Bayesian Persuasion in Credit Ratings, the Credit Cycle, and the Riskiness of Structured Debt

Alexander David       Maksim Isakin*

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*Both authors are from the University of Calgary, Haskayne School of Business. We thank Curtis Eaton and seminar participants at the Canadian Economic Association Meetings and the Alberta Finance Institute Conference for helpful comments. Address: 2500 University Drive NW, Calgary, Alberta T2N 1N4, Canada. E. Mail (David): adavid@ucalgary.ca. E.Mail (Isakin) misakin@ucalgary.ca.
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Abstract

We present a new theoretical model that sheds light on why CDO tranche spreads widen during credit crunch periods. In the model, firms’ risk taking is endogenous and credit ratings arise from an investigation process that is designed to maximize the proportion of firms with high ratings (Bayesian persuasion). We show that the rating agency changes rating standards over the business cycle. If the economy enters a recession, the deteriorating quality of fundamentals implies that debt issued in booms may not be incentive compatible with low-risk behavior. In this case, the rating agency undertakes a more stringent rating investigation to increase the precision of ratings and hence to reduce the cost of capital with good ratings. Highly rated firms can only realize the benefits of more precise ratings by calling existing debt and issuing lower cost debt. This may not be possible during a credit crunch, and hence the resulting high risk strategy by firms in such periods implies that senior tranches, which are nearly riskless at the time of issuance, get seriously impacted. We find support for this hypothesis in the data.
1 Introduction

A typical structured debt product such as a collateralized debt obligation (CDO) is a large pool of economic assets with a prioritized structure of claims (tranches) against this collateral. These instruments have made it possible to repackage credit risks and produce claims with significantly lower default probabilities and higher credit ratings than the average asset in the underlying pool. The structured finance market demonstrated spectacular growth during the decade before the financial crisis of 2007/08 but almost dried up following massive downgrades and defaults of highly rated structured products during the crisis (see Coval, Jurek and Stafford (2009b)). In an influential paper, Coval, Jurek and Stafford (2009a) argue that investors did not adequately price the risk in senior CDO tranches prior to the financial crisis (see also Collin-Dufresne, Goldstein and Yang (2012) and Wojtowicz (2014)). In this paper, we suggest an alternative view on the collapse of this market, which is based on the dynamic information content of credit ratings and the occurrence of credit crunches.

Following the work by these above authors, we study the time series of spreads on tranches on the Dow Jones North American Investment Grade Index of credit default swaps, which are shown in Figure 1. The “senior” tranche (top-left panel) represents the 15 to 100 percent loss attachment points (these securities only suffer losses if the loss on the entire collateral pool exceeds 15 percent of the underlying capital), while the “equity” tranche (top-right panel) represents the 0 to 3 loss attachment points. While both spread series rose rapidly during the financial crisis, the rise in the senior tranche spread was more spectacular, from only about 10 basis points (b.p.) before the crisis, to above 230 b.p. at its peak. The equity tranche by comparison, only roughly doubled from its pre-crisis level of 1175 b.p to 2700 b.p. at its highest point. Post-crisis (2012-2014), the senior tranche spread was still 27 b.p., while the equity tranche spread returned to its pre-crisis level.

The bottom-left panel of Figure 1 shows the growth in credit as a ratio of investment for nonfinancial corporate businesses. As seen, credit growth during the financial crisis fell drastically, bottomed out in mid-2009, and despite a substantial recovery by 2012, remained substantially below its peak in 2007. The decline in credit growth during the crisis is consistent with media and policy reports of the unwillingness of banks to lend in this period. Even recently it is estimated that banks have hoards of cash, which they are not lending out. The bottom-right panel shows that earnings growth of S&P 500 firms, also fell rapidly during the crisis, although it recovered quite spectacularly by 2011. Lines 1 and 3 of Table 1 show that in univariate regressions, credit growth is inversely related to tranche spreads, and explains 22 and 58 percent of the variation in the senior and equity tranches, respectively.
Taken together, credit growth and earnings growth explain 78 and 65 percent of the variation in the senior and equity tranches, respectively, and each variable is highly significant in each regression. It is noteworthy that despite the presence of a macroeconomic factor, credit growth additionally impacts tranche spreads, which is one of the main features of our model.

There are three crucial ingredients in our model. First, we endogenize the risk of the firms using an asset substitution mechanism. In particular, firms optimally choose their risk based on the amount of debt that they need to service. Second, we introduce imperfect credit ratings using the Bayesian persuasion concept, which we discuss more completely below. This concept implies that the rating agency changes the intensity of its investigation of firms’ credit quality with the goal of maximizing the proportion of firms with high credit ratings. Finally, credit availability in the model can be in “on” or “off” states.

These features form a mechanism that amplifies and propagates macroeconomic shocks and can create catastrophic risk observed in the prices of CDO tranches (see Collin-Dufresne et al. (2012)). According to this mechanism the rating agency produces a noisy signal (ratings) that allows the firms to borrow at the cost compatible with low-risk behaviour, i.e. the credit ratings abate the moral hazard problem just enough to induce low-risk behavior in current economic conditions. In a sense, this puts the firms on the edge of low-risk and high-risk technologies and if economic conditions change the firms could switch to risky behavior. To prevent this switching the rating agency steps in and produces more precise signal (ratings). The new ratings can decrease the cost of borrowing for the firms to maintain low-risk behavior, if they can call existing debt. However, if credit availability is off, then, firms cannot call existing debt and will continue to choose high risk projects.

We incorporate these features into a three-period model to study how the information in credit ratings evolve over the business cycle and the pricing consequences of these dynamics. We apply our model to explain risk and pricing dynamics of CDO tranches. In particular we study the relative pricing of senior and equity tranches through macro and credit availability states. Our model implies that if credit rating agencies choose myopic rating standards (as described above), then equity tranches lose value if the economy enters recessions, but senior tranches additionally get impacted when credit availability is off, so that a large number of firms fail to call their existing high cost debt and hence take on riskier projects with higher failure rates. Indeed, as seen in the bottom panels of Figure 1, the senior tranche spread declined in the second half of 2009 as credit growth rose closer to normal levels while equity tranche spreads remained elevated far longer.
Our paper builds on the coordinating role of rating agencies in driving better investment decisions by firms as in Boot, Milboum and Schmeits (2006) and Manso (2013). In both papers the models reveal multiplicity of equilibria and the credit rating agency plays a coordinating role. In their work, ratings lower the cost of finance specially since certain classes of investors are forced by institutional rigidities to invest in highly rated securities. We instead build on the concept of Bayesian persuasion (exemplified in a litigation context in Kamenica and Gentzkow (2011)), in which the precisions of the ratings are controlled by the rating agencies investigation process. In good times, the agencies allow some degree of contamination of the good ratings class by conducting a less thorough examination of firms credit quality, but still ensuring that the overall cost of capital of the mix of firms is low enough to induce the low risk project choice by high quality firms. In periods of deteriorating fundamentals, the quality of the ratings are improved to weed out bad firms from the high rating class, so that once again good quality firms still purse low risk projects. Overall, the procedure maximizes the amount of debt with high ratings. It is important to note that the time varying quality of ratings is distinct from alternative rating agency behaviors such as misreporting and ratings inflation (see e.g. Fulghieri, Strobl and Xia (2014)), which might also have played a role in financial crisis.

