Optimal Obsolescence

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Technological change is examined in a model of capital production to show that “creative destruction” can occur as an outcome of firm’s optimizing behaviour, regardless of market structure. Capital systems are made up of components that are all necessary for each system to operate and each component has uncertainty with respect to its durability. For different types of technological change agents make a corresponding decision about whether to continue to use the original capital system (if it is still alive) or to scrap it and build a new capital system which embodies the new technology. Each system has transitional probabilities for scrapping that depend on the size of the present value of the vintage. As technology improves, the optimizing level of durability, and thus the optimal stock of embodied services increases for the new capital system. Yet simultaneously the probability of scrapping an old system over any given time interval increases. Thus, the larger is the improvement in technology, the greater is the chance of scrapping the old system before its physical service life has ended. Over some time horizon of unforeseen and rapid technological change investment in new capital systems could be increasing while at the same time old capital systems are scrapped at a faster rate.

The personal computer revolution that began in the 1970s and continues to pervade the world economy into the new Millennium is an illustration of the phenomenon that the faster a technology improves the more quickly it renders old versions of itself obsolete. The manifestation of this in the personal computer market, where entirely new systems currently purchased are being rendered obsolete within months by new innovations and subsequent systems, is striking. Consumers and producers have developed expectations of the rate of obsolescence and have taken this process into account. Yet in many cases these expectations of
this rate have been too low and investments have been undertaken in systems that have subsequently been scrapped prematurely.

It is often the case that capital systems are scrapped before their physical life is over due to changes in technology because new technologies typically imply changes in the entire structure of a capital system. A model of the underlying behaviour that leads to scrapping of capital systems in the face of technological change is developed here, which follows Carlaw (2003 forthcoming). He develops a model of optimal durability for a single capital system, which shows that a non-convexity in the production feasibility set arises from the assumptions of complementarity and uncertainty. The non-convexity leads agents to build indivisible expected physical life (or durability) into capital systems.

These non-convexities are distinct from the ones discussed in Milgrom and Roberts (1990) in that they are derived from the sufficient condition of complementarity and uncertainty, whereas Milgrom and Roberts assume non-convexities exist based on observation. Milgrom and Roberts describe a set of observed non-convex relations that result in indivisibilities and non-concave maximands such as those between CAD/CAM technologies, marketing strategies and product range. To deal with these they employ a mathematical apparatus which includes supermodular functions (the arguments for which are complements) that can be utilized for performing comparative statics without the usual marginal analysis. As they note “supermodularity has no necessary relation to the concavity of convexity of the function” (517). “Nor, in the context of production functions, does supermodularity carry implications for returns to scale” (517).

The choices of durability and obsolescence in this paper differ from those reviewed in Waldman (2003). The choice of durability in the literature reviewed by Waldman starting with Swan (1970 and 1971) Sieper and Swan (1973) right through to Waldman (1996a) and Hendel and Lizzeri (1999) is set in a monopolistic market framework and market structure and not the technology of production determines durability. Furthermore, durability is a single dimension continuous choice
variable in all of these models. In contrast here durability is derived from the characteristics of technology and therefore not dependent on market structure. The durability of the capital system is a combination of the durability of potential many components each with different physical dimension. The literature on the choice of choice obsolescence discussed by Waldman focuses on Coase’s (1972) time inconsistency problem where the durable goods producing monopolist cannot credibly commit to not selling durable goods in future periods and thus reduces their own profitability as a consequence. Fundenberg and Tirole (1998) provide a model that circumvents the problem partially by allowing quality. Thus new and used become imperfect substitutes.

Waldman (1993 and 1996b) studies the problem from the perspective of R&D choice that renders old units obsolete. Again the analysis in this paper differs in that agents in the model are making decision with respect to the durability and obsolescence of a capital system in the face of unforeseen technological change and causes new version to be superior to old and these decisions are independent of market structure. The agent will face these decisions regardless of whether they are a monopolist or a perfect competitor.

The findings of Carlaw (2003 forthcoming) support Jovanavic and Stolyrov’s (2000) assumption that optimal reinvestment is lumpy (non-convex). In their model this lumpiness results from the assumed difference between the resale value of an existing input and its purchase price. In the model presented here investments are lumpy because there is an explicit non-convexity in the production feasibility set for capital systems. Agents seek to maximize the net present value of a series of capital systems, replacing old systems as they wear out. An investment plan is chosen such that the expected life of each capital system is optimal in the sense that the system is expected to wear out just as a replacement system is built. The old system is replaced before it wears out as a consequence of unexpected technological changes. When new technologies are introduced the investment plan associated with the old technology is no longer relevant and
agents may systematically write off those investments in favour of new and better capital systems.

Three types of technological change are examined. First changes in the net value of service flows that capture technological changes that save on inputs such as labour or energy at each point in time are analysed. Second, changes in the resource cost of producing component durability that capture such improvements as better materials in the process technology for durability are examined. Third, reductions in the rate of depreciation of a capital system which increase the capital system’s ability to maintain service delivery through time are considered. For each of these types of technological change the agent must make a corresponding decision about whether to continue to use the original capital system (if it is still alive) or to scrap the original system and build a new capital system, which embodies the new technology. In each case as technology improves the optimizing level of durability and thus the optimal stock of embodied services increases for the new capital system. Yet simultaneously the probability of scrapping an old system over any given interval of time increases. Thus, the larger is the improvement in technology, the greater is the chance of scrapping the old system before its physical service life has ended. Over some time horizon of unforeseen and rapid technological change this implies that investment in capital systems exploiting the new technology could be increasing, while at the same time the capital systems currently in use are scrapped at a faster rate rendering prior investments obsolete at faster rates.

Such behaviour is consistent with Peter Howitt’s (1998) treatment of the so-called productivity slowdown that has accompanied the technological revolution associated with the PC. In his view a rapid rate of technological improvement implies an increased rate of obsolescence and a period of increased investment in new capital.

Some relevant observations about technological uncertainty are presented in section I. The basic framework that models the capital investment decision of producers is developed in section II.
Section III discusses a generic technological change that can allow a firm to enter a perfectly competitive market. Section IV provides analysis of three types of technological change that may serendipitously fall on a firm already producing in the market. Section V concludes.

I. The Nature of Capital Systems

Capital systems comprise components that are complementary with each other and have durability resilience that are subject to uncertainty arising from the environment in which they operate. Examples of such capital systems include transportations systems characterized by spatially dispersed nodes, production systems characterized by vertically and horizontally related stages of production and complex capital goods themselves. Components are the individual pieces of capital systems and are interrelated in production use. These components are complementary to each other in the sense that the performance system and its components is enhanced by the inclusion of a given component. Often a system will not function in without curtain components. There is uncertainty generated from the environment in which capital goods operate, with respect to durability and resilience. There is also uncertainty with respect to how technology will change and the form that technological change will take.

Integrated capital systems such as the Boeing 777 and personal computers are made up of many interrelated components. An implication of the interrelated structure of capital is that components are complementary to one another, as well as to the integrated capital good itself. For the purposes of this paper technological complementarities occur when one component of capital is either necessary to, or enhances the service flow from, the set of components that make up the integrated capital good. These complementary relationships range from the extreme of a component being necessary for the construction of a capital good to a range of weaker versions where the component merely enhances other components to varying degrees. The model presented uses only the strong form of complementarity.
At a general level, any discussion about capital goods that embody technology must address the issue of how producers and users of technology and capital deal with the unknown. General problems of technological uncertainty, however, go beyond the scope of this paper. Two forms of uncertainty are examined here and one is actually modeled as a problem of risk. Agents face uncertainty with respect to rate and magnitude of technological change and with respect to the durability of a capital system’s components. The uncertainty of how long a component of capital will last in terms of the number of shocks or “hits” the component can withstand is modelled using Poisson distributions.

