has yet to be delivered. Miceli (1994) and Garoupa (1996) view parole as a principal–agent problem where early release is used to promote good behaviour in prison. Scoones (2007) looks at the deterrence value and the cost reduction benefits of statutory releases which are almost entirely independent of individual’s characteristics and behaviours. Our approach has the advantage of allowing us to attack the problem of rehabilitation, since behaviour in prison is a signal of post release behaviour. Pogrebin et al. (1986) cite four primary predictors of recidivism: the nature of the offense, the inmate’s prior record, personal history, and institutional behavior. Although parole authorities take all of these factors into account in their decisions, Carroll and Mondrick (1976) find that the best predictor of parole decisions is inmates’ institutional behavior. Parole authorities update their beliefs about an inmates’ potential parole success on the basis of behavior observed in prison. In particular, ‘good’ behavior is taken to be a strong indicator of rehabilitation, and therefore, parole success.

We start by investigating the case where rehabilitation is completely exogenous, so nothing inherent about the inmate changes over his time in prison. There are two types of inmates: those who will re-commit and those who will not. If an inmate will not re-commit, it is desirable to release them since incapacitation (and not deterrence) is the sole goal of imprisonment. We then extend our analysis to the case where prisoners can invest effort in the hope of rehabilitation. There is empirical evidence, Myers (1983) or Grogger (1991), which suggests that incarceration by itself is not particularly effective in rehabilitation. However, other studies like Witte (1977) and Lattimore et al. (1990) show success when inmates fully participate in various rehabilitation programs, such as vocational training programs. These facts combined

\[2\] For an early discussion of the different potential benefits of parole see Avio (1973).

\[3\] Other studies like Fabel and Leung (1995) or Meier (1999) look at the deterrence aspect of parole, while Ehrlich (1990) takes the rehabilitation process as given, and analyzes recidivism in a general equilibrium context.

\[4\] Many other criminologists and economists like Carr-Hill and Carr-Hill (1972), Schmidt and Witte (1989), or Worthington et al. (2000) have tried to predict recidivistic probabilities with limited success, using different methodologies and combinations of the aforementioned characteristics.

\[5\] However, as discussed in Gottfredson and Gottfredson (1988), ‘good’ institutional behavior in many studies is not significantly related to parole success. Perhaps authorities reward cooperative behavior with parole, as opposed to viewing cooperative behavior as a signal of rehabilitation. However, throughout this paper, we assume that parole authorities use institutional behavior as a predictor of future parole success.
suggest that effort on the part of an inmate is important.

Authorities are assumed to have prior beliefs about an inmate’s probability of being rehabilitated which they update as they observe the inmate’s behavior over time. Information is acquired by observing whether the inmate has any incidents of bad behavior in the past period. Incidents can be interpreted as a signal of rehabilitation, where ‘bad’ behavior is correlated with being a recidivist. The incidents we have in mind, can range from violation of prison rules to non-compliance with respect to a particular training program.

This paper develops the following simple proposition about release times. Given two inmates about whom the authorities have identical beliefs about their prospects for rehabilitation, the state will release first the inmate with the shorter sentence. A simple option argument explains why even though the state has identical beliefs, it should release first the inmate with the shorter sentence. For every inmate, the state essentially has an option with time until expiry equal to the length of the sentence. The state exercises the option by releasing the inmate and collecting the positive dividends if the individual is rehabilitated, and the negative dividends if he is a recidivist. The state retains the option by keeping the inmate in prison.

Ceteris paribus, the longer the sentence, the more valuable it is to hold onto the option rather than exercising it. The expected annual benefit earned from releasing an inmate is not affected by the sentence length, but the value of information gleaned from observing an inmate’s behavior in prison is greater, the longer the sentence. This is because the benefits from decisions based on additional observations can be exploited for longer. This is easiest to see for an inmate with one year remaining in his sentence: any learning about such an inmate’s prospects for rehabilitation has no value because the inmate must then be released. If, instead, the sentence is longer, then this information has value because the state can condition its release decision on whether the inmate appears reformed.

The analysis suggests that a move towards longer sentences will be associated with a greater reluctance by the state to grant parole. Indeed, the average increase in time served may exceed the increase in sentence. In particular, an increase in sentence not only delays release directly, but may also lead the state to observe behavior inconsistent with rehabilitation. This effect can have marked ramifications on the incentives of inmates to exert effort on rehabilitation. This reduction in effort reinforces the effect by leading parole officials to be less optimistic about the inmate they are dealing with.
This points naturally to the fact that with prisoner decisions at the aggregate level affecting parole board beliefs and these beliefs affecting the decision to undertake costly rehabilitative effort there exists a possibility of multiple prison equilibria; we will refer to this phenomena as different prison cultures. Some with optimistic beliefs about prisoners’ rehabilitation status that reinforce the incentive for effort, and some with pessimistic beliefs that imply there is no return to effort. We explore this within the context of our model. We also consider the possibility that restricting parole board discretion in order to increase deterrence could ultimately have large implications for the incentives to undertake rehabilitation.

In the next section of the paper, we discuss the basic model of parole decisions. We argue that time served prior to release increases in conjunction with sentence length. In Section 3, we consider the decision of an inmate to undertake rehabilitative effort and show how this varies with sentence length. In section 4 we investigate the presence of multiple equilibria, and in Section 4, we discuss judicial discretion and parole eligibility.

2 The Basic Model

Consider an individual who has been convicted of some crime and sentenced to a maximum term of $T$ years in prison. There are two types of prisoners: “rehabilitated” prisoners who have learned their lesson and will not return to crime once released, and “non-rehabilitated” prisoners who will continue down the road of crime when released. The cost of holding any prisoner is $B$. The cost of releasing a non-rehabilitated inmate who will return to a life of crime is $C'$, where $C' > B > 0$. Without loss of generality, we will assume that it is costless to release a rehabilitated inmate. Thus, instantaneous costs are given by

$$
\begin{pmatrix}
IN & B & B \\
OUT & C' & 0
\end{pmatrix}.
$$

It is useful to re-examine these payoffs by looking at the costs and benefits of release. There is a benefit, $B$, of releasing a rehabilitated inmate thereby avoiding the costs of imprisonment and a cost $C = C' - B$ of releasing a non-rehabilitated inmate.
The parole board has a prior belief $\pi_0$ that a given inmate is rehabilitated. We assume that $\pi_0B < (1 - \pi_0)C$, otherwise the inmate would be released at time zero. If the inmate has not been reformed, then with instantaneous probability $\lambda_n > 0$, he will be involved in a prison incident at any given point in time. If rehabilitated, the inmate is less likely to misbehave in prison, therefore, the probability of involvement in a prison incident is $\lambda_r < \lambda_n$. Throughout, except where explicitly noted, we assume that the same parameters characterize all inmates. In particular, differences in sentence lengths reflect the capricious nature of the legal system, rather than different priors over the prospects for rehabilitation. We also assume that the parole board does not discount the future.

