Capacity Constraints in Durable Goods Monopoly: Coase and Hotelling*

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Abstract

We examine the effect of a capacity constraint on the profits of a durable goods monopoly (DGM) in a two-period model when rationing is efficient. For sufficiently high discount factors, output *rises* through time without a constraint and a constraint increases the DGM profits: it restricts production in the second period, thereby lowering expectations of future prices and the incentive for intertemporal substitution. For lower values of the discount factor, output falls through time without a constraint. Regardless of the discount factor a constraint that binds in both periods may also be profitable. It reduces output in both periods, leading consumers to expect higher prices in the second period and thus increasing market power and prices in the first period.

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1 Introduction

Coase (1972) observed that a durable good monopolist (DGM) has an incentive to practice intertemporal price discrimination because the loss in the asset value of units of the durable good sold in earlier periods is borne not by the monopolist in the current period when she lowers price and increases sales, but by the purchasers from earlier periods. Because the DGM cannot commit not to practice intertemporal price discrimination Coase conjectured that consumers have an incentive to engage in intertemporal substitution, and that if the costs of waiting vanished rational expectations of lower prices would force the DGM to produce the competitive quantity at the competitive price "in the twinkling of an eye": the DGM would not have *any* market power.

Bulow (1982) was one of the first formal investigations of the Coase conjecture.[1] Bulow considered a two-period model with rational consumers and a profit maximizing DGM, each of whom recognizes that the DGM faces the time inconsistency problem highlighted by Coase. Bulow found that these expectations reduced the DGM's profits relative to the case where it could commit to future prices, but that profits were still positive even in the no-commitment case, suggesting that the two-period model gave the durable goods monopolist some degree of commitment relative to the case considered by Coase. Bulow also conjectured that a capacity constraint, which restricted the quantity the DGM could sell in any period, would provide it with a credible mechanism whereby it might regain some of its market power, since it reduced the ability of consumers to avoid high first period prices by waiting to purchase the good. Yet in discussing the effect of a capacity constraint, Bulow used not his two-period model, but instead an infinite-horizon model with an infinitely durable good and with the period of commitment shrinking to zero.

The rationale for why a capacity constraint *should* restore some of the DGM's monopoly power in a *two-period model* is that a binding capacity constraint in the second period should reduce the ability of consumers to engage in intertemporal substitution by waiting to purchase when prices fall in the second period. Yet, the two period models that followed Bulow have concluded that for rationing to increase the DGM profits, rationing must be *inefficient*—i.e., there is some possibility that those with the highest willingness-to-pay will not have their demands met. van Cayseele (1991) showed that if the monopolist can limit

[1]The Coase Conjecture has been formalized and its robustness assessed in a series of papers, including Stokey (1981), who was responding to a working paper version of the Bulow model, Bond and Samuelson (1984), Gul, Sonneschein, and Wilson (1986), Kahn (1986), Ausubel and Deneckere (1989), Bagnoli, Salant, and Swierzbinski (1989), Dudey (1995), and McAfee and Wiseman (2008).

its output below the quantity that would be demanded in the *second* period, consumers would anticipate that they might not be able to buy in the second period, which reduces the willingness of some high-valued consumers to engage in intertemporal substitution. Similarly, Denicolo and Garella (1999) show that a binding capacity constraint in the *first* period of a two-period model when there is random rationing results in some consumers with a higher willingness to pay remaining in the market in the second period. As a result there is less incentive for the monopolist to reduce its price in the second period, mitigating the incentive for high willingness to pay consumers to substitute intertemporally. However, with efficient rationing, a capacity constraint is not profitable in either the models of van Cayseele or Denicolo and Garella.

This paper makes three main contributions. The first is to show that in his two-period model Bulow implicitly changed the nature of the economic asset under consideration, and as a result a capacity constraint can never increase the profits of the DGM. Bulow's specification for demand, which is used by others, including Tirole (1988, p. 81) and other textbook treatments (e.g., Church and Ware (2000, pp. 135-138)), involves an inconsistent treatment of the durable good. In Bulow's specification, a consumer who purchases the good in the first period owns an economic asset that produces benefits for two periods. But a consumer who purchases the good in the second period owns an asset that produces benefits only in that second period. Thus the good purchased in the second period in Bulow's specification differs fundamentally from the good purchased in the first period and waiting to purchase the good until the second period means forgoing half the (undiscounted) flow of use benefits. This makes the threat of intertemporal substitution by consumers particularly effective since their willingness to pay is substantially reduced: a sale in the second period will be much less profitable than the same unit sold to the same consumer in the first period. As a result, in Bulow's specification, output in period one is greater than output in period two for all discount factors δ and a capacity constraint is unprofitable because its effect is to reduce sales in the first period, which is precisely when consumers are willing to pay the most for the good.

In contrast, we consider a model in which the durability of the of the durable good is consistent—i.e., whether purchased in the first period or the second period the good's durability is the same. With consistent durability, we show, similar to Bulow, that for a discount factor greater than zero and less than one, the non-capacity constrained subgame perfect Nash equilibrium profits are less than static monopoly profits. The reason is that Coasian expectations and the buyers' intertemporal Hotelling constraint—that at the margin they must be indifferent to the period of purchase—results in intertemporal substitution by consumers that disciplines the market power of the DGM.[2] Unlike Bulow's specification, however, with consistent durability the reduction in profits is increasing in the discount factor until it equals a critical value, at which point the effect of Coasian expectations and the Hotelling constraint become less and less effective at reducing the market power of the DGM. Above this critical level ($\delta > 2/3$) the DGM finds it more and more profitable for consumers to substitute consumption to the second period and when the discount factor is above this critical value, second period production is greater than first period production (which never happens in Bulow's specification). Indeed when the discount factor equals 1, the DGM only sells the static monopoly output in the second period—and nothing in the first period— and earns "full", that is static, monopoly profits. This in itself is surprising, since the closer the discount factor is to one, the greater the incentive of the customers of the DGM to substitute intertemporally. But as the discount factor rises, so to increases the willingness of the DGM to wait consistent durability. In Bulow's specification, in contrast, intertemporal substitution reduces the profits of the DGM even when the discount factor is one, since the good sold in the second period is less durable than the good sold in the first period.

Our second contribution is to show when, and how, a capacity constraint can raise profits for a DGM when rationing is efficient.[3] With consistent durability, absent a capacity constraint, second period production in the subgame perfect Nash equilibrium exceeds first period production when the common discount factor is sufficiently high ($\delta > 2/3$). A binding capacity constraint therefore raises consumers' expectations of the second period price (a Coase effect) and hence, via the Hotelling no-arbitrage condition, raises the profit maximizing price in the first period. Indeed, this effect on expectations introduces a discontinuity in the profit function when the capacity constraint just binds: reductions in first period output no longer result in an increase in second period output and a decrease in second period price.[4] As a result the DGM reduces its first period output and both the first and second period prices rise. As the capacity constraint tightens, expectations of second period sales are reduced increasing first period sales, while second period price also rises in response to

[2] Hörner and Kamien (2004) first drew the analogy to the intertemporal no-arbitrage constraint faced by Coase's (1972) DGM and the intertemporal no-arbitrage constraint faced by Hotelling's (1931) monopsony buyer of an exhaustible resource.