I follow Lipsey (2000) in noting that much of capital good production is done with inputs that are completely divisible. I also adopt the arguments of Alchian (1959) and Eaton and Lipsey (1997) that capital production is an endogenous process that exploits a non-convexity in the production feasibility set. Lipsey (2000) argues that such a non-convexity comes from the engineering relations of technologies to each other which include the three dimensional nature of the world and the physical characteristics of materials. I capture these engineering relations in the assumptions derived from the styled facts that components of capital must work in concert with each other (i.e., they are complementary in Leontief’s sense) and there is uncertainty with respect to durability or resilience of components. These assumptions are sufficient to generate a non-convexity that results in scale effects. The exploitation of these scale effects by economic agents in turn generates what appear to be indivisibilities ex post.

II. The Capital Investment Decision

Poisson distributions are used to describe the probabilities of survival for components in a capital system for given durabilities. In choosing durability the agent must decide how many hits (i.e., strains on the capital system) a component can withstand before it fails. For example, starting an automobile engine creates a number of engineering strains on the physical structure of the engine and over a given interval of time the engine can be expected to fail with some probability.
**Probabilities of System Survival**

The probability that exactly \( j \) hits occur for a particular component during a time interval of length \( T \) is:

\[
(1) \quad \frac{e^{-\lambda T} (\lambda T)^j}{j!}
\]

Assume that component one dies at shock \( N_1 \) and that component two dies at shock \( N_2 \). Then the probability of survival to date \( T_i \) (for \( i = 1, 2 \)) is:

\[
(2) \quad \Pr(\text{Survival at } T_i | N_i) = \sum_{j_i=0}^{N_i-1} \frac{e^{-\lambda_i T_i} (\lambda_i T_i)^{j_i}}{j_i!}
\]

**The Capital System as a Stock of Embodied Services**

The agent optimally chooses the total volume of expected output for the integrated system by choosing the durability of components, which in turn implies choosing the number of hits that each component can withstand. The total expected stock of capital services is:

\[
(3) \quad K = h^\rho \left[ e^{-(\rho+\delta)T} \left[ \Pr(\text{Survival at } T | N_1, N_2) \right] dT \right]^{\beta}
\]

The capital system may only function in conjunction with “off the shelf” components. Thus, \( h \) is included to represent such a component with given durability. The inclusion of \( h \) implies that there is scarcity built into the capital system such that if a superior technology is introduced the agent will never operate an old system in parallel with a new system even where the old system has some remaining physical life because the \( h \) component can only be integrated into one system as a time. An example given at the outset is that a person can only operate one computer at a time.\(^\text{10}\) For the analysis that follows simplify by setting \( h = 1 \) and \( \beta = 1 \). Then the stock of embodied services in the capital system is:
\[ K = \int_0^\infty e^{-(\rho + \delta)T} \left[ \Pr(\text{Survival at } T \mid N_1, N_2) \right] HT, \]

where \( \rho \) is the pure discount factor and \( \delta \) is a parameter that represents the rate at which capital services depreciate as the system is used. The capital system generates a service flow of one unit in every period it is alive, which is depreciated and discounted according the date since it was put into use.

The total stock of embodied services is found from the probability distribution for the time of breakdown of the integrated system. The strict or Leontief complementarities among the components of the integrated system imply that the lifespan of the system is the minimum of the lifespans of the two components. That is,

\[ T = \min(T_1, T_2). \]

where \( T \) is the date of failure for the overall system and \( T_1 \) and \( T_2 \) are the failure dates for the individual components. Thus, the probability that the integrated system survives beyond \( T \) is the joint probability that both components survive beyond \( T \). I assume that the failure dates of the two components are independently distributed (i.e., they are subject to uncorrelated shocks).

\[ \Pr(\text{system survives at } T \mid N_1, N_2) = \left[ \sum_{\lambda_1=1}^{N_1} \frac{e^{-\lambda_1 T} (\lambda_1 T)^h}{j_1!} \right] \left[ \sum_{\lambda_2=1}^{N_2} \frac{e^{-\lambda_2 T} (\lambda_2 T)^{j_2}}{j_2!} \right] \]

Appendix A shows the derivation of the stock of embodied services in 6 arrived at when 5 is substituted into 3.

\[ K = \sum_{j_1=1}^{N_1-1} \sum_{j_2=1}^{N_2-1} \frac{(j_1 + j_2)!}{j_1! j_2!} \frac{\lambda_1^h \lambda_2^{j_2}}{j_1!} \frac{\lambda_1^h \lambda_2^{j_2}}{j_2!} \frac{1}{(\rho + \delta + \lambda_1 + \lambda_2)^{h+j_1+j_2+1}} \]
**Gross Present Value of the Capital System**

Consider a single capital system that is used until some component fails and will not subsequently be replaced (or repaired). The value of the expected physical output associated with this capital system is expressed as follows

\[(7) \quad \tilde{V}(N_1, N_2) = bK\]

where \(b = (P - W)\). \(P\) is the constant gross value of the output flows generated by the system at each point in time and \(W\) is the constant labour cost associated with producing this output at each point in time. Labour is used in a fixed proportion to the capital stock, and units are chosen so that this proportion equals unity.

**Costs**

Investment in capital will take the form of purchasing durability for each of the components of capital. For simplicity and to emphasize that non-convexities are not being imposed exogenously, the marginal cost of durability for each component is assumed to be constant.

\[(8) \quad C(N_1, N_2) = w_1N_1 + w_2N_2\]

\(N_i\) is the number of hits required to render component \(i = (1, 2)\) of the capital system useless.

It will be useful in the following analysis to employ the concepts of marginal and average costs. For this purpose define the total expected output of a capital system to be identical with the expected size of the capital stock (i.e., choose output units so that \(y = K\)). Then the total cost associated with output \(y\) is

\[(9) \quad TC(y) = w_1N_1(y) + w_2N_2(y)\]

where \(N_1(y)\) and \(N_2(y)\) are cost minimizing component durabilities associated with output \(y\). It is then possible to define marginal and average costs in the usual way:
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\begin{equation}
MC(y) = \frac{dTC(y)}{dy}
\end{equation}

and

\begin{equation}
AC(y) = \frac{w_1 N_1(y) + w_2 N_2(y)}{y}
\end{equation}

Note that \(TC(y)\) is only defined for certain values of \(y\) because \(N_1(y)\) and \(N_2(y)\) must be integers. Therefore, strictly speaking \(MC(y)\) will not be well defined since \(TC(y)\) is not differentiable. However, rates of change in total cost can still be calculated for suitably chosen discrete changes in \(y\).

