Prediction and Inference in the Hubbert-Deffeyes Peak Oil Model *

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Abstract

World oil production has grown at an annual rate of 4.86% since 1900. Yet, the Hubbert-Deffeyes 'peak oil' (HDPO) model predicts that world oil production is about to enter a sustained period of decline. This paper investigates the empirical robustness of these claims. I document that the data for the HDPO model shows that the ratio of production-to-cumulative-production is decreasing in cumulative production, and that the rate of decrease is itself decreasing. The HDPO model attempts to fit a linear curve through this data. To do so, Hubbert and Deffeyes are forced to either exclude early data or to discount the validity of discoveries and reserves data. I show that an HDPO model which includes early data systematically under-predicts actual cumulative production and that the data also rejects the hypothesis that the fit is linear. These findings undermine claims that the HDPO model is capable of yielding meaningful measures of ultimately recoverable reserves.

Key Words: Peak Oil, Exhaustible Resources, Exploration and Development
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"*Day by day it becomes more evident that the Coal we happily possess in excellent quality and abundance is the mainspring of modern material civilization*" (p. 1).

"*[T]he growing difficulties of management and extraction of coal in a very deep mine must greatly enhance its price.*" (p. 7).

"*[T]here is no reasonable prospect of any relief from a future want of the main agent of industry. It cannot be supposed we shall do without coal more than a fraction of what we do with it*" (p. 9).

W. S. Jevons (1866), "*The Coal Question*"

1 Introduction

The experience of the two oil price shocks in the 1970s and the more recent one 2008 leave little doubt that oil is a resource for which a decline in production could prove very costly.[1] That such a decline is both inevitable and imminent is the claim behind a number of recent publications on "peak oil," whose name is derived from the time path of U.S. oil production, which reached its maximum ('peaked') in 1970. The original peak oil analysis is due to Hubbert (1956, 1982), who in 1956 predicted that U.S. oil production would peak in 1970, but it has had a number of proponents over the years with the most prominent recent advocate being Deffeyes (2003, 2005).[2] These authors warn of the dire consequences of a rapidly approaching decline in world oil production. Like Jevons, these authors cannot imagine how "we shall do without [oil] more than a fraction of what we do with it." The fact that U.S. oil production did indeed peak, as did British coal production in 1913 (Mitchel, 1988, at pp. 248-49), suggests that the phenomenon of a "peak" in world oil production may be real.[3] Even the most optimistic observer believes that oil production may eventually peak—either because oil is a finite resource (the peak oil rationale) or because a superior substitute will eventually be found, as has occurred at every other historical energy transition

[1]See Blanchard and Gali (2007), Hamilton (2009a,b), Smith (2009) and Kilian (2009), *inter alia* for discussions of the causes and effects of disruptions to oil production on the macroeconomy. Dvir and Rogoff (2009) attribute higher growth and volatility of oil prices since 1970 to rapid demand growth coupled with market power by Middle Eastern suppliers.

[2]See also Warman (1972); Hall and Cleveland (1981); Fisher (1987); Campbell and Laherrère (1998); Kerr (1981, 1998, 2010); Witze (2007); Heinberg and Fridley (2010); Campbell (1997); Roberts (2004); Goodstein (2005); Simmons (2006); Heinberg (2007); Tertzakian (2007).

[3]These estimates are increasingly being taken seriously by policy makers and by serious economists. For example, the U.S. Energy Information Agency (EIA) has produced estimates of when oil will peak (Wood and Long, 2000). The International Energy Agency's (IEA) 2009 Oil Market Market Report states that conventional oil production is "projected to reach a plateau sometime before 2030" (quoted in "The IEA puts a date on peak oil production," (*Economist*, Dec. 10, 2009). See also Hamilton (2012).

(Fouquet and Pearson, 1998, 2006). But the question is whether the Hubbert-Deffeyes peak oil (HDPO) model has anything to say about the timing and consequences of such a peak.

The peak oil model claims to explain the time paths of production, discoveries and proved reserves of oil. To economists, these are the result of an equilibrium between present and future suppliers and demanders of oil (Hotelling, 1931).[4] But the peak oil analysis is not an economic model. Like models extending back to Malthus (1798), Jevons (1866), and Meadows et al. (1972), the peak oil model has no prices guiding investment and consumption decisions.[5] Rather, the HDPO model maps the time paths of production and discoveries based upon an observed negative correlation between cumulative production and the ratio of current production-to-cumulative-production. Then, by linear extrapolation, the HDPO model solves for the level of cumulative production—the ultimately recoverable reserves—where production is driven to zero.

This paper asks whether the HDPO model is capable of generating meaningful predictions about ultimately recoverable reserves. Since the ultimately recoverable reserves of crude oil is unknowable, I test the hypothesis of whether the HDPO model, using only historical data, predicts a level of ultimately recoverable reserves that are as large observed cumulative production or discoveries at the end of 2010. To implement this, I use the historical data that would have been available in 1900, 1950 or 1975 to estimate ultimately recoverable reserves using both the linear HDPO methodology (where the ratio of production-to-cumulative production is linear in cumulative production) and the quadratic HDPO methodology (where production is quadratic in cumulative production). Then I compare those estimates with observed cumulative production and discoveries through 2010. I find that the HDPO out-of-sample estimates of ultimately recoverable reserves made 30 years or more into the future yield estimates of ultimately recoverable reserves that are less than observed cumulative production or discoveries in 2010. These results are found for both U.S. and world production, as well as at less aggregated levels for 24 U.S. states, for 44 countries, and even for a sample of U.S. fields. Similar results are also found using data on discoveries, first at the U.S. and world levels of aggregation, then at country and state levels, and finally at the level of discoveries of super-giant fields. In addition, I find that the HDPO model suffers the same out-of-sample prediction problem when applied to other resources such as coal and to a sample of 78 other minerals. Estimates of ultimately recoverable reserves from the HDPO model are nearly always too low, even when compared only with observed future production.

The reason these results occur with such high frequency is that the data upon

[4]Economists are not all that successful in predicting the time paths of prices and production. Hotelling's model predicts that prices should be rising across time, but real prices fell between 1870 and 1970. See Hamilton (2012) for a discussion of the limitations of these models.

[5]See Nordhaus (1973), Gordon (1973), Bradley (1973), Rosenberg (1973), Nordhaus (1974), and Simon (1996), *inter alia*, for economic criticisms of this earlier literature.

which the linear HDPO model predictions are made is inherently non-linear. Data show a negative correlation between the ratio of production-to-cumulative-production and cumulative production or between the ratio of discoveries-to-cumulative-discoveries and cumulative discoveries. But these plots also show that that the slope of the curve is diminishing as cumulative production or discoveries increases. Since early data, when filtered through the HDPO model, predict an estimate of ultimately recoverable reserves that is less than observed cumulative production, Hubbert and Deffeyes are forced to exclude that data in their curve-fitting. They appear do so on the basis of simple 'eye-ball' methods. Deffeyes (2006, p. 36), for example, states "The graph of U.S. production history settles down to a pretty good straight line after 1958." But it is not too surprising that this occurs. When cumulative production is large the denominator of the ratio of production-to-cumulative-production swamps variations in production, so the data appear linear. Thus, the second contribution of the paper is to document that the data rejects the HDPO empirical specifications. I test this by adding higher order polynomials of cumulative production to the right-hand-side of regressions, and test whether the data rejects their inclusion. The data also reject the null hypothesis that these terms have zero coefficients in the HDPO specifications in a battery of tests using different geographic and historical samples as well as for a broad sample of other mineral commodities. This suggests that the HDPO model is fundamentally misspecified.

An important implication of this research is that the HDPO model may be incapable of distinguishing between the implied HDPO data-generating process and other plausible alternative data generating processes. I show that other data-generating processes, such as constant production or production which grows exponentially or geometrically, each which have starkly different predictions about ultimately recoverable reserves than the HDPO model, produce data plots similar to those observed in the data. I then ask whether the HDPO model is capable of identifying a process in which production never vanishes, by applying the HDPO model to data on agricultural production in the United States. Agricultural production, like production of oil and most exhaustible resources, has been increasing for nearly all commodities. But agricultural production is sustainable, so cumulative production is unbounded. I find that the HDPO model, however, predicts that agricultural production too has an imminent peak. Since such a conclusion is both logically and empirically refutable, this questions whether the HDPO model can differentiate between production from a finite and soon to peak production process and one in which cumulative production may grow without bound.

