Strategic Interactions in Two-Sided Market Oligopolies

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Abstract

Strategic interactions between two-sided platforms depend not only on whether their decision variables are strategic complements or substitutes as for one-sided firms, but also and crucially so on whether or not the platforms subsidize one side of the market in equilibrium. For example, with prices being strategic complements across platforms, we show that a cost-reducing investment by one firm may have a positive effect on its rival’s profits and a negative effect on its own profits when one side is subsidized in equilibrium. By contrast, if platforms make positive margins on both sides, the same investment has the regular, expected effects. Our analysis implies that the strategy space and the logic of competitive advantage are fundamentally different in two-sided markets relative to one-sided markets.

Keywords: Two-Sided Markets, Two-Sided Platforms, Strategic Complements, Strategic Substitutes, Competitive Advantage.

JEL Classifications: L1, L2, L4, L8

1 Introduction

Fudenberg Tirole (1984) (hereafter FT) and Bulow Geanakoplos and Klemperer (1985) (hereafter BGK) have proposed a typology of strategic interactions in one-sided markets oligopolies. They analyze a competition game preceded by an investment stage. Three factors are shown to determine whether an incumbent will over-invest or under-invest: whether the objective of the incumbent is to accommodate or to deter entry, whether actions in the competition game are strategic complements or substitutes and whether investment increases or decreases rivals’ profits (cf. Tirole 1988).

In this paper, we show that the possibility of subsidization of one side in a two-sided market can lead to fundamentally new (and somewhat surprising) strategic configurations in oligopoly. For instance, we show that a cost-reducing investment by a two-sided platform may be a successful entry accommodation strategy and at the same time raise the profits of its rival (in a one-sided market, cost reductions by one firm unambiguously hurt its competitors). The intuition behind this result is as follows. A cost-reduction by one of the platforms gives it a competitive advantage relative to its rival, so that in the new equilibrium its prices on both sides will be lower than its rival’s and it will steal customers from its rival on both sides of the market relative to the initial

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equilibrium. However, the ratio between the number of customers stolen from the subsidized side and the number of customers stolen from the other (profitable) side may be sufficiently high so that the rival is happy to get rid of both, and sufficiently low so that the more cost-effective platform can serve them profitably at the new equilibrium prices. In other words, the cost reduction allows a "price-rebalancing" act by both platforms\textsuperscript{1}, which may end up being beneficial to both of them - in stark contrast with the one-sided case.

An important application of this result concerns tying in the spirit of Whinston (1990): a two-sided platform A also has a monopoly power over another product M which is homogenously valued by all customers on one side of A. Tying M and the purchase of the platform on this side of A then acts as a commitment to price aggressively by raising the opportunity cost of a foregone sale. In the pricing game that follows, it has the same effect as a reduction of the marginal cost of distribution of A on the side of the market which buys M, relative to rival two-sided platforms. In a one-sided market with price competition and homogenous valuations of the tying good, tying is always a "top dog" strategy: it decreases rivals’ profits while increasing one’s own. By contrast, the result mentioned above implies that in a two-sided market, tying can be part of a "fat cat" strategy: a profitable way to accommodate entry while at the same time being "soft" (i.e. benefitting rivals as well).

More broadly, our analysis reveals that the set of strategic configurations in a two-sided market is strictly larger than in a one-sided market - not in terms of the nature of the strategies but in terms of the conditions under which they emerge. This is due to the fact that in two-sided markets, the sign of the strategic effect – that determines whether the incumbent will over- or under-invest – can no longer be entirely determined by the effect on rivals’ profits as in one-sided markets.

This conclusion lends significant substance and support to the contention that two-sided markets have new implications, both from a strategy and from a public policy perspective. Indeed, the recent and burgeoning literature on two-sided markets is built on this premise, but the main argument put forward up to now is that two-sided platforms are different because they might end up subsidizing one side of the market in order to recoup on the other. This argument has received significant attention from antitrust scholars (e.g. Evans (2003), Wright (2004)), which have pointed out several implications, such as for instance that below-cost pricing in a market may not be indicative of predation, but of a two-sided pricing strategy, which can be profitable regardless of the presence of competition. Our paper takes a step beyond exhibiting this type of skewed two-sided pricing structure by deriving deeper implications of this feature for strategic interactions among platforms and a systematic categorization of strategies in two-sided markets oligopolies (which we show can be very different from one-sided markets).

\textbf{Relation to the literature.} This paper contributes to the recent two-sided markets literature, initiated by Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003). Most of this literature has analyzed how platforms might solve the chicken and egg problem associated

\textsuperscript{1}Cf. Rochet and Tirole (2006).
with two sided-markets and focused almost exclusively on the conditions which determine which (if any) side is subsidized and how much. We study the implications of the two-sided pricing game between two platforms on strategic investment choices that must be made prior to the pricing game. To illustrate the conclusions of our general analysis, we borrow from Armstrong (2006)’s two-sided Hotelling competition setting, but we extend his model by allowing for "hinterlands" - this extension is absolutely necessary in order to exhibit results which are different from the ones occurring in one-sided markets.

There are three papers in the two-sided market literature that focus on tying. Rochet and Tirole (2003b) provide an economic analysis of the tying practice initiated by payments card associations Visa and MasterCard in which merchants who accept their credit cards were forced also to accept their debit cards. They show that in the absence of tying, the interchange fee between the merchant’s and the cardholder’s banks on debit is too low and tends to be too high on credit compared to the social optimum. Tying is shown to be a mechanism to rebalance the interchange fee structure and raise social welfare. Choi (2007) analyzes the welfare effects of tying in a model of competition between two-sided platforms (connecting consumers and content providers), when one or both sides can multihome. In his model, tying simply allows one of the platforms to reach all consumers by bundling the platform product in question with another product that all consumers need (the motivating example is the tying of Windows Media Player to the Windows Operating System, which every PC user has). The impact of tying on social welfare depends on whether consumers can multihome or not, but in all cases, tying unambiguously hurts the rival platform.

Amelio and Jullien (2007) consider a setting in which two-sided platforms would like to set prices below zero on one side of the market in order to solve the demand coordination problem, but are constrained to set non-negative prices. Tying can then serve as a mechanism to introduce implicit subsidies on one side of the market in order to solve the aforementioned coordination failure. As a result, tying can raise participation on both sides and can benefit consumers in the case of monopoly platform. In a duopoly context tying also has a strategic effect on competition. Contrary to the monopoly case, tying may not be ex-post and/or ex-ante optimal for a contested platform. Moreover, the competing platform benefits from it if the equilibrium implicit subsidy is large enough. We also obtain this result, although as a particular case of a broader setting.

The remainder of the paper is organized as follows. In Section 2, we briefly review the typology introduced by FT for one sided-markets. In Section 3, we lay out a very general two-sided market oligopoly setting, derive the corresponding characterization of the strategic space and explain the key differences relative to the one-sided environment. In Section 4, we provide specific examples and analyze the implications for tying. We conclude in Section 5.
2 One-sided markets

This section provides a brief summary of the framework and animal strategy nomenclature introduced by FT; it draws heavily on Tirole (1988), to which the reader is referred for more details.

There are two firms, an incumbent, A, and an entrant, B. The timing of the duopoly game comprises three stages. In stage 1, firm A chooses a variable $K_A$ – for example a capacity, an investment in product quality or in a cost reducing technology. In stage 2, firm B observes $K_A$ and decides whether or not to enter. In stage 3, the stage game depends on whether or not $B$ has entered at stage 2. If $B$ has not entered at stage 2, $A$ chooses $x_A$ – which could be a price or a quantity – to maximize profits as a monopolist. If $B$ has entered then $A$ and $B$ simultaneously choose $x_A$ and $x_B$.