[3] In the models of both van Cayseele (1991) and Denicolo and Garella (1999) durability is consistent.

[4] The change in profits from an increase in first period output is zero; but with the capacity constraint, the change in profits from an increase in first period output is negative, evaluated at the level of capacity that just binds in the second period.

the decrease in first period sales. So even though output and profits in the second period are reduced, total profits increase.[5]

Perhaps more surprisingly, continuing to tighten the capacity constraint is profitable even when the constraint binds in *both* periods, at least for sufficiently high discount factors ($\delta > 1/2$). Decreasing capacity decreases output in both periods. As a result, the increase in second period price from decreasing capacity is much larger, which by the Hotelling no-arbitrage constraint results in a much higher first period price as well. Hence there is greater incentive to reduce capacity when it binds in both periods. Indeed, while decreasing output in the first period has a negative effect on profits, decreasing capacity—which results in a decrease in output in both periods—has a positive effect on profits. Therefore, relative to binding only in the first period, the effect of a binding constraint in both periods is to reverse the marginal profitability of capacity relative to the marginal profitability of output in the first period.[6] As the constraint tightens, the effect is to raise price in both periods, thereby increasing profits. Eventually, however, the opportunity cost of reducing sales in the first period becomes too large and further reductions in capacity no longer increase profits. This opportunity cost is decreasing in the discount factor: giving up profits on first period units to increase market power in the second period and increase the price in the first period is not profitable for small discount factors.

Our third contribution is to show why a capacity constraint which induces efficient rationing works to increase the profits of the DGM when the first period output exceeds second period output in the non-capacity constrained equilibrium. Denicolo and Garella (1999) showed that when a capacity constraint binds only on first period output, that it cannot increase the profits to the DGM. In their model, a necessary, but not sufficient condition for a capacity constraint to be profitable is that the DGM chooses in the second period to sell only to consumers who were (inefficiently) rationed in the first period, ensuring that second period prices are higher than first period prices. We too find that when efficient rationing by a capacity constraint binds only in the first period that the DGM is made worse off with a capacity constraint. We show, however, that a binding capacity constraint in the first period can be profitable with efficient rationing, but only if it *also* binds in the second period and if the discount factor is sufficiently high ($\delta > 1/2$). For a binding capacity constraint

[5]Notice that a DGM may have something to gain from rationing in the second period because of the effect on first period profits, contrary to the assertion by Denicolo and Garella "that in the second and final period the monopolist has nothing to gain from rationing." (1999, p. 46)

[6]The change in profits from an increase in first period output is positive; but the change in profits from an increase in first period capacity is negative, evaluated at the level of capacity that just binds in both the first and second periods.

in the first period to be profitable it does not need to result in second period prices being higher than first period prices. Instead its effect works at the margin, a binding constraint in both periods raises expectations of higher prices in the second period, which raises prices in the first period, leading to higher profits even though sales in both periods are decreased. We also show that there is always a profitable capacity constraint that binds only in the second period when the discount factor is sufficiently large ($\delta > 2/3$) that output in the second period exceeds output in the first period in the unconstrained subgame Perfect Nash equilibrium.[7] Neither of these cases were considered by Denicolo and Garella.

Finally, our finding that the magnitude of the discount factor matters is itself interesting. In both Bulow and McAfee and Wiseman (2008), who prove Bulow's conjecture regarding the effect of a capacity constraint in an infinite horizon game, the only issue relating to the magnitude of the discount factor is whether it is greater than zero. McAfee and Wiseman show that by limiting its investment in capacity at the beginning of the game, the monopolist can preserve its market power and some, but not all, of the "static" monopoly profits, as the period of commitment goes to zero. A low level of capacity acts as a commitment device ensuring that prices are high, output "dribbles" out, and prices fall slowly, thereby inducing high value consumers to buy early. They also show, however, that even if the monopolist could augment its capacity, it would not because the size of the market remaining shrinks over time. When the period of commitment goes to zero, a capacity constraint works for *any* positive discount rate. If the period of commitment is bounded away from zero, however, the magnitude of the discount factor matters since whether production is rising or falling over time depends on the discount factor, as does the profitability of a capacity constraint. Economic examples where the period of commitment is bounded away from zero in durable goods are the vintages of automobiles and appliances where new models are released annually. Thus, the question of how the discount factors affects the profitability of capacity constraints in these situations is interesting, as such questions do not arise when the period of commitment goes to zero.

The remainder of the paper is organized as follows. Section 2 shows how the nature of the asset differs between the Bulow formulation and a formulation in which the durability of the good is independent of the period in which it is purchased. Section 3 derives the

[7] Our result that a binding capacity constraint in the second period increases profits for sufficiently large discount factors is similar to the result of van Cayseele, though he assumes no discounting and that in the second period at least some consumers with a low valuation are able to purchase before any high valuation types that did not buy in the first period. As with the analysis of Denicolo and Garella, we show that a capacity constraint can be profitable when rationing is efficient because of its effect on expectations at the margin.

no-commitment equilibrium in a DGM model that nests both the consistent durability and the Bulow declining durability assumptions. Section 4 derives the main results regarding capacity constraints in the DGM model. Section 5 concludes.

2 Consumer Preferences and Durability

2.1 Identifying the Inconsistency in Bulow's Specification

A buyer who buys an infinitely durable good such as land purchases an asset which generates a flow of utility over the life of the asset. Let $D(S_t)$ denote the one period current value of the flow of marginal utility from a stock of size $S_t = \sum_{s=1}^{t} q_s$ of the durable good, where q_t is the additional quantity purchased in each period and $S_0 = 0$. Naturally, $D(S_t)$ is decreasing in cumulative purchases, S_t. With a one period ahead discount factor of δ, and prices $p_t, t = 1, 2, 3, \ldots$, the consumer chooses $q_t, t = 1, 2, 3, \ldots$ to maximize

$$V_B^\infty = \sum_{t=1}^{\infty} \delta^{t-1} \left[\int_{S_{t-1}}^{S_t} \frac{D(z)}{1-\delta} \mathrm{d}z - p_t q_t \right]. \tag{1}$$

Taking the prices as given, the consumer maximization implies that the following no-arbitrage equation must hold along the equilibrium path:

$$p_t = D(S_t) + \delta p_{t+1}, \qquad t = 1, 2, 3, \ldots \tag{2}$$

This says that the price the consumer is willing to pay in period t is the sum of the flow of marginal utility over period t to $t+1$, $D(S_t)$, plus the discounted value of what he is willing to pay in period $t+1$, δp_{t+1}.[8]

Suppose, instead, that what is finite is the number of periods in which purchases can be made as in Bulow (1982). We still assume, however, that the utility earned by purchasing