**The Optimal Investment Plan**

Producers must plan for the eventuality that the current capital system fails since each capital system faces the possibility of not surviving over a given period of time. The producer will replace a failed system with an identical system if technology is the same. At each point in time for which the current asset survives it yields an instantaneous payoff of \(b\). The value of an investment plan is the present value of the stream of instantaneous payoffs plus the net present value of a replacement system in the event that the old system fails. The gross present value for a system producing \(y\), after the initial costs are sunk and given a sequence of replacements involving identical capital systems is as follows:

\begin{equation}
V(y) = b \int_0^\infty e^{-(\rho + \delta)T} \Pr(K \text{ alive } | T) dT \\
+ \int_0^\infty \left[ V(y) - C(y) \right] e^{-\rho T} \Pr(K \text{ alive } | T) \Pr(\text{death } | T) dT
\end{equation}

where \(\Pr(\text{death } | T)\) is the instantaneous probability of death for the old system. The second term in equation (12) is equal to zero in perfect competition (and where no technological improvement
takes place). In this case, the value of an investment plan is equal to the quasi rents associated with the existing capital system.\textsuperscript{14}

Equation (12) says that the gross present value over the infinite investment horizon is the discounted value of flow of services from the current asset once the costs of the asset are sunk (the first term) plus the net value of services from its subsequent replacement when the first asset dies (the second term).\textsuperscript{15} Although it abuses notation slightly because there is a discrete mapping from hits to output, this will sometimes be represented as:

\[
V(N_1, N_2) = b \int_0^\infty e^{-(\rho + \delta)T} \Pr(K \text{ alive} | T) dT + \int_0^\infty [V(N_1, N_2) - C(N_1, N_2)] e^{-\rho T} \Pr(K \text{ alive} | T) \Pr(\text{death} | T) dT
\]

(13)

with the understanding that the output \( y = K \) corresponds to the value of the capital stock determined by \((N_1, N_2)\) as in equation (6).

It is clear from equation 12 that a firm faced with fixed technology in a perfectly competitive market will never scrap an existing capital system until it dies. This is so since perfect competition as defined in section III implies that the expected net present value of each system to be built will be just equal to zero. Thus, as long as the system is alive it yields positive quasi rents (the first term in equation 12) and the net present value of the replacement system is zero (the second term in equation 12).

Exploiting the stationarity of equation 12, \( V(y) \) can be solved for.

\[
V(y) = \frac{b \int_0^\infty e^{-(\rho + \delta)T} \Pr(K \text{ alive} | T) dT - C(y) \int_0^\infty e^{-\rho T} \Pr(K \text{ alive} | T) \Pr(\text{death} | T) dT}{1 - \int_0^\infty e^{-\rho T} \Pr(K \text{ alive} | T) \Pr(\text{death} | T) dT}
\]

(14)
The probability that the current capital system is still alive at $T$ has already been calculated in equation 5. This leaves the hazard rate associated with the death of the current system to be calculated. See appendix B for the calculation of the hazard rate and the definition of $\Omega(N_1, N_2)$.

\[
(15) \quad V(N_1, N_2) = \frac{b \sum_{j_1=1}^{N_1} \sum_{j_2=1}^{N_2} \left( \frac{j_1 + j_2}{j_1! j_2!} \right) \left( \frac{\lambda_1^{j_1} \lambda_2^{j_2}}{(\rho + \delta + \lambda_1 + \lambda_2)^{j_1+j_2+1}} \right) - C(N_1, N_2) \Omega(N_1, N_2)}{1 - \Omega(N_1, N_2)}
\]

The gross value of a capital system when each failing asset is to be replaced with an identical asset depends on durabilities chosen for each asset in the plan. Note that the stationarity of the investment plan is being exploited so that the assumption $(N_1, N_2)$ is identical for each capital system can be employed whenever a replacement decision is made.

The firm will optimize the net present value of the capital system by choice of durability in each component. The optimizing decision is:

\[
(16) \quad \max_{(N_1, N_2)} V(N_1, N_2) - C(N_1, N_2) = \frac{b \sum_{j_1=1}^{N_1} \sum_{j_2=1}^{N_2} \left( \frac{j_1 + j_2}{j_1! j_2!} \right) \left( \frac{\lambda_1^{j_1} \lambda_2^{j_2}}{(\rho + \delta + \lambda_1 + \lambda_2)^{j_1+j_2+1}} \right)}{1 - \Omega(N_1, N_2)} - \frac{C(N_1, N_2) \Omega(N_1, N_2)}{1 - \Omega(N_1, N_2)} - w_1 N_1 - w_2 N_2
\]

It is difficult to solve analytically for the values of $N_1$ and $N_2$ that maximize the objective function in 16, because the choice variables appear in the limits of summation and thus the factorial expressions for the equation. However, maximizing values of $N_1$ and $N_2$ can be chosen for the system when parameter values are specified to calibrate the model. For example the parameter values which will be chosen for the base case in section IV below are:

\[
\begin{align*}
    b &= 6.0483026 \\
    \lambda_1 &= \lambda_2 = 4 \\
    \rho &= 0.05 \\
    \delta &= 0.2 \\
    w_1 &= w_2 = 0.5
\end{align*}
\]
and these parameters imply that the maximizing durabilities for each component are \( N_1 = N_2 = 6 \). The resulting value of \( K \) is 0.992014 and the net present value of output for a given capital system is 0. The prices are chosen so that in this base case the characteristics of long run competitive equilibrium are present, or that the optimizing choice of durability for the capital system is such that the net present value of a capital good built at the firm MES of expected total volume of output will just equal zero. The expected total volume of output for a given capital system in the investment stream is:

\[
y = \sum_{j_1=1}^{N_1-1} \sum_{j_2=1}^{N_2-1} (j_1 + j_2)! \left( \frac{(4.25)^{j_1+j_2}}{(8.25)^{j_1+j_2}} \right)
\]

In this example, firm MES occurs at \( y = K = 0.992014 \) and \( N_1 = N_2 = 6 \), which is the durability at the minimum of the average cost curve defined in equation 11.16.

As has been noted already the optimal durability for the components of any replacement capital system is the same as that of the old when technology does not change. In such cases there is a scrapping boundary which is defined by the number of hits each component can withstand before it fails. Table 1 shows\(^\text{17}\) that without any technological change the scrapping boundary for the capital system that maximizes the net present value of the stream of investments is just the one where one or both of the components physically fail. This is because the quasi rent is always positive for the current system and the net present value of the replacement system is zero.

There is a transitional probability of failure for the old system, which can be calculated as follows.

\[
\Pr(\text{scrapped at } T' | N_1 = 6 = N_2) = 1 - \left[ \sum_{k_1=0}^{N_1-1} \frac{e^{-\lambda_1 T'} (\lambda_1 T')^{k_1}}{k_1!} \right] \left[ \sum_{k_2=0}^{N_2-1} \frac{e^{-\lambda_2 T'} (\lambda_2 T')^{k_2}}{k_2!} \right]
\]

This probability of system failure is plotted against time in figure 1.\(^\text{18}\) The probability of the old capital system failing and being replaced with an identical system increases as time progresses.
III. Market Structure

Assume the market is perfectly competitive so that each producer is small relative to the market. Also assume that the producer of capital uses that capital to produce output for the market. Assume further that any technological change that a single entrant might experience is such that the producer is still small relative to the market (i.e., the entrant will remain a price taker). This situation is illustrated in figure 2.

Only one entrant firm is allowed. The entrant firm is small relative to the market so the equilibrium price remains at $P^*$. The long-run supply (LRS) curve is not flat throughout. Since the price associated with the MES for the new technology is lower than the market-clearing price the LRS is just the marginal cost curve for the entrant up to $y'$. Thereafter it is flat at the MES of the producers using the old technology, ignoring integer problems associated with the number of firms.

The demand curve for the market is defined in terms of the discounted and depreciated expected total volume of output. This is not the usual static demand curve represented in the standard model of perfect competition. In particular demand is not explicitly defined over a service flow per period of time. It is defined over the whole volume of service per capital system. The importance of the discount rate is highlighted here. Because the construction of capital implies a deferral of consumption into the future and the actual consumption of the services of capital is spread out through time, there is a price of time that must be made explicit in the model. Therefore both supply and demand functions incorporate the discount rate as a parameter.