While there have been attempts to explain the peak oil phenomenon, notably by Holland (2008). Smith (2011), and Hamilton (2012), there has been little empirical work testing the peak oil model. Exceptions include Brandt (2006), Considine and Dalton (2008) who test hypotheses regarding the time paths of production. Brandt focused exclusively on the time paths of production but considered models in which the time path could be symmetric or asymmetric. Like Brandt, I test hypotheses using oil production from various regions (U.S. states, countries, and the world as a whole). However, Brandt did not explicitly test Hubbert's logistic specification, nor did he examine discoveries data, nor did he examine the predictive powers of the HDPO model. The approach I take follows Nehring (2006a,b,c) and Maugeri (2009), each of whom argues that predictions of ultimately recoverable reserves even at the field and basin level typically rise over time. Unlike those authors, I use the HDPO model to compare the estimates of ultimately recoverable reserves with observed cumulative production.

The remainder of the paper is organized as follows. Section 2 presents the assumptions and methods of the HDPO model, as well as the hypotheses I intend to test. Section 3 presents the empirical evidence of how well the HDPO model does using data on crude oil production from various historical samples and levels of disaggregation. Section 4 repeats this exercise for discoveries data. Section 5 presents empirical evidence on the HDPO model when applied to coal, other mineral resources, and to agricultural production. Section 6 concludes.

2 The Hubbert-Deffeyes Peak Oil (HDPO) Model

The peak oil model was developed by Hubbert (1956, 1982) and has been recently extended by Deffeyes (2003, 2005). Hubbert (1982, equation (27), at p. 46) assumed the following relationship between production, Q_t, and cumulative production to period t, $X_t = \sum_{s=0}^{t-1} Q_s$:

$$\frac{Q_t}{X_t} = r_q \left(1 - \frac{X_t}{K}\right). \tag{1}$$

In the HDPO model, K corresponds to the ultimately recoverable reserves—the maximum possible cumulative production and discoveries—since when $X_{\bar{T}_Q} = K$, the right-hand-side of (1) equals zero; thus exhaustion occurs at time $\bar{T}_Q$. The value of r_q corresponds to the predicted level of the initial ratio of production-to-cumulative-production. Equation (1) is a logistic growth function.[6] The logistic function produces time paths in which X_t follows an 'S'-shaped time path, increasing first at an increasing rate and then increasing at a decreasing rate, and

[6]The logistic growth function is used to model dynamics of biological populations, and underlies the most famous bioeconomic model, the Gordon-Schaefer model (Gordon, 1954). In those models, Z_t is the stock of the species, and $\dot{Z}_t \equiv dZ_t/dt$ is the rate at which the stock increases. The logistic curve is written as $\dot{Z}_t/Z_t = r(1 - Z_t/K)$, where r is the maximum intrinsic growth rate of the species (higher for fast growing species such as insects than for slow growing species such as whales), and where K is the "carrying capacity" of the species, which corresponds to one of two stable biological steady-states (the other being the extinction state, $Z = 0$).

Q_t follows a bell-shaped path.

$$Q_t = r_q X_t \left(1 - \frac{X_t}{K}\right). \tag{2}$$

This specification is the basis of the bell-shaped curve over cumulative production which peak oil advocates generally show filled from the left to the point of current cumulative production, with the implication that the remaining production is the area yet to be filled in under the curve.

Hubbert also assumed that a logistic relationship exists for discoveries:

$$\frac{D_t}{C_t} = r_d \left(1 - \frac{C_t}{K}\right). \tag{3}$$

Here, D_t denotes new discoveries in period t, and $C_t = \sum_{s=0}^{t-1} D_s$ denotes cumulative discoveries to period t. Again, when $C_{\bar{T}_D} = K$, discoveries cease. This occurs at time $\bar{T}_D \leq \bar{T}_Q$. Thus (3) has only one new parameter, r_d, since K is common to both (1) and (3) because cumulative discoveries must equal cumulative extraction at the moment that oil is exhausted. As with production, a quadratic version of the discoveries equation can be written:

$$D_t = r_d C_t \left(1 - \frac{C_t}{K}\right). \tag{4}$$

Finally, proved reserves, R_t, at the beginning of year t are given by

$$R_t = C_t - X_t. \tag{5}$$

One set of discoveries data used below are derived from proved reserves data, using the following identity:

$$D_t = C_t - C_{t-1} = R_t + X_t - R_{t-1} - X_{t-1}. \tag{6}$$

Since proved reserves are revised when economic and geologic information changes, this source of discoveries data is subject to great variation and some dispute. Thus, I also use a second set of data on discoveries which uses all of the information available in 2009 about the size of discoveries for a subset of fields. The data on discoveries calculated from proved reserves is important, however, since both production and exploration decisions are made based on the information available at the time.

The parameter K is related to another important feature of the peak oil model. Because of the logistic specification, the 'peak' in production occurs at time $\hat{T}_Q$ when $X_{\hat{T}_Q} = K/2$ and the peak in discoveries occurs at time $\hat{T}_D$ where

$C_{\hat{T}_D} = K/2$.[7] Deffeyes (2003, 2005) predicts the imminence of the date of the peak in world crude oil production on the basis that cumulative production is approaching his estimate of $K/2$. Thus, the estimation of K is central to the peak oil model both for its interpretation as ultimately recoverable reserves and for the interpretation of $K/2$ as the size of cumulative production and discoveries whereby production and discoveries begin to decline.

Hubbert (1982) considered the logistic specification as the "simplest case" (pp. 44-55), but it forms the empirical basis of the Hubbert and Deffeyes peak oil predictions. According to Hubbert,

> "We may accordingly regard the parabolic form as a sort of idealization for all such actual data curves, just as the Gaussian error curve is an idealization of actual probability distributions" (Hubbert, 1982, at p. 46).

2.1 The 'Peaks' in the HDPO Model

Fig. 1 plots the time paths of U.S. crude oil production, crude oil proved reserves, and crude oil discoveries (with the latter smoothed by a 5 year moving average) for the period 1900-2010.[8] U.S. production data begins in 1859, but U.S. reserves data are available only since 1900.[9] The central feature of these data are that production, discoveries, and proved reserves all peaked in approximately 1970.[10] Even though discoveries are smoothed by a 5 year moving average, it is clear that there is much greater volatility in discoveries, with between six or eight cycles in discoveries since 1900.

Fig. 2 shows the time paths of world production, proved reserves, and discoveries for the period 1948-2010.[11] Unlike the U.S. graph, neither world reserves nor world production have yet peaked.[12] However, like the U.S. data, there are at least four peaks in the smoothed discoveries data since 1950, approximately

[7]In the biological interpretation of the logistic function, $K/2$ is the stock that produces the maximum growth in the population, so is the stock level that corresponds to the maximum sustainable harvest.

[8]The annual values for discoveries are plotted below in Fig. 5, and are discussed there.

[9]Data for the period 1948 forward are published biannually in the American Petroleum Institute's *Basic Petroleum Databook*. Pre-1948 production data come from the American Petroleum Institute, "Petroleum Facts and Figures (1959 Centennial Edition)."

[10]The major spike in discoveries and proved reserves around 1970 is due to the addition of the Prudhoe Bay field to U.S. reserves.

[11]Like the U.S., world production data are available from 1859 forward. However, world reserves data are not available prior to 1948. World production data are from *World Oil* (the August "Annual Review Issue"); data prior to 1956 are from the August 15, 1956 (pp. 145-147) "World Crude Oil Production, by Country, by Years", and World reserves data, 1952-2008, is from the *Oil and Gas Journal*'s annual "World Production Report." World reserves data for 1948-1951 are from the American Petroleum Institute's "Basic Petroleum Databook" (August 2009).