If an inmate has been involved in $j$ incidents in his first $t$ years, the state then updates and places probability

$$\pi_t(j) = \frac{\pi_0 \lambda_r^j e^{-\lambda_r t}}{\pi_0 \lambda_r^j e^{-\lambda_r t} + (1 - \pi_0) \lambda_n^j e^{-\lambda_n t}}$$

(2.1)
on the possibility that the inmate is rehabilitated at time $t$.

Over the very last $\Delta t$ moments of an inmate’s sentence, it is optimal for the state to release the inmate if and only if the expected benefits from doing so are positive, i.e. if and only if $\pi_{T-\Delta t}(j)B - (1 - \pi_{T-\Delta t}(j))C \geq 0$. Since information gathered over the final stretch $\Delta t$ of the sentences cannot be used because the inmate has to be released anyway, there is no option value of keeping an inmate longer. More generally, the state weighs the expected benefits from release against the option value of continued incarceration.

The expected payoff to society of releasing an inmate with a $T$–year sentence and $j$ prison incidents at time $t$ of his sentence, with $T - t$ remaining, is then given by

$$V^t_T(j) = (T - t)[\pi_t(j)B - (1 - \pi_t(j))C] = (T - t)[\pi_t(j)(B + C) - C].$$

(2.2)

Define $W^t_T(j)$ to be the option value of keeping such an inmate in prison longer, and let $Z^t_T(j) = \max\{V^t_T(j), W^t_T(j)\}$ be the expected value to society of taking the more attractive alternative — release or continued imprisonment. Here,

$$W^t_T(j) = \left[\pi_t(j)e^{-\lambda_r \Delta t} + (1 - \pi_t(j))e^{-\lambda_n \Delta t}\right]Z^{t+\Delta t}_T(j)$$

$$+ \left[\pi_t(j)\lambda_r e^{-\lambda_r \Delta t} + (1 - \pi_t(j))\lambda_n e^{-\lambda_n \Delta t}\right]Z^{t+\Delta t}_T(j + 1).$$

(2.3)

Note that we assume that a recidivist is never returned to prison. Including a probability of this does not alter our results qualitatively, and therefore, we abstract from this possibility.
Note that $W_T^T(j)$ incorporates the value of decisions based on additional observation: the future decision to release will be based in part on whether the inmate had an additional incident during the time interval $[t, t + \Delta t]$. Moreover, in the last moment of a sentence, the option value of incarceration is 0: $W_T^T(j) = Z_T^T(j) = 0, \forall j$.

We begin with a useful lemma which shows that the more likely an inmate is rehabilitated, the more attractive it is for the state to release him:

**Lemma 1** Consider two inmates, Upperbar and Underbar, about whom the state has beliefs for rehabilitation of $\bar{\pi}_t > \bar{\pi}_t$. If it is optimal to release inmate Underbar, then it is optimal for the state to also release inmate Upperbar, about whom the state is more optimistic.

**Lemma 2** If an inmate is not released at time $t$ and has an incident in $[t, t + \Delta t]$ of his sentence, then he will not be released at $t + \Delta t$.

Intuitively, if an inmate is released after an incident, then his release would be non-contingent and would generate positive expected benefits to society. It would then be optimal to release the inmate earlier in order to exploit further these expected benefits. The following proposition states that if two inmates with the same number of prison incidents and time remaining in the sentence, the one with the shorter sentence is released only if the one with the longer sentence is.

**Proposition 1** Suppose that at time $t$ an inmate with a sentence of length $T$ and $j$ prison incidents would be released. Then, an otherwise identical inmate with a sentence length $T + k$ and $j$ incidents at time $t + k$, $k > 0$ would be released.

Intuitively, the state has more optimistic beliefs about the chances for rehabilitation of the inmate with the longer sentence because he has had fewer incidents per unit of time served. Both inmates have the same time remaining in their sentence, so the additional learning from continued observation is the same. Hence, it is more attractive to release the inmate who has served more time.

The next proposition captures the crux of the option argument. Ceteris paribus, the expected immediate benefit earned from releasing any inmate is the same, but the value of the information gleaned from observing an inmate’s behavior in prison is greater for a prisoner with a longer sentence. This is because the benefits from decisions based on the better information can be exploited for longer.
Proposition 2 Consider two inmates who face sentences of different lengths, but are otherwise identical. If the inmate with the longer sentence is released, then so will the inmate with the shorter sentence.

Consequently, a $\Delta T$ increase in sentence length prevents an inmate from being released at time $t$, and will forces such a prisoner to serve an additional $\Delta t < \Delta T$ units of time without prison incident before being released.\footnote{The fact that $\Delta t < \Delta T$ is formerly shown in the proof of Lemma 4.} The expected increase in time served by rehabilitated and non-rehabilitated inmates will differ. In particular, suppose that when an inmate is rehabilitated his prison record is always clean ($\lambda_r = 0$). This assumption has the advantage of simplifying the analysis considerably; one single reported prison incident automatically forces the prisoner to serve the full sentence. Under such an assumption, a rehabilitated prisoner only remains incarcerated for the additional $\Delta t$, after which he is released with certainty. However, a non-rehabilitated criminal would serve on average an extra

$$[e^{-\lambda_n t} \Delta t + (1 - e^{-\lambda_n t}) \Delta T] + [e^{-\lambda_n \Delta t} (1 - e^{-\lambda_n t})(T + \Delta T - t)]$$

before being released. For an inmate who was on the cusp of being released at time $t$, any incidents that occur during the additional $\Delta t$ force this inmate, who would have been released otherwise, to serve the full sentence instead. We will refer to this as the “sorting effect” of longer sentences. The second term in bracket describes this sorting effect, which is driven by the probability of being victim of an incident during the additional $\Delta t$, and by the additional incarceration period $(T + \Delta T - t)$.

Under the original sentence $T$, a non-rehabilitated inmate who was victim of an incident prior to the early release date $t$, was force to serve the full sentence. Now, such an individual will be forced to serve an extra $\Delta T$ unit of time in prison. Similarly, inmates who are released early now have to serve an extra $\Delta t$ before being released. We refer to this as the “incapacitation effect” of longer sentences, and is represented by the first term in bracket. This incapacitation effect simply accounts for the facts that inmates are kept in prison longer no mater if there are released early or not.