Besides providing an analysis of the dynamics of CDO tranche prices, our model also explains why there are multiple downgrades of corporate debt in recessions. It is already well understood that credit ratings are “through-the-business-cycle” (see e.g. Treacy and Carey (2000), David (2008)). According to this hypothesis, each firm is rated not based on its current conditions, but rather on its average probability of default through good and bad years of the business cycle. Therefore, increases in default risk in recessions should not necessarily lead to downgrades. We show that a deterioration of economic conditions may result in multiple downgrades in an equilibrium model. In our model, downgrades are a necessary measure aimed to “cleanse” the class of firms with high ratings, reduce the cost of capital for these firms, and prevent them from engaging in risky behavior. This cleansing effect is similar to the tightening of loan standards by bank loan officers in recessions as seen from the survey results in Figure 2.

The remainder of the paper follows the following plan: Section 2 introduces the model. Section 3 analyzes the equilibria in the model with two different credit rating standards. Section 4 provides results on the pricing of CDO tranches. Section 5 presents empirical results. Section 6 concludes.

1We therefore have a coordination game with strategic complementarity as for example in Milgrom and Roberts (1990) and Cooper (1999).
2 Model

The economy has three periods \((t = 0, 1, 2)\) with a continuum of risk-neutral firms, and investors, and a monopolistic credit rating agency (CRA). State of the economy is publicly observable and determined by business-cycle state \(m\) and credit availability \(c\). The business-cycle state can be either \(B\) (boom) or \(R\) (recession). This state directly affects firms’ cash flows and the success probabilities of their projects. The credit state can be either \(A\) (credit-on or credit resources are available) or \(N\) (credit-off or credit crunch). This state determines whether the firms are able to raise debt. Overall, the state of the economy takes one of four regimes \((BA, BN, RA, RN)\) and follows a stationary Markov process with a \(4 \times 4\) transition matrix \(\Lambda = (\lambda_{ij}^{mc})\) where \(\lambda_{ij}^{mc}\) is the probability of transition from state \(mc\) into state \(ij\). At \(t = 0\) the economy is in state \(mA\), i.e. the credit is available and the business-cycle state is \(m \in B, R\). For simplicity, we assume no discounting.

Firms There are two types of firms: good and bad. In each period every good firm chooses between two one-period projects: low risk \(LR\) and high risk \(HR\). A bad firm can only implement the \(HR\) project. There are no switching costs and a good firm could choose different projects in the first and second periods. Each project has a binary outcome, success or failure, with success probabilities \(p^m\) and \(q^m\) for the \(LR\) and \(HR\) projects respectively where \(m \in \{B, R\}\) is the business-cycle state. We assume that \(p^B \geq p^R\), \(q^B \geq q^R\) and \(p^m \geq q^m\) in each state \(m \in \{B, R\}\). If a project succeeds, it has net return on capital \(u^m\) where \(m \in \{B, R\}\). If it fails, it destroys fraction \(d^m\) of the capital, i.e. it has negative net return. We assume that the price of capital is unity and physical capital can always be converted into financial. The firms pay no dividends. Thus, if in period \(t \in \{0, 1\}\) a firm has capital \(K_t\), in period \((t + 1)\) and state \(m \in \{B, R\}\) its capital is either \(K_{t+1} = (1 + u^m)K_t\) or \(K_{t+1} = (1 - d^m)K_t\) depending on the outcome of the project. If a good firm chooses the \(LR\) project it incurs an unobservable cost of effort \(e^m\), which depends on business-cycle state \(m \in \{B, R\}\). We assume that the choice of the project is not contractable. Since observable outcomes \(u^m\) and \(d^m\) are the same for both projects and level of effort \(e^m\) is unobservable (and irrelevant to capital formation), investors and the CRA cannot (ex-post) infer the true type of the project. For this reason they cannot deduce the true type of the firm albeit they update their beliefs about the type of the firm (discussed below).

We assume that at \(t = 0\) each firm has capital in place \(\hat{K}\) and raises debt \(D_0\) issuing two-period zero-coupon callable bond with call price \(H\). Therefore, total capital at \(t = 0\) is \(K_0 = \hat{K} + D_0\). If at \(t = 1\) credit is available, each firm can refinance its debt. In this case, the firm redeems the existing
two-period bond and issue a new one-period bond in order to finance call price \( H \). In the case of default, the debt holders incur a bankruptcy cost which is fraction \( \delta \) of the residual value.

**CRA** The level of investors’ beliefs about the type of a firm affects the cost of capital for good firms (described below). If prior beliefs are low, the debt burden may urge good firms to choose the \( HR \) project. In this case, the CRA may influence the beliefs via rating the firms. We assume that at \( t = 0 \) the CRA evaluates all firms and assigns either a good (\( G \)) or a bad (\( B \)) rating to each firm. The ratings are not precise in the sense that the \( G \)-rating could be assigned to a bad firm or the \( B \)-rating could be assigned to a good firm (although the latter is never optimal). We assume that the CRA’s evaluation method is described by two publicly observable parameters: probability of assigning the \( G \)-rating conditional on the firm being good \( \mu \equiv \mathbb{P}[G|\text{good}] \) and probability of assigning the \( G \)-rating conditional on the firm being bad \( \pi \equiv \mathbb{P}[G|\text{bad}] \). At \( t = 1 \) the CRA may adjust firms’ ratings applying similar evaluation procedure (with other parameters) to previously rated firms. If the CRA changes the ratings, the investors update their beliefs. Under new beliefs \( G \)-rated firms may refinance their debt at lower cost.

We assume that the CRA attempts to issue as many \( G \)-ratings as possible. This assumption is consistent with the widespread view that the issuer-pays business model adopted by most of the nationally recognized statistical rating organizations (NRSROs) lead to rating inflation (for example, see Bar-Isaac and Shapiro (2011), Bolton, Freixas and Shapiro (2012), Fulghieri et al. (2012), Kartasheva and Yilmaz (2012), Harris, Opp and Opp (2013), and Cohn, Rajan and Strobl (2013)). In particular, the CRA designs its evaluation method choosing conditional probabilities \( \mu \) and \( \pi \) to maximize the unconditional probability of assigning the good rating \( \mathbb{P}[G] = \hat{\alpha}\mu + (1 - \hat{\alpha})\pi \) where \( \hat{\alpha} \) is prior beliefs such that the level of posterior investors’ beliefs motivates good firms to choose the \( LR \) project. Following the evaluation, the CRA always reveals the results of the evaluation in full. The commitment to a certain rating standard defined in terms of conditional probabilities \( \mu \) and \( \pi \) is justified by the fact that NRSROs are prohibited from making ratings fees contingent on assigned ratings. We assume that the CRA adopts the logic of Bayesian persuasion (see Kamenica and Gentzkow (2011) for discussion). The posterior investors’ beliefs are represented by the probabilities that the firm is good conditional on observing \( G \)- and \( B \)-ratings, \( \mathbb{P}[\text{good}|G] \) and \( \mathbb{P}[\text{good}|G] \) respectively. There are two extreme cases. First, the CRA can perfectly separate good and bad firms choosing \( \mu = 1 \) and \( \pi = 0 \). Second, the CRA can produce completely uninformative ratings assigning \( G \)-rating to all firms, i.e. choosing \( \mu = 1 \) and
\( \pi = 1 \). If what follows, we denote \( \alpha_0 \) and \( \alpha_1 \) beliefs that a firm is good if it gets the \( G \)-rating at \( t = 0 \) and if it retains the \( G \)-rating at \( t = 1 \) respectively.