[12]World oil production has fallen relative to the previous year in 31 of the past 150 years. The most sustained declines occurred in 1980-1983 and in 1930-32. Since 2000, there have been 3 years (2001, 2002, and 2007) in which world oil production declined. However, there were six years of decline in the 1980s and four years of decline in the 1930s.

every twenty years. Note also that world proved reserves have nearly doubled since 1985. This number is often disputed by peak oil advocates such as Campbell (1997), Deffeyes (2005) and Simmons (2006), who have argued that OPEC members have increased their reserves estimates to increase their share of the quota within OPEC.

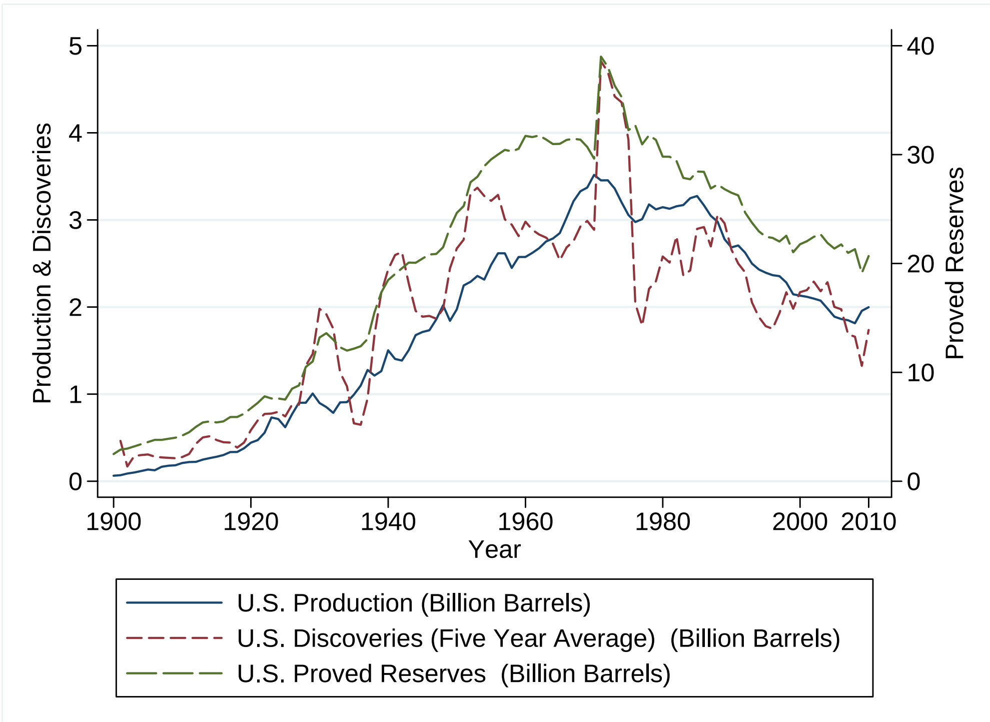


Figure 1: Time Paths of U.S. Crude Oil Production, Discoveries and Proved Reserves, 1900-2010

Deffeyes (2005) also produces a graph of what he calls 'hits' for discoveries of giant oil fields (those greater than 100 million barrels of oil equivalent). Figure 3 shows the pattern of discoveries for a comparable data set, that of so-called 'supergiant' fields (those greater than 500 million barrels of oil equivalent) by region based on data from Horn (2003) updated through 2009. These fields together account for over 60% of world discoveries. World discoveries of supergiant fields peaked in the 1960s. But if one looks at North America, there are several peaks. The one in the 1930s corresponds to when the lower-forty-eight U.S. states peaked (*cf.* Deffeyes, 2005, at p. 138).

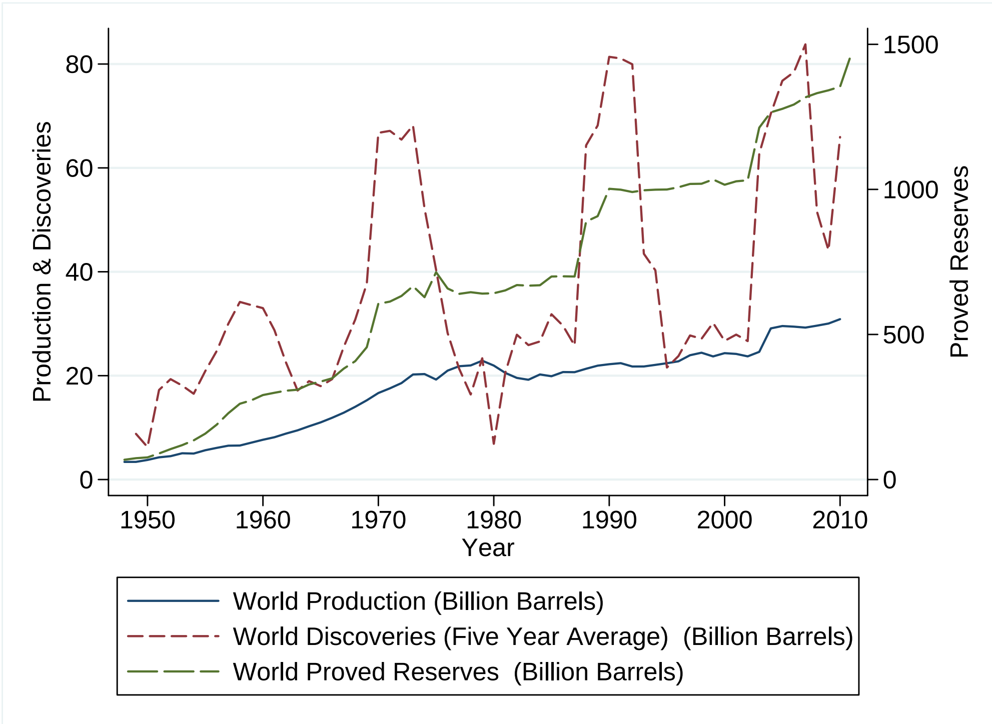


Figure 2: Time Paths World Crude Oil Production, Discoveries and Proved Reserves, 1948-2010

2.2 The HDPO Linear Empirical Specification

Since X_t and C_t are each increasing over time, the quadratic versions of the HDPO model, (2) and (4), are estimated on graphs similar to those Figs. 1-3. But Deffeyes uses the the linear relationship given in (1) and (3) to produce estimates of ultimately recoverable reserves.

Fig. 4 shows the relationship between Q_t/X_t and X_t using annual data from U.S. and world crude oil production, respectively, and Fig. 5 shows the corresponding plots of D_t/C_t and C_t for U.S. and world oil discoveries. The horizontal axis is the cumulative production, X_t, in Fig. 4 and cumulative discoveries, C_t, in Fig. 5, each measured in billions of barrels of oil produced. The vertical axis is the ratio Q_t/X_t in Fig. 4 and D_t/C_t in Fig. 5. The units of this ratio are $1/t$ since both Q_t and D_t are barrels per year while X_t and C_t are measured in of barrels. Observations at each decade are marked with the year. Both time and cumulative production (discoveries) are rising moving from left to right.