The assumptions about competition in the market imply that firms are profit maximizing price takers. Thus, in long run perfectly competitive equilibrium with technology held constant they operate at minimum efficient scale (MES) and make exactly zero profit. Where technology differs between firms, all firms will still be profit maximizing price takers, however, the firm with the
superior technology will be operating at an output level above its MES and making positive profit. In section IV that follows all of the improvements in technology will be cases where the average and marginal cost curves shift down and to the right relative to the old technology, as shown in figure 3.

Figure 3 shows that for an incumbent firm the optimal amount of output (and thus by definition the size of the capital stock) is \( y_0 \) when the market price is \( P^* \). Thus, each firm operating with the “old” technology makes exactly zero profit. A new technology user can make positive profit, and maximizes that profit at \( y^* \) where \( P^* = MC^* \) where as the MES expected total output occurs at \( y_1 \). The profit accruing to the entrant is represented as the shaded area.

There are two experiments that can now be conducted with respect to how a new technology is introduced. Given the assumptions of a competitive market, only the second case is relevant to the issue of technological obsolescence. However the first is mentioned as it provides a distinction between Schumpeterian creative destruction and an alternative form of obsolescence.

In the first case assume the incumbent firms producing in the market are using the “old” technology and one entrant firm has the blueprint for the new technology. Because this entrant is small relative to the market demand it can enter and not influence the market price. Yet, it will make more than zero profit since it has a superior technology. Thus, it will enter employing the capital system which optimizes its investment plan using the new technology. Obsolescence does not occur in this case. However, if competition is imperfect, the usual form of Schumpeterian creative destruction can take place. The innovating firm will be able to capture a share of the market away from existing firms and thus render part of the existing capital stock obsolete.

In the second case, one of the incumbent firms experiences serendipitous technological learning and must make a decision whether to scrap its old capital investment plan in favour of a new one that builds and employs a stream of capital systems which embody the superior technology.
Obsolescence will occur in this case regardless of market competition and Schumpeterian creative
destruction is a special case (under imperfect competition) of this type of obsolescence.

IV. Changes in Technology for an Existing Firm

This section examines what happens to the investment plan of agents as unforeseen technological
change introduces new capital assets. Three ways of introducing exogenous technological change
into the model are examined here. First, changes in \( b \) represent changes in the value of services
delivered at each date when a capital good is alive. A new capital good that delivers more capital
services than an old one will have a higher \( b \) associated with it because the difference between \( P \)
and \( \bar{w} \) has increased. Second, changes in \( w_1 \) and \( w_2 \) represent changes in the marginal cost of
component durability. Decreases in \( w_1 \) and \( w_2 \) will have the effect of making durability cheaper
and the capital will be built to last longer. Third, decreases in depreciation, \( \delta \), represent increases
in the efficiency with which services are delivered over the life of a given capital system.

The analysis here deals only with cases where innovations imply that bigger is better and the rate
at which capital systems embodying the new technology get thrown away actually increases, even
while their value and durability is increasing. The implication of the analysis is that the economic
life of the capital assets will decrease, in spite of their potentially longer physical life, because
they are being rendered obsolete quickly by continuing technological change that generates better
versions of capital.¹⁹

Optimal Reinvestment in Physical Capital

In order to model what happens when an incumbent firm experiences serendipitous technological
learning a new optimal investment plan must be established. Of particular relevance to the new
investment decision is how much durability remains in the existing capital system and what is the
current state of technology (i.e., has technology changed in unforeseen ways since the original
investment).
The optimal durability of a capital system was established in section II by the maximization of the objective function in equation 16. However, when technology changes, the optimal present value of a capital system changes. This change in value may induce the firm to switch from its current capital system to a new capital system even if the old system is not physically dead.

Since each case of technological change examined here results in an unambiguously better capital system it is clear that one limit on when a firm switches capital systems, given the new technology, is the time when it would have replaced the existing system in the investment plan where technology is held constant, shown in section II.

To determine if the old system is scrapped two new investment plans must be defined. In the first the firm continues to use the old capital system replacing it in the future possibly before it physically dies with a series of capital systems using the new technology. In the second, the firm scraps the old system immediately and replaces it with a series of systems using the new technology. The comparison the firm makes is the gross present value of the current system to the maximized net present value of a system derived from an investment plan using the new system.

If the net present value of the new system is smaller than the present value of the quasi rent associated with operating the old capital system to term, the firm will wait until the old system fails before switching to the new technology.

Replacement of an old system will thus occur when the following condition is satisfied:

\[ V_i(N_1', N_2') - \left[ w_1N_1' + w_2N_2' \right] \geq \hat{V}(N_1, N_2), \]

where

\[ \hat{V}(N_1, N_2) = b_0 \sum_{j_1=1}^{N_1-1} \sum_{j_2=1}^{N_2-1} \frac{\lambda_1^{j_1} \lambda_2^{j_2}}{j_1! j_2!} \left( \rho + \delta_0 + \lambda_1 + \lambda_2 \right)^{j_1+j_2+1} - C(N_1', N_2') \Omega(N_1', N_2') \]

\[ \frac{1 - \Omega(N_1', N_2')}{1 - \Omega(N_1'', N_2'')}. \]
\( \hat{V} \) is derived from the investment plan that uses the current capital system and replaces it with the new technology at some point in the future, possibly before the current system wears out. The notation for number of hits is altered slightly here. Now \( N_i = N_i^0 - M_i \) for \( (i = 1,2) \), where \( N_i^0 \) is the original durability built into the current system, and \( M_i \) is the number of hits that have occurred on the \( i^{th} \) component of the current system since it was put into use. The \( N_i' \)'s represent ex ante optimizing choices of durability for the new capital good. Note if \( N_i' = N_i \) (i.e., the technology has not changed) then \( \hat{V} \) is just the quasi rent associated with the old system since the net present value of a new system is zero. If this is not the case \( \hat{V} \) includes that quasi rent plus the positive net present value of the replacement systems using the new technology.

The gross present value of a plan using new technology immediately is:

\[
\left(21\right) \quad V_1(N_1', N_2') = \frac{b_0 \sum_{j_1=1}^{N_1-1} \sum_{j_2=1}^{N_2-1} \frac{\lambda_1^{j_1} \lambda_2^{j_2}}{j_1! j_2!} (\rho + \delta_1 + \lambda_1 + \lambda_2)^{j_1+j_2+1}}{1 - \Omega(N_1', N_2')} - C(N_1', N_2') \Omega(N_1', N_2').
\]

It is now possible to express inequality 19 in terms of the \( M_i \)'s describing the number of hits that have occurred since the original capital good was built and put into service. Unfortunately this inequality is again difficult to manipulate analytically because the variables of interest appear as the limits of summation in equations 20 and 21, and thus are part of the factorial expressions for those equations. However, once again for a given set of parameter values it is possible to show by numerical simulation what \( M_i \)'s induce immediate investment using the new technology.

**Numerical simulation**

As was noted at the beginning of this section, three experiments are contemplated. Changes in \( b \) represent improvements in the services delivered as each date when the capital system is alive.

Changes in the input prices for durability represent improvements in technology one step back in
the production system. Changes is the physical depreciation rate represent improvements in technology where the capital system where out at a slow physical rate. For each of these cases of technological change below the optimal durability and thus present value of service flow from the system is larger. This implies that there will be technological changes for which scrapping of the old system will occur before one or both of the components physically fails. In such cases the scrapping boundary (defined in terms of hits on the components of the existing system) will tolerate fewer hits on the components of the existing system than it was physically built to withstand.

Appendix C provides the details of the parameter changes and the implications of these changes for the optimal size of $y$ and $N_1$ and $N_2$, as well as the details of how to calculate the transitional probabilities shown in the figures.