In stage 3, if both firms are active, firm i’s profits are denoted by $\Pi^i(K_A, x_A, x_B)$. Denote by $(x_A^*(K_A), x_B^*(K_A))$ the Nash equilibrium in stage 3 given $A$’s choice of $K_A$ in stage 1. There are two cases to consider. If entry deterrence is the most profitable strategy for $A$ then its choice of $K_A$ is driven by $\Pi_B$. Firm $A$ will choose $K_A$ so as to just deter entry by $B$: $\Pi_B(K_A, x_A^*(K_A), x_B^*(K_A)) = 0$ (1)

By contrast, if entry accommodation is the most profitable strategy for firm $A$, then its choice of $K_A$ is driven by $\Pi_A$. In this case, $K_A$ is chosen so as to maximize $\Pi_A(K_A, x_A^*(K_A), x_B^*(K_A))$:

$$\frac{d\Pi_A}{dK_A} = \frac{\partial \Pi_A}{\partial K_A} + \frac{\partial \Pi_A}{\partial x_B} \frac{dx_B^*}{dK_A} = 0$$

(2)

In both cases, the influence of $K_A$ on firm B’s profits is given by:

$$\frac{d\Pi_B}{dK_A} = \frac{\partial \Pi_B}{\partial K_A} + \frac{\partial \Pi_B}{\partial x_A} \frac{dx_A^*}{dK_A}$$

(3)

Assuming by symmetry that $\text{sign} \left( \frac{\partial \Pi_A}{\partial x_B} \frac{dx_B^*}{dK_A} \right) = \text{sign} \left( \frac{\partial \Pi_B}{\partial x_A} \frac{dx_A^*}{dK_A} \right)$, we have:

$$\text{sign} \left( \frac{\partial \Pi_A}{\partial x_B} \frac{dx_B^*}{dK_A} \right) = \text{sign} \left( \frac{\partial \Pi_B}{\partial x_A} \frac{dx_A^*}{dK_A} \right) \text{sign} \left( \frac{dx_A^*}{dK_A} \right)$$

where $\frac{dx_A^*}{dK_A} > 0$ ($< 0$) if the variables $(x_A, x_B)$ are strategic complements (substitutes).

The focus of the analysis is on the strategic effects. Direct effects would exist even if firm $B$ did not observe $K_A$ (open loop solution). Therefore, the sign of the strategic effect - under both entry deterrence and accommodation - determines whether firm $A$ should over- or under-invest relative to the level of investment which would prevail in the open loop equilibrium. FT adopt the convention

\[\text{At the stage 3 Nash equilibrium values } (x_A^*(K_A), x_B^*(K_A)) \text{ we have: } \frac{\partial \Pi_A}{\partial x_A} = \frac{\partial \Pi_B}{\partial x_B} = 0.\]
that investment makes firm A "tough" ("soft") if the strategic effect of $K_A$ on firm B’s profits is negative (positive). We will maintain this convention throughout our paper. They then define the following strategies: (i) "top dog", be big (i.e. overinvest) in order to look tough (aggressive); (ii) "lean and hungry look", stay small (i.e. underinvest) in order to look tough (aggressive), (iii) "puppy dog": stay small in order to look soft (inoffensive) and (iv) "fat cat": be big in order to look soft (inoffensive). These four strategies turn out to be sufficient for fully describing firm A’s desired behavior in all cases, as shown in the following table:

<table>
<thead>
<tr>
<th>Actions are:</th>
<th>Investment $K_A$ is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic</td>
<td>Tough</td>
</tr>
<tr>
<td>complements</td>
<td>A: Puppy Dog</td>
</tr>
<tr>
<td>D: Top Dog</td>
<td>D: Lean and Hungry</td>
</tr>
<tr>
<td>Strategic</td>
<td>A: Top Dog</td>
</tr>
<tr>
<td>substitutes</td>
<td>D: Top Dog</td>
</tr>
</tbody>
</table>

Table 1

where A stands for entry accomodation and D for entry deterrence.

We now turn to a simple application, to which we will refer in the next sections of the paper. Assume that the actions $x_i$ are prices - $x_i \equiv p_i$ for $i = A, B$ - and $K_A$ is an investment which reduces the marginal cost $c_A$ of firm A. Also, assume that prices are strategic complements (this is the case with linear demand models): $\frac{dp_i^*}{dp_{-i}} > 0$ for $i \in \{1, 2\}$. Denoting by $n_i(p_A, p_B)$ the demand for firm $i$ – with the property that $\frac{\partial n_i}{\partial p_{-i}} > 0 > \frac{\partial n_i}{\partial p_i}$, for $i \in \{1, 2\}$ – we have:

$$\frac{d\Pi^B}{dK_A} = \frac{d}{dK_A} ((p_B - c_B) n_B) = (p_B^* - c_B) \left( \frac{\partial n_B}{\partial p_A} \frac{dp_A^*}{\partial p_A} \frac{dc_A}{dc_A} \frac{dK_A}{dK_A} \right) < 0$$

where the first term is positive because firm B’s profits have to be positive in equilibrium in order for firm B to be active. The third term is positive as a result of the first order condition for profit maximization for firm A$^3$. Thus, a cost-reducing investment by firm A in a one-sided market can only be tough. This is quite intuitive: a marginal cost reduction by one firm can only hurt its rival in a one-sided context, irrespective of whether competition is in prices or quantities. As we will see in the next section, things change radically in a two-sided context.

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$^3$Assuming the second order conditions are satisfied, i.e. the second derivative of A’s profits with respect to $p_A$ is negative.
3 Two-sided markets

We now move to a two-sided context, in which the two firms A and B are competing not for just one type of customers, but for two interrelated groups of customers 1 and 2. Each platform \( i \in \{A, B\} \) chooses 2 actions \( p_i^1 \) and \( p_i^2 \), which correspond to sides 1 and 2 respectively. Although in principle these could be any strategic actions, we will focus on the case in which they correspond to prices that the platforms have to set on both sides of the market.4

Two-sidedness is captured by assuming the demand a platform faces on each side is decreasing in the price it charges to that side, increasing in the price charged by the rival platform on the same side, increasing in the realized demand on the other side of the same platform and decreasing in the realized demand on the other side of the rival platform. In short, the demand for platform \( i \) on side \( j \) is given by:

\[
N_i^j = \hat{n}_i^j (p_i^j, p_{-i}^j, N_{-i}^j, N_{-j}^j)
\]

with \( \frac{\partial \hat{n}_i^j}{\partial p_i^j} < 0 < \frac{\partial \hat{n}_i^j}{\partial p_{-i}^j} \) and \( \frac{\partial \hat{n}_i^j}{\partial N_{-i}^j} < 0 < \frac{\partial \hat{n}_i^j}{\partial N_{-j}^j} \).

The timing of the game is the same as in the previous section. In stage 1, firm A chooses investment \( K_A \). In stage 2, firm B observes \( K_A \) and decides whether or not to enter. In stage 3: if B has not entered at stage 2, then A chooses \( p_1^A \) and \( p_2^A \) as a monopolist; if B has entered then A and B simultaneously choose \( p_i^1 \) and \( p_i^2 \), \( i \in \{A, B\} \).

Throughout the rest of the paper we will make the following four assumptions in order simplify the analysis (the substance of our conclusions is not affected).

Assumption 1 (non-singularity) The functions \( n_i^j \) are well-behaved so that for any quartet of prices \( \{p_i^k\} \), the four equations (4), determine a unique, stable, configuration of demands:

\[
N_i^j = n_i^j (p_i^k)
\]

where \( \frac{\partial n_i^j}{\partial p_i^j} < 0 < \frac{\partial n_i^j}{\partial p_{-i}^j} \) and \( \frac{\partial n_i^j}{\partial p_{-j}^j} < 0 < \frac{\partial n_i^j}{\partial p_{-i}^j} \).

The profits of platform \( i \) can then be written as:

\[
\Pi^i = (p_1^i - c_1^i) n_1^i + (p_2^i - c_2^i) n_2^i
\]

where \( c_j^i \) is the marginal costs of platform \( i \) on side \( j \).

Assumption 2 (concavity) Each platform’s profits are concave in both of its prices holding the rival platform’s prices constant and there always exists a unique Nash equilibrium in the stage 3 pricing game (which depends on the value of \( K_A \) chosen in stage 1 and on platforms’ costs \( c_j^i \)), denoted by \( \{p_j^*\} \) with \( i \in \{1, 2\}, j \in \{A, B\} \).

4In fact, we are unaware of any model of two-sided markets in which platforms compete in variables other than prices.
Assumption 3 (strategic complementarity) Prices are strategic complements: \( \frac{\partial p_j^*}{\partial c_{j'}} > 0 \) and equilibrium prices for each platform are increasing in its own costs:

\[
\frac{\partial p_j^*}{\partial c_{j'}} > 0 \text{ for all } i \in \{1, 2\}, \ j, j' \in \{A, B\}
\]  

Note that this assumption implies that \( \frac{dp^B}{dK_A} = \frac{\partial p^B}{\partial p^A} + \frac{\partial p^A}{\partial p^B} \frac{dp^B}{dK_A} < 0 \) for \( i = 1, 2 \).

In a one-sided market, the corresponding condition to (5) is automatically verified and can be seen at the most general level as an application of the monotone comparative statics results in Milgrom and Shannon (1994). These principles, however, cannot be invoked in the two-sided context analyzed in this section.