Since $\Delta t < \Delta T$, non-rehabilitated inmates always suffer more from an increase in sentence. This difference in expected time served is mainly due to the differences in sorting effects, and will dictate the choice of rehabilitation effort by inmates as we will see in the next section. The difference in sorting effect is generated by the fact that rehabilitated inmates are less likely to be involved in prison incident.
It is also interesting to see that an increase in legislated sentences can produce an even longer expected incarceration period for non-rehabilitated inmates. This would mean that a one month increase in legislated sentences could force inmates to serve more than an additional month in prison in expectation. The larger the sorting effect is, the more likely the increase in expected incarceration period will exceed the increase in legislated sentences.

To be able to formalized those intuitions, we first need to show the existence of an optimal release date for any given number of incidents. Define \( \hat{t}(j) \) as the optimal release date for an inmate with \( j \) prison incidents. The following proposition guarantees the existence and uniqueness of \( \hat{t}(j) \).

**Lemma 3** An inmate with \( j \) incidents is released at \( \hat{t}(j) \), where \( \hat{t}(j) \) solves for \( V_{T}^j(j) = W_{T}^j(j) \) if such a \( \hat{t}(j) \) exists, and \( \hat{t}(j) = T \) otherwise.

Early release dates \( \hat{t}(j) \) depend on the number of incidents an inmate experiences. According to lemma 2, it is obvious that incidents postpone release, or in other words \( \hat{t}(j) \) is strictly increasing in \( j \). Moreover, proposition 2 implies that \( \hat{t}(j) \) is an increasing function of the sentence \( T \), so longer sentences delay early release. Let \( E[P_s] \) be the expected stay in prison for a individual of rehabilitation status \( s \in \{r, n\} \). Also, define \( q_s(t) \) as the probability an individual of rehabilitation status \( s \) is released at time \( t \). For example, an inmate would be released at \( \hat{t}(0) \) only if he has not experienced an incident up to that point. Similarly, an inmate is released at \( \hat{t}(1) \) if and only if he experiences one incident prior to \( \hat{t}(0) \), but no others before \( \hat{t}(1) \). More generally, an inmate is released at \( \hat{t}(j) \) if he had \( j \) incidents at time \( \hat{t}(j) \), and at least \( y + 1 \) incidents at \( \hat{t}(y) \), \( y = 0, 1, \ldots, j - 1 \). The expected stay in prison for an inmate of type \( s \) can be defined as

\[
E[P_s] = \sum_{j=0}^{\infty} q_s(\hat{t}(j)) \min\{\hat{t}(j), T\},
\]

with \( \min\{\hat{t}(j), T\} \) allowing for the possibility that an inmate will not benefit from an early release at all. Since non-rehabilitated individuals face a higher probability of being involved in prison incidents, they should expect longer incarceration periods, so \( E[P_n] > E[P_r] \). The variation in an inmate’s expected incarceration period due to changes in sentence \( T \) is given by

\[
\frac{\partial E[P_s]}{\partial T} = \sum_{j=0}^{\infty} q_s(\hat{t}(j)) \frac{\partial \min\{\hat{t}(j), T\}}{\partial T} + \sum_{j=0}^{\infty} \frac{\partial q_s(\hat{t}(j))}{\partial t} \frac{\partial \hat{t}(j)}{\partial T} \min\{\hat{t}(j), T\}.
\]
The first term represents the incapacitation effect of longer sentences, and is obviously positive. For a given probability of being released after $j$ incidents, inmates have to spend more time in prison since each $\hat{t}(j)$ is pushed farther away into the future. The same applies for inmates who do not benefit from an early release since $T$ itself increases. The second term represents the sorting effect, and is also positive. For every release date $\hat{t}(j)$, the probability of being released at that date changes. More specifically, the probability that an inmate is released after $j$ incidents is reduced. To be released at time $\hat{t}(0)$, an inmate needs to have no incidents up to that point. When $\hat{t}(0)$ increases, it becomes less likely that the inmate will actually make it to $\hat{t}(0)$ with no incidents. Obviously, it has to be associated with an increase in the probability of being released after $\hat{t}(0)$. Similarly, to be released at $\hat{t}(1)$, an inmate can only experience one incident prior to that date. If $\hat{t}(1)$ increases, it becomes less likely to happen, and so on. With longer sentences, both the probability of being released early is reduced, and early release dates are pushed further into the future. Overall, an inmate can expect to experience a longer incarceration period.

Proposition 3 Following an increase in sentence $T$, the expected incarceration period for a non-rehabilitated inmate is larger than a rehabilitated inmate whenever $\frac{\lambda_n}{\lambda_r} \geq \frac{\pi_0}{1-\pi_0}$. If the condition is not satisfied, rehabilitated inmates may suffer a larger increase in their expected incarceration period due to the sentence increase.

Corollary 1 In the special case, when rehabilitated inmates never have incidents ($\lambda_r = 0$), non-rehabilitated inmates always suffer a larger increase in expected incarceration from a sentence increase.

To understand the intuition behind Lemma 4, it is useful to look at the incapacitation effect and the sorting effect separately. According to the sorting effect, inmates are required to display longer periods of time without incidents to earn early release. Because non-rehabilitated inmates are more likely to be involved in prison incidents, the sorting effect always hurts them more. To put it simply, more time allows for better sorting. At first glance, one can easily have the impression that the incapacitation effect is the same for both types of inmates. After all, release dates $\hat{t}(j)$ vary in the same way for both rehabilitated and non-rehabilitated inmates. However, the probability of being released after $j$ incidents differs between individuals, and therefore, non-rehabilitated inmates are less likely to benefit from early release. If early release date $\hat{t}(j)$ was to be increased by more than a later release date $\hat{t}(j + i)$,
rehabilitated inmates could suffer more from the increase in sentences compared to their non-rehabilitated counterparts. The condition stated in Lemma 4 guarantees that it is not the case. Consequently, this condition is sufficient, but by no means necessary. To understand where this condition comes from, one needs to realize that small increases in \( \hat{t}(j) \) are the result of updating. Holding beliefs constant, imagine that the sentence was to increase by one month, then granting an early release one month later would be optimal. However, as time passes, the authority become more optimistic about an inmate rehabilitation prospects, and will consequently be inclined to keep an inmate in prison for less than the full month. When prison incidents are a good signal of rehabilitation (high \( \lambda_\text{nr} \)), most of the updating is done early. The initial beliefs also play a role. Bayes’ rule is less responsive when the prior is close to zero or close to one. If a parole board starts with very optimistic belief (\( \pi_0 \) close to one), time without incident does not alter beliefs by much, meaning that \( \hat{t}(0) \) would increase by almost a full month following a one month increase in \( T \). However, if an inmate was to be involved in a few incidents, updating would have a large impact on beliefs, and subsequently \( \hat{t}(j) \) could increase by significantly less than a full month. Note that when \( \pi_0 < \frac{1}{2} \), the condition is always satisfied.