Technological change can take the form of increased value in the services delivered at every date that the capital asset is alive. For example, more fuel-efficient automobiles deliver higher net service flows since they are absolutely saving in energy resources. Computer numeric controlled machining delivers the same service measured in terms of output of cut metal but is absolutely saving in terms of wasted metal because it can map a series of cuts simultaneously and do the cuts more precisely. In general any innovation that allows for a higher value of delivered services relative to variable resource costs will be reflected as an increase in the net value of services delivered at a given date.

Increases in the parameter $b$ reflect increases in the value of the asset relative to the variable costs in use (i.e., increases in the difference between $P$ and $\tilde{w}$). Technological change can take the form of increased service flow delivery, or savings on other variable factors per unit of service delivered or both.

The analysis is benchmarked with the case of no technological change shown in section three and the parameter $b$ is then varied are follows:
\begin{align*}
b_0 &= 6.0483026 & b_1 &= 7.5 \\
b_2 &= 8 & b_3 &= 10
\end{align*}

The details of this simulation are provided in appendix C with the main results summarized below.\textsuperscript{20} Table 2 shows that there are negative values when the net present value of the new capital system is subtracted from the quasi rents associated with the old system and thus there are some positive values of $N_1$ and $N_2$ where the system is scrapped before in physically fails.

The old system is not scrapped unless a sufficient number of hits have occurred such that $N_1$ and $N_2$ correspond to negative values in table 2. There is a transitional probability that the system is not scrapped in some future time interval if the current $N$'s are associated with the bold positive numbers in table 2.\textsuperscript{21} Figure 4 plots the probability that the old system is scrapped against time for these two cases of technological change. Maintaining the assumption that $N_1^0 = N_2^0 = 6$, the probability that the old system is scrapped is higher as the value of the replacement system increases and each probability increases as the date for the comparison is mover further into the future. Table 3 shows that the scrapping boundary has disappeared; the old system would be scrapped even if it has just been constructed and no hits have yet occurred. Thus the probability of scrapping the old capital system when the new technology becomes available is one, regardless of the number of hits that have occurred for each component.

Another way in which technological change can occur is in the production process that produces durability in the components of a capital system. An example of such technological change is the effect that innovations in the blast furnace had on the quality of steel. The use of the more abundant high quality steel in turn allowed for the creation of more durable components in a wide range of manufactured goods for which steel is a material input. More generally any improvement in the quality of materials used for the manufacture of components will allow for a resource cost saving in the creation of durability for those components.
Once again this type of change can be accommodated in the framework developed in section II. Reductions in $w_1$ and $w_2$ reflect improvements in efficiency in the creation of inputs used to create durability in the components of the capital system. There are a number of avenues to explore in terms of altering the input cost of component durability including asymmetric changes. While such changes are possible in the framework developed in section II, for simplicity, only symmetric reductions in the cost of durability are examined. Such changes would occur when the resource experiencing the technological improvement is an input in the durability-creating process of all components in the capital system. The set of changes are:

$$
\begin{align*}
&w_1^0 = w_2^0 = 0.5 & w_1^1 = w_2^1 = 0.4 \\
&w_1^2 = w_2^2 = 0.35 & w_1^3 = w_2^3 = 0.2
\end{align*}
$$

Table 4 shows that the scrapping boundary for lower input prices shifts down to the right. In this case it possible that the capital system is scrapped before it physically fails. The transitional probabilities of not scrapping the capital system in a subsequent time period are shown in figure 5. The probability of scrapping the old system is everywhere higher for the larger reduction in input costs and each probability increases as time progresses.

Table 5 shows that for a sufficient reduction in input cost of durability the scrapping boundary lies outside the durability for the components of the original capital system. In this case the probability of scrapping is one for the old capital system faced with the new technology, even starting from a situation where the old technology has not suffered any hits.

A third way in which technological change can be introduced into the model is through the efficiency with which the capital system can deliver services through time. Many new vintages of capital exhibit improved efficiency in the delivery of their services. For example, as the quality of services delivered through time improved over different vintages of automobile from the 1970s
through to the 1990s they have been built to last longer. Evidence of this can be found in the progressively longer and more comprehensive warranties that are sold with new cars. A five year 150 thousand mile warranty is commonplace for a new automobile sold in 1999, whereas it was unheard of in the 1960’s. Improvements in the efficiency with which capital service are delivered are modelled as reductions in $\delta$. Decreases in $\delta$ imply that the service delivered by a capital system do not depreciate as much (deterministically over time while the system lasts). $\delta$ is changed in two ways from its base case value.

$$\delta_0 = 0.2 \quad \delta_1 = 0.05 \quad \delta_2 = 0$$

The details of these changes in the parameter $\delta$ are provided in appendix E. Table 6 shows that the system may be scrapped before it physically fails when $\delta_1 = 0.05$.

Table 7 shows that the scrapping boundary has moved to the right and down but the probability of scrapping the original capital system does not go to one even for a physical depreciation rate of zero. Figure 6 plots the transitional probabilities of scrapping for $\delta_1 = 0.05$ and $\delta_2 = 0.00$ on fixed dates further in the future.

V. Implications and Conclusions

For each of the types of technological change presented the probability of scrapping the old system increases as the size of the technological improvement embodied in the next capital system increases. The implication of this is that in a world where there are continuous technological improvements occurring, the rate at which existing capital is scrapped increases.

Each of the three types of technological change imposed on the system had a different reason underlying the change in optimal durability and scrapping. For the first type of technological improvement there was no change in the underlying production function and physical returns to
scale of producing the capital system. However, the resources used to construct the capital system were made more valuable by the change in $b$. The optimal durability increased for each improvement in technology, but so did the rate of scrapping of old systems.

For the second type of technological improvement there was once again no change in the production function for the capital system. Rather there was an improvement in technology one stage back that made producing durability for the capital system cheaper. This resulted in increased durability and increased scrapping of old systems.

For the third type of technological improvement the physical returns to scale in the production function increased because the delivery of services deteriorated less severely as the capital system aged. The underlying production function shifted up in this case and the long run average cost curve shifted down.

For this last type of technological change it is important to note the distinction between physical depreciation of the capital system represented by $\delta$ and the rate of obsolescence associated with capital investment. Standard treatments of depreciation and obsolescence simply include obsolescence as a component of depreciation to determine the amount of investment that is required to offset these processes. Howitt (1998) is an instance in the literature where the distinction has been made. The analysis here reveals that depreciation and obsolescence need not work in the same direction and there is no a priori reason why they should be treated in the same manner. In fact where technological change takes the form of reducing physical depreciation in new systems, it can actually increase the rate of obsolescence of old systems and the two effects work in opposition to each other in determining some perfectly foresighted rate of capital replacement.

This has implications for theories of economic growth driven by endogenous technological change. The optimizing process that generates technological change may also generate increases in obsolescence. Thus, there is a trade off in the maximization procedure between more
technology and the destruction of value in existing capital systems that goes beyond
Schumpeter’s creative destruction. In this case the trade off is potentially internal to the agent
making the investment in new technology and new capital systems. The external element of
creative destruction occurs in addition to this.

The optimal investment plan creates a lumpy time profile for the costs of a capital system. In the
world of no technological change replacement systems are purchased at the time old systems fail.
The scale effect inherent in the production function means that these costs will be spaced apart in
time, creating the lumpiness in the firm’s investment flow. Where technological change occurs,
this lumpiness will be aggravated by the fact that the size of investment also changes. This result
provides a rationale for the assumptions of Jovanovic and Stolyarov (2000), who assume lumpy
reinvestment. The rationale provided here is that there is a scale effect in the production of
durability. The fact that changes in technology can lead to scrapping of sunk investments also
provides a rationale for Jovanovic and Stolyarov’s assumption that there is a difference between
purchase and resale prices of inputs for production.