Assumption 4 (symmetry) For \( K_A = 0 \) and the corresponding Nash equilibrium, we have:

\[
\frac{\partial \Pi^i}{\partial p_j^*} (\{p^*_{i,k}\}) = \frac{\partial \Pi^{-i}}{\partial p_j^*} (\{p^*_{i,k}\})
\]

Indeed, instead of solving the last two stages of the game for all values of \( K_A \), we will only perform a local comparative statics analysis in \( K_A \) around \( K_A = 0 \), which is sufficient for our purposes.

Finally, from now on we will focus on the case in which \( K_A \) is a cost-reducing investment on side 1 for platform \( A \):

\[
\frac{dc_1^A}{dK_A} < 0 \quad \text{and} \quad \frac{dc_2^A}{dK_A} = \frac{dc_1^B}{dK_A} = \frac{dc_2^B}{dK_A} = 0
\]

Equations (5) imply then:

\[
\frac{dp_j^*}{dK_A} = \frac{dp_j^*}{dc_1^A} \cdot \frac{dc_1^A}{dK_A} < 0
\]

for \( i \in \{1, 2\}, \ j \in \{A, B\} \).

3.1 Strategic interactions with variable prices on both sides

As in the one-sided context, we are interested in determining the strategic effects of \( K_A \) on the rival platform \( B \)'s profits as well as on platform \( A \)'s own profits. The notion of under/over-investment is relative to a situation in which only the direct effects exists. Hence our interest lies in signing \( \frac{d\Pi^A}{dK_A} - \frac{\partial \Pi^A}{\partial K_A} \) and \( \frac{d\Pi^B}{dK_A} - \frac{\partial \Pi^B}{\partial K_A} \).

Using the envelope theorem, the sign of the strategic effect on platform \( B \)'s profits is:

\[
sign \left\{ \frac{d\Pi^B}{dK_A} - \frac{\partial \Pi^B}{\partial K_A} \right\} = sign \left\{ \frac{\partial \Pi^B}{\partial p_j^*} \frac{dp_j^*}{dK_A} + \frac{\partial \Pi^B}{\partial p_j^*} \frac{dp_j^*}{dK_A} \right\}
\]

\[
= sign \left\{ \sum_{j \in \{1, 2\}} \left( p_j^B - c_j^B \right) \frac{\partial n_j^B}{\partial p_j^*} + \left( p_j^B - c_j^B \right) \frac{\partial n_j^B}{\partial p_j^*} \right\} \frac{dp_j^*}{dK_A}
\]  

(6)
The sign of the strategic effect of \( K_A \) on firm A’s own profits is:

\[
\text{sign} \left\{ \frac{d\Pi^A}{dK_A} - \frac{\partial \Pi^A}{\partial K_A} \right\} = \text{sign} \left\{ \frac{\partial \Pi^A}{\partial p_1^B} \frac{dp_1^B}{dK_A} + \frac{\partial \Pi^A}{\partial p_2^B} \frac{dp_2^B}{dK_A} \right\} = \text{sign} \left\{ \frac{\partial \Pi^B}{\partial p_1^A} \frac{dp_1^A}{dK_A} + \frac{\partial \Pi^B}{\partial p_2^A} \frac{dp_2^A}{dK_A} \right\}
\]

\[
= \text{sign} \left\{ \sum_{j \in \{1, 2\}} \left( (p_1^B - c_1^B) \frac{\partial n_i^B}{\partial p_j^A} + (p_2^B - c_2^B) \frac{\partial n_i^B}{\partial p_j^A} \right) \frac{dp_j^B}{dK_A} \right\}
\]

(7)

These two expressions constitute the core of our analysis. Note that even when prices are strategic complements (which we will assume below), the respective signs of the strategic effects for firms A and B are disconnected. This is a crucial difference with the case of one sided markets. Indeed, in BGK and FT, which analyze the same kind of strategic interactions, but in one-sided oligopolies, all the action is in the terms \( \frac{dp_i^A}{dK_A} \) and \( \frac{dp_j^B}{dK_A} \), the signs of which depend crucially on whether the "actions" \( (p_i^A, p_j^B) \) are strategic complements or substitutes. By contrast, here the action comes from the sign of the terms \( \frac{\partial n_i^B}{\partial p_j^A} \), which can be positive or negative even when the "actions" \( (p_i^A, p_j^B) \) are in fact prices and therefore strategic complements. In BGK and FT these terms are positive if the actions are prices (and negative if the actions are quantities).

The set of strategic configurations in a two-sided market is strictly larger than in a one-sided market - not in terms of the nature of the strategies but in terms of the conditions under which they emerge. Indeed, even with pricing being strategic complements (cf. assumption 3), equations (6) and (7) imply that all four configurations are possible:

(1) \( \text{sign} \left\{ \frac{dn^A}{dK_A} - \frac{\partial n^A}{\partial K_A} \right\} > 0 \) and \( \text{sign} \left\{ \frac{dn^B}{dK_A} - \frac{\partial n^B}{\partial K_A} \right\} > 0 \); (ii) \( \text{sign} \left\{ \frac{dn^A}{dK_A} - \frac{\partial n^A}{\partial K_A} \right\} > 0 \) and \( \text{sign} \left\{ \frac{dn^B}{dK_A} - \frac{\partial n^B}{\partial K_A} \right\} < 0 \); (iii) \( \text{sign} \left\{ \frac{dn^A}{dK_A} - \frac{\partial n^A}{\partial K_A} \right\} < 0 \) and \( \text{sign} \left\{ \frac{dn^B}{dK_A} - \frac{\partial n^B}{\partial K_A} \right\} > 0 \); (iv) \( \text{sign} \left\{ \frac{dn^A}{dK_A} - \frac{\partial n^A}{\partial K_A} \right\} < 0 \) and \( \text{sign} \left\{ \frac{dn^B}{dK_A} - \frac{\partial n^B}{\partial K_A} \right\} < 0 \).

Under entry accomodation, these cases lead to the following strategies for platform A: (i) fat cat; (ii) top dog; (iii) puppy dog; (iv) lean and hungry look.

By contrast, in a one-sided market with strategic variables being strategic complements, the optimal strategy for entry accomodation by A can only take two forms: fat cat and puppy dog (cf. Table 1). This is because in a one-sided market with strategic complement actions, the two strategic effects of \( K_A \) necessarily have the same sign.

Consequently, table 1 above needs to be adjusted in order to allow for the full range of strategic scenarios in a two-sided market. We will say that investment \( K_A \) is self-serving (self-harming) for firm A if the strategic effect on A is positive (negative). We then have the following table corresponding to the case where the actions are strategic complements and A finds it optimal to accomodate entry by B:

<table>
<thead>
<tr>
<th>Case</th>
<th>Fat Cat</th>
<th>Top Dog</th>
<th>Puppy Dog</th>
<th>Lean and Hungry Look</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>True</td>
<td>False</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>(ii)</td>
<td>True</td>
<td>False</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>(iii)</td>
<td>False</td>
<td>True</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>(iv)</td>
<td>False</td>
<td>True</td>
<td>False</td>
<td>False</td>
</tr>
</tbody>
</table>

5And while still maintaining the condition that B’s profits are positive.
First, note that it is possible that (6) is positive, i.e. $K_A$ has a positive effect on platform B’s profits – so that $K_A$ is "soft" – without violating the condition that B’s profits are positive in equilibrium:

$$ (p_B^1 - c_1^B) n_1^B + (p_B^2 - c_2^B) n_2^B > 0 $$

The possibility for a cost reduction by one platform to increase the profits of its rival is novel. This situation never occurs in one-sided markets – as we showed in the previous section. In a two-sided market setting, it is possible that in equilibrium the platforms subsidize one side of the market – say side 2 – and recoup the losses on the other side: $p_B^{2*} - c_2^B < 0$ and $p_B^{1*} - c_1^B > 0$. Note that for $K_A$ to be soft in this case it is necessary that:

$$ \frac{\partial n_2^B / \partial p_A^j}{n_2^B} > \frac{\partial n_1^B / \partial p_A^j}{n_1^B} $$  \hspace{1cm} (8) $$_{}$$

for at least one $j \in \{1, 2\}$. This condition has an intuitive interpretation. Softness is more likely to occur when the "business stealing" effect on the subsidized side $(n_2^B)^{-1} (\partial n_2^B / \partial p_A^j)$ is higher relative to the business stealing effect on the profitable side $(n_1^B)^{-1} (\partial n_1^B / \partial p_A^j)$.