We will now use a numerical example to better illustrate the interaction between sentences and expected prison stays. We again assume for simplicity, that rehabilitated inmates never get involved in prison incidents. Under such assumption, an inmate will be released after spending \( \hat{t} \) in prison without incident, where \( \hat{t} \) is given by 
\[
V_T^\hat{t}(0) = W_T^\hat{t}(0).
\]
A single incident will force an inmate to serve the full sentence.

**Example:** Take the case of an individual who gets arrested and is found guilty of auto theft in the United States. According to the Bureau of Justice Statistics, more than two thirds of released prisoners accused of property crimes get re-arrested within three years, so we will set \( \pi_0 = 30\% \). According to the Federal Sentencing Commission the median prison sentence for auto theft in 1995 was eighteen months, it has increased to thirty months in 2005. Consequently, we look at the impact of an increase in \( T \) from \( T = 18 \) to \( T = 30 \). The U.S. Courts Office estimates a monthly imprisonment cost of around $1900, while the probation supervision cost is approximately $300. Consequently, we set \( B = 1600 \). The FBI estimates that in 2000, the average value of a stolen motor vehicle was $6,682, and that the recovery rate of stolen motor vehicles

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9See [http://www.ussc.gov/linktojp.htm](http://www.ussc.gov/linktojp.htm) for references.

10See [http://www.uscourts.gov/ttb/may04ttb/costs/index.html](http://www.uscourts.gov/ttb/may04ttb/costs/index.html) for references. The imprisonment cost is consistent with Levitt (1996).
is 62.2%.\footnote{See http://www.fbi.gov/pressrel/pressrel01/cius2000.htm for references.} This corresponds to an average loss of $4000 per theft. If we assume that a non-rehabilitated individual steals a car every month, we set $C = 4000$.\footnote{Again, Levitt (1996) uses very similar numbers with a cost of auto theft of $4000, and an average of 15 crimes a year.}

Take a relatively un-informative environment where non-rehabilitated inmates are not involved in prison incidents significantly more frequently than rehabilitated ones, with $\lambda_n = 0.2$ for example. This could be an environment where prison officers do not gather detailed information about prisoners’ behaviour. A car thief with no prison incidents would be released after 12 months under the initial 18 month sentence, while the same individual would be released after 15.1 months under a 30 month sentence. A rehabilitated inmate would spend an additional 3.1 months in jail, while a non-rehabilitated inmate would spend an additional 9.9 months on average.

Alternatively, $\lambda_n = 0.8$ describes a relatively informative environment. An inmate would only need to serve 4.7 months of an 18 month sentence without any transgressions before being released. If the sentence was to increase to 30 months, an inmate would need to spend 5.5 months. A rehabilitated inmate would only have to spend an additional 0.8 months in prison following this 12 month increase in sentence. However, non-rehabilitated inmates would spend an additional 12.4 months in prison on average, which is more than the 12 month increase in the sentence. This is consistent with the condition derived when holding $\alpha$ exogenous; large $\lambda_n$ creates substantial sorting effects where the extra scrutiny can force non-rehabilitated inmates to suffer from a significant increase in time served.

The expected time served depends on the ease with which the state can distinguish a recidivist. As $\lambda_n - \lambda_r$ increases, the time an inmate is required to serve decreases. This is because as $\lambda_n - \lambda_r$ decreases, the option of gaining more information becomes more valuable since information is harder to disentangle. The relationship between $\lambda_n$ and minimal time served is described in figure 1 when $\lambda_r$ is zero, and the sentence is 30 months. Also, when information is hard to extract, the condition in lemma 4 becomes more likely to be violated. Despite the fact that in both examples non-rehabilitated individuals suffer more from a increase in sentence, the difference is less pronounced when information is harder to disentangle, as in the first case.
Figure 1

Probability of incident for bad inmate

Release time

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
3 Rehabilitative Effort

We now look at the incentive for an inmate to work on becoming rehabilitated. Rehabilitation is obviously a long and time-consuming process, but modeling it that way would pose many additional difficulties, without providing many additional insights. Consequently, we assume that immediately after sentencing, inmates choose whether or not to engage in costly effort to rehabilitate themselves. Denote by \( e \in \{0, 1\} \) an inmate’s choice of effort. Such effort if undertaken has a private cost \( \theta \), and increases the probability of rehabilitation. Without loss of generality, we assume that an inmate who does not provide effort is never rehabilitated. On the other hand, if an inmate chooses effort \( e = 1 \), rehabilitation occurs with probability \( \mu \). Inmates are assumed to be risk neutral, and do suffer a disutility \( D \) per unit of time incarcerated. An individual released at time \( \tau \) would suffer a total disutility

\[
\int_0^\tau e^{-\rho t} D \, dt = \frac{[1 - e^{-\rho \tau}]}{\rho} D,
\]

where \( \rho \geq 0 \) is the rate of time preference. Note that \( \rho = 0 \) represents an infinitely patient individual.

For simplicity, we restrict ourselves to the case where rehabilitated inmates are never involved in prison incidents (\( \lambda_r = 0 \)). Inmates who are involved in any incident prior to an appropriately defined \( \hat{t}(0) \) are forced to serve the full sentence \( T \), while all other inmates are released at \( \hat{t}(0) \). An inmate who decides not to provide effort \( e = 0 \) will suffer a total expected cost \( \Omega(0) \), which is given by:

\[
\Omega(0) = e^{-\lambda_n \hat{t}(0)} \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} D + (1 - e^{-\lambda_n \hat{t}(0)}) \frac{1 - e^{-\rho T}}{\rho} D.
\]

Alternatively, the total expected disutility for an individual who provides effort \( e = 1 \) is defined by

\[
\Omega(1) = \left[ \mu + (1 - \mu)e^{-\lambda_n \hat{t}(0)} \right] \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} D + (1 - \mu)(1 - e^{-\lambda_n \hat{t}(0)}) \frac{1 - e^{-\rho T}}{\rho} D + \theta.
\]

An inmate will provide effort only when \( \Omega(0) - \Omega(1) \geq 0 \), or equivalently when:

\[
\theta \leq \mu(1 - e^{-\lambda_n \hat{t}(0)}) \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} \right] D. \tag{3.1}
\]
When rehabilitation is purely exogenous, the frequency of prison incidents is the only information at the disposal of a parole board to assess whether or not an individual is rehabilitated. Moreover, each individual’s rehabilitation status can be evaluated independently. The same cannot be said with endogenous effort choice. For example, if the environment is such that effort \( e = 1 \) is preferable for all inmates, the parole board should hold in equilibrium an initial belief \( \pi_o = \mu \). Also, choices made by one individual may no longer be independent of other inmates decisions. If the vast majority of inmates are believed to have chosen \( e = 1 \), the parole board should have a relatively optimistic belief about any given inmates’ rehabilitation status, and this in turn this will influence effort choices.