**Investors** We denote \( \hat{\alpha}_0 \) investors’ prior beliefs that a firm is good at \( t = 0 \). After observing the ratings investors update their beliefs using the Bayes law. In particular, at \( t = 0 \) they calculate

\[
\alpha_0 \equiv P[\text{good}|G] = \frac{\hat{\alpha}_0 \mu}{\hat{\alpha}_0 \mu + (1 - \hat{\alpha}_0) \pi},
\]

\[
\chi_0 \equiv P[\text{good}|B] = \frac{\hat{\alpha}_0 (1 - \mu)}{\hat{\alpha}_0 (1 - \mu) + (1 - \hat{\alpha}_0)(1 - \pi)}.
\]

Moreover, since the survival probabilities of the \( LR \) and \( HR \) projects are different, the investors rationally learn the type of the firms observing the outcomes of their projects at \( t = 1 \) and \( t = 2 \). Intuitively, since the default probability of the bad firms is higher than that of the good firms (conditional on choosing the \( LR \) project) if a firm survives by \( t = 1 \) the investors put greater probability mass to the good type. In particular, the level of beliefs in period \( t \) where \( t \in \{1, 2\} \) in business-cycle state \( m \) before observing new ratings (if any) becomes

\[
\hat{\alpha}_t = \frac{\alpha_{t-1} p^m}{\alpha_{t-1} p^m + (1 - \alpha_{t-1}) q^m} > \alpha_{t-1}.
\]

We assume that credit markets are perfectly competitive. Thus, in equilibrium the investors require return that yields them zero expected profit.

**Sequence of Events** At \( t = 0 \) the CRA issues ratings for the firms. If the CRA chooses \( \mu \geq \pi \), the investors’ beliefs that a firm is good increases for \( G \)-rated firms and decreases for \( B \)-rated firms. In what follows, we focus on the firms that obtain the \( G \)-rating. The investors’ beliefs for these firms increase from prior \( \hat{\alpha}_0 \) to \( \alpha_0 \). After obtaining a rating, each firm issues a two-period bond and start a project. If good firms choose the \( LR \) project at \( t = 0 \), the investors observe whether a firm succeeded or failed and update their beliefs according to (3). We denote \( \hat{\alpha}_1^S \) and \( \hat{\alpha}_1^F \) the levels of beliefs after learning (3) that a \( G \)-rated firm is good if it succeeded and failed at \( t = 0 \) respectively. If at \( t = 0 \) good firms choose the \( HR \) project, \( \hat{\alpha}_1^S = \hat{\alpha}_1^F = \alpha_0 \). At \( t = 1 \) the CRA may adjust the rating of the firms. If the CRA changes the ratings, the investors update their beliefs according to (1) and (2). We denote \( \alpha_1^S \) and \( \alpha_1^F \) the updated beliefs that a \( G \)-rated firm is good if it succeeded and failed at \( t = 0 \) respectively. If there is no re-rating, \( \alpha_1 = \hat{\alpha}_1 \). Since the bond is callable, if at \( t = 1 \) credit is available the firms can refinance their debt. In the case of refinancing, at \( t = 1 \) in business-cycle state \( m \) the firm issues a one-period bond with face value \( F_{t2} \) to raise the amount equal call price \( H \). Using these proceeds they
redeem the existing two-period bond. Regardless of refinancing, at \( t = 1 \) the firms start new projects as at \( t = 0 \). At \( t = 2 \) the firms redeem their debt. Figure 3 summarizes the sequence of events.

3 Equilibrium

This section describes a perfect Bayesian equilibrium that arises in the model. We consider two rating standards that can be adopted by the CRA. First, we examine the case of the myopic rating standard. In this case, at \( t = 0 \) the CRA assigns ratings to the firms and the investors update their beliefs to the level at which good firms choose the \( LR \) project at \( t = 0 \). However, at \( t = 0 \) the CRA ignores firms’ choice at \( t = 1 \). At \( t = 1 \), conditional on the realized state of the economy, the CRA may increase the rating precision if under the existing level of beliefs good firms choose the \( HR \) project. It is worth noting that it is not always possible to incentivize the firm to choose the \( LR \) project. Second, we analyze the case of the farsighted rating standard. In this case, at \( t = 0 \) the CRA increases investors’ beliefs such that good firms choose the \( LR \) project at \( t = 0 \) and \( t = 1 \) (perhaps, after re-rating) if such a level of beliefs exists. In general, the farsighted rating standard is more stringent than myopic standard and, therefore, it results in more precise ratings and lower cost of capital for good firms.

Definition 1 Suppose the CRA follows the myopic/farsighted rating standard. An equilibrium is defined as the set of strategies of the CRA, firms, and investors such that:

1. the firms choose optimally between \( LR \) and \( HR \) projects at \( t = 0 \) and \( t = 1 \) in each state and decide whether to refinance their debt at \( t = 1 \) in the states when the credit market is on;

2. the investors have rational beliefs \( \alpha_i^t(h_i^t) \) about firm \( i \) in period \( t \) where \( h_i^t \) is the history of successes and failures of firm \( i \).

3.1 Myopic Rating Standard

Under the myopic rating standard, in each period the CRA chooses the rating standard to motivate good firms to choose the \( LR \) project in that period. In particular, at \( t = 0 \) the CRA increases the beliefs to the level at which the firm chooses the \( LR \) project at \( t = 0 \). At this moment the CRA ignores firms’ choice at \( t = 1 \) and feasibility to induce the \( LR \) project at \( t = 1 \) through the re-rating.
Equity value  We use the dynamic programming principle to determine a good firm’s equity value. Due to limited liability, at $t = 2$ the good firm’s equity value is $E_2(K_2) = (K_2 - F)^+$ where $K_2$ the accumulated capital, and $F$ is the face value of debt to be repaid. At $t = 1$ if credit is available each firm decides whether to refinance its debt in order to maximize the equity value. Since in the case of refinancing the firm raises call price $H$ issuing a one-period bond with face value $F_{12}$, the refinancing is possible only if $H$ is below the debt capacity (provided in Lemma ??). If refinancing is possible, the firm refines its debt if $F_{12} > F_{02}$.