Equation (8) holds the key to interpreting this surprising result. By the envelope theorem, a small change in $K_A$ impacts $\Pi_B$ only through $A$’s equilibrium prices. Given that $K_A$ reduces $c_A^1$, we know that $A$’s prices will be lower in the new equilibrium on both sides ($\frac{dp_A^1}{dK_A}$, $\frac{dp_A^2}{dK_A} < 0$), which means that $A$ steals customers from $B$ on both sides of the market. However, if the proportion of customers stolen by $A$ from $B$ on the loss-making side is sufficiently high relative to the proportion of customers stolen on the profit-making side, this might increase $B$’s profits. Getting new customers on both sides in these proportions can be profitable for $A$ because it has now gained some competitive advantage relative to $B$ ($\frac{dc_A^1}{dK_A} < 0$). Inspecting (7), it should be clear however that the strategic effect of $K_A$ on $A$’s own profits can be negative.

**Tying.** Tying can be analyzed as a reduction in marginal costs on one side of the market. Configuration 4 above is then a case in which tying makes the incumbent soft and is profitable at
the same time. In other words, tying becomes a fat cat strategy. We have therefore uncovered the possibility that tying can be a profitable entry-accomodation strategy, in stark contrast with the case of one sided markets.

To put this result in perspective, it is worth reviewing the logic of Whinston (1990)’s argument. Whinston considers a model with two firms 1 and 2. Firm 1 produces two different goods A and B₁. Production of good A is monopolized by firm A. Goods B₁ and good B₂ are imperfect substitutes: the demand for good Bᵢ is given by \( xᵢ(P_{B₁}, P_{B₂}) \) with \( xᵢ ≥ 0 \) if \( i ≠ j \) and \( xᵢ < 0 \) if \( i = j \). Firm 2 however, needs to pay an entry cost \( K₂ \) in order to operate. In addition to its pricing decisions, firm 1 can offer bundles of its products. Two cases have to be distinguished. The commitment case occurs when firm 1 commits to a specific bundle of products before entry and pricing decisions are made. The no-commitment case occurs when bundling decisions are made at the same time as pricing decisions.

Whinston’s main result corresponds to the case that is closest to our setup, where valuation for the tying good A is homogenous. Whinston shows that tying is useless in the no-commitment case: firm 1 makes sure that all customers purchase good A and the equilibrium is equivalent to the equilibrium of the independent pricing game where bundling is prohibited.

In the commitment case tying has the same effect on best response functions as a reduction in marginal cost for firm 1: every sale of good B₂ comes with the extra benefit of a sale of good A, thus lowering the effective marginal cost of the B₁ sale. Tying therefore acts as a commitment by firm 1 to be more aggressive. If firm 2 remains active, this reduces profits for both firms. Tying is then a self-deafeating strategy to accomodate entry. On the other hand however, tying can result in foreclosure whereby firm 2 decides not to pay the fixed cost of entry. Firm 1 then monopolizes the market for good B. The benefit for firm 1 is reduced competition for good B. The potential loss comes from the fact that firm 1 will be a monopolist who can only offer a bundle. Thus, the presence of a large number of consumers who dislike product B₁ may make a commitment to bundling unprofitable, even when it leads to exclusion. The conclusion is that in this context, tying is a top dog strategy that is always unprofitable to accomodate entry, and can sometimes be profitable to deter entry.⁶

⁶Whinston then investigates the consequences of relaxing the assumption that the valuation for good A is homogenous across consumers.

The results are modified as follows. First and most related to this paper, a commitment to tying need not always result in foreclosure: it can be the case that firm 2’s profits increase as a result of a commitment to tying by firm 1. The reason is twofold. First, enough consumers may find A unattractive so that the margin on every bundle sale is lower than the margin on an independent sale of good B₁ (it is always higher in the homogenous case) – the monopoly power of firm 1 is thus too weak. Second, the elasticity of bundle sales to the price of the bundle is no longer identical to that arising in market B: indeed, when B₁ and B₂ are nearly homogenous, tying essentially transforms the nearly homogenous market B into a vertically differentiated market, potentially raising firm 2’s profits.

Second, tying can be a profitable strategy even in the absence of an ability to commit, and when it is, it may lower firm 2’s profitability.
3.2 Strategic interactions with prices fixed on one side

It is interesting to briefly compare the results above with a situation in which platforms’ margins on one side of the market, say side 2, are exogenously fixed, i.e. \( p_2^A - c_2^A = p_2^B - c_2^B \equiv \pi_2 \). This may be interpreted as a context in which regulation, common practice or other institutions put prices on one side of the market beyond the control of the two platforms. Then each platform \( i \) only has one variable to choose - \( p_i^1 \) - and the demand functions can be written as \( N_j^i = n_j^i (\{p_i^A, p_i^B\}) \).

The strategic effect of \( K_A \) on platform B is:

\[
\frac{d\Pi_B}{dK_A} = \frac{\partial \Pi_B}{\partial p_i^1} \frac{dp_i^A}{dK_A} = \left[ (p_i^B - c_i^B) \frac{\partial n_i^B}{\partial p_i^A} + \pi_2 \frac{\partial n_i^B}{\partial p_i^A} \right] \frac{dp_i^A}{dK_A}
\]

whereas the strategic effect on platform A’s own profits is:

\[
\frac{\partial \Pi_A}{\partial p_i^1} \frac{dp_i^B}{dK_A} = \frac{\partial \Pi_B}{\partial p_i^1} \frac{dp_i^B}{dK_A} \frac{dp_i^A}{dK_A} = \left[ (p_i^B - c_i^B) \frac{\partial n_i^B}{\partial p_i^A} + \pi_2 \frac{\partial n_i^B}{\partial p_i^A} \right] \frac{dp_i^B}{dp_i^A} \frac{dp_i^A}{dK_A} = \frac{d\Pi_B}{dK_A} \frac{dp_i^B}{dp_i^A}
\]

The first expression above implies that the possibility of \( K_A \) being soft survives. If for example \( \pi_2 < 0 \) and \( (n_i^B)^{-1} (\partial n_i^B / \partial p_i^A) > (n_i^B)^{-1} (\partial n_i^B / \partial p_i^A) \), then we can have simultaneously \( \frac{d\Pi_B}{dK_A} > 0 \) and \( \Pi_B > 0 \) in equilibrium. However, given that \( \text{sign} \left( \frac{\partial \Pi_A}{\partial p_i^1} \frac{dp_i^B}{dK_A} \right) = \text{sign} \left( \frac{d\Pi_B}{dK_A} \right) \text{sign} \left( \frac{dp_i^B}{dp_i^A} \right) \), if \( \frac{dp_i^B}{dp_i^A} > 0 \) – which is always the case with linear demands and prices fixed on one side – then the two strategic effects have the same sign, just like in a one-sided market. Hence, by fixing prices on one side, the set of possible optimal strategies for entry accommodation for platform A is reduced back to \{puppy dog, fat cat\}.

4 Examples

In this section we use specific models to prove that the results anticipated by the general analysis above are indeed possible. In particular, for each example, our goal is to exhibit the possibility of a small cost reduction by platform A being "soft" (i.e., increasing platform B’s profits), while prices are strategic complements across platforms. An additional benefit of going through these examples is that they allow us to gain further intuition regarding the two-sided strategic interactions we are illustrating.

4.1 Fixed prices on one side

We begin with the simpler case, in which platform prices – or markups – are fixed on one side of the market. In this case, we can write without loss of generality the profits of platform \( i \in \{A, B\} \) as:

\[
\Pi_i = (p_i^i - c_i^i) n_i^i (p_i^i, p_i^A) + \pi_2 f (n_i^1, n_i^1)
\]
where \( n^i_1 (p^i_1, p^j_1) \) is decreasing in \( p^i_1 \) and increasing in \( p^j_1 \); \( f (n^i_1, n^j_1) \) is increasing in \( n^j_1 \) and decreasing in \( n^i_1 \).