While it is clear that parole board beliefs affect inmate incentives, we think it is implausible that inmates’ take into account the affect their actions may have on the formation of those beliefs. As a consequence, we will provide a very simple model of belief formation. To be able to properly understand how those beliefs are formed, we assume that there exists a unitary mass of inmates all starting at time zero of a similar sentence of length \( T \). A proportion \( a \) of those inmates are assumed to be rehabilitated. In other words, their rehabilitation status is purely exogenous. Conversely, a proportion \( z \) of all inmates are assumed not to be rehabilitated. Only a proportion \( 1 - a - z \) of the inmates actually makes the effort decision described above by equation (3.1).

One important and intuitive conclusion about the interaction between sentence length and effort provision can be drawn. When sentences are too short, the gain of providing effort is small relative to its cost. Put differently, early release is not that valuable when sentences are short. The next proposition captures this intuition.

**Proposition 4** When the sentences \( T \) are short enough, inmates find it advantageous not to provide rehabilitative effort.

Surprisingly, the length of sentences as an ambiguous impact on effort provision. Obviously, as the legislated sentences become longer, the benefit of being released earlier increases. But at the same time, early release \( \hat{t}(0) \) is pushed farther away in the future. This makes investment less attractive since inmates are incarcerated for longer no matter what. Take the example of an inmate who starts imprisonment, and has to decide whether or not to invest in rehabilitative effort. If he picks \( e = 1 \), he pays the associated cost \( \theta \) at time \( t_0 \). The length of the sentence has no impact on such a cost, but it influences future release decisions. In the near future, rehabilitated
and non-rehabilitated inmates suffer from longer sentences, because $\hat{t}(0)$ increases. However, with longer sentences, more sorting can be done. Rehabilitated inmates gain by avoiding those long sentences, however those gains only materialize in the distant future. In other words, everybody suffers at first, but ultimately it is the non-rehabilitated inmates that pay the higher price of longer $T$. If inmates do not discount the future, what matters is the expected incarceration period, so longer sentences will encourage rehabilitation. However, if prisoners heavily discount the future, longer sentences may discourage rehabilitation. We summarize this with the following proposition:

**Proposition 5** Arbitrarily long sentences will always give inmates who don’t discount the future ($\rho = 0$) the incentive to provide rehabilitative effort if $a > 0$. When inmates discount the future heavily, an increase in the sentence may discourage the provision of rehabilitation effort.

If $\pi_0 = 0$, then no agents will ever receive early release. Note that if $\pi_0 = 0$, then no agents will ever receive early release $\hat{t}(0) = T$, so no effort will ever be undertaken. Assuming $a > 0$ is sufficient to eliminate this case. In this proposition, we see that adding discounting has fundamentally changed the insight of Corollary 1. With $\lambda_r = 0$, longer sentences always keep non-rehabilitated inmates in prison for longer on average compared to rehabilitated individuals, but that is not enough to guarantee that longer sentences will increase rehabilitation. When agents discount heavily, increasing sanctions can actually reduce an agent’s desire to undertake rehabilitative effort, even though such rehabilitation would certainly result in early release. Note that if inmates were to use hyperbolic discounting, excessively long sentences would be even more likely to discourage effort.

### 3.1 Prison Culture

We now investigate the potential for multiple equilibria in rehabilitation outcomes. The existence of multiple equilibriums in crime rate has received considerable attention in the literature,\(^{13}\) but the idea of different “prison cultures” has to our knowledge been unexplored by economists. It seems natural to imagine that in certain prisons

\(^{13}\)Like in Sah (1991), Murphy, Shleifer and Vishny (1993) or Burdett, Lagos and Wright (2003) to name a few.
or even within a prison in different wards, it could be the case that different attitudes toward rehabilitation and participation in training programs could exist and be self-enforcing. In our simple environment, a good and a bad prison culture could be defined different equilibria with varying rates of rehabilitative efforts. It interesting to note that the notion of prison culture can be generated without assuming exogenous externalities or without assuming endogenous social sanctions or norms among inmates. In our environment, it will arise purely from the interaction between beliefs and inmates’ effort decisions.

In equilibrium, beliefs will be determined by the values of $a$ and $z$ and also by the equilibrium effort choices of agents. In the special case, where $a + z \to 1$, and so virtually all inmates’ effort decisions are exogenously predetermined, beliefs depend only on the frequency of prison incidents, as in the previous section. In that case, the probability $\pi_t(0)$ that a parole board places on an inmate with no incident at time $t$ being rehabilitated is given by:

$$\pi_t(0) = \frac{a}{a + ze^{-\lambda t}}.$$ 

With any beliefs, Lemma 3 defines $\hat{t}(0)$, and equation (3.1) gives the conditions under which inmates are willing to provide rehabilitation effort $e = 1$. An increase in the cost of effort $\theta$ lowers the likelihood of rehabilitation. More “pleasant” imprisonment (low $D$) or inmates caring less about the future (high $\rho$) both lower the incentive to undertake effort. When effort plays less of a role in rehabilitation outcomes (low $\mu$), effort provision is also less likely.

To understand the role of parole board’s initial beliefs, we consider what happens when parole board becomes more pessimistic by considering an increase in $z$ holding $a + z \to 1$ constant. This implies that the proportion of individuals who are not rehabilitated exogenously increases. A parole board would then hold less optimistic beliefs about any inmates’ rehabilitation status, and would tend to release inmates later. An increase in $\hat{t}(0)$ has two opposite effects on the incentive to undertake effort. The first one is associated with the incapacitation effect. Inmates benefit less from any early release, so effort provision is less attractive. On the other hand, inmates are subject to more scrutiny, so providing effort is more important. Only if this sorting effect is more important than the incapacitation effect will inmates be more likely to provide effort.

In an environment where $a + z < 1$, different prison cultures can co-exist. A good prison culture would be characterized by optimistic beliefs, and good rehabilitative
behaviours. For example, if all $1 - a - z$ inmates who actually make effort choices, choose $e = 1$, the belief $\pi^1_t(0)$ that an individual with no incident is rehabilitated would be given by:

$$\pi^1_t(0) = \frac{a + (1 - a - z)\mu}{a + (1 - a - z)\mu + ze^{-\lambda_n t}}.$$  

With these beliefs, an individual with no incident would be released at $\hat{t}^1(0)$. Similarly, if all $1 - a - z$ inmates choose $e = 0$, individual with no incident are believed to be rehabilitated with a smaller probability:

$$\pi^0_t(0) = \frac{a}{a + (1 - a)e^{-\lambda_n t}}.$$  

An individual with no incident would be then released at $\hat{t}^0(0) > \hat{t}^1(0)$.  