[8] The continuous time equivalent in Hörner and Kamien (2004) is:

$$\dot{p}_t = r \left[p_t - \frac{D(S_t)}{r} \right] \leq 0.$$

Observe that written in this form the no-arbitrage relationship implies that the difference between the price and the present value of the stream of future utility the consumer earns falls at the rate of interest since $D(S_t)/r \geq p_t$ to induce the consumer to purchase.

the durable good is an infinite discounted stream. Then for two purchase periods,

$$V_B^2 = \int_0^{q_1} \frac{D(z)}{1-\delta} \mathrm{d}z - p_1 q_1 + \delta \left[\int_{q_1}^{q_1+q_2} \frac{D(z)}{1-\delta} \mathrm{d}z - p_2 q_2 \right]. \tag{3}$$

The optimal choices for q_1 and q_2 imply the following:

$$p_2 = \frac{D(q_1+q_2)}{1-\delta} \qquad \text{and} \qquad p_1 = D(q_1) + \delta p_2. \tag{4}$$

It is clear that the no-arbitrage equation, $p_1 = D(q_1) + \delta p_2$, is of the same form as (2). The introduction of a terminal period introduces a second condition which pins down inverse demand in the final period: $p_2 = D(q_1 + q_2)/(1 - \delta)$. This is simply the present value, at the margin, of the flow of services of the infinitely durable good.

Bulow (1982), however, does not use (4), but instead

$$p_2 = D(q_1 + q_2), \qquad \text{and} \qquad p_1 = D(q_1) + \delta p_2. \tag{5}$$

The difference is that the price of the durable good in the second period is based only on the flow of services provided in the second period. Hence Bulow's formulation assumes not only that the number of purchase periods is finite, but also that the durability of the good varied depending upon the period in which it was purchased. Specifically, his two-period model assumed that the good provided two periods of utility if purchased in the first period but only one period of utility if purchased in the second period. Thus Bulow's consumer implicitly maximized:

$$\hat{V}_B^2 = \int_0^{q_1} (1+\delta) D(z) \mathrm{d}z - p_1 q_1 + \delta \left[\int_{q_1}^{q_1+q_2} D(z) \mathrm{d}z - p_2 q_2 \right],$$

where the $\hat{V}_B^2$ notation is used to distinguish between the case where the good is infinitely durable in each period and where it is finitely durable of decreasing lengths in each successive period. So what differs in Bulow's specification is not the intertemporal no-arbitrage equation, which is of the form of (2), but the valuation the consumer places on the good purchased in the last period.

2.2 A More General Formulation

In this section we introduce a formulation that nests consistent durability and Bulow's declining durability. We assume the utility of the single forward-looking, non-strategic buyer is given by:

$$V_B^2 = \theta_1 \int_0^{q_1} D(S)\mathrm{d}S - p_1 q_1 + \delta \left[\theta_2 \int_{q_1}^{q_1+q_2} D(S)\mathrm{d}S - p_2 q_2 \right], \tag{6}$$

where q_i and p_i are purchases and price in period $i = 1, 2$, and $D(S)$ is the marginal benefit of consumption given total purchases S. The parameters $\theta_1 \geq \theta_2$ measure the durability of the good purchased in period t. We assume $\theta_1 \geq 1 + \delta$ and $\theta_2 \geq 1$. When $\theta_t = 1$, the good provides services in only one period; when $\theta_t > 1$ the good generates utility flow in some periods that follow. If the durability of the good is consistent through time, we write $\theta_1 = \theta_2 = \theta$. This nests Bulow's specification, where $\theta_1 = 1 + \delta$ and $\theta_2 = 1$, the infinitely durable specification, where $\theta_1 = \theta_2 = 1/(1-\delta)$, and other consistent durability specifications such as a good that produces benefits for two periods, $\theta_1 = \theta_2 = 1 + \delta$.

From (6), inverse demand for purchases in first and second periods are:

$$p_1 = \theta_1 D(q_1) + \delta\theta_2 (D(q_1 + q_2) - D(q_1)) \tag{7}$$

and

$$p_2 = \theta_2 D(q_1 + q_2). \tag{8}$$

Willingness to pay in the second period equals the marginal benefit in the second period scaled by durability. Willingness to pay in the first period equals the marginal benefit in the first period scaled by durability less the capital loss in the second period if price falls.

Combining (7) and (8) we have

$$\delta[\theta_2 D(q_1) - p_2] = [\theta_1 D(q_1) - p_1]. \tag{9}$$

This is the Hotelling no-arbitrage condition for the buyer. It requires that the surplus at the margin in the first period equal the discounted surplus if the marginal unit were purchased in the second period. Expectations regarding p_2 are therefore a key determinant of demand in the first period, which is the key insight of the Coase Conjecture.

3 Equilibrium in the Two-Period DGM without Capacity Constraints

Following Bulow, assume that the DGM values a dollar in the future at the same rate, δ, as the buyer and faces zero marginal cost of production. The DGM's profits are thus

$$\pi = p_1 q_1 + \delta p_2 q_2, \tag{10}$$

where p_1 is given by (7) and p_2 is given by (8).

3.1 Full Commitment Nash Equilibrium

If the DGM could commit to an output path, then profit maximization of (10) involves simultaneously solving for q_1 and q_2 subject to the constraints that prices satisfy the no-arbitrage equations (7) and (8). The first-order-necessary conditions for the second period is

$$\theta_2 \delta \left[D(q_1 + q_2) + (q_1 + q_2) D'(q_1 + q_2) \right] = 0. \tag{11}$$

Assuming commitment, in the second period, the DGM sets marginal revenue from *total* production equal to marginal cost (which, again, is zero). The profit maximizing condition for first period output is:

$$\begin{aligned} (\theta_1 - \delta\theta_2) \left[D(q_1) + q_1 D'(q_1) \right] &= -\theta_2 \delta \left[D(q_1 + q_2) + (q_1 + q_2) D'(q_1 + q_2) \right] \\ \left[D(q_1) + q_1 D'(q_1) \right] &= 0, \end{aligned} \tag{12}$$

where the second equality uses (11). For both (12) and (11) to hold $q_2^c = 0$, where the 'c' superscript denotes the full-commitment equilibrium. Thus under full commitment the DGM produces the static monopoly output (given by (12)) in the first period and nothing in the second period.

3.2 No-Commitment Subgame Perfect Nash Equilibrium

In the second period in the subgame perfect Nash equilibrium, the DGM takes as given its sales in the first period. Since the consumer owns the output q_1 purchased in the first period, the DGM chooses q_2 to maximize its second period sales profits:

$$\pi_2 = p_2 q_2, \tag{13}$$

where p_2 is given by (8). Optimal second period sales q_2^* maximize (13), which implies setting marginal revenue equal to marginal cost (which is zero by assumption):

$$D(q_1 + q_2^*) + q_2^* D'(q_1 + q_2^*) = 0. \tag{14}$$

In the no-commitment equilibrium, the DGM expands output in the second period to set marginal revenue based on the second period residual demand equal to marginal cost. In doing so she recognizes that the loss on inframarginal units on first period sales is borne by purchasers in the first period. The loss on inframarginal units for the DGM in the second period is only $q_2 D'(q_1+q_2)$, not $(q_1+q_2)D'(q_1+q_2)$ as in the condition under full-commitment, (11).