We want to show that the strategic effect of a decrease in \( c^i_1 \) on \( \Pi^B \) can be positive, i.e. 
\[
\frac{\partial \Pi^B}{\partial p^i_1} (p^{B*}_1, p^{A*}_1) < 0.
\]
This is equivalent to\(^7\):
\[
(p^{B*}_1 - c^B_1) + \pi_2 \left[ \frac{\partial f}{\partial n^B_1} + \frac{\partial f}{\partial n^A_1} \frac{\partial n^B_1}{\partial p^B_1} \left( \frac{\partial n^B_1}{\partial p^B_1} \right)^{-1} \right] < 0
\]
(9)
The first order condition for \( p^B_1 \) yields:
\[
(p^{B*}_1 - c^B_1) + \pi_2 \left[ \frac{\partial f}{\partial n^B_1} + \frac{\partial f}{\partial n^A_1} \frac{\partial n^A_1}{\partial p^B_1} \left( \frac{\partial n^B_1}{\partial p^B_1} \right)^{-1} \right] = -n^B_1 \left( \frac{\partial n^B_1}{\partial p^B_1} \right)^{-1} > 0
\]
(10)
Thus, in order for our example to work, we need to have \( \frac{\partial f}{\partial n^i_1} < 0 \) and either:
\[
\pi_2 > 0 \text{ and } \left| \frac{\partial n^A_1}{\partial p^A_1} \frac{\partial n^B_1}{\partial p^B_1} \right| < \left| \frac{\partial n^B_1}{\partial p^B_1} \frac{\partial n^A_1}{\partial p^A_1} \right|
\]
(11)
or:
\[
\pi_2 < 0 \text{ and } \left| \frac{\partial n^A_1}{\partial p^A_1} \frac{\partial n^B_1}{\partial p^B_1} \right| > \left| \frac{\partial n^B_1}{\partial p^B_1} \frac{\partial n^A_1}{\partial p^A_1} \right|
\]
(12)
Stability of the demand system in \( (p^A_1, p^B_1) \) requires \( \left| \frac{\partial n^A_1}{\partial p^A_1} \frac{\partial n^B_1}{\partial p^B_1} \right| > \left| \frac{\partial n^B_1}{\partial p^B_1} \frac{\partial n^A_1}{\partial p^A_1} \right| \), hence we must have \( \pi_2 < 0^9 \).

Most models found in the two-sided market literature up to now – Armstrong (2006) in particular – have a symmetric linear structure which implies \( \left| \frac{\partial n^A_1}{\partial p^A_1} \frac{\partial n^B_1}{\partial p^B_1} \right| = \left| \frac{\partial n^B_1}{\partial p^B_1} \frac{\partial n^A_1}{\partial p^A_1} \right| = \left| \frac{\partial n^A_1}{\partial p^A_1} \right| = \left| \frac{\partial n^A_1}{\partial p^A_1} \right| \). Hence they cannot satisfy our conditions (11) or (12). We therefore need to depart from the symmetry assumption and look to obtain the following type of expression:
\[
n^i_1 (p^i_1, p^j_1) = N - p^i_1 + \gamma p^j_1
\]
with \( 0 < \gamma < 1 \). In other words, we need the elasticity of a platform’s demand in its own price to be strictly higher in absolute value than the elasticity in the rival platform’s price. This can be achieved by adding "hinterlands" on side 1 to the two-sided Hotelling model considered in Armstrong (2006).

In the appendix we provide the detailed micro-foundations of this model, which allows us to

---

\(^{7}\)Recall that, with fixed prices on one side, \( \frac{\partial \Pi^A}{\partial c^i_1} > 0 \) whenever \( \Pi^A \) is concave in \( p^A_1 \).

\(^{8}\)All terms are evaluated at \( (p^{B*}_1, p^{A*}_1) \).

\(^{9}\)Note that the requirement \( \frac{\partial f}{\partial n^i_1} < 0 \) rules out multihoming by all side 2 agents, since in that case demands for each platform on side 2 would be independent of each other (assuming away economies of scale in multihoming) and hence \( f \) would only depend on \( n^B_1 \).
obtain:

\[ n_1^i = N_1 - p_1^i + \gamma p_1^j \]

\[ n_2^i = \frac{N_2}{2} + \frac{N_2 \alpha_2}{2t_2} (n_1^i - n_1^j) \]

\[ \Pi^i = (p_1^i - c_1^i) n_1^i + \pi_2 n_2^i \]

(13)

where \( N_1 > 0, N_2 > 0, t_2 > 0, \alpha_2 > 0, \pi_2 < 0 \) (necessary as we have seen above) and \( \gamma \in [0, 1] \) are all constants.

The linearity of demand in prices implies that platform \( i \)'s profits are concave in \( p_1^i \), which in turn implies \( \frac{\partial \Pi^A}{\partial c_1^A} > 0 \), and that platform prices are strategic complements, i.e. \( \frac{\partial p_1^i}{\partial p_1^j} > 0 \) for all \( i \neq j \in \{A, B\} \) (so that assumptions 1-4 are satisfied).

In the appendix we prove:

**Proposition 1** A necessary and sufficient condition for a cost reduction by platform \( A \) to be profit enhancing for platform \( B \) – (9) above – is:

\[ N_1 - c_1 (1 - \gamma) - |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) \frac{2 (1 - \gamma)}{\gamma} < 0 \]

(14)

The condition in the text of Proposition 1 can be compared to the condition for \( n_1 \) to be positive in equilibrium\(^{10}\):

\[ N_1 - c_1 (1 - \gamma) - |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) (1 - \gamma) > 0 \]

(15)

and to the condition for platform profits to be positive in equilibrium:

\[ \left[ N_1 - c_1 (1 - \gamma) + |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) \right] \left[ N_1 - c_1 (1 - \gamma) - |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) (1 - \gamma) \right] > |\pi_2| \frac{N_2}{2} \]

(16)

It is then easily verified that the last three conditions can hold simultaneously for a range of parameter values. Note that \( \gamma < 1 \) is absolutely necessary in making it possible that (14) and (15) hold at the same time.

To gain some intuition, note that (14), (15) (16) are clearly satisfied when \( \gamma = 0 \) and \( N_1 \) is sufficiently large relative to \( N_2 \). In this case, (13) yields \( n_1^i = N_1 - p_1^i \) and so there is no competition between platforms on side 1. Meanwhile, on side 2 platforms are "forced" to lose money and each gains unprofitable side 2 customers in proportion to its market share advantage relative to its rival on side 1. Consequently, platform \( A \) is always happy to lose market share on side 2 (since that

\(^{10}\)We use the equilibrium expressions of \( p_1 \) and \( n_1 \) provided in the proof of Proposition 1 in the appendix.
entails no loss in market share on side 1) and this happens whenever platform B’s marginal cost \( c_1^B \) decreases because it leads to lower \( p_1^B \), higher \( n_1^B \) and higher \( n_2^B \). The condition that \( N_1 \) is sufficiently large relative to \( N_2 \) simply ensures that the market of unprofitable customers (side 2) is sufficiently small relative to the market of profitable customers (side 1) so that the platforms can make positive profits in equilibrium.

The result is thus straightforward when the two platforms are sufficiently differentiated so that they don’t compete on side 1. When competition among them on side 1 becomes more intense (i.e. \( \gamma \) increases), it becomes less and less likely that a cost reduction by one platform on side 1 will benefit its rival. This is because each platform is now ambivalent about losing customers on side 2: although that rids it of unprofitable customers, it also decreases its market share of profitable side 1 customers. One can confirm this intuition by noting that the left hand side of (14) is increasing in \( \gamma \) and strictly positive for \( \gamma = 1 \). Therefore, for \( \gamma \) high enough, a cost reduction by one platform always hurts its rival, as standard intuition suggests.

### 4.2 Fully two-sided pricing

Let us now turn to the general case, when prices on both sides of the market are flexible. An investment that reduces platform A’s marginal cost \( c_1^A \) is soft if and only if (recall 6):

\[
\left[ (p_1^{B*} - c_1^B) \frac{\partial n_1^B}{\partial p_1^B} + (p_2^{B*} - c_2^B) \frac{\partial n_2^B}{\partial p_1^B} \right] \frac{dp_1^{A*}}{dc_1} + \left[ (p_1^{B*} - c_1^B) \frac{\partial n_1^B}{\partial p_2^B} + (p_2^{B*} - c_2^B) \frac{\partial n_2^B}{\partial p_2^B} \right] \frac{dp_2^{A*}}{dc_1} < 0
\]

which means we want at least one of the two terms in-between square brackets to be negative. Assume then that:

\[
(p_1^{B*} - c_1^B) \frac{\partial n_1^B}{\partial p_1^B} + (p_2^{B*} - c_2^B) \frac{\partial n_2^B}{\partial p_1^B} < 0
\]

or, equivalently

\[
(p_1^{B*} - c_1^B)^2 + (p_2^{B*} - c_2^B)^2 \frac{\partial n_2^B}{\partial p_1^B} \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} < 0
\]

(17)

Just like in the previous example, the main issue is also satisfying the first order conditions. In particular, the first order condition in \( p_1^B \) is:

\[
(p_1^{B*} - c_1^B) + (p_2^{B*} - c_2^B) \frac{\partial n_2^B}{\partial p_1^B} \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} = -n_1^B \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} > 0
\]

(18)