Technological change can manifest itself in a number of ways. The three possibilities examined
here support the Schumpeterian conclusion that unforeseen improvements in technology can
increase the rate at which existing capital is scrapped, and thus increase the rate of obsolescence.
However, the obsolescence modelled here does not lead to Schupmeterian creative destruction,
since there is no strategic interaction among firms and each firm optimally chooses to scrap old
capital.

As technological improvement occurs the rate of obsolescence increases, leading to the result that
new capital systems may get bigger in a value sense while old systems get scrapped more
quickly. Furthermore, improvements caused by a reduction in depreciation increase the optimal
durability and expected total volume of output, but also increase the rate of obsolescence. Thus,
more of the sunk cost of capital investment may get written off as technology increases rapidly.
This is not the same as Schumpeterian creative destruction where one agent’s actions have adverse effects on another agent. However, if the assumption of perfect competition is relaxed creative destruction can occur in this framework. In this case producers are aware of the trade off between building more durability into a system and the fact that it is more likely to be scrapped in the near future. The result provides support for Howitt’s argument that periods of rapid technological change are accompanied by higher rates of obsolescence and this can lead to a real productivity slowdown in the aggregate for changes in general purpose technologies.

The results of increased system value accompanied by both less durability and a higher rate of scrapping, shown for the first two types of technological change, is consistent with observations in the personal computer market where the expected economic life of a computer has declined rapidly at the same time that the physical life and per period rate of service flow has increased. Computers are able to deliver vastly increased power per unit of time in use relative to a few years ago, yet they are being scrapped more rapidly and built with less physical durability.
### Tables

#### Table 1
**Scraping boundary with no technological change**

<table>
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<tr>
<th>N1/N2</th>
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#### Table 2
**Change in net value of capital services (case 2)**

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#### Table 3
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#### Table 4
**Change in input prices**

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**Change in input prices**

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#### Table 6
**Change in depreciation**

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Table 7
Change in depreciation $\delta = 0$

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Figures

Figure 1

Figure 4

Figure 2

Figure 5

Figure 3

Figure 6
Appendix A.

Equation 5 can be inserted directly into the capital stock equation.

\[ K = \int_{0}^{\infty} e^{-(\rho+\delta)T} \sum_{j_1=1}^{N_1-1} \sum_{j_2=1}^{N_2-1} \left[ e^{-\lambda_1 T} \left( \frac{\lambda_1 T}{j_1!} \right)^{j_1} e^{-\lambda_2 T} \left( \frac{\lambda_2 T}{j_2!} \right)^{j_2} \right] dT \]

Rewriting the expression for capital to collect terms under the integral and moving terms not dependent on time outside the integral yields:

\[ K = \sum_{j_1=1}^{N_1-1} \sum_{j_2=1}^{N_2-1} \frac{\lambda_1^{j_1} \lambda_2^{j_2}}{j_1! j_2!} \int_{0}^{\infty} e^{-(\rho+\delta+\lambda_1+\lambda_2)T} (T)^{j_1+j_2} dT \]

The integral inside the expression can be evaluated as follows. (See Gradshteyn and Ryzhik, 1980: 92):

\[ K = \sum_{j_1=1}^{N_1-1} \sum_{j_2=1}^{N_2-1} \frac{\lambda_1^{j_1} \lambda_2^{j_2}}{j_1! j_2!} \int_{0}^{\infty} e^{-(\rho+\delta+\lambda_1+\lambda_2)T} (T)^{j_1+j_2} dT = \frac{(j_1+j_2)!}{(\rho+\delta+\lambda_1+\lambda_2)} \left[ \frac{1}{(\rho+\delta+\lambda_1+\lambda_2)} \right]^{j_1+j_2} \]

Substituting A.3 into A.2 gives the expected total stock of physical capital services for the system.

Appendix B.

There are three cases where there is a positive instantaneous probability of death for the old system that is not zero or one.

- $N_1 - 1$ hits have occurred on component 1 while fewer than $N_2 - 1$ hits have occurred on component 2
- $N_2 - 1$ hits have occurred on component 2 while fewer than $N_1 - 1$ hits have occurred on component 1
- Both components have only one hit remaining

The probability associated with each of these cases is dealt with in turn.
Case 1:

[B.1a] \( \Pr(K \text{ alive } | T) \Pr(\text{inst. death } | T) = \frac{e^{-\lambda_1 T} (\lambda_1 T)^{N_1-1}}{(N_1-1)!} \sum_{j_1=0}^{N_1-2} \frac{e^{-\lambda_2 T} (\lambda_2 T)^{j_1}}{(j_1)!} (\lambda_1 + \lambda_2) \)

where \( \lambda_1 \) is the instantaneous probability of death (Hogg and Craig, 1978: 99-100).

Case 2:

[B.1b] \( \Pr(K \text{ alive } | T) \Pr(\text{inst. death } | T) = \frac{e^{-\lambda_1 T} (\lambda_2 T)^{N_2-1}}{(N_2-1)!} \sum_{j_2=0}^{N_2-2} \frac{e^{-\lambda_1 T} (\lambda_1 T)^{j_2}}{(j_2)!} (\lambda_1 + \lambda_2) \)

where \( \lambda_2 \) is the instantaneous probability of death.

Case 3:

[B.1c] \( \Pr(K \text{ alive } | T) \Pr(\text{inst. death } | T) = \frac{e^{-\lambda_1 T} (\lambda_1 T)^{N_1-1}}{(N_1-1)!} \cdot \frac{e^{-\lambda_2 T} (\lambda_2 T)^{N_2-1}}{(N_2-1)!} (\lambda_1 + \lambda_2) \)

where \( (\lambda_1 + \lambda_2) \) is the instantaneous probability of death.

Each of these expressions can now be substituted into the integral expression used in equation 14 and the integral evaluated as follows:

[B.2]

\[
\int_0^\infty e^{-\rho T} \Pr(K \text{ alive } | T) \Pr(\text{inst death } | T) dT = \sum_{j_1=0}^{N_1-2} \frac{\lambda_1^{N_1-1}}{(N_1-1)!} \frac{\lambda_2^{j_1}}{j_1!} \int_0^\infty e^{-(\rho + \lambda_1 + \lambda_2) T} T^{N_1-1+j_1} dT \\
+ \sum_{j_2=0}^{N_2-2} \frac{\lambda_2^{N_2-1}}{(N_2-1)!} \frac{\lambda_1^{j_2}}{j_2!} \int_0^\infty e^{-(\rho + \lambda_1 + \lambda_2) T} T^{N_2-1+j_2} dT \\
+ \frac{\lambda_1^{N_1-1}}{(N_1-1)!} \frac{\lambda_2^{N_2-1}}{(N_2-1)!} (\lambda_1 + \lambda_2) \int_0^\infty e^{-(\rho + \lambda_1 + \lambda_2) T} T^{N_1+N_2-2} dT
\]

Once again the three integral expressions can be evaluated using the solution from Gradshteyn and Ryzhik, 1980: 92.