**Proposition 6** For some value of effort cost $\theta$, there exist two equilibria, one in which all $1 - a - z$ inmates choose effort $e = 1$ and another in which all $1 - a - z$ inmates choose effort $e = 0$ if and only if Condition I is satisfied, where Condition I is given by:

$$\left[1 - e^{-\lambda_n \hat{t}^1(0)}\right]\left[e^{-\rho \hat{t}^1(0)} - e^{-\rho T}\right] > \left[1 - e^{-\lambda_n \hat{t}^0(0)}\right]\left[e^{-\rho \hat{t}^0(0)} - e^{-\rho T}\right].$$

**Corollary 2** Condition I is satisfied if and only if the incapacitation dominates the sorting effect.

In order to have to co-existence of two equilibria of this sort, optimistic beliefs have to be consistent with high effort and pessimistic beliefs with low effort. The sorting effect where time allows for agents to be more carefully separted is an effect which always operates in the direction of encouraging more effort. The low effort equilibrium has greater time for sorting. If the sorting effect dominates, then in that case when $\hat{t}(0)$ is further away and there is more time for sorting agents should have a greater, not a lesser incentive to undertake effort in that equilibrium. As a consequence, what is crucial for these two equilibria to co-exist is that the other effect, the incapacitation effect, dominates. Agents gain from being released earlier and this encourages effort in the optimistic equilibrium. The longer delay in the pessimistic equilibrium diminishes this benefit and it is not offset by the longer time for sorting.
4 Parole Eligibility

The objective we attribute to a parole board is simple; parole boards care about releasing rehabilitated inmates. This objective completely abstracts from the deterrence value of longer incarceration periods, as well as any justice issues associated with early release. This objective can easily conflict with the objectives of judges who may also care about deterrence. Parole boards may, in fact, end up undermining the effort of judges by releasing inmates early. Despite the fact that it is \textit{ex post} beneficial to release a rehabilitated inmate early, it may increase crime \textit{ex ante}.\footnote{A closely related time consistency issue was discussed in Boadway et al (1996).}

As a way to mitigate this problem, authorities may want to restrict the discretion of parole boards by prescribing a minimum time be served before granting parole. For example, both the US and Canada impose that one third of a sentence must be completed before being eligible for full parole.\footnote{Inmates can be eligible for what is called day parole before one third of their sentence is completed.} Taking this restriction as given, we will look at its impact on both parole board release decisions and more importantly on inmates’ rehabilitation effort choices. Using the model previously laid out, and restricting ourselves to the special case where $\lambda_r = 0$ and $a + z \to 1$, we will look at a policy where inmates only become eligible for parole after serving a minimum proportion of $\gamma$ of their total sentences.

Prior to $\gamma T$, no one can be released, even if the benefit of parole exceeds the value of continued incarceration. After such a date, the parole board will release an inmate without any incidents only if $V_T^{\gamma T}(0) \geq W_T^{\gamma T}(0)$. If such a condition is not satisfied, the constraint is simply not binding and incarceration will continue until $\hat{t}(0)$. We will consequently restrict our analysis to environment where $\gamma T$ is sufficiently large so that $\hat{t}(0) < \gamma T$. Inmates’ decisions of whether or not to invest in rehabilitative effort follow the same pattern as before, and as long as $\Omega(0) - \Omega(1) \geq 0$ investment will take place. When inmates only become eligible for parole after serving a minimum $\gamma T$ of incarceration, $\Omega(0) - \Omega(1)$ is now given by

$$\Omega(0) - \Omega(1) = \mu(1 - e^{-\lambda_n \gamma T}) \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \gamma T}}{\rho} \right] D - \theta.$$

An increase in the eligibility requirement $\gamma$ will influence inmates’ decision of whether or not to invest in rehabilitation effort. Rehabilitated inmates are more likely...
to be in a position where this eligibility requirement is binding, so longer requirement will tend to lower incentives for provision of effort. However, the sorting effect works in opposite direction. Additional scrutiny due to delayed early releases provides more incentive to undertake effort. Increasing $\gamma$ acts in the same way as increasing $z$ above, both push $\hat{t}(0)$ further away.

**Proposition 7** An increase in the eligibility requirement $\gamma$ from $\gamma^1$ to $\gamma^2 > \gamma^1$, will increase the incentive to undertake rehabilitation effort if and only if Condition II is satisfied, where Condition II is given by:

$$
\left[1 - e^{-\lambda_n^1 T} \right] \left[ e^{-\rho T} - e^{-\rho^1 T} \right] > \left[1 - e^{-\lambda_n^2 T} \right] \left[ e^{-\rho^2 T} - e^{-\rho T} \right].
$$

**Corollary 3** Condition II is satisfied if and only if the sorting effect dominates the incapacitation effect.

By restricting discretion and delaying early release, this result mirrors that of the previous section. Delays release can only serve to encourage more rehabilitation if the sorting effect dominates. By delaying release, the parole board has more time for to learn about the inmates rehabilitation status. This is an effect that encourages rehabilitative effort. However, by delaying release, the benefit of early release is reduced (the incapacitation effect) which diminishes effort incentives. Only if the first force is stronger than the second will effort increase. While it is not obvious which of these effects should or will dominate, more discounting always makes effort less appealing.

**5 Concluding Comments**

This paper demonstrates that the option value of keeping an inmate in prison is more valuable the longer the sentence. Simply put, the value of information gleaned from observing an inmate’s behavior is greater the longer the period of time affected by decisions based on it. This leads the state to release an inmate with a shorter sentence before one with a longer sentence, even though the state has identical beliefs about their prospects for rehabilitation. In turn, the increase in expected time served can exceed the sentence increase. To the extent that inmates whom the state believes are less likely to be rehabilitated receive longer sentences, this effect is reinforced.
We have also shown how increased sentences can dissuade agents from attempting to rehabilitate themselves, but that under some conditions maximal sentence will encourage rehabilitation. When information is easy to disentangle, meaning that non-rehabilitated individual are vastly more often involved in prison incidents, maximal sentence are likely to be desirable. However, if prison incidents are a poor signal of rehabilitation, maximal sentence can reduce rehabilitative effort. This offers another reason for the non-optimality of maximal sentences.

Given parole boards may fail to internalize the deterrence benefits of punishment severity, there may be rationale for discretion restrictions such as mandatory minimum sentences, or other forms of parole limitation. However, this form of increased severity is not costless since it may result in less rehabilitation, and this may be the case when a simple increase in sentence length would not have such undesirable effect.