To simplify things somewhat, assume that\(^{11} \frac{\partial n_2^B}{\partial N_1^B} = -\frac{\partial n_1^B}{\partial N_2^B} > 0 \). Then:

\[
\frac{\partial n_2^B}{\partial p_1^B} \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} = \frac{\partial n_2^B}{\partial N_1^B} \left( 1 + \frac{\partial n_1^A}{\partial p_1^A} \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} \right) \quad \text{and} \quad \frac{\partial n_2^B}{\partial p_1^B} \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} = \frac{\partial n_2^B}{\partial N_1^B} \left( 1 + \frac{\partial n_1^A}{\partial p_1^A} \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} \right)
\]

\(^{11}\text{This holds for example when platforms compete a la Hotelling with no hinterlands on side 2.}\)
Stability requires \( \left| \frac{\partial n_i^A}{\partial p_1^A} \frac{\partial n_i^B}{\partial p_1^B} \right| \leq \left| \frac{\partial n_i^B}{\partial p_1^B} \frac{\partial n_i^A}{\partial p_1^A} \right| \), therefore, in order to satisfy (17) and (18) simultaneously, we must have:

\[
p_{2*}^B - c_2^B < 0 \quad \text{and} \quad \left| \frac{\partial n_1^A}{\partial p_1^A} \left( \frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} \right| < \left| \frac{\partial n_1^B}{\partial p_1^B} \left( \frac{\partial n_1^A}{\partial p_1^A} \right)^{-1} \right|
\]

Again, this condition cannot be satisfied in the Armstrong (2006) model. Just like in the previous example, we need to have:

\[
\frac{\partial n_1^A}{\partial p_1^B} = \frac{\partial n_1^B}{\partial p_1^A} = - \gamma \frac{\partial n_1^A}{\partial p_1^B} = - \gamma \frac{\partial n_1^B}{\partial p_1^A}
\]

with \( 0 < \gamma < 1 \)\(^{12}\). In this case:

\[
\frac{\partial n_2^B}{\partial p_1^B} \left( \frac{\partial n_1^B}{\partial p_1^A} \right)^{-1} = \frac{\partial n_2^B}{\partial p_1^B} \left( 1 + \frac{1}{\gamma} \right) = \frac{\partial n_2^B}{\partial p_1^B} \left( \frac{\partial n_1^B}{\partial p_1^A} \right)^{-1}
\]

To obtain this, we posit the following demand functions:

\[
n_i^1 = \frac{1}{2} + \frac{u_i^1 - u_i^2}{2t_1} + \frac{x_1}{2t_1} (V_1 + u_i^1) \quad (19)
\]

\[
n_i^2 = N_2 \left[ \frac{1}{2} + \frac{u_i^2 - u_j^2}{2t_2} \right] \quad (20)
\]

with:

\[
u_i^1 = \alpha_1 n_i^2 - p_i^1 \quad \text{and} \quad u_i^2 = \alpha_2 n_i^1 - p_i^2
\]

Note that we have:

\[
n_A^2 + n_B^2 = N_2 \quad \text{and} \quad n_A^1 + n_B^1 = 1
\]

Relegating the tedious calculations in the appendix, we have:

**Proposition 2** There exists a range of parameters \((\alpha_1, \alpha_2, t_1, t_2, x_1, V_1, N_2)\) for which an investment that slightly decreases platform \(i\)'s marginal cost \(c_i^1\) is soft, i.e. increases the profits of platform \(j \neq i\), \(i, j \in \{A, B\}\).

To get some more intuition for this result, let us imagine that the initial situation is a symmetric equilibrium where prices are given by \(p_1\) and \(p_2\) and market shares are given by \(n_1\) and \(n_2\), with \(p_1 - c_1 > 0\) and \(p_2 - c_2 < 0\). Then

\[
d\pi^i - dc_i^i n_1 = dp_i^1 n_1 + dp_i^2 n_2 + (p_1 - c_1)dn_1^i + (p_2 - c_2)dn_2^i
\]

Note that under our assumptions, \(dp_i^1 < 0\) and \(dp_i^2 < 0\) so that

\[
dp_i^1 n_1 + dp_i^2 n_2 < 0 \quad \text{for all} \ i.
\]

\(^{12}\)In the Armstrong (2006) model, \(\gamma = 1\).
Hence a necessary condition for $\pi^i - dc^i_i n_1$ to be positive for all $i$, and hence for tying to be a fat cat strategy is that

$$ (p_1 - c_1)dn^i_1 + (p_2 - c_2)dn^i_2 > 0 \text{ for all } i. \quad (22) $$

Clearly, if the total market size is fixed on each side, then $dn^i_j = -dn^{-i}_j$ for all $i, j$ so that

$$ (p_1 - c_1)dn^i_1 + (p_2 - c_2)dn^i_2 = -(p_1 - c_1)dn^{-i}_1 - (p_2 - c_2)dn^{-i}_2 $$

and (22) cannot be verified. Tying being a fat cat strategy requires an expansion in market sizes. Indeed, suppose that on side 1, total market size is not fixed. Denote by $dn_1$ the total expansion in market size. Then we have

$$ (p_1 - c_1)dn^i_1 + (p_2 - c_2)dn^i_2 = -(p_1 - c_1)dn^{-i}_1 - (p_2 - c_2)dn^{-i}_2 + (p_1 - c_1)dn_1 $$

with $(p_1 - c_1)dn_1 > 0$. Therefore, if the market size expansion is strong enough, (22) can be verified.

5 Conclusion

We have shown that strategic interactions in two-sided market duopolies are fundamentally different from those in one-sided markets. The possibility of cross-subsidization between the two sides leads to novel and counterintuitive results. In particular, a cost-reducing investment by one firm can both increase the profits of its rivals and be desirable for the firm undertaking the investment. Thus, increasing competitive advantage through cost advantage for one platform may end up benefiting both platforms (in a one sided market, cost reductions by one firm always hurt its rivals). The fundamental reason is that in the event one side of the market is heavily subsidized, the platform gaining a cost advantage helps the rival platform by stealing customers on both sides in proportions that it was unprofitable for the latter to serve. This means that a cost-reducing investment can be part of a fat cat strategy under entry accommodation, which is never the case in a one-sided market, as shown by FT.

A prominent application is tying, which in a one-sided market is unambiguously tough. By contrast, our analysis implies that it can actually be soft in a two-sided market, which means that tying may in fact benefit both the tying firm and its rival. This can for example mean that when Microsoft is tying Windows Media Player into Windows (which can simply be interpreted as leveraging a distribution cost advantage on the consumer side over Real Networks and other rivals), everyone could possibly benefit. Thus, even before worrying about social welfare, an antitrust tying claim may be invalidated on grounds of lack of harm to the competition.
References


6 Appendix

Micro-foundations leading to the demand system (13)

We assume that on side $i$ there is a Hotelling segment of consumers with linear transportation costs $t_i$, standalone valuation for either platform $V_i$ and additional utility $\alpha_i$ per customer on side $j$ of the same platform, with $i \neq j \in \{1, 2\}$. We assume the total mass of these customers is $1$ on side $1$ and $M_2$ on side $2$. Furthermore, on side $1$, each platform faces a downward-sloping demand of "loyal" customers (i.e. who are never interested in the rival platform), with standalone valuation $V_1$, utility per side $2$ customer on the same platform $\beta_1$ and transportation costs $\frac{2t_1}{x}$. This yields the following expression of for platform $i$’s demands:

$$n_i^i = \frac{1}{2} + \frac{\alpha_i (n_i^i - n_j^i) + (p_i^i - p_j^i)}{2t_1} + \frac{x}{2t_1} (V_1 + \beta_1 n_i^i - p_i^i)$$

$$n_i^j = M_2 \left( \frac{1}{2} + \frac{\alpha_2 (n_i^i - n_j^i)}{2t_2} \right)$$

where $\alpha_1, \alpha_2, x, V_1, \beta_1, t_1, t_2 > 0$.