Also, at $t = 1$ the good firm chooses between the LR and HR projects to maximize the equity value under optimal refinancing decision. As a result, at $t = 1$ in state $mc$ the firm’s equity value is

$$E_{mc}^1(K_1) = \max \left\{ \sum_{m' \in \{B,R\}} \lambda_{mc}^{m'} p^{m'} ((1 + u^{m'})K_1 - F)^+ + (1 - p^{m'})((1 - d^{m'})K_1 - F)^+ - e^m, \right.$$  

$$\sum_{m' \in \{B,R\}} \lambda_{mc}^{m'} q^{m'} ((1 + u^{m'})K_1 - F)^+ + (1 - q^{m'})((1 - d^{m'})K_1 - F)^+ \right\} ,$$ (4)

where $F = \min \{F_{02}, F_{12}\}$ if credit is available, and $F = F_{02}$ otherwise.

At $t = 0$ in state $mA$ (the business-cycle is in state $m$ and the credit is available), the good firm chooses the project to maximize the equity value

$$E_{mA}^0(K_0) = \max \left\{ \sum_{m' \in \{B,R\}} \lambda_{mA}^{m'} (p^{m'} E_1((1 + u^{m'})K_0) + (1 - p^{m'})E_1((1 - d^{m'})K_0) - e^m), \right.$$  

$$\sum_{m' \in \{B,R\}} \lambda_{mA}^{m'} (q^{m'} E_1((1 + u^{m'})K_0) + (1 - q^{m'})E_1((1 - d^{m'})K_0)) \right\} .$$ (5)

Bad firms including those with the G-rating always implement the HR project. Similar to the good firms, if refinancing is possible, the bad firms refinance their debt at $t = 1$ if $F_{12} > F_{02}$. Face values $F_{02}$ and $F_{12}$ depend on the investors’ beliefs about the type of the firm in the period of bond issuance. In turn, the investors’ beliefs depend on the firm’s rating history and the history of successes or failures.

CRA  The CRA issues ratings to every firm at $t = 0$. At $t = 0$, after observing the ratings, the investors update their beliefs except a special case when all firms receive G-ratings. At $t = 1$ the investors update their beliefs based on the outcomes of the projects. Then the CRA may adjust the ratings and induce another update of investors’ beliefs. The following lemma shows the CRA’s optimal
choice of the rating standard that increases the level of the investors’ beliefs from prior level $\theta$ to target level $\theta'$. 

**Lemma 1** *The optimal rating standard prescribes*

\[
\mu = 1 \\
\pi = \frac{\theta(1 - \theta')}{\theta'(1 - \theta)}. \tag{6}
\]

Solution (6) and (7) results in posterior beliefs such that the investors are certain that a firm is bad if it has the $B$-rating. This is in accordance with Proposition 4 in Kamenica and Gentzkow (2011) stating that if an optimal signal induces beliefs that leads to a worst for a sender action, the receiver is certain of her action. Expression (7) shows that conditional probability $\pi$ is decreasing in $\theta'$, that is higher posterior beliefs require less noisy ratings. At the same time, $\pi$ is increasing in $\theta$ meaning that higher prior beliefs allow the CRA to choose a looser rating standard. Lemma 1 describes the optimal standard for rating all firms at $t = 0$ and the optimal standard for re-rating the firms with the $G$-rating at $t = 1$. Since the firms with the $B$-ratings are all bad, the CRA never changes their ratings at $t = 1$.

The CRA adopts the following logic at $t = 1$. If under existing beliefs (after possible refinancing) the $G$-rated firms choose the $LR$ project, the CRA keeps the ratings unchanged. Otherwise, the CRA tries to increase the beliefs to the level such that $G$-rated firms refinance their debt and choose the $LR$ project. If such a level exists, the CRA chooses rating parameters $\mu$ and $\pi$ according to Lemma 1 to increase the beliefs to level $\alpha_{mA}^*$, the minimum level of beliefs such that

\[
\sum_{m' \in \{B, R\}} \lambda_{me}^{m'} ((1 + u^{m'})K_1 - F_{12}^m(\alpha_{mA}^*))^+ + (1 - p^{m'})(1 - d^{m'})K_1 - F_{12}^m(\alpha_{mA}^*)^+ - e^m,
\]

\[
\geq \sum_{m' \in \{B, R\}} \lambda_{me}^{m'} ((1 + u^{m'})K_1 - F_{12}^m(\alpha_{mA}^*))^+ + (1 - q^{m'})(1 - d^{m'})K_1 - F_{12}^m(\alpha_{mA}^*)^+), \tag{8}
\]

where business-cycle state $m \in \{B, R\}$. If the $G$-rated firms choose the $HR$ project under the highest level of beliefs, $\mathbb{P}[\text{good}|G] = 1$, the CRA increases the level of beliefs to unity.

Similarly, the rating at $t = 0$ is as follows. If under the prior level of beliefs good firms choose the $LR$ project the CRA assigns the $G$-rating to all firms. Otherwise, the CRA tries to increase the beliefs to the level such that the good firms choose the $LR$ project. If such a level exists, the CRA chooses rating parameters $\mu$ and $\pi$ according to Lemma 1 to increase the beliefs to level $\alpha_0$, the
minimum level of beliefs such that

\[
\sum_{m' \in \{B,R\}, c' \in \{A,N\}} \lambda_{mc}^{m'c'} \left( p^{m'} E_1 \left( (1 + u^{m'}) K_0 \right) + (1 - p^{m'}) E_1 \left( (1 - d^{m'}) K_0 \right) - e^m \right),
\]

\[
\geq \sum_{m' \in \{B,R\}, c' \in \{A,N\}} \lambda_{mc}^{m'c'} \left( q^{m'} E_1 \left( (1 + u^{m'}) K_0 \right) + (1 - q^{m'}) E_1 \left( (1 - d^{m'}) K_0 \right) \right)
\]

(9)

**Debt value** Due to limited liability if at maturity the value of firm’s capital is less than the face value of the bond, the value of the debt is the value of capital less bankruptcy cost. Therefore, at \( t = 2 \) the value of a bond with face value \( F \) belonging a firm with capital \( K_2 \) is

\[
D_2(K_2, F) = \begin{cases} 
F & \text{if } F \leq K_2 \\
(1 - \delta)K_2 & \text{if } F > K_2.
\end{cases}
\]

(10)