Several important features appear in the U.S. production data shown in Fig. 4. First, on average, the ratio Q_t/X_t has been declining as X_t rises. Second, the data show that the relationship between Q_t/X_t and X_t is is highly nonlinear. That is, the rate of decline in Q_t/X_t relative to X_t is much greater when X_t is

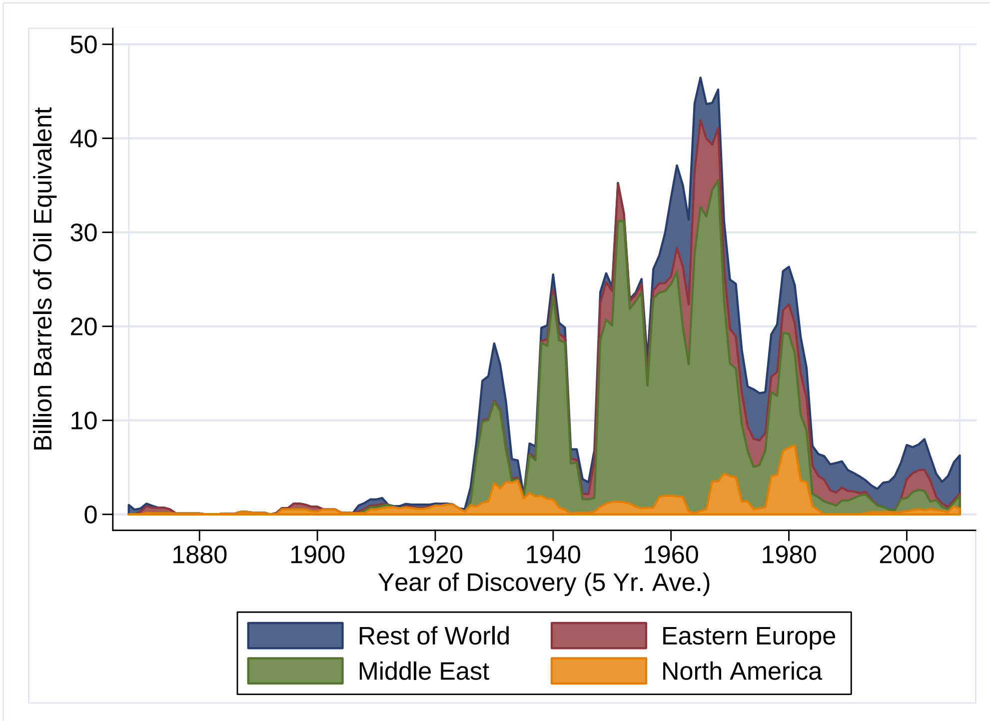


Figure 3: Discoveries from Super-Giant Fields by Region, 1860-2009

small than it is when X_t is large. However, in the U.S., for the period 1958-2008, the rate of decline in Q_t/X_t as X_t increases is closely approximated by the linear regression line corresponding to Deffeyes' "pretty good straight line," which I have fit using Deffeyes' preferred sample of 1958-2010 data. As the peak oil estimate of ultimately recoverable reserves are found by projecting the regression line to the horizontal axis, it is clear that Deffeyes linear prediction of $K = 230$ billion barrels implies an imminent collapse in U.S. production. However, *all* of the U.S. data prior to 1958 lies above Deffeyes' regression line, and data prior to 1930 show a sharp rise in the Q_t/X_t ratio. This rise is even larger than is shown, because data for which $Q_t/X_t > 0.15$ lie above the plot region of the graph — thus all production 1859-1883 is excluded from the graph. According to Deffeyes,

> "Many of the points before 1958 line are above the trend—*not because production was too high, but because it happened too early.*" (2005 p. 37, emphasis added).

Unfortunately, he does not explain either why production before 1958 happened too early or why it is now happening when it should. But if data in the distant past was not in equilibrium, why are we to believe that data in the recent past are no different?

The plot for world oil production in Fig 4(b) again shows Deffeyes' regression line—this time estimated using his preferred sample of 1983 forward. The plot of world production is decidedly less linear than the graph of U.S. production in Fig. 4(a), but it too shows a tendency for all of the data before Deffeyes' preferred sample to lie above the linear regression line, and for values of Q_t/X_t prior to 1930 to increase asymptotically.

The exclusion of data which does not fit their theory is central to the Hubbert-Deffeyes peak oil model. Hubbert (1956), when making his prediction that U.S. production would peak around 1970, omitted all production data prior to 1930. (He never states this explicitly, but any regression using data prior to 1930 results in an estimated value of ultimately recoverable reserves, $\hat{K}$, that is substantially lower than 170 billion barrels, which implies an earlier predicted date of the peak.) Thus, to predict the peak in U.S. production in 1970 Hubbert used data between 1930 and 1960, excluding the 70% of production data before 1930. To predict U.S. ultimately recoverable reserves, Deffeyes uses data from 1958 forward, excluding the hundred years of data before 1958 (66%). Ironically, Deffeyes, when estimating $\hat{K}$, omits the 1930-1960 data upon which the central claim to scientific prediction – the peak in U.S. production in 1970 – that the HDPO model rests.

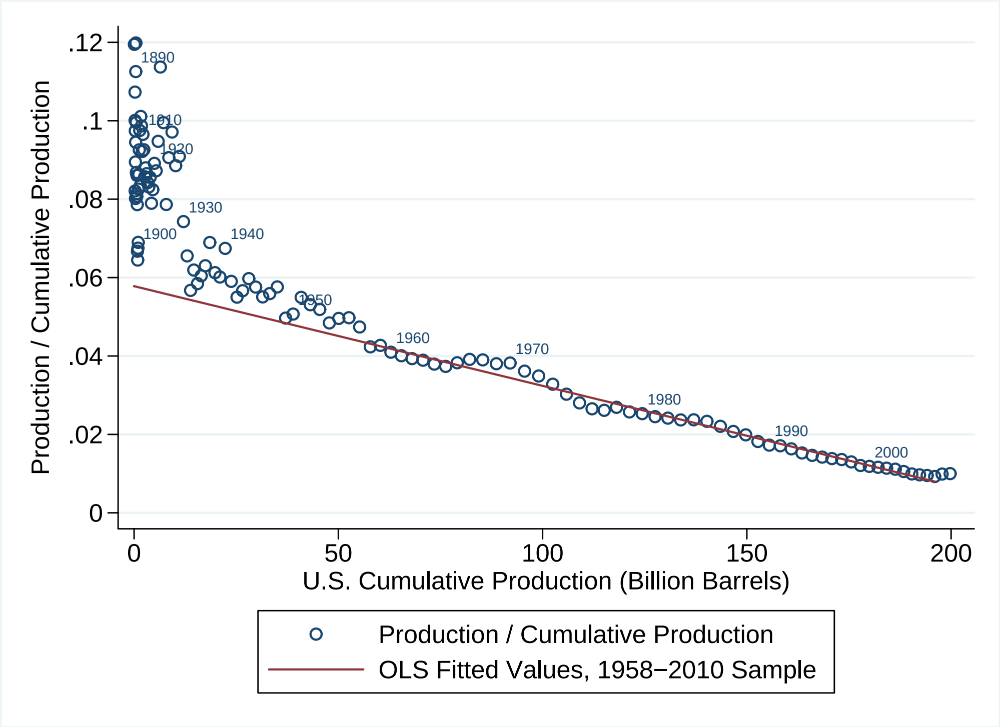
To see the problems this causes, I simultaneously estimate the HDPO model for the two U.S. samples, using a dummy variable, Z_{1960}, whose value is zero before 1960 and one after 1960, to allow for different intercept and slope estimates across the samples. The regression results (with Newey-West lag 5 standard errors in parentheses) and reserves in trillions (10^{12}) of barrels is

$$Q_t/X_t = \underset{(.0017)}{.0692} - \underset{(.0415)}{.4171}\ X_t - \underset{(.0023)}{.0112}\ Z_{1960} + \underset{(.043)}{.1615}\ Z_{1960}X_t$$