Finally, we have abstracted from any incentives inmates have to behave in prison and appear rehabilitated, like in Garoupa (1996) or Miceli (1994). In the context of our model, inmates with longer sentences would have a greater incentive to appear rehabilitated. This is because they would get more time off for “good behavior”. Since this would make it harder for the state to distinguish rehabilitated inmates from recidivists, this, too, would lead the state to make inmates with longer sentences serve more time. More generally, this moral hazard has ambiguous consequences. Since those with longer sentences have to behave for longer, it may not pay for recidivists to do so, so that it may be easier for the state to distinguish those who are reformed. One can only conclude that, ceteris paribus, those who receive longer sentences must serve more time before release.
6 References


punishment?,” NBER working paper 6361.


7 Appendix

Proof of Lemma 1: Inmate Upperbar will be released only if $V_T^t - W_T^t \geq 0$. We can rewrite $V_T^t - W_T^t = E[Z_T^{t+\Delta t}]$. Give that $Z_T^{t+\Delta t} = \max\{V_T^{t+\Delta t}, W_T^{t+\Delta t}\}$, it implies that $V_T^t - W_T^t \geq V_T^t - E[Z_T^{t+\Delta t}]$. Since $\bar{\pi} > \pi$, we than know that $V_T^t - W_T^t \geq V_T^t - E[Z_T^{t+\Delta t}]$. Since Underbar is released at $t$, it implies that $V_T^t \geq E[Z_T^{t+\Delta t}]$. Consequently, $V_T^t - W_T^t \geq E[Z_T^{t+\Delta t}] - E[V_T^{t+\Delta t}] \geq 0$, so Upperbar is also released. ■

Proof of Lemma 2: If an inmate is not released at $t$, then $V_T^t(j) < W_T^t(j)$. If the inmate is released at $t + \Delta t$ after an additional incident, then from lemma 1, he would be released even if he did not have an incident. But then

$$0 < W_T^t(j) = \left[\pi_t(j)e^{-\lambda_r \Delta t} + (1 - \pi_t(j))e^{-\lambda_n \Delta t}\right] V_T^{t+\Delta t}(j),$$

$$+ \left[\pi_t(j)\lambda_r e^{-\lambda_r \Delta t} + (1 - \pi_t(j))\lambda_e e^{-\lambda_n \Delta t}\right] V_T^{t+\Delta t}(j + 1),$$

$$W_T^t(j) = (T - t - \Delta t)[\pi_t(B + C) - C] < V_T^t(j) = (T - t)[\pi_t(B + C) - C].$$

Connecting the inequalities, we obtain $V_T^t(j) > W_T^t(j)$, so that it would have been optimal to release the inmate at $t$, a contradiction. ■

Proof of Proposition 1: Obviously $\pi_{t+k}(j) > \pi_t(j)$, so $V_{T+k}^t > V_T^t(j)$. The result then follows from lemma 1. ■

Proof of Proposition 2: Define $\tilde{W}_{T+k}^t(j)$ to be the value of continuing to incarcerate the inmate sentenced to term $T + k$ at $t$ of his sentence when subsequent (potentially sub-optimal) decisions to release are those made were his sentence but $T$ years (in particular, all are released at $T$). An inmate with a $T$–year sentence and $j$ incidents is released at $t$ if and only if $V_T^t(j) \geq W_T^t(j)$. An inmate with a $(T + k)$–year sentence,
$k > 0$ and $j$ incidents is released at $t$ if and only if $V_{T+k}^l(j) \geq W_{T+k}^l(j)$. So, suppose that the proposition is not true, i.e. that

$$V_{T+k}^l(j) \geq W_{T+k}^l(j),$$

but

$$V_T^l(j) < W_T^l(j).$$

Subtracting yields

$$V_{T+k}^l(j) - V_T^l(j) = k[\pi_t(j)(B + C) - C] > W_{T+k}^l(j) - W_T^l(j)$$

$$\geq \hat{W}_{T+k}^l(j) - W_T^l(j) = k[\pi_t(j)(B + C) - C],$$

a contradiction. ■

**Proof of Lemma 3:** At time $t$, it is preferable to keep an individual with $j$ incidents in prison longer if $V_T^l(j) - W_T^l(j) < 0$, and it is preferable to release the individual when $V_T^l(j) - W_T^l(j) > 0$. Given proposition 2, we know that $V_T^l(j) - W_T^l(j)$ is strictly increasing in $t$. Consequently, we can solve for a unique optimal release date $\hat{t}(j) \in [0, T]$ for an inmate with $j$ incidents, and $\hat{t}(j)$ is given by $V_T^{\hat{t}(j)}(j) - W_T^{\hat{t}(j)}(j) = 0$. Note that if $V_T^l - W_T^l > 0$ the inmate will be released at $t = 0$, and if $V_T^l(j) - W_T^l(j) < 0$ the individual will only be released at $T$. Given lemma 2, $\hat{t}(j)$ is increasing in $j$. ■

**Proof of Proposition 3:** The difference in the change in expected incarceration period for a non-rehabilitated inmate compared to a rehabilitated one is given by:

$$\frac{\partial}{\partial T} \left( E[P_n] - E[P_r] \right) = \left[ \sum_{j=0}^{\infty} q(\hat{t}(j), b) \frac{\partial}{\partial T} \min\{\hat{t}(j), T\} - \sum_{j=0}^{\infty} q(\hat{t}(j), g) \frac{\partial}{\partial T} \min\{\hat{t}(j), T\} \right]$$

$$+ \left[ \sum_{j=0}^{\infty} \frac{\partial q(\hat{t}(j), b)}{\partial t} \frac{\partial}{\partial T} \min\{\hat{t}(j), T\} - \sum_{j=0}^{\infty} \frac{\partial q(\hat{t}(j), g)}{\partial t} \frac{\partial}{\partial T} \min\{\hat{t}(j), T\} \right].$$

The first term represents the differences in incapacitation effects, while the second term represents the difference in sorting effects. We will first show recursively that the difference in sorting effect is always positive. Imagine an individual who would be released at $\hat{t}(0)$. An increase in $T$ triggers an increase in $\hat{t}(0)$, forcing the inmate to spend extra time in prison, regardless of his or her type. The probability that an inmate would be released early with no incidents is reduced relative to the probability of being released later on. Since, non-rehabilitated individuals have a higher instantaneous probability of being involved in an incident, this extra scrutiny will hurt
them more. The same applies for all releases $j \in \{1, 2, 3, \ldots\}$ incidents. Consequently, 
$$
\sum_{j=0}^{\infty} \frac{\partial \tilde{\Omega}(j, b)}{\partial t} \frac{\partial \tilde{\Omega}(j)}{\partial j} \min\{\tilde{t}(j), T\} > \sum_{j=0}^{\infty} \frac{\partial \tilde{\Omega}(j, g)}{\partial t} \frac{\partial \tilde{\Omega}(j)}{\partial j} \min\{\tilde{t}(j), T\}.
$$