We denote \( P_t^{m'c'}(K_1) \in \{p^{m'}, q^{m'}\} \) the success probability under the good firm’s optimal choice of the project in period \( t \) if in period \((t+1)\) the economy is in state \( m'c' \). Then the value of a bond with face value \( F \) issued at \( t = 1 \) in state \( mc \) with \( c = A \) by a \( G \)-rated firm with capital \( K_1 \) is

\[
D_1^{mc}(K_1, F) = \sum_{m' \in \{B,R\}, c' \in \{A,N\}} \lambda_{mc}^{m'c'} \left( \alpha_1^{mc} P_1^{m'c'}(K_1) + (1 - \alpha_1^{mc}) q^{m'} \right) D_2((1 + u^{m'})K_1, F)
\]

\[
+ \left( \alpha_1^{mc} \left( 1 - P_1^{m'c'}(K_1) \right) + (1 - \alpha_1^{mc}) (1 - q^{m'}) \right) D_2((1 - d^{m'})K_1, F).
\]

(11)

The first line in the RHS of (11) is the expected value of the debt if a firm succeeds in the project initiated at \( t = 1 \). Since the investors believe that a firm is good with probability \( \alpha_1^{mc} \) and the success probabilities of a good and a bad firms are \( P_1^{m'c'} \in \{p^{m'}, q^{m'}\} \) and \( q^{m'} \) respectively, the total probability of success is \( \alpha_1^{mc} P_1^{m'c'} + (1 - \alpha_1^{mc}) q^{m'} \). Similarly, the second line of (11) is the expected value of the debt if the firm fails in the project initiated at \( t = 1 \). We denote \( \hat{D}_1^{mc}(K_1, F) \) the value of a bond with face value \( F \) belonging to a \( B \)-rated firm with capital \( K_1 \). Since a \( B \) rated firm is always bad this value is given by (11) when \( \alpha_1^{mc} = 0 \) and \( P_1^{m'c'} = q^{m'} \). Equations (11) also provides the value of a two-period bond with face value \( F = F_{02} \) if the firm does not refinance at \( t = 1 \).

If a firm refinances its debt at \( t = 1 \), it issues a one-period bond with face value \( F_{12}^{mA} \) which solves \( D_1^{mc}(K_1, F_{12}^{mA}) = H \) with \( D_1^{mc} \) defined by (11). It is worth mentioning that a firm may be unable to refinance its debt if amount \( H \) is greater than its debt capacity. If a good firm chooses the \( LR \) project, the face value is decreasing in \( \alpha_1^{mc} \) because if the investors believe that a firm is more likely to be good they require a lower return on its bond.
The value of the two-period bond at \( t = 0 \) depends on firms’ optimal decision on refinancing at \( t = 1 \) in each state and good firms’ optimal choice of the project at \( t = 0 \) and \( t = 1 \) in each state. We let \( R^{mc}(K_1) \) be the expected at \( t = 1 \) in state \( mc \) value of the repayment of a firm with capital \( K_1 \) that received \( G \)-rating at both \( t = 0 \) and \( t = 1 \). Thus, \( R^{mc}(K_1) = H \) if such firms optimally call their two-period bonds back at \( t = 1 \) and \( R^{mc}(K_1) = D^{mc}(K_1, F_{02}) \) otherwise. In a similar fashion, we denote \( \hat{R}^{mc}(K_1) \) the expected at \( t = 1 \) in state \( mc \) value of repayment of a bad firm with capital \( K_1 \) that received \( G \)-rating at \( t = 0 \) and \( B \)-rating at \( t = 1 \). In particular, \( \hat{R}^{mc}(K_1) = H \) if bad firms optimally call their two-period bonds back at \( t = 1 \) and \( \hat{R}^{mc}(K_1) = \hat{D}^{mc}(K_1, F_{02}) \) otherwise. With these notations the value of the two-period bond at \( t = 0 \) in state \( mA \) is

\[
P_{0mA}^{mA}(K_0) = \sum_{m',c' \in \{B,R\}} \lambda_{mA}^{m'c'} \left( (\alpha_0 P_{0}^{m'c'} + (1 - \alpha_0)q^{m'} \pi_1^{m'c'}) R^{mc}((1 + u^m')K_0, F_{02}) + (1 - \alpha_0)q^{m'}(1 - \pi_1^{m'c'}) \hat{R}^{mc}((1 + u^m')K_0, F_{02}) + (\alpha_0(1 - P_{0}^{m'c'}) + (1 - \alpha_0)(1 - q^{m'} \pi_1^{m'c'}) \hat{R}^{mc}((1 - d^m')K_0, F_{02}) + (1 - \alpha_0)(1 - q^{m'})(1 - \pi_1^{m'c'}) \hat{R}^{mc}((1 - d^m')K_0, F_{02}) \right) \tag{12}
\]

where \( \pi_1^{m'c'} \) and \( \hat{\pi}_1^{m'c'} \) are the probabilities that the firm retains the \( G \)-rating at \( t = 1 \) in state \( m'c' \) in the cases when the firm’s project initiated at \( t = 0 \) succeeds and fails respectively. The first line in the RHS of (12) is the expected value of the two-period bond of the firm that receives the \( G \)-rating at \( t = 0 \), succeeds at \( t = 0 \) and retains the \( G \)-rating at \( t = 1 \). The second line represent the expected value of the bond of the firm that receives the \( G \)-rating at \( t = 0 \), succeeds at \( t = 0 \) but gets downgraded at \( t = 1 \). Similarly, the third and fourth lines give the expected values of the bonds of the firms that fail at \( t = 0 \).

### 4 Securitized Debt

In this section we apply the model to price the tranches of the CDO. We assume that the CDO’s collateral pool consists of a large number of callable two-period bonds issued by \( G \)-rated firms at \( t = 0 \). In section 5 we adapt the results for pricing a synthetic CDO with the underlying pool which comprises a portfolio of credit default swaps (CDS) associated with these bonds. The pricing mechanism for the synthetic CDO tranches is explored in Coval et al. (2009a) and Gibson (2005). We assume that conditional on the state of the economy the losses on the bonds in the pool are independent.

We denote \( \bar{L}_0^{i}(m_1c_1, m_2c_2) \) the random state-contingent loss on the \( i \)th bond in the pool at \( t = 0 \) conditional on state \( m_1c_1 \) at \( t = 1 \) and state \( m_2c_2 \) at \( t = 2 \). At \( t = 0 \) the random state-contingent
loss in the pool conditional on state \( m_{1c1} \) at \( t = 1 \) and state \( m_{2c2} \) at \( t = 2 \) is

\[
\tilde{L}_0(m_{1c1}, m_{2c2}) \equiv \frac{1}{N} \sum_{i=1}^{N} \tilde{L}^i_0(m_{1c1}, m_{2c2}).
\]  

(13)

The strong law of large numbers implies that if the number of bonds \( N \) in the collateral pool goes to infinity conditional on the state average loss in the pool converges almost surely to the conditional expected value \( \mathbb{E}[\tilde{L}^i_0(m_{1c1}, m_{2c2})|m_{1c1}, m_{2c2}] \).