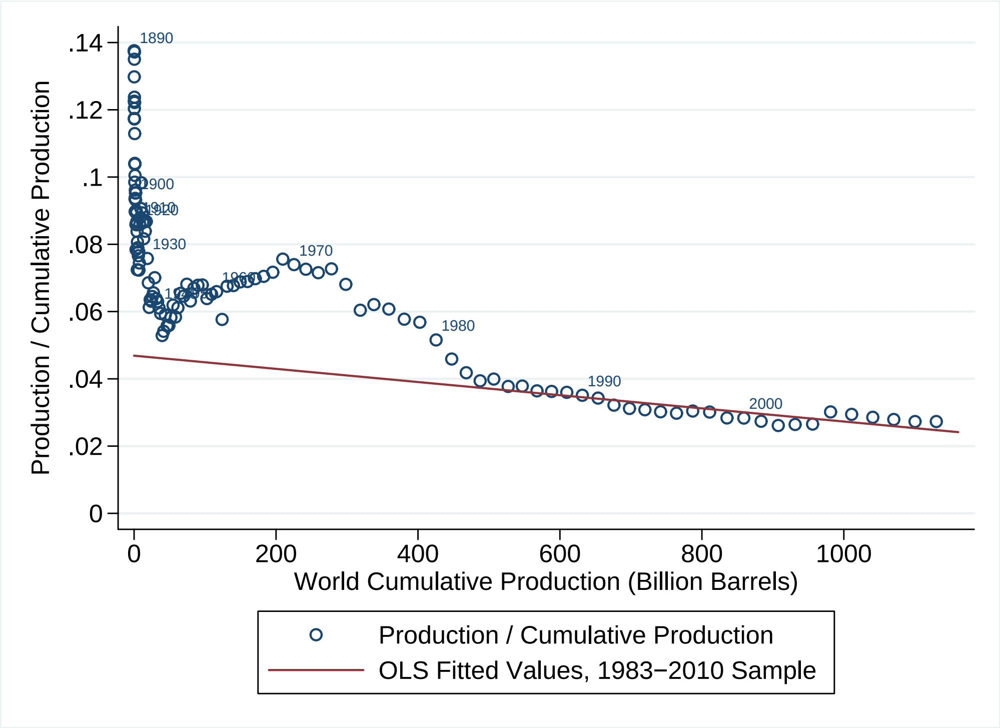
The F-statistic on the test that Hubbert's estimate of ultimately recoverable reserves, $\hat{K} = .0692/.4171 = 165.9$ billion (10^9) barrels, is equal to Deffeyes' estimate of $\hat{K} = (.0692 - .0112)/(.4171 - .1615) = 226.9$ billion (10^9) barrels, is $F(1, 74) = 22.19$, which has a p-value less than 0.01. Thus the data reject that Deffeyes' and Hubbert's predictions of ultimately recoverable reserves for the U.S. are the same.[13] Furthermore, when estimating the HDPO model for the world, Deffeyes not only excludes Hubbert's data, but he also excludes the twenty-three years before 1983 as well. While the normal scientific method is to refine a theory to explain the data, Hubbert and Deffeyes refine the data to fit their theory.

Fig. 5 shows the peak oil analysis of crude oil discoveries for the U.S., 1900-2008, and for the world, 1948-2008, the full periods for which the data are available. Unlike the production data, I do not include a regression line for this data as Deffeyes does not report an estimate from this data. The discoveries data

[13] These results are robust to splitting the sample anywhere between 1956 and 1960.

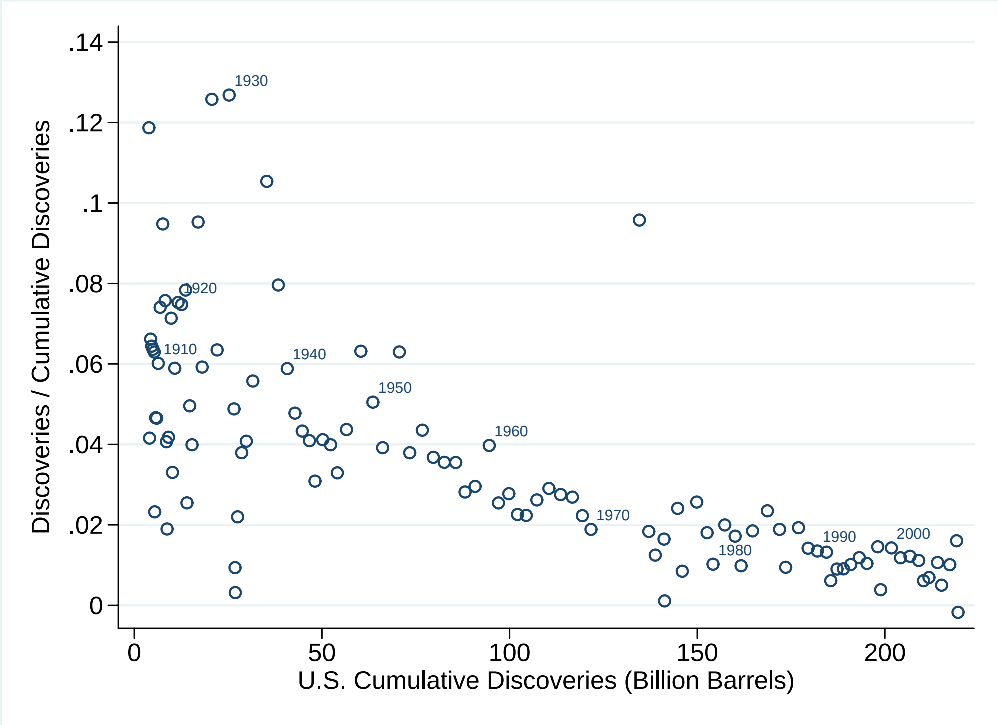


(a) U.S. Crude Oil Production, 1883-2010

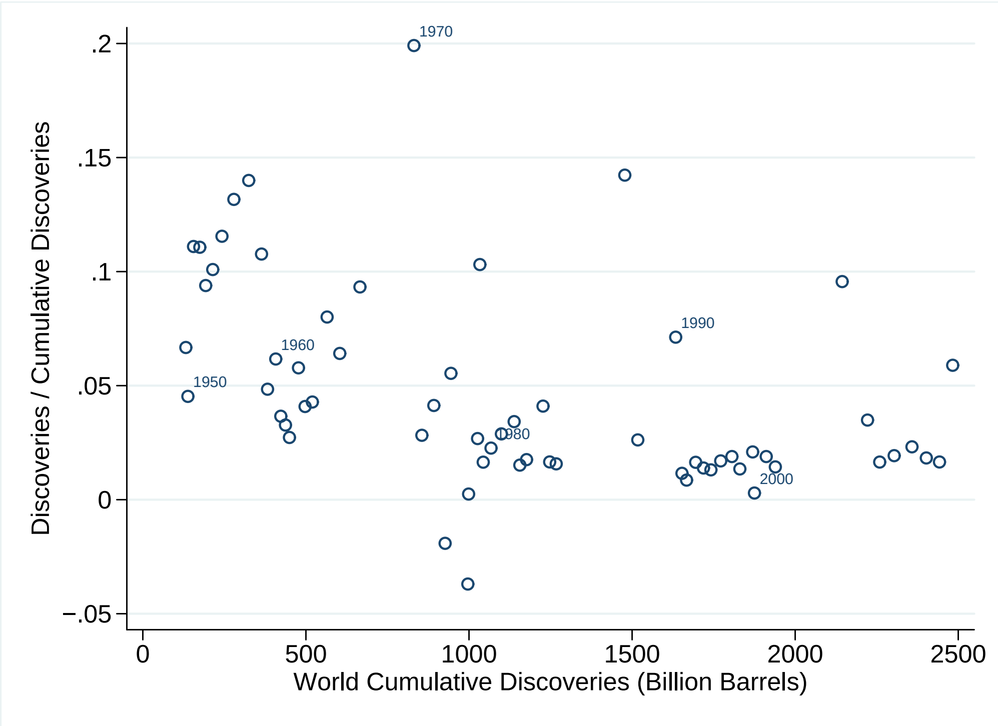


(b) World Crude Oil Production, 1883-2010

Figure 4: Hubbert-Deffeyes Peak Oil Graphs of U.S. and World Crude Oil Production



(a) U.S. Crude Oil Discoveries, 1900-2010



(b) World Crude Oil Discoveries, 1948-2010

Figure 5: Hubbert-Deffeyes Peak Oil Graphs of U.S. and World Crude Oil Discoveries