If second period marginal revenue is decreasing in second period output and first and second period output are strategic substitutes (both of which we assume), then it follows that $dq_2^*/dq_1 \in (-1, 0)$.[9] Thus while the profit maximizing second period output is decreasing in first period output, the decrease in second period output from an increase of one unit of first period output is less than one and total output rises.

In the first period in the no-commitment subgame perfect Nash equilibrium the DGM forecasts that it will profit maximize in the second period and sell $q_2^*(q_1)$ defined by (14). First period profits to be maximized by choice of q_1 are

$$\pi = p_1 q_1 + \delta\theta_2 D[q_1 + q_2^*(q_1)]q_2^*(q_1),$$

where $D[q_1 + q_2^*(q_1)]q_2^*(q_1)$ is maximized second period profits and the optimization is subject

[9] Formally we assume $d^2\pi_2/dq_2^2 < 0$ and $d^2\pi_2/dq_2 dq_1 < 0$. Outputs in the two periods will be strategic substitutes if the reduction in marginal profit from the fall in the price in the second period from an increase in q_1 (and hence the gain from the marginal unit) exceeds the effect on marginal profit of any decrease in the loss on inframarginal units. As long as demand is not too convex this will be true.

to first period inverse demand, (7). Equilibrium first period quantity q_1^* solves:

$$
\begin{aligned}
(\theta_1 - \delta\theta_2)\left[D(q_1^*) + q_1^* D'(q_1^*)\right] &= -\delta\theta_2\left[D(q_1^* + q_2^*) + (q_1^* + q_2^*)D'(q_1^* + q_2^*)\right]\left[1 + \frac{dq_2^*}{dq_1}\right] \\
&= -\delta\theta_2 q_1^* D'(q_1^* + q_2^*)\left[1 + \frac{dq_2^*}{dq_1}\right], \qquad (15)
\end{aligned}
$$

where the second equality uses the envelope theorem to account for how the DGM behaves in the second period from (14). The effect q_1 has upon second period profits takes account of the effect an increase in q_1 has upon $q_2^*(q_1)$ and incorporates consumers' expectations of q_2^*.

We refer to the term on the right-hand-side of (15) as the *Coaseian expectations effect*. It reflects an extra negative effect of expanding first period output in the no commitment (or subgame perfect Nash equilibrium) relative to the full commitment equilibrium (compare (15) to (12)). It equals the reduction in the second period price at $t = 2$ from an expansion in first period output discounted back times first period output. Thus it reflects a lower price in the first period due to the intertemporal arbitrage condition (Hotelling) when consumers expect total output in the second period to increase and hence reduce the second period price (Coase). As a result first period output in the no-commitment equilibrium is less than in the full-commitment equilibrium; second period output is greater; and so too is total output.

3.3 Linear Inverse Demand and Consistent Durability

Table 1 shows the equilibrium prices, profits, and outputs for the no-commitment and full-commitment equilibrium assuming linear inverse demand for services given by $D(S) = \alpha - \beta S$, where α and β are each positive parameters, for the general case, $\theta_1 \geq \theta_2$, the case of consistent durability, $\theta_1 = \theta_2 = \theta$, and for Bulow's specification, $\theta_1 = 1 + \delta$ and $\theta_2 = 1$.[10]

3.3.1 Full-Commitment vs. No-Commitment

As expected based on the comparison between (12) and (15) output is lower in the first period, higher in the second period, and total output higher under no-commitment relative to full-commitment. First period price is lower under the no-commitment equilibrium for

[10]Denicolo and Garella's (1999) specification with common discount factors yields profits in the no-commitment equilibrium which equal the consistent durability case if $\alpha = \beta = \theta = 1$. If instead of a continuum of consumers with unit demand for the good and willingness to pay of v distributed uniformly over the interval $[0, 1]$, there was single buyer with $v(S_t) = \theta(\alpha - \beta S_t)$ the equilibrium expressions in Table 1 for consistent durability would also characterize the no-commitment equilibrium to Denicolo and Garella.

Full-Commitment Nash Equilibrium $\theta_1 \geq \theta_2 \geq 1$	No-Commitment Subgame Perfect Nash Equilibrium: In General $\theta_1 \geq \theta_2 \geq 1$	Consistent Durability $\theta_1 = \theta_2 \equiv \theta > 1$	Bulow $\theta_1 = 1 + \delta$, $\theta_2 = 1$
$q_1^c = \frac{\alpha}{2\beta}$	$q_1^* = \frac{2\alpha(\theta_1 - \delta\theta_2)}{\beta(4\theta_1 - 3\delta\theta_2)}$	$q_1^* = \frac{2\alpha(1-\delta)}{\beta(4-3\delta)}$	$q_1^* = \frac{2\alpha}{\beta(4+\delta)}$
$q_2^c = 0$	$q_2^* = \frac{\alpha(2\theta_1 - \delta\theta_2)}{2\beta(4\theta_1 - 3\delta\theta_2)}$	$q_2^* = \frac{\alpha(2-\delta)}{2\beta(4-3\delta)}$	$q_2^* = \frac{\alpha(2+\delta)}{2\beta(4+\delta)}$
$p_1^c = \frac{\alpha\theta_1}{2}$	$p_1^* = \frac{\alpha(2\theta_1 - \delta\theta_2)^2}{2(4\theta_1 - 3\delta\theta_2)}$	$p_1^* = \frac{\alpha\theta(2-\delta)^2}{2(4-3\delta)}$	$p_1^* = \frac{\alpha(2+\delta)^2}{2(4+\delta)}$
$p_2^c = \frac{\alpha\theta_2}{2}$	$p_2^* = \frac{\theta_2\alpha(2\theta_1 - \delta\theta_2)}{2(4\theta_1 - 3\delta\theta_2)}$	$p_2^* = \frac{\theta\alpha(2-\delta)}{2(4-3\delta)}$	$p_2^* = \frac{\alpha(2+\delta)}{2(4+\delta)}$
$\pi^c = \frac{\alpha^2\theta_1}{4\beta}$	$\pi^* = \frac{\alpha^2(2\theta_1 - \delta\theta_2)^2}{4\beta(4\theta_1 - 3\delta\theta_2)}$	$\pi^* = \frac{\theta\alpha^2(2-\delta)^2}{4\beta(4-3\delta)}$	$\pi^* = \frac{\alpha^2(2+\delta)^2}{4\beta(4+\delta)}$

Table 1: Linear Equilibrium.

all $\delta > 0$, though at $\delta = 0$ they are equal since both maximize first-period static monopoly profits. The second period price is always higher under the full-commitment equilibrium since the DGM shuts down sales in the second period.