[B.3]
\[ \int_0^\infty e^{-\rho T} \Pr(K \text{ alive } | T) \Pr(\text{inst death } | T)dT = \sum_{j_2=0}^{N_2-2} \frac{(N_1 + j_2 - 1)!}{(N_1 - 1)!j_2!} \frac{\lambda_1^{N_1-1} \lambda_2^{j_2}}{(\rho + \lambda_1 + \lambda_2)^{N_1+j_2}} \lambda_1 \]
\[ + \sum_{j_1=0}^{N_1-2} \frac{(N_2 + j_1 - 1)!}{(N_2 - 1)!j_1!} \frac{\lambda_1^{N_1-1} \lambda_2^{N_2-1}}{(\rho + \lambda_1 + \lambda_2)^{N_2+j_1}} \lambda_2 \]
\[ + \frac{(N_1 + N_2 - 2)!}{(N_1 - 1)!(N_2 - 1)!} \frac{\lambda_1^{N_1-1} \lambda_2^{N_2-1}}{(\rho + \lambda_1 + \lambda_2)^{N_1+N_2-1}} \lambda_1 \lambda_2 \]

The solution in equation B.3 can now be substituted back into equation B.1. But first simplify notation in the following way:

\[ \Omega(N_1, N_2) = \sum_{j_2=0}^{N_2-2} \frac{(N_1 + j_2 - 1)!}{(N_1 - 1)!j_2!} \frac{\lambda_1^{N_1-1} \lambda_2^{j_2}}{(\rho + \lambda_1 + \lambda_2)^{N_1+j_2}} \lambda_1 \]
\[ + \sum_{j_1=0}^{N_1-2} \frac{(N_2 + j_1 - 1)!}{(N_2 - 1)!j_1!} \frac{\lambda_1^{N_1-1} \lambda_2^{N_2-1}}{(\rho + \lambda_1 + \lambda_2)^{N_2+j_1}} \lambda_2 \]
\[ + \frac{(N_1 + N_2 - 2)!}{(N_1 - 1)!(N_2 - 1)!} \frac{\lambda_1^{N_1-1} \lambda_2^{N_2-1}}{(\rho + \lambda_1 + \lambda_2)^{N_1+N_2-1}} \lambda_1 \lambda_2 \]  

Substituting B.4 into equation 14 yields equation 15.
Appendix C

The base case parameters are used here as a benchmark.

\[ b_0 = 6.0483026 \quad \rho = 0.05 \quad \lambda_1 = \lambda_2 = 4 \]
\[ \delta = 0.2 \quad w_1 = w_2 = 0.5 \]

The set of parameter variations that are considered one at a time are in table C.1

Table C.1

<table>
<thead>
<tr>
<th>Parameter changes</th>
<th>( y' )</th>
<th>( N'_1 = N'_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b' = 6.5 )</td>
<td>1.1511</td>
<td>7</td>
</tr>
<tr>
<td>( b_1 = 7.6 )</td>
<td>1.4482</td>
<td>9</td>
</tr>
<tr>
<td>( b_2 = 8 )</td>
<td>1.8418</td>
<td>12</td>
</tr>
<tr>
<td>( b_3 = 10 )</td>
<td>2.072</td>
<td>14</td>
</tr>
<tr>
<td>( w_1 = w_2 = 0.49 )</td>
<td>0.992014</td>
<td>6</td>
</tr>
<tr>
<td>( w_1^1 = w_2^1 = 0.4 )</td>
<td>1.448</td>
<td>9</td>
</tr>
<tr>
<td>( w_1^2 = w_2^2 = 0.35 )</td>
<td>1.841</td>
<td>12</td>
</tr>
<tr>
<td>( w_1^3 = w_2^3 = 0.2 )</td>
<td>2.636</td>
<td>20</td>
</tr>
<tr>
<td>( \delta = 0.19 )</td>
<td>0.998</td>
<td>6</td>
</tr>
<tr>
<td>( \delta_1 = 0.05 )</td>
<td>2.726</td>
<td>15</td>
</tr>
<tr>
<td>( \delta_2 = 0.00 )</td>
<td>5.524</td>
<td>29</td>
</tr>
</tbody>
</table>

For \( b_0 = 6.0483026 \) the present value of the quasi rent depends on how many hits each component of the capital system has sustained in the time interval since the old capital system was put into service. The original expected volume of services was \( y = 0.992014 \) and the durability of each component was 6 hits. For \( b' = 6.5 \) the maximum net present value is

\[
(C.1) \quad V' - \left[ w_1 N'_1 + w_2 N'_2 \right] = 0.867902 .
\]

and occurs at \( y' = 1.1511 \) and \( N'_1 = N'_2 = 7 \). The rewritten inequality V.1 is negative in states of the world where it pays to scrap the old and build the new.

\[
(C.2) \quad 0 \geq \hat{V} - \left\{ V' - \left[ w_1 N'_1 + w_2 N'_2 \right] \right\} .
\]
When \( b_1 = 7.5 \) the maximum net present expected value of output for the new system is 3.140696 and occurs at \( y' = 1.448152 \) and \( N_1' = N_2' = 9 \). When \( b_3 = 8 \) the maximum net present expected value of output for the new system is 4.331691 and occurs at \( y' = 1.841763 \) and \( N_1' = N_2' = 12 \). Table 2 shows that there are now values of \( N_1 \) and \( N_2 \) such that inequality V.9 holds and thus there is a scrapping boundary. Also, this boundary shifts closer to the original values of component durability as \( b \) is increased from \( b_2 \) to \( b_3 \). Thus there is a larger probability associated with scrapping the old system as \( b \) increases.

The transitional probabilities that the system is not scrapped at some future time interval are computed as follows. Suppose the old system has just been constructed and no hits have yet occurred so \( N_1^0 = N_2^0 = 6 \) with \( M_1 = M_2 = 0 \). Assuming \( b_1 = 7.5 \) as in the first panel of table 3.3 for a fixed future date \( T' \) let event \( F_2 \) be \( 3 \leq N_1 \leq 6 \) and \( 2 \leq N_2 \leq 6 \). Let event \( G_2 \) be \( 2 \leq N_1 \leq 6 \) and \( 3 \leq N_2 \leq 6 \). The probability that the old system has not yet been scrapped at \( T' \) is:

\[
\Pr(F_2 \cup G_2) = \Pr(F_2) + \Pr(G_2) - \Pr(F_2 \cap G_2)
\]

\[
= \sum_{j_1=0}^{3} \frac{e^{-\lambda_1 T'} (\lambda_1 T')^{j_1}}{j_1!} \sum_{j_2=0}^{4} \frac{e^{-\lambda_2 T'} (\lambda_2 T')^{j_2}}{j_2!} + \sum_{j_3=0}^{4} \frac{e^{-\lambda_2 T'} (\lambda_2 T')^{j_3}}{j_3!} \sum_{j_4=0}^{3} \frac{e^{-\lambda_3 T'} (\lambda_3 T')^{j_4}}{j_4!}
\]

\[
- \sum_{j_5=0}^{3} \frac{e^{-\lambda_5 T'} (\lambda_5 T')^{j_5}}{j_5!} \sum_{j_6=0}^{2} \frac{e^{-\lambda_6 T'} (\lambda_6 T')^{j_6}}{j_6!} \frac{e^{-\lambda_7 T'} (\lambda_7 T')^{j_7}}{j_7!}
\]

For \( b_2 = 8 \) only one event needs to be defined. Let event \( F_3 \) be \( 3 \leq N_1 \leq 6 \) and \( 3 \leq N_1 \leq 6 \). Then starting from \( N_1^0 = N_2^0 = 6 \), the probability of not scrapping prior to date \( T' \) is simply \( \Pr(F_3) \).

For \( b_3 = 10 \) the probability of scrapping is one when the new technology is introduced. In this case the maximum net present expected value of output is 10.32286 and occurs at \( y' = 2.071992 \) and \( N_1' = N_2' = 14 \).
When the input prices are changed from 0.5 to 0.49 the maximum net present value of output for the new technology is 0.225703 and occurs at \( y' = 0.992014 \) and \( N'_1 = N'_2 = 6 \).