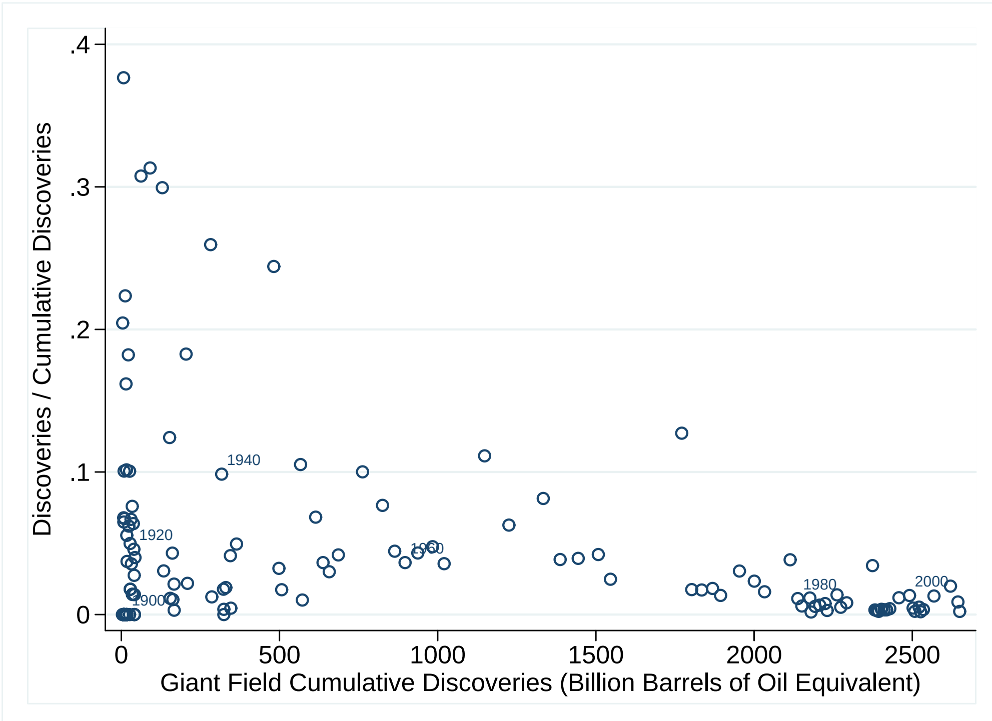


Figure 6: Hubbert-Deffeyes Peak Oil Graph of World Giant Field Discoveries

exhibit much greater variance than the production data in Fig. 4. Fig. 5(a) shows that the addition of the 9 billion barrel Prudhoe Bay field in 1971 equalled almost 10% of U.S. discoveries to that point, and that the 1930 discovery of the East Texas field accounted for an even larger share of cumulative discoveries to that date. In Fig. 5(b), there are 13 years since 1948 in which current world discoveries were greater than 9% of cumulative discoveries to that date. In addition, there are also two years (1973 and 1975) in which world net discoveries are negative due to downward revisions of prior estimates of proved reserves.[14]

As with the production data, there is a negative correlation between the ratio of discoveries-to-cumulative-discoveries, D_t/C_t, and cumulative discoveries, C_t, as is implied by the peak oil model. But, like the production data, the D_t/C_t ratio also appears to have a 'kink' around 1930 in the U.S. data, although the kink is less distinct than with the production data. Data on world discoveries does not extend back into the 1930s, so whether such a kink exists is indeterminate.

A kink is observed, however, in the super giant field discoveries data in Fig.

[14]In 1973 and 1975, net world discoveries, calculated from proved reserve changes, are negative because downward revisions of previous discoveries were larger than claimed new discoveries in those years. There is no instance where this happened with U.S. aggregate discoveries, however, every state and every other country experienced negative net discoveries in at least one year when discoveries are calculated from proved reserves.

6. The super giant field data differs from the U.S. and world discoveries data in Fig. 5 in three ways. First, the super giant fields data uses 2009 estimates of reserves for each field from Horn (2003, as updated), attributing all of the reserves subsequently determined to be on the field to the year in which the field was discovered. As field reserve data are most often revised upwards (e.g., Maugeri (2009), Nehring (2006b,a,c)), a discovery in the super giant fields data include all subsequent revisions as occurring in the year of discovery, while for the data in Fig. 5, the revisions are added in the year the revisions are made. Second, the super giant fields data include only discoveries in which the field has greater than 500 million barrels of oil equivalent. Third, the super giant fields data include natural gas and condensate fields, converting these into oil equivalent units, while the discoveries data in Fig. 5 contains only data on oil and condensate fields.

2.3 The HDPO Data-Generating Process

The key empirical relationships in the linear HDPO model are that the ratio of production-to-cumulative-production has a positive intercept and is linearly decreasing in cumulative production and that the ratio of discoveries-to-cumulative discoveries has a positive intercept and is linearly decreasing in cumulative discoveries. In the quadratic HDPO model, these imply that the curve $Q = r_q X(1 - X/K)$ is bell-shaped. The question is whether there are other data-generating process (DGP) which also have the negative correlation as in the linear HDPO model, and if so, how do the implied ultimately recoverable reserves under these processes differ from the HDPO specification?

At least three other DGPs also yield a negative correlation between Q/X and X. First, suppose that Q_t is simply a *constant*,

$$Q_t = Q_0 + \epsilon_t,$$

where $Q_0 > 0$ and ϵ_t is identically distributed and independently drawn white noise. Then the ratio Q/X is declining in X, since $X_t \approx tQ_0$, since $E(\int_{s=0}^{t} \epsilon_s) = 0$, which implies that $Q/X = 1/t$. This plot, too, settles down to a pretty good straight line as X becomes sufficiently large. But as $t \to \infty$, X is unbounded.

Second, suppose that Q_t grows *exponentially*, so that

$$Q_t = \bar{Q}e^{\phi t} + \epsilon_t \tag{7}$$

at some rate ϕ, where ϕ may be either positive (a growing process) or negative (a declining process), and where $\bar{Q} > 0$. Then $X_t \approx \int_0^t \bar{Q}e^{\phi s}\mathrm{d}s = \frac{\bar{Q}}{\phi}(e^{\phi t} - 1)$. Observe that $X_t > 0$ for all $t > 0$ no matter whether $\phi > 0$ or $\phi < 0$, since the sign of $\bar{Q}/\phi$ and the sign of $e^{\phi t} - 1$ are each negative when $\phi < 0$ and positive when $\phi > 0$. Then $Q_t/X_t = \frac{\phi}{1-e^{-\phi t}}$, implying that $d(Q/X)/dt = \frac{-e^{-\phi t}\phi^2}{(1-e^{-\phi t})^2} < 0$

and that $dX/dt = Q > 0$. Thus, plotting Q/X against X again results in a negative correlation. However, Q/X grows without bound as $X \to 0$.[15]

Third, suppose that Q_t grows according to a *geometric* function, so that

$$Q_t = t^{\theta} + \epsilon_t, \tag{8}$$

for $\theta > -1$, where Q_t is increasing when $\theta > 0$ and is decreasing when $\theta < 0$. In this case, for $X(1) = 1/(1+\theta)$, $X_t \approx t^{(1+\theta)}/(1+\theta)$, and $Q_t/X_t = (1+\theta)/t$. Thus, X_t is increasing over time and Q_t/X_t is decreasing over time at a decreasing rate. Therefore, plotting Q/X against X again results in a negative correlation. Again, Q/X grows without bound as $X \to 0$.

Each of these alternative DGP's have the property that the ratio Q/X is decreasing in X, so each are capable of generating plots similar to those in Figs. 4 and 6. Indeed, each of these, in contrast to the HDPO process, also is consistent with the rise in Q/X as $X \to 0$. But each of these DGP has a very different relationship between Q and X as well. With the constant function, Q is independent of X, and with the exponential or geometric DGP, Q is either increasing or deceasing in X. The linear HDPO DGP, therefore, could be forced to fit data produced by other DGPs with sufficient refinement of the data sample, but these other DGPs have very different implications regarding ultimately recoverable reserves. Furthermore, it is clear that determining which of these fits the data requires looking at both the relationship between Q/X and X and the relationship between Q and X.