Full-commitment equilibrium profits are greater than the no-commitment equilibrium profits, *provided* $0 < \delta < 1$ since

$$\pi^c - \pi^* = \frac{\alpha^2\delta\theta(1-\delta)}{4\beta(4-3\delta)} > 0. \tag{16}$$

The reason is the ability and willingness of consumers to engage in intertemporal substitution.

3.3.2 Consistent Durability

It is interesting to see how the effectiveness of intertemporal substitution to reduce the market power of the DGM depends on the discount factor. We consider this for the case of consistent durability, i.e., $\theta_1 = \theta_2$. For $\delta = 0$ or $\delta = 1$ the two profit levels are identical. When $\delta = 0$, neither consumers nor the DGM care about the future. Thus, the DGM chooses the static monopoly equilibrium in the first period in both the full- and no-commitment equilibria. When $\delta = 1$, both consumers and the DGM are infinitely patient. Thus, in the no-commitment equilibrium, the DGM sets first period output at zero and then chooses the static monopoly equilibrium in the second period. It is easy to show that (16) is concave in δ, increasing in δ for $\delta < 2/3$, and decreasing in $\delta > 2/3$. Similarly, p_1^* is convex in the discount factor, falling as it rises to 2/3, then rising with the discount factor for discount factors greater than 2/3.

Coasian expectations and buyers' intertemporal Hotelling constraint—that at the margin they must be indifferent to the period of purchase—results in intertemporal substitution by consumers that disciplines the market power of the DGM. The reduction in profits is increasing in the discount factor until the discount factor equals its critical value of 2/3, at which point the effect of Coasian expectations and the Hotelling constraint become less and less effective at reducing the market power of the DGM. Above this critical level the DGM finds it more and more profitable for consumers to substitute consumption to the second period. Indeed when the discount factor equals 1, the DGM only sells in the second period and earns "full" monopoly profits.

Table 1 also shows the difference between the SPNE with consistent durability and Bulow's specification. In Bulow's specification, the DGM is able to obtain the full-commitment profit level only when $\delta = 0$. This is because when $\delta = 1$, the DGM in Bulow's specification receives higher profits at the static monopoly equilibrium in the first period than in the second period, since demand is for a good that provides greater services in the first period.

Comparing q_1^* with q_2^* for the consistent durability case shows that[11]

$$q_2^* > q_1^* \qquad \text{if, and only if,} \qquad \delta > \frac{2}{3} \equiv \hat{\delta}. \tag{17}$$

For the Bulow assumptions, however, the critical value is $\hat{\delta} = 2$, which implies that there is no value of $\delta \in [0, 1]$ such that $q_2^* > q_1^*$. This is not surprising: it is never in the interest of the DGM to have sales occur in the second period since willingness to pay in the second period is based only on a single period's worth of services, not two periods as in the first period.

[11]In the general case, $\hat{\delta} = 2\theta_1/3\theta_2 \geq 2/3$.

4 Effect of a Capacity Constraint on DGM Profits in the No-Commitment Equilibrium

Now, let turn to the central question of the paper: Under what circumstances can a capacity constraint on sales restore—at least partially—the market power of the DGM in the no-commitment equilibrium? A capacity constraint places an upper bound on how much can be sold in each period.[12] The timing of the game is identical to the unconstrained no-commitment game, except that now there exists an initial stage in which the DGM chooses its capacity. We assume that capacity cannot be subsequently altered and explore the incentives of the DGM to adopt a binding capacity constraint, i.e., one that restricts its production below the no commitment equilibrium output in at least one period.

4.1 Effect of A Capacity Constraint when Output Is Increasing in the No-Commitment Equilibrium: $q_2^* > q_1^*$

We begin with the case where the second period output is greater than the first period output. This case seems most likely to result in a capacity constraint increasing profits because it most obviously severs the ability of consumers to avoid paying the higher first period price by restricting sales in the second period and hence reduces their ability and willingness to engage in intertemporal substitution.

Let K_A denote the value of capacity such that the constraint just binds in the second period: $K_A = q_2^*$. For values of $K > K_A$, the capacity constraint does not bind. When $K \leq K_A$, however, the capacity constraint binds on second period output. Since the constraint binds in the second period, second period price and profits are

$$p_2(K) = \theta_2 D(q_1 + K) \qquad \text{and} \qquad \pi_2(K) = \theta_2 D(q_1 + K)K \qquad \text{when} \qquad K \leq K_A.$$

Since q_2 is restricted to being K, it follows that marginal second period profits are positive: $\partial\pi_2/\partial q_2 = D(q_1 + K) + KD'(q_1 + K) > 0$.

The first period price is again given by (7), but with second period sales constrained to equal K:

$$p_1 = (\theta_1 - \delta\theta_2)D(q_1) + \delta\theta_2 D(q_1 + K).$$

[12] Throughout, we assume that capacity (K) is costless. Making capacity costly gives the DGM an additional reason to restrict capacity, which reduces the optimal level of capacity, K^*, relative to the case where capacity is used simply to gain a strategic advantage.

With the capacity constraint, consumers expect a higher second period price $p_2 = \theta_2 D(q_1 + K)$ and hence by this arbitrage constraint are willing to buy more in the first period. First period profits are:

$$\pi = (\theta_1 - \delta\theta_2)D(q_1) + \delta\theta_2(q_1 + K)D(q_1 + K). \tag{18}$$

Suppose that the constraint just binds: $K = K_A$. In that case, second period marginal profits are just equal to zero, which implies that the first period output that maximizes (18) at $K = K_A$ satisfies

$$(\theta_1 - \delta\theta_2)\left[D(q_1) + q_1 D'(q_1)\right] = -\delta\theta_2 q_1 D'(q_1 + K_A) \tag{19}$$

Comparing (15) to (19), the right-hand side of (19) is greater than (15) because in (15) the expression on the right-hand side of (19) is multiplied by $1 + dq_2^*/dq_1$ which is less than 1. Hence with a binding capacity constraint in the second period the negative effect of an increase in output in the first period from intertemporal arbitrage is *larger*. The reason is that unlike the no-commitment case when there is a capacity constraint the DGM does not reduce its output in the second period when it increases its output in the first period. Hence the effect on second period prices from an increase in first period output is larger at $K = K_A$ and as a result the DGM reduces first period output. The capacity constraint changes the marginal profitability of increasing first period output at $K = K_A$ from zero to less than zero. This implies that the DGM produces less in the first period compared to the case where the capacity constraint does not bind, which implies he will exercise greater market power when $q_2 = K_A$.

If the DGM produces less in the first period, it can now credibly signal that sales in the second period will not increase. Since second period sales are unchanged at $K = K_A$, the reduction in first period sales means that the second period price rises (since the residual demand has a higher intercept given that q_1 is lower), and the first period price also rises since q_1 decreases and consumers expect higher prices in the second period. Thus, the moment the constraint binds, the DGM's profits increase, and this can be traced to the effect the capacity constraint has on the expectations of consumers.