We now need to deal with the first term; the difference in incapacitation effects. When $T$ increases, both types of inmates face the same changes in $\hat{t}(j)$, but the probability of being released at any of those $\hat{t}(j)$ is different. Non-rehabilitated individuals are facing higher probabilities of being released at later dates including date $T$. Two conditions need to be satisfied to guaranty that non-rehabilitated inmates will remain on average in prison for longer. First, we need for $\frac{\partial \tilde{\Omega}(j)}{\partial t}$ to be increasing in $j$. If it was not the case, rehabilitated inmates could suffer more from an increase in sentences $T$ because they are more likely to be release after a small number of incidents (low $j$). Similarly, we need $\frac{\partial \tilde{\Omega}(j)}{\partial T} \leq 1$, for all $j$.

This second condition is always satisfied. Holding beliefs constant, when $T$ increases, the value of releasing an inmate at $\tilde{t}(j)$ become smaller relative to the option value of keeping the inmate longer. For constant beliefs $\pi_t(j)$, delaying release by $\Delta \tilde{t}(j) = \Delta T$ would reestablish the equality between the two values. Since the inmate has still only $j$ incidents, but spent longer time in prison, the belief $\pi_t(j)$ has to increase. This implies that $\Delta \tilde{t}(j) < \Delta T$, and consequently $\frac{\partial \tilde{\Omega}(j)}{\partial t} \leq 1$ for all $j$.

To know when the first condition is satisfied, we need to acknowledge that larger changes in one’s rehabilitation beliefs leads to smaller increase in $\hat{t}(j)$. Consequently, a sufficient condition for $\frac{\partial \tilde{\Omega}(j)}{\partial t}$ to be non-decreasing in $j$ is that $\frac{\partial^2 \pi_t(j)}{\partial t \partial j} \leq 0$. Partial differentiation of $\pi_t(j)$ with respect to $t$ and $j$, reveals that $\frac{\partial^2 \pi_t(j)}{\partial t \partial j}$ is negative whenever $\pi_0 \lambda_r e^{-\lambda_r t} \leq (1 - \pi_0) \lambda_n e^{-\lambda_n t}$. A sufficient condition for this condition to itself be satisfied is that $\frac{\lambda_r}{\lambda_n} \geq \frac{\pi_0}{1 - \pi_0}$. Note that for the special case where $\lambda_r = 0$, there exist only one early release date $\hat{t}(0)$, so this condition is always satisfied.

**Proof of Proposition 4:** When $T = 0$, we know that for any $\theta > 0$ it implies that $\Omega(1) \geq \Omega(0)$. Consequently, inmates will choose $e = 0$. Obviously, $\Omega(0) - \Omega(1)$ could be either increasing or decreasing in $T$. In either of the two cases, there exist a neighborhood around $T = 0$ where effort $e = 0$ will be chosen.

**Proof of Proposition 5:** Longer sentences increase the incentive to choose $e = 1$, as long as $\Omega(0) - \Omega(1)$ is increasing with sentence $T$. Note that if $\pi_0 = 0$, then no agents will ever receive early release and this $\hat{t}(0) = T$, so no effort will ever be undertaken. Assuming $a > 0$ is sufficient to eliminate this case.

Suppose all agents set $e = 1$ and define the corresponding beliefs and $\hat{t}(0)$. Now
consider whether as $T$ increases agents have an incentive to set $e = 1$:

$$\frac{\partial}{\partial T} \left[ \Omega(0) - \Omega(1) \right] = \mu \lambda_n e^{-\lambda_n \hat{t}(0)} \frac{\partial \hat{t}(0)}{\partial T} \left[ \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} - \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} \right] D$$

$$- \mu \left( 1 - e^{-\lambda_n \hat{t}(0)} \right) \left( e^{-\rho T} - e^{-\rho \hat{t}(0)} \frac{\partial \hat{t}(0)}{\partial T} \right) D.$$

The first term on the right hand side represents the difference in sorting effects, and like in Proposition 3 it is always positive. The relative strength of the incapacitation effects is given by the second term. Whenever $\rho = 0$, only the length of the prison stay matters, so this relative strength is exactly as in Proposition 4. Given that $\lambda_r = 0$, this second term would also be positive in that case. When $\rho$ is strictly positive, $e^{-\rho \hat{t}(0)} \frac{\partial \hat{t}(0)}{\partial T}$ may be larger than $e^{-\rho T}$. Consequently, if sentence are arbitrary long, inmates will always invest in rehabilitative effort whenever $\rho = 0$. If $\rho$ is sufficiently large, $\Omega(0) - \Omega(1)$ may not be monotonically increasing in $T$, and excessively large sentence may not imply that inmates will choose $e = 1$. Given this non-monotonicity, at such values of $T$, the correct beliefs will be less optimistic and corresponding value of $\hat{t}(0)$ will be higher, which will reinforce the incentive to not undertake effort.

**Proof of Proposition 6:** Given early release date $\hat{t}^1(0)$, all $1 - a - z$ inmates will choose $e = 1$ if and only if:

$$\theta \leq \mu \left[ 1 - e^{-\lambda_n \hat{t}^1(0)} \right] \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}^1(0)}}{\rho} \right] D.$$

Similarly, for a given early release date $\hat{t}^0(0)$, all $1 - a - z$ inmates will choose $e = 1$ if and only if:

$$\theta > \mu \left[ 1 - e^{-\lambda_n \hat{t}^0(0)} \right] \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}^0(0)}}{\rho} \right] D.$$

Consequently, it is possible to find a range of value for $\theta$ that satisfied those two equations simultaneously if and only if Condition I is satisfied.

**Proof of Corollary 2:** Since $\hat{t}^0(0) > \hat{t}^1(0)$, Condition 1 implies that $\bar{\theta}$ must be decreasing in $\hat{t}$, where:

$$\bar{\theta} = \mu \left[ 1 - e^{-\lambda n \hat{t}} \right] \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}}}{\rho} \right] D.$$

Since the incapacitation effect is negative and the sorting effect is positive, $\bar{\theta}$ is increasing in $\hat{t}$ if and only if the incapacitation
effect is stronger. ■

Proof of Proposition 7: Similar to Proposition 6. ■

Proof of Corollary 3: Similar to Corollary 2. ■