2.4 Estimation of the HDPO Model

If the linear specifications in (1) and (3) correctly model the discovery and production process, then by adding white noise errors, ϵ_{qt} and ϵ_{dt}, to the specifications with Q_t/X_t and D_t/C_t as the dependent variables and with X_t and C_t as the single regressor, each equation can be estimated as follows:

$$Q_t/X_t = \beta_{q0} + \beta_X X_t + \epsilon_{qt} \tag{9}$$

$$D_t/C_t = \beta_{q0} + \beta_C C_t + \epsilon_{dt}, \tag{10}$$

where $\beta_{i0} = r_i$, $\beta_X = r_q/K$, $\beta_C = r_d/K$ and ϵ_{it} are statistical residuals, $i = q, d$. The estimate of K from each equation is

$$\hat{K}_{Q/X} = -\hat{\beta}_{q0}/\hat{\beta}_X, \qquad \text{and} \qquad \hat{K}_{D/C} = -\hat{\beta}_{d0}/\hat{\beta}_C, \tag{11}$$

[15] Important economic examples of exponential output equations are the Hotelling (1931) model with linear demand and constant marginal cost and growth models such as Stiglitz (1974). In both cases, oil production exponentially declines over time.

where $\hat{\beta}_{i0}$, $i = q, d$, and $\hat{\beta}_X$ and $\hat{\beta}_C$ are the estimated coefficients from (9) and (10), and the subscripts on K refer to the dependent variable of the equation from this estimate of K is derived. This is the method used in Deffeyes (2003, 2005). Throughout, I refer to these as the linear "production" and "discoveries" models, although the dependent variable in each is the ratio of production-to-cumulative-production or discoveries-to-cumulative-discoveries.

An alternative specification of the HDPO model are the quadratic equations in (2) and (4). These equations can be estimated as follows:

$$Q_t = \gamma_{q0} + \gamma_X X_t + \gamma_{X^2}(X_t)^2 + \nu_{qt} \tag{12}$$

$$D_t = \gamma_{d0} + \gamma_C C_t + \gamma_{C^2}(C_t)^2 + \nu_{dt}, \tag{13}$$

where ν_{qt} and ν_{dt} are the unexplained residuals. In this specification, the dependent variable is production and discoveries. The parameters estimates in this specification have the interpretations that $\gamma_X = \beta_{q0} = r_q$, $\gamma_{X^2} = \beta_X = r_q/K$, $\gamma_C = \beta_{d0} = r_d$ and $\gamma_{C^2} = \beta_C = r_d/K$. In addition, the parameters γ_{q0} and γ_{d0} are each zero in the quadratic HDPO model. Thus, one advantage of this specification is that the additional test of the hypotheses that $\gamma_{q0} = 0$ and $\gamma_{d0} = 0$ occurs.

One other feature of the quadratic specification is of note. The estimate of K is the solution to a quadratic equation:

$$\begin{aligned} \hat{K}_Q &= \frac{-\hat{\gamma}_X}{2\hat{\gamma}_{X^2}} \pm \frac{1}{2\hat{\gamma}_{X^2}}\sqrt{\hat{\gamma}_X^2 - 4\hat{\gamma}_{q0}\hat{\gamma}_{X^2}}, \\ \hat{K}_D &= \frac{-\hat{\gamma}_C}{2\hat{\gamma}_{C^2}} \pm \frac{1}{2\hat{\gamma}_{C^2}}\sqrt{\hat{\gamma}_C^2 - 4\hat{\gamma}_{d0}\hat{\gamma}_{C^2}}. \end{aligned} \tag{14}$$

Thus, one problem with this specification is that K_Q or K_D may be an imaginary number.

The models in (9), (10), (12) and (13) are estimated using annual data. Cumulative production and cumulative discoveries are based on January 1 estimates in year t, while production and discoveries are those that occur January 1-December 31 in year t. Because the data is highly autocorrelated across time and perhaps subject to unknown heteroskedasticity, the models are estimated using the Newey-West (Bartlett kernel) estimator, with a bandwidth of five. This estimator provides point estimates of the slope and intercept parameters which are identical to those from ordinary least squares, but the standard errors for these estimates are robust for arbitrary heteroskedasticity and up to fifth order autocorrelation. One asterisk indicates statistical significance at the five percent confidence level ('*') and two asterisks indicates statistical significance at the one percent confidence level ('**').

2.5 The HDPO Hypothesis Tests

I report the results of four types of hypothesis tests. For each test, I report the two-sided significance level of the test statistic.

First, I report the estimated coefficients of the parameters in (9) and (10). The HDPO model implies:

HDPO Hypothesis 1. *(i) In the linear production and discoveries models, the intercepts β_{q0} and β_{d0} are each positive and the slope parameters β_X and β_C are each negative; (ii) in the quadratic production and discoveries models, the intercept parameters γ_{q0} and γ_{d0} are each zero, the slope parameters γ_X and γ_C are each positive, and the slope parameters γ_{X^2} and γ_{C^2} are each negative.*

For the parameters other than γ_{q0} and γ_{d0}, a rejection of this hypothesis occurs either when a sign is wrong according to the HDPO model or when the coefficient is of the correct sign but is not statistically different from zero. For the parameters γ_{q0} γ_{d0}, a rejection of this hypothesis occurs when the coefficient is statistically different from zero.

The condition under which Q/X is decreasing over time is given by

$$\frac{d(Q/X)}{dt} = \frac{Q}{X}\left[\frac{\dot{Q}}{Q} - \frac{Q}{X}\right] < 0. \tag{15}$$

Thus, so long as the rate of change in Q, $\dot{Q}/Q$, is less than the rate of change in X, $\dot{X}/X = Q/X$, this will be decreasing over time. This means that the growth in production must be less than growth in cumulative production to satisfy HDPO Hypothesis 1. Since this relationship occurs in a large number of DGPs, I expect this hypothesis to be broadly supported, except in times of extraordinary growth in production, such as occurred in the U.S. at the beginning of the 20$^{\text{th}}$ Century.

For each specification, I also report a test that the estimate of ultimately recoverable reserves, $K_{Q/X}$, $K_{D/C}$, K_Q or K_D, is equal to currently observed cumulative production, X_T, or cumulative discoveries, C_T:

HDPO Hypothesis 2. *Ultimately recoverable reserves are at least as large as observed cumulative production and discoveries: $K_{Q/X} \geq X_T$, $K_Q \geq X_T$, $K_{D/C} \geq C_T$, and $K_D \geq C_T$.*

In the tables below, I report the estimated values of the ratios K/X_T and K/C_T, which are expressed in percentage terms, so that $K_{Q/X}/X_T = {}^{**}85$, for example, implies that the point estimate of ultimately recoverable reserves from the linear specification is 85 percent of currently observed cumulative production, and is statistically different from zero at the 1% confidence level. Out-of-sample predictions on K that are statistically less than X_T or C_T are taken as evidence that the HDPO model fails to predict estimated ultimately recoverable reserves, since ultimately recoverable reserves are at least as great as cumulative production or discoveries. While the linear models always produce a value of K except when the slope coefficient is exactly zero, the quadratic models may not predict a value of $\hat{K}$ at all if the estimate of $\hat{K}$ is an imaginary number. Failure to produce a value of $\hat{K}$ is also interpreted as a rejection of HDPO Hypothesis 2.

Third, the HDPO model predicts that the linear relationships in (1) and (3) or the quadratic relationships in (2) and (4) capture the essential features of the data. The simplest test of this hypothesis is the following:

HDPO Hypothesis 3. *The inclusion of higher powers of X_t or C_t, into the linear or quadratic production and discoveries models has no statistical effect.*

For each linear regression, I report the test statistic and its statistical significance for the test of the null hypothesis that $\beta_{X^2} = 0$ in the augmented linear production model, $Q_t/X_t = \beta_{q0} + \beta_X X_t + \beta_{X^2} X_t^2 + \epsilon_{qt}$, or that $\beta_{C^2} = 0$ in the augmented linear discoveries model, $D_t/C_t = \beta_{d0} + \beta_C C_t + \beta_{C^2} C_t^2 + \epsilon_{dt}$. This tests whether the linear specification is rejected for each sample. Similarly, for each quadratic specification, I report a test of the hypothesis that $\gamma_{X^3} = 0$ in in the quadratic production model, $Q_t = \gamma_{q0} + \gamma_X X_t + \gamma_{X^2}(X_t)^2 + \gamma_{X^3} X^3 + \nu_{qt}$, and a test of the hypothesis that $\gamma_{C^3} = 0$ in the quadratic discoveries model, $D_t = \gamma_{d0} + \gamma_C C_t + \gamma_{C^2}(C_t)^2 + \gamma_{C^3} C^3 + \nu_{dt}$. These tests do not exhaust the universe of possible alternative models to the HDPO model, but a rejection of the null hypothesis is taken as evidence that the HDPO model is misspecified.