Solving this demand system for $(n_i^i, n_i^j)$ as functions of prices only, we obtain:

$$n_i^i = M_1 - \theta p_i^i + \lambda p_j^j$$

with:

$$M_1 = \frac{t_1 + xV_1 + \beta_1 M_2}{2t_1}$$

$$\theta = \frac{(1 + x) \left( 1 - \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4t_1 t_2} \right) + \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4t_1 t_2}}{4t_1 \left( \frac{1}{2} - \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4t_1 t_2} \right)}$$

$$\lambda = \frac{\left( 1 - \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4t_1 t_2} \right) + (1 + x) \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4t_1 t_2}}{4t_1 \left( \frac{1}{2} - \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4t_1 t_2} \right)}$$

We need to impose:

$$\frac{1}{2} > \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4t_1 t_2}$$

which not surprisingly implies:

$$0 < \lambda < \theta$$

To simplify things, we let then $\gamma = \frac{\lambda}{\theta} < 1$ and $N_1 = \frac{M_1}{\theta}$; $N_2 = \frac{M_2}{\theta}$, since we can factor all profits by $\theta$ without changing anything.
So everything is as if we had:

\[
\begin{align*}
n_1^i &= N_1 - p_1^i + \gamma p_1^i \\
n_2^i &= \frac{N_2}{2} + \frac{N_2 \alpha_2}{2t_2} (n_1^i - n_2^i)
\end{align*}
\]

with \( \gamma < 1 \). □

**Proof. (Proposition 1)** The first order condition ((10) in the text) at the symmetric equilibrium \((p_1^A = p_1^B = p_1 \text{ and } n_1^A = n_1^B = n_1)\) can be written as:

\[
p_1 - c_1 + \pi_2 \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) = n_1 > 0
\]

At the symmetric equilibrium:

\[
n_1 = N_1 - (1 - \gamma) p_1
\]

Plugging in the first order condition above, we obtain:

\[
p_1 = \frac{c_1}{2 - \gamma} + \frac{N_1 - \pi_2 \frac{N_2 \alpha_2}{2t_2} (1 + \gamma)}{2 - \gamma}
\]

The condition for a cost reduction by platform \( A \) to be profit enhancing for platform \( B \) ((9) above) is then:

\[
p_1 - c_1 + \pi_2 \frac{N_2 \alpha_2}{2t_2} \frac{\gamma + 1}{\gamma} < 0
\]

Replacing \( p_1 \) with its expression as a function of the model parameters, this condition is equivalent to (14). □

**Proof. (Proposition 2)** Solving (19), (20) and (21) for \( n_1^i \) and \( n_2^i \), \( i = A, B \), we obtain:

\[
\begin{align*}
n_2^A &= \frac{N_2}{2} + \frac{\alpha_2 (2 + x_1) N_2 (p_1^B - p_1^A) + 2t_1 (p_2^B - p_2^A)}{4t_1 t_2 - 2 \alpha_1 \alpha_2 (2 + x_1) N_2} \\
n_2^B &= \frac{N_2}{2} + \frac{\alpha_2 (2 + x_1) N_2 (p_1^A - p_1^B) + 2t_1 (p_2^A - p_2^B)}{4t_1 t_2 - 2 \alpha_1 \alpha_2 (2 + x_1) N_2} \\
n_1^A &= \frac{t_1 + V_1 x_1 + \frac{\alpha_1 N_2}{2}}{2t_1} + \frac{\alpha_1 (2 + x_1) (p_2^B - p_2^A)}{4t_1 t_2 - 2 \alpha_1 \alpha_2 (2 + x_1) N_2} \\
&\quad + \frac{4t_1 t_2 + \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2 \alpha_1 \alpha_2 (2 + x_1) N_2} \times \frac{p_1^B}{2t_1} - \frac{4t_1 t_2 (1 + x_1) - \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2 \alpha_1 \alpha_2 (2 + x_1) N_2} \times \frac{p_1^A}{2t_1}
\end{align*}
\]

\(^{13}\)Recall that \( \pi_2 < 0 \) so that \( |\pi_2| = -\pi_2 > 0 \).
\[ n_1^B = n_2^A = \frac{t_1 + V_1x_1 + \frac{\alpha_1 N_1}{2}}{2t_1} + \frac{\alpha_1 (2 + x_1) \left( p_2^A - p_2^B \right)}{4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_1} \]

\[ + \frac{4t_1 t_2 + \alpha_1 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_2} \times \frac{p_1^A}{2t_1} - \frac{4t_1 t_2 (1 + x_1) - \alpha_1 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_2} \times \frac{p_1^B}{2t_1} \]

In order to simplify notation for the calculation of the pricing equilibrium, let:

\[ \gamma_1 \equiv \frac{\alpha_1 (2 + x_1)}{4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_2} \quad ; \quad \gamma_2 \equiv \frac{\alpha_2 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_2} \]

\[ \delta \equiv \frac{2t_1}{4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_2} \quad ; \quad \varepsilon \equiv \frac{4t_1 t_2 + \alpha_1 x_1 (2 + x_1) N_2}{2t_1 [4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_2]} \]

\[ N_1 \equiv \frac{t_1 + V_1x_1 + \frac{\alpha_1 N_2}{2}}{2t_1} \quad ; \quad 1 + u \equiv \frac{4t_1 t_2 (1 + x_1) - \alpha_1 x_1 (2 + x_1) N_2}{4t_1 t_2 + \alpha_1 x_1 (2 + x_1) N_2} \]

Clearly \( u > 0 \) since we must have \( 4t_1 t_2 - 2\alpha_1 x_1 (2 + x_1) N_2 > 0 \). We can now write:

\[ n_1^A = \frac{N_1}{2} + \gamma_1 \left( p_2^B - p_2^A \right) + \varepsilon p_1^B - \varepsilon (1 + u) p_1^A \]

\[ n_1^B = \frac{N_1}{2} + \gamma_1 \left( p_2^A - p_2^B \right) + \varepsilon p_1^A - \varepsilon (1 + u) p_1^B \]

\[ n_2^A = \frac{N_2}{2} + \gamma_2 \left( p_1^B - p_1^A \right) + \delta (p_2^B - p_2^A) \]

\[ n_2^B = \frac{N_2}{2} + \gamma_2 \left( p_1^A - p_1^B \right) + \delta (p_2^A - p_2^B) \]

Recall that platform profits are:

\[ \Pi^A = \left( p_1^A - c_1^A \right) n_1^A + \left( p_2^A - c_2^A \right) n_2^A \]

\[ \Pi^B = \left( p_1^B - c_1^B \right) n_1^B + \left( p_2^B - c_2^B \right) n_2^B \]

We start from a symmetric situation with \( c_1^A = c_1^B = c_1 \) and \( c_2^A = c_2^B = c_2 \) and consider a slight decrease in \( c_1^A \). Using the demand expressions above, the first order conditions in prices are:

\[ \left( p_1^A - c_1^A \right) \times \varepsilon (1 + u) + \left( p_2^A - c_2^A \right) \gamma_2 = \frac{N_1}{2} + \gamma_1 \left( p_2^B - p_2^A \right) + \varepsilon p_1^B - \varepsilon (1 + u) p_1^A \]  \( (23) \)

\[ \left( p_1^A - c_1^A \right) \times \gamma_1 + \left( p_2^A - c_2^A \right) \delta = \frac{N_2}{2} + \gamma_2 \left( p_1^B - p_1^A \right) + \delta (p_2^B - p_2^A) \]  \( (24) \)

\[ \left( p_1^B - c_1^B \right) \times \varepsilon (1 + u) + \left( p_2^B - c_2^B \right) \gamma_2 = \frac{N_1}{2} + \gamma_1 \left( p_2^A - p_2^B \right) + \varepsilon p_1^A - \varepsilon (1 + u) p_1^B \]  \( (25) \)

\[ \left( p_1^B - c_1^B \right) \times \gamma_1 + \left( p_2^B - c_2^B \right) \delta = \frac{N_2}{2} + \gamma_2 \left( p_1^A - p_1^B \right) + \delta (p_2^A - p_2^B) \]  \( (26) \)

We can use (25) and (26) to determine platform B’s best response prices \( \left( p_1^B, p_2^B \right) \) as a function of platform
\[ \begin{align*} 
p_1^B &= \frac{1}{4\delta \varepsilon (1 + u) - (\gamma_1 + \gamma_2)^2} \times \left\{ 2\delta \frac{N_2}{2} - (\gamma_1 + \gamma_2) \frac{N_2}{2} + [2\delta \varepsilon (1 + u) - (\gamma_1 + \gamma_2) \gamma_1] c_2^B + \delta (\gamma_2 - \gamma_1) c_1^B \right\} \\
p_2^B &= \frac{1}{4\delta \varepsilon (1 + u) - (\gamma_1 + \gamma_2)^2} \times \left\{ 2\varepsilon (1 + u) \frac{N_2}{2} - (\gamma_1 + \gamma_2) \frac{N_2}{2} + \varepsilon (\gamma_1 - \gamma_2) c_1^B \right\} \\
&\quad + \varepsilon [\gamma_2 (1 + 2u) - \gamma_1] p_1^A + [2\varepsilon \delta (1 + u) - \gamma_1 (\gamma_1 + \gamma_2)] p_2^A \right\} 
\end{align*} \]