When the marginal cost of durability for each component is changed to \( w^1_i = w^1_j = 0.4 \), the maximum net present value of a new capital system is 2.630597 and occurs at \( y' = 1.448152 \) and \( N'_1 = N'_2 = 9 \). For the second price change, to \( w^2_i = w^2_j = 0.35 \), the maximum net present value of a capital system is 4.340304 and occurs at \( y' = 1.841763 \) and \( N'_1 = N'_2 = 12 \). Table 6 shows the values of \( N_1 \) and \( N_2 \) that satisfy inequality C.2.

The transitional probabilities of not entering the scrapping boundary in a subsequent time period can be calculated depending on the number of hits realized at \( T \), starting from a situation where a system involving the old technology has just been installed (\( N^0_1 = N^0_2 = 6 \)). The probability of scrapping is just the probability associated with \( N_1 \geq 2 \) and \( N_2 \geq 2 \) for the first price change, and \( N_1 \geq 3 \) and \( N_2 \geq 3 \) for the second price change. Figure 5 plots the transitional probabilities of scrapping for each price change.

When the marginal costs of durability are changed to \( w^3_i = w^3_j = 0.2 \) the maximum net present value of output is 11.36883 and occurs at \( y' = 2.636416 \) and \( N'_1 = N'_2 = 20 \).

When \( \delta \) is changed from its original value of 0.2 to 0.19 the new technology has a maximum net present value of 0.068807, which occurs at \( y' = 0.998062 \) and \( N'_1 = N'_2 = 6 \). Thus, the scale of operation with the new technology does not change for this small decrease in the depreciation rate. When \( \delta_2 = 0.05 \) the system will be scrapped before it physically breaks down. Table 6 shows there is now a scrapping boundary. For \( \delta_2 = 0.05 \) the maximum net present value is 2.256621 and occurs at \( y' = 2.726171 \) and \( N'_1 = N'_2 = 15 \).

Once again in this case there are transitional probabilities of not scrapping associated with the change in \( \delta \). These are reflections of the state of number of hits that have occurred at \( T \) and
the scrapping boundary shown in table 6. The transitional probability of not scrapping is the probability associated with the event \( N_1 \geq 2 \) and \( N_2 \geq 2 \).

When \( \delta_2 = 0.00 \) the scrapping boundary, shown in table 7, shifts down and to the right making it more likely that the system is scrapped early. But the probability of scrapping the original capital system does not go to one even for a physical depreciation rate of zero. The maximum net present value is 5.942234 and occurs at \( y' = 5.524225 \) and \( N_1' = N_2' = 29 \).

The transitional probabilities of scrapping are positive but not equal to one and can be calculated as follows. Let event \( F_3 \) be \( 5 \leq N_1 \leq 6 \) and \( 4 \leq N_2 \leq 6 \). Let event \( G_3 \) be \( 4 \leq N_1 \leq 6 \) and \( 5 \leq N_2 \leq 6 \). Then the probability of scrapping the system is

\[
\Pr(F_3 \cup G_3) = \Pr(F_3) + \Pr(G_3) - \Pr(F_3 \cap G_3)
\]

(C.4) 

This is shown in figure 6.


End Notes

1 Joseph Schumpeter with his vision of creative destruction in the ongoing process of technological growth originally made the observation that innovation renders technology obsolete (Schumpeter, 1939/64: 62-77).


3 This section is drawn from Carlaw (2000).

4 Carlaw (2000) provides discussion about and examples for a variety of capital systems to which the model relates.

5 For example the 777 has Pratt and Whitney 4084 engines, a fuselage, an undercarriage and a navigation system (Sabbagh, 1995). The components themselves consist of many sub-components and these sub-components are made up from sub-sub-components, and so on. In this view, capital systems may be broken down into more and more detailed descriptions, to the level of the raw materials used to build them (Lipsey, Bekar and Carlaw, 1998a).

6 The term complementarity is usually attributed to Hicks. (Eatwell, J. et. al (1987): 547-8.) The kinds of complementarities analyzed by Hicks refer to a change in factor demands, in response to a change in a given input price. If the price of one factor increases, the demand for complementary factors goes down.

7 This is in contrast to the standard treatment of indivisible capital investment which assumes exogenous indivisibilities (e.g., Tirole, 1998).

8 For many economists the observation that non-convexities come from an engineering relationship is sufficient evidence that indivisibilities are exogenous, since the study of engineering lies outside economics. This attitude dates back to the work of Bohm Bawerk (1889). It is important to note however, that while the actual physical relationship is an engineering one, its exploitation is clearly within the sphere of economic studies, and it is the exploitation of the engineering relationship that generates the indivisibility, not nature itself.

9 The model draws on Carlaw (2002).

10 In presentations of this paper some people have argued that they are able to operate more than one computer at a time but the upper limit on their capacity is finite. I simplify by limiting the off-the-shelf component to one system.

11 In some circumstances it may be optimal to repair a failed system. Conditions where such behaviour does and does not occur can be derived from the model. This, however, is the subject of another paper and so the assumption that the system is replaced when it fails is maintained throughout the analysis here. This means there is an implicit assumption that the conditions for throwing away the old machine are always realized.

12 Labour is used to represent all of the variable inputs, such as power (e.g., electricity, gasoline, etc) that are used in the production process, which exploits the services of the capital asset.

13 A detailed description of the costs associated with producing durable capital is provided in Carlaw (2000). The standard production cost duality holds under the assumption of constant marginal costs of component durability. Carlaw (2000) discusses of some conditions under which this type of duality does not hold.

14 Section IV provides a new definition of the investment plan when the second term is not equal to zero (i.e., when technological improvement takes place, but production is still perfectly competitive).

15 The term in the second integral, the probability of the old system surviving to T multiplied by its instantaneous probability of death, is commonly referred to as the hazard rate (Green 1993: 717).

16 Once again note that $N(y)$ is well defined only for values of $y$ such that $y = y(N_1, N_2)$ for some pair of integers $(N_1, N_2)$. Since $y$ is monotonic (increasing) in $N$, it is possible to define $N(N)$ as the inverse of $y(N)$. However this only works if $N_1$ is held constant while $N_2$ varies or vice versa. More generally for any given $y$ there are many $(N_1, N_2)$ pairs that produce it (like an isoquant) so $N(N)$ and $N(N)$ must be cost minimizing.

17 All tables are shown at the end of the text.

18 All figures are shown at the end of the text.

19 It is important to note that all technological innovations do not necessarily imply that bigger, in terms of the stock of embodied services, is better. Some innovations such as electricity for machine tools imply that a smaller stock of embodied service is optimal.

20 In each case of technological change there are parameter increases that will result in no change in the scrapping boundary. These are not shown here but are discussed in the appendixes.

21 The calculations for the probabilities are shown in appendix C.

22 The details of the effect of these changes are discussed in appendix D.
See appendix D for the calculation of the transitional probabilities.

There are parameterizations of the model where sufficiently large reductions in depreciation do result in a probability of scrapping equal to one. But for the particular base case parameterization chosen this will not happen.

In this and every subsequent table the positive, black, bold face numbers represent instances where the system is not scrapped and negative, red, italicized numbers represent instances where the system is scrapped.

There is also a negligibly small probability that both components get hit in the instant that is ignored here.

There are parameterizations of the model where sufficiently large reductions in depreciation do result in a probability of scrapping equal to one. But for the particular base case parameterization chosen this will not happen.