For $K < K_A$, the optimal first period output, $q_1^*(K)$, given that the capacity constraint binds second period production at $q_2^*(K) = K$ solves:

$$\begin{aligned}(\theta_1 - \delta\theta_2)\left[D(q_1^*) + q_1^* D'(q_1^*)\right] &= -\delta\theta_2\left[D(q_1^* + K) + (q_1^* + K)D'(q_1^* + K)\right] \\ &= -\delta\theta_2\left[D(q_1^* + K) + KD'(q_1^* + K) + q_1^* D'(q_1^* + K)\right] \end{aligned}\tag{20}$$

Compared to (15), there are now two important differences in (20). First, the effect just noted that because $q_2^*(K) = K$, the term dq_2^*/dq_1, which is between zero and one in (15), is zero in (20); second, because $q_2^*(K)$ is restricted to equal K, $D(q_1^* + K) + KD'(q_1^* + K) > 0$ in (20) rather than zero as in (15). This second effect means that as q_1 is increased the revenue gain from the value provided in the second period of a marginal unit in the first period ($\delta\theta_2 D(q_1^* + K)$, see (7)), exceeds the loss to second period profits from a lower second period price ($\delta\theta_2 D'(q_1^* + K)K$). This is an additional positive effect of an increase in first period output, that offsets the negative effect associated with the change in valuation of first period output from a lower price in the second period, the Coasian expectations effect identified previously.[13]

The net effect on first period output from lowering K below K_A is:

$$\frac{dq_1^*(K)}{dK} = -\delta\theta_2 MR'(q_1^* + K) / \left[(\theta_1 - \delta\theta_2)MR'(q_1^*) + \delta\theta_2 MR'(q_1^* + K)\right],$$

which is negative since the numerator is positive and the denominator negative. The numerator is positive and the denominator negative because marginal revenue ($MR(S) = D(S) + SD'(S)$) is decreasing in S. As K continues to be lowered, q_1 rises and q_2 falls until capacity reaches a value (K_B) where it just binds in both periods.

Thus, in the interval $[K_B, K_A)$, as K is reduced, $q_1^*(K)$ increases and $q_2^*(K) = K$ decreases, which causes the equilibrium to move towards the full-commitment equilibrium in which q_1^c is equal to the quantity that maximizes static monopoly profits and in which $q_2^c = 0$. This suggests that profits are increasing as K decreases. That this occurs can be seen by differentiating profits given by (18) with respect to K, taking account of how $q_1^*(K)$ is affected:

$$\begin{aligned} d\pi/dK &= (\partial\pi/\partial q_1) dq_1^*(K)/dK + \partial\pi/\partial K \\ &= \delta\theta_2 \left[D(q_1 + K) + (q_1 + K)D'(q_1 + K)\right] < 0, \end{aligned} \tag{21}$$

where the second equality uses the fact that $q_1^*(K)$ solves $\partial\pi/\partial q_1 = 0$, so that only the $\partial\pi/\partial K$ term is not zero on the right-hand-side. To see that (21) is negative, observe that bringing everything to the left-hand-side of (20) and subtracting $\theta_1 \left[D(q_1 + K) + (q_1 + K)D'(q_1 + K)\right]$

[13] See discussion above after (15).

from each side yields

$$(\theta_1 - \delta\theta_2)\left\{[D(q_1) + q_1 D'(q_1)] - [D(q_1 + K) + (q_1 + K)D'(q_1 + K)]\right\}$$
$$= -\theta_1 \left[D(q_1 + K) + (q_1 + K)D'(q_1 + K)\right].$$

The expression on left-hand-side is the difference between marginal revenue evaluated at q_1 and at $q_1 + K$. This difference is positive since marginal revenue is decreasing in output. Thus, the expression on the second line must be positive, which, because $\theta_1 \geq \theta_2 \geq 1$, implies that $d\pi/dK < 0$ in (21).

The effect of tightening the capacity constraint is two-fold: profits rise because of the effect that the constraint has on expectations, leading to decreased output and higher profits in the first period.[14] However, the cost of this is reduced profits in the second period. However, for $K > K_B$ a tightening of the constraint always increases profits.

4.2 Effect of A Capacity Constraint when Output Is Decreasing in the No-Commitment Equilibrium: $q_1^* \geq q_2^*$

Consider the case where $\delta \leq \hat{\delta}$, so that absent a capacity constraint, the no-commitment equilibrium is characterized by $q_1^* \geq q_2^*$. This occurs under Bulow's assumptions and it occurs more generally as long as $\delta \leq \hat{\delta}$. When output is decreasing over time, the capacity constraint, if it binds, first restricts sales in period one since $q_1^* \geq q_2^*$.

Let K_C denote the value of K such that the capacity constraint just binds in the first period, but does not bind in the second period. Thus, at K_C, first period output equals K_C and second period output satisfies (14), since output in the second period is unconstrained. Therefore, it follows that $dq_2^*/dq_1 \in (-1, 0)$ and there is not a discontinuity in the marginal profitability of first period output.

For $K < K_C$, $q_1^*(K) = K$ and $q_2^*(K) < K$ implies that first period profits are

$$\pi(K) = (\theta_1 - \delta\theta_2)D(K)K + \delta\theta_2 \left(K + q_2^*(K)\right) D\left(K + q_2^*(K)\right),$$

where $q_2^*(K)$ maximizes π_2, given that $q_1^* = K$, and where p_1 and p_2 are given by (7) and (8), respectively, evaluated at $q_1 = K$. Thus, when the constraint binds in period one but not period two, the DGM's first-order-necessary condition in q_1 implies that first period marginal

[14] In the linear case first period price is unchanged as K falls.

profits are positive:

$$(\theta_1 - \delta\theta_2)\left[D(K) + KD'(K)\right] + \delta\theta_2 KD'(K + q_2^*)\left[1 + \frac{dq_2^*}{dq_1}\right] > 0,$$

Thus, lowering K causes profits to fall. This is the result of Denicolo and Garella (1999) that a capacity constraint with efficient rationing that binds on first period output only makes the DGM worse off. Here we simply note that our result means that, at least locally, a capacity constraint makes the DGM worse off. But as K decreases, $q_2^*(K)$ increases.[15] Thus, there exists a value K_D such that the constraint binds in both periods at K_D: $q_1^*(K_D) = q_2^*(K_D) = K_D$.

4.3 Effect of a Capacity Constraint that Binds in Both Periods

When the capacity constraint binds in both periods, the DGM profits are:

$$\pi(K) = (\theta_1 - \delta\theta_2)D(K)K + \delta\theta_2 2KD(2K) \text{ for } \begin{cases} K \leq K_B \text{ and } \delta > \hat{\delta} \\ \qquad\text{or} \\ K \leq K_D \text{ and } \delta \leq \hat{\delta} \end{cases} \tag{22}$$

The capacity constraint is only binding if it is less than K_B or K_D (depending on whether $q_1^* >$ or $< q_2^*$, which depends on whether $\delta >$ or $< \hat{\delta}$.