Therefore, prices are strategic complements across platforms \((\frac{\partial k}{\partial p_j}) > 0\) for all \(i, j \in \{1, 2\}\) and \(k \neq l \in \{A, B\}\) if and only if the following 5 conditions hold:

\[
\begin{align*}
4\delta \varepsilon (1 + u) - (\gamma_1 + \gamma_2)^2 &> 0 \\
2\delta \varepsilon (\gamma_1 + \gamma_2) &> 0 \\
\gamma_1 - \gamma_2 &> 0 \\
\gamma_2 (1 + 2u) - \gamma_1 &> 0 \\
2\varepsilon (1 + u) - \gamma_1 (\gamma_1 + \gamma_2) &> 0
\end{align*}
\]

Taking the derivative in \(c_1^A\) of both sides of the equality in (23), (24), (25) and (26) above, we obtain the following system of 4 equations with 4 unknowns (\(\frac{dp_i^A}{dc_1^A}, \frac{dp_i^A}{dc_1^A}, \frac{dp_i^B}{dc_1^A}, \frac{dp_i^B}{dc_1^A}\)):

\[
\begin{align*}
\frac{dp_1^A}{dc_1^A} \times 2\varepsilon (1 + u) + \frac{dp_2^A}{dc_1^A} \times (\gamma_1 + \gamma_2) - \frac{dp_1^B}{dc_1^A} \times \varepsilon - \frac{dp_2^B}{dc_1^A} \times \gamma_1 &= \varepsilon (1 + u) \\
\frac{dp_1^A}{dc_1^A} \times (\gamma_1 + \gamma_2) + \frac{dp_2^A}{dc_1^A} \times 2\delta - \frac{dp_1^B}{dc_1^A} \times \gamma_2 - \frac{dp_2^B}{dc_1^A} \times \delta &= \gamma_1 \\
\frac{dp_1^B}{dc_1^A} \times 2\varepsilon (1 + u) + \frac{dp_2^B}{dc_1^A} \times (\gamma_1 + \gamma_2) - \frac{dp_1^A}{dc_1^A} \times \varepsilon - \frac{dp_2^A}{dc_1^A} \times \gamma_1 &= 0 \\
\frac{dp_1^B}{dc_1^A} \times (\gamma_1 + \gamma_2) + \frac{dp_2^B}{dc_1^A} \times 2\delta - \frac{dp_1^A}{dc_1^A} \times \gamma_2 - \frac{dp_2^A}{dc_1^A} \times \delta &= 0
\end{align*}
\]

Taking the difference between the first and third equalities, then between the second and fourth, we obtain:

\[
\begin{align*}
\left(\frac{dp_1^A}{dc_1^A} - \frac{dp_1^B}{dc_1^A}\right) \times \varepsilon (3 + 2u) + \left(\frac{dp_2^A}{dc_1^A} - \frac{dp_2^B}{dc_1^A}\right) \times (2\gamma_1 + \gamma_2) &= \varepsilon (1 + u) \\
\left(\frac{dp_1^A}{dc_1^A} - \frac{dp_1^B}{dc_1^A}\right) \times (\gamma_1 + 2\gamma_2) + \left(\frac{dp_2^A}{dc_1^A} - \frac{dp_2^B}{dc_1^A}\right) \times 3\delta &= \gamma_1
\end{align*}
\]

Similarly, summing first and third, then second and fourth, we also have:

\[
\begin{align*}
\left(\frac{dp_1^A}{dc_1^A} + \frac{dp_1^B}{dc_1^A}\right) \times \varepsilon (1 + 2u) + \left(\frac{dp_2^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A}\right) \times \gamma_2 &= \varepsilon (1 + u) 
\end{align*}
\]
\[ \left( \frac{dp_1^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A} \right) \times \gamma_1 + \left( \frac{dp_2^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A} \right) \times \delta = \gamma_1 \]

Solving the two systems above, we finally obtain:

\[ \frac{dp_1^A}{dc_1^A} - \frac{dp_2^B}{dc_1^A} = \frac{3\delta \varepsilon (1 + u) - \gamma_1 (2\gamma_1 + \gamma_2)}{3\delta \varepsilon (3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} \]

\[ \frac{dp_2^A}{dc_1^A} - \frac{dp_2^B}{dc_1^A} = \frac{\gamma_1 \varepsilon (3 + 2u) - (\gamma_1 + 2\gamma_2) \varepsilon (1 + u)}{3\delta \varepsilon (3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} = \frac{\varepsilon [\gamma_1 (2 + u) - \gamma_2 (2 + 2u)]}{3\delta \varepsilon (3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} \]

\[ \frac{dp_1^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A} = \frac{\delta \varepsilon (1 + u) - \gamma_1 \gamma_2}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} \]

\[ \frac{dp_2^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A} = \frac{\gamma_1 \varepsilon (1 + 2u) - \gamma_1 \varepsilon (1 + u)}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} = \frac{\gamma_1 \varepsilon u}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} \]

We then immediately get the two terms we are interested in:

\[ \frac{dp_1^A}{dc_1^A} = \frac{1}{2} \left[ \frac{3\delta \varepsilon (1 + u) - \gamma_1 (2\gamma_1 + \gamma_2)}{3\delta \varepsilon (3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} + \frac{\delta \varepsilon (1 + u) - \gamma_1 \gamma_2}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} \right] \quad (28) \]

\[ \frac{dp_2^A}{dc_1^A} = \frac{1}{2} \left[ \frac{\varepsilon [\gamma_1 (2 + u) - \gamma_2 (2 + 2u)]}{3\delta \varepsilon (3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} + \frac{\gamma_1 \varepsilon u}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} \right] \quad (29) \]

We want both of these terms to be positive.

Finally, we also have to determine the symmetric equilibrium \( p_i^A = p_i^B = p_i \) and \( n_i^A = n_i^B = n_i \) for \( i = 1, 2 \) and \( c_i^A = c_i^B = c_1 \). It suffices to use the four first order conditions above (23, 24, 25, 26) in order to obtain:

\[ p_1 \times \varepsilon (1 + 2u) + p_2 \times \gamma_2 = \frac{N_1}{2} + c_1 \times \varepsilon (1 + u) + c_2 \times \gamma_2 \]

\[ p_1 \times \gamma_1 + p_2 \times \delta = \frac{N_2}{2} + c_1 \times \gamma_1 + c_2 \times \delta \]

which we can immediately solve to get:

\[ p_1 - c_1 = \frac{-\delta \varepsilon u}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} + \frac{N_1 \delta - N_2 \gamma_2}{2 [\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2]} \]

\[ p_2 - c_2 = \frac{\gamma_1 \varepsilon u}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} + \frac{N_2 \varepsilon (1 + 2u) - N_1 \gamma_1}{2 [\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2]} \]

We can now write all the conditions that need to be satisfied in order for an equilibrium in which a slight reduction in \( c_1^A \) increases B’s profits while prices are strategic complements across platforms to exist.

Using the expressions of \( n_i^k \) \( i \in \{1, 2\} \), \( k \in \{A, B\} \) as functions of \( (p_1^A, p_2^A, p_1^B, p_2^B) \), the softness condition is:

\[ \varepsilon (p_1 - c_1) + \gamma_2 (p_2 - c_2) \frac{dp_1^A}{dc_1^A} + \gamma_1 (p_1 - c_1) + \delta (p_2 - c_2) \frac{dp_2^A}{dc_1^A} < 0 \]
which can be rewritten as:

\[(p_1 - c_1) \left( \varepsilon \frac{dp_1^A}{dc_1^A} + \gamma_1 \frac{dp_2^A}{dc_1^A} \right) + (p_2 - c_2) \left( \gamma_2 \frac{dp_1^A}{dc_1^A} + \delta \frac{dp_2^A}{dc_1^A} \right) < 0 \]  

(30)

At the same time, just like in the previous example, we need to make sure that the equilibrium \( n_1 \) is positive:

\[ n_1 = \frac{1}{2} + \frac{x_1}{2t_1} \left( V_1 + \alpha_1 \frac{N_2}{2} - p_1 \right) > 0 \]  

(31)

Finally, platforms must make positive profits in equilibrium:

\[(p_1 - c_1) \left[ \frac{1}{2} + \frac{x_1}{2t_1} \left( V_1 + \alpha_1 \frac{N_2}{2} - p_1 \right) \right] + (p_2 - c_2) \frac{N_2}{2} > 0 \]  

(32)

Together with (27), (28) and (29), (30), (31) and (32) complete the set of necessary conditions. Tedious calculations reveal that all of these conditions can be satisfied for a range of parameter values (\( \varepsilon, \delta, u, \gamma_1, \gamma_2, N_1, N_2 \)) or of the primitive parameters (\( \alpha_1, \alpha_2, t_1, t_2, x_1, V_1, N_2 \)).