At \( t = 0 \) the pool consists of \( G \)-rated firms; some of them are good and make a choice between projects, some firms are bad and implement the \( HR \) project. At \( t = 1 \) the pool consists of eight classes of firms that may behave differently. First dichotomy is due to success or failure in implementing projects started at \( t = 0 \): at \( t = 1 \) some firms in the pool have increased their capital, some – have decreased. Second division occurs due to the adjustment of ratings: at \( t = 1 \) the pool includes both \( G \)- and \( B \)-rated firms. Third, the \( G \)-rated firms may still be good or bad and may implement different projects. Each of these eight classes makes the refinancing decision independently. We let \( \tilde{L}^i_1(m_{2c2}, K_1, \alpha_1) \) be the conditional on state \( m_{2c2} \) expected at \( t = 1 \) loss on the bond of a firm with capital \( K_1 \) and investors’ belief that the firm is good \( \alpha_1 \). If the firm calls its bond at \( t = 1 \), \( \tilde{L}^i_1(m_{2c2}, K_1, \alpha_1) = F_{02} - H \), otherwise if a firm implement a project with probability of success \( P^{m_{2c2}}_1(K_1, \alpha_1) \in \{p^m, q^m\} \),

\[
\tilde{L}^i_1(m_{2c2}, K_1, \alpha_1) = \left( \alpha_1^{m_{1c1}} P^{m_{2c2}}_1(K_1, \alpha_1) + (1 - \alpha_1^{m_{1c1}}) q^{m_{2}} \right) \left( F_{02} - D_2((1 + u^{m_{2}}) K_1, F_{02}) \right),
\]

\[
+ \left( \alpha_1^{m_{1c1}} (1 - P^{m_{2c2}}_1(K_1, \alpha_1)) \right),
\]

\[
+ \left( 1 - \alpha_1^{m_{1c1}} \right) \left( 1 - q^{m_{2}} \right) \left( F_{02} - D_2((1 - d^{m_{2}}) K_1, F_{02}) \right).
\]  

(14)

The structure of the pool is determined by the investors’ beliefs \( \alpha_0 \) and the success probability under the good firm’s optimal choice of the project \( P^{m_{1c1}}_0 \). Therefore, conditional on state \( m_{1c1} \) at \( t = 1 \) and state \( m_{2c2} \) at \( t = 2 \) the expected at \( t = 0 \) loss in the pool is

\[
\tilde{L}_0(m_{1c1}, m_{2c2}) = \mathbb{E}[\tilde{L}^i_0(m_{1c1}, m_{2c2})|m_{1c1}, m_{2c2}]
\]

\[
= \left( \alpha_0 P^{m_{1c1}}_0 + (1 - \alpha_0) q^{m_{1}} \tilde{\pi}^{m_{1c1}} \right) \tilde{L}^i_1(m_{2c2}, (1 + u^{m_{1}}) K_0, \alpha_1^{m_{1c1}})
\]

\[
+ \left( 1 - \alpha_0 \right) q^{m_{1}} \left( 1 - \tilde{\pi}^{m_{1c1}} \right) \tilde{L}^i_1(m_{2c2}, (1 + u^{m_{1}}) K_0, 0)
\]

\[
+ \left( \alpha_0 (1 - P^{m_{1c1}}_0) + (1 - \alpha_0)(1 - q^{m_{1}}) \tilde{\pi}^{m_{1c1}} \right) \tilde{L}^i_1(m_{2c2}, (1 - d^{m_{1}}) K_0, \tilde{\alpha}_1^{m_{1c1}})
\]

\[
+ \left( 1 - \alpha_0 \right)(1 - q^{m_{1}})(1 - \tilde{\pi}^{m_{1c1}}) \tilde{L}^i_1(m_{2c2}, (1 - d^{m_{1}}) K_0, 0)
\]  

(15)
Thus, if the number of bonds $N$ in the pool goes to infinity, the conditional on the state $mc$ at $t = 1$ CDF of the loss is

$$
\Phi_{L_0}(\theta|m_1c_1, m_2c_2) = \lim_{n \to \infty} \mathbb{P}[\tilde{L}_n(m_1c_1, m_2c_2) \leq \theta|m_1c_1, m_2c_2] = \begin{cases} 
0 & \text{if } \theta < \tilde{L}_0(m_1c_1, m_2c_2) \\
1 & \text{if } \theta \geq \tilde{L}_0(m_1c_1, m_2c_2).
\end{cases} \quad (16)
$$

Furthermore, by the law of total probability the CDF of the unconditional loss $\tilde{L}_0$ is a step function

$$
\Phi_{L_0}(\theta) = \mathbb{P}[\tilde{L}_0 \leq \theta] = \sum_{m_1 \in \{B,R\}} \lambda^{m_1c_1}_{m_0A} \sum_{m_2 \in \{B,R\}} \sum_{c_2 \in \{A,N\}} \lambda^{m_2c_2}_{m_1c_1} \Phi_{L_0}(\theta|m_1c_1, m_2c_2), \quad (17)
$$

where $m_0A$ is the state at $t = 0$.

Since tranches are derivative claims on the underlying portfolio, the loss on a tranche can be expressed as a function of the loss in the pool. Consider a tranche with a lower loss attachment point of $A^L$ and an upper loss attachment point of $A^U$. If the loss in the pool is less than $A^L$ the loss on the tranche is zero, if the loss in the pool is greater than $A^U$ the loss on the tranche equals the size of the tranche $(A^U - A^L)$. In general, for loss in the pool $\tilde{L}_0(m_1c_1, m_2c_2)$ the loss on the tranche with attachment points $A^L$ and $A^U$ conditional on state $m_1c_1$ at $t = 1$ and state $m_2c_2$ at $t = 2$ is

$$
\tilde{L}_0^{A^L, A^U}(m_1c_1, m_2c_2) = \max(\tilde{L}_0(m_1c_1, m_2c_2) - A^L, 0) - \max(\tilde{L}_0(m_1c_1, m_2c_2) - A^U, 0) \quad (18)
$$

Given the state price densities $\{s^{mc}\}$, conditional on state $m_2c_2$ at $t = 2$, the value of the tranche with loss attachment points $A^L$ and $A^U$ at $t = 1$ is

$$
V^{A^L, A^U}_1(m_1c_1) = \sum_{m_2 \in \{B,R\}} \sum_{c_2 \in \{A,N\}} \lambda^{m_2c_2}_{m_1c_1} s^{m_2c_2}(A^U - A^L - \tilde{L}_0^{A^L, A^U}(m_1c_1, m_2c_2)). \quad (19)
$$

At $t = 0$ the value of the tranche with loss attachment points $A^L$ and $A^U$ is

$$
V^{A^L, A^U}_0 = \sum_{m_1 \in \{B,R\}} \sum_{c_1 \in \{A,N\}} \lambda^{m_1c_1}_{m_0A} s^{m_1c_1} V^{A^L, A^U}_1(m_1c_1). \quad (20)
$$

The yield on the tranche with the lower and upper loss attachment points $A^L$ and $A^U$ respectively can be calculated as

$$
Y^{A^L, A^U} = \left[ \frac{A^U - A^L}{V^{A^L, A^U}_0} \right]^{1/2} - 1 \quad (21)
$$
Panel A of Figure 4 represent a hypothetical cumulative distribution function of the loss in the CDO pool under the myopic rating standard. The CDF of the loss has four jump points which reflect one-period systematic risk of switching from state $mc$ at $t = 1$ to any state at $t = 2$. The part of the collateral that absorbs losses from 0 to $L^{BA}$ is worthless because it goes bankrupt almost surely even when economy moves to the state of the boom with available credit. At the same time, the part of the pool between $L^{RN}$ and 1 is risk-free. In state $RN$, bad firm fundamentals coupled with the lack of credit result in good firms taking excessive risk and boosting the average default rate. It is likely that in this state, the losses are large so that the senior tranche get affected (the precise answer depends on the tranche loss attachment point).