The optimal choice for capacity when the constraint binds in both periods is found by maximizing profits, (22), with respect to K:[16]

$$\frac{\partial\pi(K)}{\partial K} = (\theta_1 - \delta\theta_2)[D(K) + KD'(K)] + 2\delta\theta_2[D(2K) + 2KD'(2K)]. \tag{23}$$

4.3.1 $q_2^* > q_1^*$ ($\delta > \hat{\delta}$)

For the DGM to have an incentive to reduce K below K_B (23) must be negative at $q_1^*(K) = K_B$, i.e., when the constraint just binds in both periods. Recall that the optimal first period

[15] Again assuming that $K = q_1$ and q_2 are strategic substitutes. See footnote 9.

[16] By inspection, it is clear that as K approaches zero, profits $\pi(K)$ vanish, as long as the choke price $D(0)$ is finite. Since marginal revenue in each period is decreasing in K, for values of K such that the constraint binds in both periods, profits are either monotonically increasing in K or there exists a value of K^* such that profits are maximized. Therefore, if there exists a value of K that satisfies $\partial\pi(K)/\partial K = 0$ and is less than K_B or K_D (whichever is relevant), then that value is unique.

output when the constraint binds in the second period is given by (20). Evaluating (20) at K_B and rearranging:

$$((\theta_1 - \delta\theta_2)\left[D(K_B) + K_B D'(K_B)\right] = -\delta\theta_2\left[D(2K_B) + (2K_B)D'(2K_B)\right]. \tag{24}$$

Substituting this into (23) gives:

$$\frac{\partial\pi(K)}{\partial K} = \delta\theta_2[D(2K) + 2KD'(2K)] \tag{25}$$

The right hand side of (25), by (24), is negative, indicating that it is profitable to reduce K. Hence there is a jump downwards in the profitability of increasing first period output when it increases output in both periods relative to when it only results in an increase in output in the first period. The relevant first order condition jumps from being positive to negative! Relative to binding only in the second period, the effect of a binding constraint in both periods is to reverse the marginal profitability of capacity relative to the marginal profitability of output in the first period.

The reason is that the effect of increasing capacity is to increase output in both the first and second period: when the constraint binds only in the second period increasing capacity only increases output in the second period. As a result when it binds in both periods the reduction in second period price is much larger—resulting in a much lower expected price in the second period, which through the Hotelling constraint results in a much lower first period price. Hence there is more incentive to reduce capacity when it binds in both periods. Indeed while decreasing output in the first period has a negative effect on profits, decreasing capacity—which results in a decrease in output in *both* periods—has a positive effect on profits. As the constraint tightens, the effect is to raise price in both periods, thereby increasing profits. However, eventually the opportunity cost of reducing sales in the first period becomes too large and further reductions in capacity no longer increase profits. This opportunity cost is decreasing in the discount factor: giving up profits on first period units to increase market power in the second period and increase the price in the first period is not profitable for small discount factors.

To summarize: For the case where $q_2^* > q_1^*$, i.e., $\delta > \hat{\delta}$ we have shown that capacity constrained profits are greater than the no commitment profits for $K_B < K < K_A$ in the previous section. In this section we have shown that profits will continue to be larger then no commitment profits for any $K^* \leq K \leq K_B$, where K^* solves (23). The profit maximizing capacity is K^* provided it is credible, i.e., less than K_B. Otherwise the profit maximizing capacity is K_B.

4.3.2 $q_1^* > q_2^*$ ($\hat{\delta} > \delta$)

There are two requirements for a capacity constraint to raise profits relative to the no commitment equilibrium. First it must be the case that a decrease in capacity increases profits at K_D even though for $K > K_D$ we showed above that decreases in K reduce profits. That is, it must be demonstrated that locally the marginal effect on profits of increasing capacity changes from being positive to negative. Second, for a capacity constraint to result in greater profits than the no commitment outcome requires that the increase in profits over the interval $[K^*, K_D]$ exceed the decrease over the interval $[K_D, q_1^*]$. For the DGM to have an incentive (locally) to reduce K below K_D (23) must be negative at $q_2^*(K) = K_D$, i.e., when the constraint just binds in both periods. Recall that the optimal second period output when the constraint binds in the second period is given by (14). Evaluating (14) at K_D and rearranging:

$$D(2K_D) + K_D D'(2K_D) = 0. \tag{26}$$

Substituting this into (23) gives:

$$\frac{\partial \pi(K)}{\partial K} = (\theta_1 - \delta\theta_2)(D(K_D) + K_D D'(K_D)) + 2\delta\theta_2 K_D D'(2K_D), \tag{27}$$

which could be greater or less than zero—the first term is positive, the second negative. It will clearly be less than zero if $K_D > K^*$.

To summarize: For the case where $q_1^* > q_2^*$, i.e., $\delta < \hat{\delta}$, we have shown that capacity constrained profits are less than the no-commitment profits for $K_D < K < q_1^*$ in the previous section. In this section we have shown that profits locally may increase for $K < K_D$ and hence that the profit maximizing capacity K^* may be greater than the no-commitment profits. We confirm these results in the next section for linear inverse demand for services in a period.

4.3.3 Linear Inverse Demand for Services

To assess when and whether $K_B > K^*$ ($K_D > K^*$) and whether $\pi(K^*) > \pi^*$ when $K_D > K^*$, we assume linear demand for services in a period $D(Q) = \alpha - \beta Q$

Setting (23) equal to zero and solving for K^* yields

$$K^* = \frac{\alpha(\theta_1 + \delta\theta_2)}{2\beta(\theta_1 + 3\delta\theta_2)}. \tag{28}$$

For the credibility of K^*, we have two cases to consider:

1. $\delta > \hat{\delta}$. In order that K^* be a feasible maximum, it must be that $K^* \leq K_B$, as K_B is the value of K such that $q_1^*(K_B) = K_B$ when $q_2^* = K_B$. Using (20), $K_B = \frac{\alpha\theta_1}{2\beta(\theta_1+\delta\theta_2)}$. Comparing K_B to K^* reveals that $K^* < K_B$ so long as $\delta > 0$ and $K^* = K_B$ when $\delta = 0$. So K^* is always credible and is the profit maximizing choice of K when $\delta > \hat{\delta}$ or $q_2^* > q_1^*$.

2. $\delta \leq \hat{\delta}$. In order that K^* be a feasible maximum, it must be that $K^* \leq K_D$, where K_D is the value of K such that $q_2^*(K_D) = K_D$ when $q_1^* = K_D$. Using (14),$K_D = \alpha/3\beta$. Comparing K^* with K_D reveals that $K^* \leq K_D$ if, and only if, $\delta > \theta_1/3\theta_2 \equiv \bar{\delta}$.