5 Empirical Analysis

In this section, we simulate the spread dynamics of the equity and senior tranches and calibrate the parameters of the model to replicate the patterns observed in the data. In doing so, we numerically solve for equilibrium rating standards and firms’ strategies and determine the equity and debt values for the cases of myopic and farsighted rating standards. In each equilibrium, we calculate the losses and spreads on the equity and senior tranches at $t = 1$ in each state.

We use a four state regime switching model to estimate the state transition matrix and filtered probabilities $\{\omega_{m,c}\}$ of each regime in each moment of time. We perform an out-of-the-sample analysis of the CDO spreads and estimate the parameters of the model based on the credit share and GDP growth over 1952:Q1 – 2004:Q2 (the spreads data start in 2004:Q3).

Figure 5 displays the probabilities of each state for the full sample 1952:Q1 – 2014:Q3, however they are in-the-sample until 2004:Q2, and are out-of-the-sample thereafter. That is, these are the probabilities that an investor would have in the latter subsample if he observed the fundamentals in 2004:Q4 - 2014:Q3. We calculate the model expected spread as

$$Y^{A^L,A^U} = \sum_{m \in \{B,R\}, c \in \{A,N\}} \omega_{mc} Y^{A^L,A^U}$$

(22)

The state transition matrix is

$$\Lambda = \begin{pmatrix}
0.931 & 0.004 & 0.041 & 0.024 \\
0.138 & 0.862 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.900 & 0.100 \\
0.108 & 0.088 & 0.100 & 0.705
\end{pmatrix}$$

(23)
The survival probabilities of the LR and HR projects in states B and R are $p^B = 91.7\%$, $q^B = 90.9\%$, $p^R = 89.3\%$, and $q^R = 74.3\%$. The potential gains/losses on the projects in states B and R are $u^B = 10.3\%$, $u^R = 7.5\%$, $d^B = 47.7\%$, and $d^R = 68.8\%$. The levels of effort are $e^B = 0.4\%$, and $e^R = 7.5\%$. The prior level of beliefs that a firm is good is 30.0\%. The call price of the two-period bond is 18.59.

We analyze the myopic rating standard. Suppose at $t = 0$ the economy starts from state $BA$. Then at $t = 0$ the CRA leaves the level of beliefs at the prior level 30.0\%, thus, $P[G|bad] = 1$, because with this level the good firms choose the LR project. At $t = 1$ in state $BA$ the CRA does not adjust the rating because the good firms choose the LR project under current investors’ beliefs of 30.2\%. At $t = 1$ in states $BN$ and $RN$ the CRA separate firms perfectly, i.e. induces $P[good|G] = 1$, because good firms engage into the HR projects.

At $t = 1$ in states $BA$ and $RA$ firms refinance their two-period bonds with one-period bonds and the good firms choose the LR project. However, in states $BN$ and $RN$ the good firms choose the HR project. With this strategy the face value of the two-period bond is 18.65. Given the level of beliefs at $t = 0$ and firms’ choice of the project, we calculate the losses in the CDO pool in each state. Since the projects have different survival probabilities in the boom and recession states and the good firms choose different projects when credit is available and in short supply, at $t = 1$ the CDF has four jumps which correspond to the switches of the economic state.

Following the tranche specifications of the CDX.NA.IG, we define the equity tranche by the upper loss attachment points equal 3\%. This tranche absorbs first 3\% of losses in the collateral pool. Also we define the senior tranche by the lower attachment point equal 15\%. This tranche has losses only if the total losses exceed 15\%. The model yields on the equity tranche at $t = 1$ in each state are $Y^{0.3}(BA) = 1133.40$, $Y^{0.3}(BN) = 69.48$, $Y^{0.3}(RA) = 1478.57$, and $Y^{0.3}(RN) = 2626.34$. The model yields on the senior tranche at $t = 1$ in each state are $Y^{15,100}(BA) = 0.57$, $Y^{15,100}(BN) = 17.60$, $Y^{15,100}(RA) = 41.01$, and $Y^{15,100}(RN) = 221.48$. In the $RN$ state, the absence of credit in the economy prevents firms from refinancing their debt which, in turn, urges good firms to switch to the HR project. This results in a significant jump in the senior tranche spread in the $RN$ state. Figure 6 displays the actual and model spreads on the senior and equity tranches. The model fits well the spread dynamics of the senior tranche. The model spread on the equity tranche explains the magnitude of the jump in 2008, however, it fails to provide an accurate spread dynamics in before 2005 and after 2009. One reason for this deficiency is that our current calibration procedure fits the average spreads in each state of the economy. Also, the significant divergence of the model and actual spreads in 2009 and
2010 can be caused by the data shortcoming. For this period we use the data on off-the-run CDX.NA.IG Series 9 due to the absence of the data on on-the-run series.

6 Conclusion

In this paper, we build a simple three period model to show how imperfect credit ratings and occurrence of credit crunches can create catastrophic risk observed in the prices of CDO tranches. There are three crucial ingredients in our model. First, we endogenize firms’ risk-taking using the asset substitution mechanism. In particular, the firms choose the riskiness of their projects based on the amount of debt that they need to service. Second, the credit rating agency changes the intensity of the investigation of firms’ credit quality to maximize the proportion of firms with high credit ratings. Finally, the credit shortage can engage firms in risky behaviour if they are unable to refinance their debt under a more precise rating standard.

We apply our model to simulate the spreads dynamics of the CDX.NA.IG tranches. In particular, we examine the pricing of senior and equity tranches through the business and credit cycles. Our model implies that if the credit rating agency chooses the myopic rating standards (as described above), then the equity tranche loses its value if the economy enters a recession. At the same time, the senior tranche gets impacted when credit availability is off, so that a large number of firms fail to refinance their existing high cost debt and hence take on riskier projects with higher failure rates. Consistent with our prediction, the empirical analysis shows that the senior tranche spread is impacted stronger than the equity tranche spread when the credit is limited. We thus shed light on the puzzling dynamics of tranche spreads that has been found in Coval et al. (2009a).

Data Appendix

We obtain monthly time series of tranche spreads on synthetic CDOs based on the DJ CDX North American Investment Grade Index (CDX.NA.IG). This index consists of an equally weighted portfolio of 125 credit default swap (CDS) contracts on US firms with investment grade debt. Our sample covers the eleven year period from September 2004 to October 2014. The data from September 2007 to October 2014 is provided by Bloomberg (CMA New York). The data from September 2004 to August 2007 is from Coval et al. (2009a).
The CDX indices roll every six months. In particular, on September 20 and March 20 new series of the index with updated constituents are introduced. After a new series is created, the previous series continue trading though liquidity is usually concentrated on the on-the-run series. An exception is series 9 introduced in September 2007 and traded till the end of 2012 together with less liquid on-the-run series. The CDX indices have 3, 5, 7 and 10 year tenors. We use 5 year CDX indices which are most liquid for most series.

We build our sample from on-the-run series except period from March 2008 to September 2010 where we use most liquid series 9. Before series 15 introduced in September 2010 the CDX index has been traded with tranches 0-3%, 3-7%, 7-10%, 10-15%, 15-30% and 30-100%. Starting from series 15 and onward, only odd series of the index are traded with tranches and the structure of tranches changes to 0-3%, 3-7%, 7-15% and 15-100%.

We focus our analysis on the equity and the most senior tranches. Since the equity 0-3% tranche is quoted as an upfront payment, we calculate the par spread using the formula $S_{0-3\%} = 500\text{ b.p.} + U/D$ where $U$ is the upfront fees and $D$ is the time to maturity of the tranche. While earlier series (before 15) have tranches 15-30% and 30-100%, there is only one tranche 15-100% for later series. To make the series consistent we create a tranche 15-100% for earlier series as the sum of tranches 15-30% and 30-100%.

We obtain credit growth at nonfinancial corporate businesses from the Federal Reserve Board’s flow of funds accounts (series FA104104005.Q), and the series on nonresidential fixed investment from the Bureau of Economic Analysis (BEA).

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2The tickers of the tranches of series 9 are CT753589 Curncy, CT753593 Curncy, CT753597 Curncy, CT753601 Curncy, CT753605 Curncy, CT753609 Curncy.

3The tickers of the tranches of series 15, 17, 19 and 21 are CY071225 Curncy, CY071229 Curncy, CY071233 Curncy, CY071237 Curncy, CY087579 Curncy, CY087583 Curncy, CY087587 Curncy, CY087591 Curncy, CY125375 Curncy, CY125380 Curncy, CY125385 Curncy, CY125390 Curncy, CY181667 Curncy, CY181672 Curncy, CY181677 Curncy and CY181682 Curncy.
Appendix A

Proof of Lemma 1  The CRA’s problem is

\[
\begin{align*}
\max_{\delta_1, \delta_2} & \quad \mathbb{P}[G] \\
\text{such that} & \quad \mathbb{P}[g|G] \geq \theta',
\end{align*}
\]

(24)

(25)

where \(\theta'\) is the target level of beliefs. By the law of total probability

\[
\mathbb{P}[G] = \mathbb{P}[G|g]\mathbb{P}[g] + \mathbb{P}[G|b]\mathbb{P}[b] = \theta\delta_1 + (1 - \theta)\delta_2.
\]

(26)

Given \(\theta\), to maximize unconditional probability \(\mathbb{P}[G]\) the CRA chooses probabilities \(\delta_1\) and \(\delta_2\) as large as possible (but not greater than one). The optimal solution follows from the fact that these variables are related by (25), i.e.

\[
\mathbb{P}[g|G] = \frac{\mathbb{P}[G|g]\mathbb{P}[g]}{\mathbb{P}[G|g]\mathbb{P}[g] + \mathbb{P}[G|b]\mathbb{P}[b]} = \frac{\theta\delta_1}{\theta\delta_1 + (1 - \theta)\delta_2} \geq \theta',
\]

(27)

or, equivalently,

\[
\delta_2 \leq \frac{\theta(1 - \theta')}{\theta'(1 - \theta)} \delta_1.
\]

(28)

Conditions (28), \(\delta_1 \leq 1\) and \(\delta_2 \leq 1\) imply that maximum values of \(\delta_1\) and \(\delta_2\) are given by (6) and (7).

\[\blacksquare\]
Table 1: What Explains CDO Tranche Spreads?

<table>
<thead>
<tr>
<th>No.</th>
<th>Senior Tranche Spread</th>
<th>Equity Tranche Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta_1$</td>
</tr>
<tr>
<td>1.</td>
<td>65.10</td>
<td>-46.45</td>
</tr>
<tr>
<td></td>
<td>[9.71]</td>
<td>[-4.76]</td>
</tr>
<tr>
<td>2.</td>
<td>73.21</td>
<td>-57.49</td>
</tr>
</tbody>
</table>

Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). The “senior” spread represents the 15 to 100 percent loss attachment points, while the “equity” tranche represents the 0 to 3 loss attachment points. We report the coefficients of the fitted regression:

$$\text{Tranche Spread}(t) = \alpha + \beta_1 \frac{\text{Credit Growth}(t)}{\text{Investment}(t)} + \beta_2 \text{Earnings Growth}(t) + \epsilon(t),$$

for the senior and equity tranches, respectively. T-statistics are in parenthesis and are adjusted by White’s procedure for heteroskedasticity.
Figure 1: Tranche Spreads, Credit Growth, and Earnings Growth

Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). The “senior” spread represents the 15 to 100 percent loss attachment points, while the “equity” tranche represents the 0 to 3 loss attachment points.
Figure 2: Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans

Source: Board of Governors of the Federal Reserve System
Figure 3: Sequence of Events

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA</td>
<td>BA/BN/RA/RN</td>
<td>BA/BN/RA/RN</td>
</tr>
</tbody>
</table>

(RE-)RATING
- The CRA produces ratings and moves investors’ belief from $\hat{\alpha}_0$ to $\alpha_0$.
- If necessary the CRA adjusts ratings so that investors update their beliefs from $\hat{\alpha}_1$ to $\alpha_1$.

FINANCING
- The firm issues a two-period bond with face value $F_{02}(\alpha_0)$.
- If the credit is available, the firm may refinance its debt. If it refinances, it pays call price $H$ and issues a one-period bond to finance the repayment.
- If refinanced at $t = 1$ the firm repays $F_{12}$; otherwise the firm repays $F_{02}$.

INVESTMENT
- The firm chooses LR/HR and invests all its capital.
- The firm chooses LR/HR and invests all its capital.
The top and bottom panels show a potential distribution of losses for the case of myopic and farsighted rating standards, respectively.
Figure 5: Probabilities of the States

Probability of High Credit and High GDP Growth

Probability of High Credit and Low GDP Growth

Probability of Low Credit and High GDP Growth

Probability of Low Credit and Low GDP Growth
Figure 6: Model and Actual Spreads on Senior and Equity Tranches

Series t15-100 and T0_3 are the actual spreads on the senior and equity tranches correspondingly. Series SENIOR1 and EQUITY1 are the model spreads on these tranches.
References