There is a final step for the case of $\delta \leq \hat{\delta}$. We must show that the DGM profits at K^* are greater than the no-commitment profits. Profits at K^* are:

$$\pi(K^*) = \frac{\alpha^2}{4\beta}\frac{(\theta_1 + \delta\theta_2)^2}{(\theta_1 + 3\delta\theta_2)}.$$

Comparing this with the profits in the no-commitment, no-capacity constraint case shown in Table 1 yields the necessary and sufficient condition that must be satisfied in order that $\pi(K^*) > \pi^*$:

$$\pi(K^*) - \pi^* = \frac{3\alpha^2\delta\theta_2(2\delta\theta_2 - \theta_1)(\theta_1 - \delta\theta_2)}{4\beta(4\theta_1 - 3\delta\theta_2)(\theta_1 + 3\delta\theta_2)} > 0$$

This implies that $\pi(K^*) > \pi^*$ if, and only if, $\delta > \delta^* \equiv \theta_1/2\theta_2$.

For $\pi(K^*) > \pi^*$ requires $\delta > \delta^* \equiv \theta_1/2\theta_2$. For K^* to be credible, i.e., less than K_D, $\delta > \theta_1/3\theta_2 \equiv \bar{\delta}$. We can conclude that K^* is the optimal choice of capacity for $\delta^* < \delta < \hat{\delta}$. Above we showed that K^* is always credible and is the profit maximizing choice of K when $\delta > \hat{\delta}$ or $q_2^* > q_1^*$.

We can summarize our results in the following proposition.

Proposition 1. *A DGM in a two-period game facing linear demand for services in a period and having zero marginal costs of production, can increase his profits relative to the no-commitment subgame perfect Nash equilibrium by imposing capacity constraint K^* for any $\delta > \delta^* \equiv \theta_1/2\theta_2 = 1/2$ (when $\theta_1 = \theta_2$).*

This result shows that Denicolo and Garella's (1999) conclusion that efficient rationing is not profitable is only true when a capacity constraint binds only in the first period. When $\delta > 2/3$, profits are increased by reducing capacity for all levels of capacity between K^* and K_A, and when $\delta > 1/2$ a restriction of capacity in both periods to K^* yields higher profits than the unconstrained no-commitment equilibrium. [17]

4.4 Bulow and Capacity Constraint

Proposition 2. *There are no values of the discount factor, δ, for which a capacity constraint increases the DGM profits in Bulow's two-period game facing linear demand having zero marginal cost.*

Recall that for the Bulow specification, $\theta_1 = 1 + \delta$ and $\theta_2 = 1$. For these values, $\delta^* = (1+\delta)/2$. For $\delta > \delta^*$ requires $\delta > 1$. Thus, although Bulow correctly intuited that a capacity constraint could increase the profits of the DGM, his two-period specification was incapable of yielding that result.

5 Discussion and Conclusions

Bulow's (1982) paper made two contributions to the economic theory of a durable goods monopolist. First, in a simple and intuitive model he formalized Coase's (1972) conjecture that rational consumers, by waiting to make purchases because they expect future prices to be lowered, would cause the monopolist to lose profits. Second, he provided the insight that a capacity constraint could increase the profits of the durable goods monopolist.

But this insight did not hold in his simple two-period model. We show that this is due to the way in which Bulow effectively altered the nature of the durable good by making it durable only if purchased in first period. We also show that when the durability of the good is independent of the period in which it is purchased, Bulow's central insight about capacity

[17] In numeric comparisons involving the Denicolo and Garella parameters (i.e., $\alpha = \beta = \theta = 1$), the DGM does at least as well using the optimal capacity constraint as opposed to the proportional rationing scheme of Denicolo and Garella.

constraints is regained, although only for sufficiently high discount factors ($\delta > 1/2$). A capacity constraint has this effect because it changes expectations (a Coase effect) and, via the Hotelling no-arbitrage condition, expectations of a higher price in the second period result in greater demand, higher prices, and thus profits in the first period. We find that the capacity constraint changes expectations in this way both when it binds in the second period and when it binds in both periods.

Our model shows that it is not necessary for there to be inefficient rationing when there is a capacity constraint. The conditions under which Denicolo and Garella rejected efficient rationing schemes hold also in the model we consider, but we find that by broadening the nature of the capacity constraint to include a constraint that binds only in the second period or which binds in both periods, that restricting capacity can be a strategy used by a DGM to restore—at least in part—-its market power and monopoly profits lost from Coasian dynamics.

References

[1] Ausubel, Lawrence M., and Raymond J. Deneckere (1989), "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, 57 (May): 511-31.

[2] Bagnoli, Mark, Stephen W. Salant, and Joseph E. Swierzbinski (1989), "Durable-Goods Monopoly with Discrete Demand," *Journal of Political Economy*, 97 (December): 1459-1478.

[3] Bond, Eric, and Larry Samuelson (1984), "Durable Good Monopolies with Rational Expectations and Replacement Sales," *Rand Journal of Economics*, 15 (3): 336-345.

[4] Boyce, John R., Jeffrey Church, and Lucia Vojtassak (2012), "Pipeline Economics: Coase and Hotelling," University of Calgary Mimeo.

[5] Bulow, Jeremy I. (1982), "Durable-Goods Monopolists," *Journal of Political Economy*, 90 (2): 314-332.

[6] Bulow, Jeremy I. (1986), "An Economic Theory of Planned Obsolescence," *Quarterly Journal of Economics*, 101 (November): 729-49.

[7] Church, Jeffrey, and Roger Ware (2000), *Industrial Organization: A Strategic Approach*, San Francisco: McGraw-Hill.

[8] Coase, Ronald H. (1972), "Durability and Monopoly," *The Journal of Law and Economics*, 15 (1): 143-149.

[9] Denicolo, Vincenzo, and Paolo G. Garella (1999), "Rationing in a Durable Goods Monopoly," *RAND Journal of Economics*, 30 (Spring): 44-55.

[10] Driskill, Robert (1997), "Durable-Goods Monopoly, Increasing Marginal Cost and Depreciation," *Economica* 64: 137-154.

[11] Dudey, Marc (1995), "On the Foundations of Dynamic Monopoly Theory," *Journal of Political Economy.* 103: 893-902.

[12] Gul, Faruk, Hugo Sonneschein, and Robert Wilson (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39 (June): 155-90.

[13] Hörner, Johannes, and Morton I. Kamien (2004), "Coase and Hotelling: A Meeting of the Minds," *Journal of Political Economy*, 112 (3): 718-723.

[14] Hotelling, Harold (1931), "The Economics of Exhaustible Resources," *Journal of Political Economy*, 39: 137-175.

[15] Kahn, Charles (1986), "The Durable Goods Monopolist and Consistency with Increasing Costs," *Econometrica*, 54 (2): 275-294.

[16] McAfee, R. Preston, and Thomas Wiseman (2008), "Capacity Choice Counters the Coase Conjecture," *Review of Economic Studies*, 75 (January): 317-332.

[17] Stokey, Nancy L. (1981), "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics*, 12 (Spring): 112-128.

[18] Tirole, Jean (1988), *The Theory of Industrial Organization*, Cambridge: MIT Press.

[19] van Cayseele, P. (1991), "Consumer Rationing and the Possibility of Intertemporal Price Discrimination," *European Economic Review,* 35: 1473-1484.