Fourth, I also report several cross-equation tests between the quadratic and linear specifications and between the production and discoveries models:

HDPO Hypothesis 4. *(i) The linear and quadratic regression parameters are equal: $\beta_0 = \gamma_X$ and $\beta_X = \gamma_{X^2}$ in the production models and $\beta_0 = \gamma_C$ and $\beta_C = \gamma_{C^2}$ in the discovery models; (ii) the linear and quadratic regression estimates of ultimately recoverable reserves are equal: $K_{Q/X} = K_Q$ and $K_{D/C} = K_D$; and (iii) the estimates of ultimately recoverable reserves are equal across discoveries and production specifications of the linear and quadratic regressions: $K_{Q/X} = K_{D/C}$ in the linear specification and $K_Q = K_D$ in the quadratic specification.*

The cross-equation hypotheses are tested using seemingly-unrelated regression methodology, which estimates the the different specifications simultaneously.[16]

These hypothesis form the empirical basis for evaluating the HDPO model.

[16]Due to a limitation in Stata's **sureg** program, however, these estimates are not corrected for autocorrelation or heteroskedasticity, as are the Newey-West estimates reported elsewhere. Thus, the tests of HDPO Hypotheses 1-3, these test statistics are based on inefficient estimators.

3 Analysis of the HDPO Model Using Production Data

I now turn to the empirical analysis. Because most of the peak oil estimates are about U.S. and world crude oil production, I begin with that data.

3.1 Crude Oil Production from the U.S. and World

Table 1 presents the results for the HDPO production model using U.S. and world crude oil production data on samples ending in 2010: 1859-2010, 1900-2010, 1931-2010, 1958-2010, and 1983-2010. The 1931 break is approximately where the 'kink' in the U.S. Q/X graph occurs; 1958 is the first year in Deffeyes' HDPO model fit for U.S. data, while 1983 is the first year Deffeyes chooses to fit the HDPO model using world data. This analysis examines how the sample selections chosen by Hubbert and Deffeyes affect their estimates of ultimately recoverable reserves. Reported are both the linear and quadratic HDPO models.

In the linear specifications, the predictions of HDPO Hypothesis 1 are found to occur and to be statistically different from zero in all ten samples. In the quadratic specifications, however, nine out of ten samples have a positive intercept, and in the 1983-2010 samples for both the U.S. and the world, the estimated slope coefficients have the wrong signs according to the HDPO model and 3 out of 4 are statistically significant. Regarding HDPO Hypothesis 2, only the U.S. linear sample beginning in 1859 produces an estimate of $\hat{K}_{Q/X}$ that is statistically less than X_{2010}. In the quadratic specification, however, the samples starting in 1983 for both the U.S. and the world cannot produce an estimate of $\hat{K}_Q$, since the point estimates result in an imaginary number. HDPO Hypothesis 3 is rejected in 8 of 10 samples. HDPO Hypothesis 4 is rejected for all 10 samples.

A supporter of the HDPO model might be satisfied with the U.S. $\hat{K}$ predictions 7 to 17% above current cumulative production and with the world predictions of additional output of between 20 to 111% above current cumulative production. But there should be concern in the 80% rejection rate of HDPO Hypothesis 3, that 90% of the quadratic samples rejected a zero intercept, and that both components of HDPO Hypothesis 4 are rejected in every sample.

Next, let us consider how the HDPO model does when forced to predict out-of-sample. Table 2 presents results for the HDPO production model using U.S. and world crude oil production data on samples which begin in 1859 in Panel A, and in 1900 in Panel B. In panel A the sample periods are 1859-1899, 1859-1930, 1859-1960, and 1859-1990, while in Panel B the four sample periods anchored instead at 1900.[17]

[17] The 1859-2008 and 1900-2008 samples in Table 1 are repeated in Panels A and B, respectively, of Table 2 to see the effect of changing samples.

In the linear specification, only the 1900-1930 U.S. and world samples reject HDPO Hypothesis 1. In the quadratic specification, the slope parameters reject HDPO Hypothesis 1 in 6 of 18 samples, including all samples ending in 1930 and for the world also the samples ending in 1960. Rejection of HDPO Hypothesis 1 is expected only during times of extraordinary expansion of production. This clearly is the case during 1900-1930, where U.S. production rose from 63 million barrels in 1900 to just over a billion barrels in 1929, a thirteen-fold increase in three decades (9% average annual growth). Similarly, world production increased ten-fold over this period. In the quadratic specifications, the parameter γ_0, which is expected to be zero by HDPO Hypothesis 1 are found to be different from zero in 80% of the samples. Regarding HDPO Hypothesis 2, in the linear specification, 8 of 10 samples ending 1960 or earlier and 3 of 4 samples ending in 1990 predict that $\hat{K}$ is less than or equal to cumulative production in 2010 for both the U.S. and the world. Similarly, in the quadratic specification, 4 of 5 samples ending in 1960 or earlier for the U.S. and 5 of 7 samples ending in 1990 or earlier for the world predict that $\hat{K}$ is less than cumulative production in 2010. HDPO Hypothesis 3 is rejected in all of the linear specification samples and in 8 of 10 of the quadratic samples starting in 1859, and in 9 of 15 linear and quadratic samples beginning in 1900. HDPO Hypothesis 4 is rejected in every sample as well.

The rejection of HDPO Hypothesis 2 that $K \geq X_T$ in 21 of 28 samples ending 1990 or earlier, and the rejection of HDPO Hypothesis 4 that $K_{Q/X} = K_Q$ in 11 of 18 samples overall suggests that the HDPO model does poorly predicting out-of-sample. Furthermore, considering the four HDPO Hypotheses together, in Tables 1 and 2 together, not one of the 56 samples satisfies all four HDPO hypotheses.

3.2 Crude Oil Production from 24 U.S. States and 44 Countries

Tables 3 and 4 report the estimates of the HDPO model on crude oil production for 24 U.S. states and 44 countries, respectively. For each state or country, regressions using the linear and quadratic specifications are run using all available data from 1859 to 1972 to estimate ultimately recoverable reserves.[18] For each specification, the results of the three HDPO Hypothesis tests are reported, where X_T in HDPO Hypothesis 2 is cumulative production through the year 2008. Also reported are cumulative production through 2008, and whether the country has ever been a member of OPEC. For those countries (no states had this issue) for which production began in the 1970s or later, out-of-sample predictions are based on data through a later year such as 1990. I exclude all Eastern bloc countries

[18] The year 1972 is generally believed to be the last year in which the U.S. government-run prorationing cartel remained the major influence over world oil prices.

for whom no separate production data existed prior to the break-up of the Soviet Union in 1991, to ensure that the out-of-sample prediction is at least 20 years into the future.

In the linear specifications, no state or country yields parameter point estimates that are incorrect in sign relative to the prediction in HDPO Hypothesis 1. However, 18 of 24 states and 23 of 44 countries have one or more coefficients which are not statistically different from zero. In the quadratic specifications, the intercept, γ_{q0}, is statistically different from zero in 10 of 24 states and in 21 of 44 countries, and while one or both of the γ_X and γ_{X^2} coefficients are either incorrect in sign or statistically insignificant in 4 out of 24 U.S. states and in 21 of the 44 countries the γ_X and γ_{X^2} coefficients that are either incorrect in sign or statistically insignificant. Overall, many more states and countries than expected reject HDPO Hypothesis 1.

In the linear specification, all 24 U.S. states and all but 4 of the 44 countries have an out-of-sample prediction that rejects the null of HDPO Hypothesis 2 that $\hat{K} \geq X_T$ in favor of the alternative hypothesis that $\hat{K} < X_T$, and not one state or country produces an out-of-sample prediction of K which is in excess of X_T. In 7 countries, furthermore, the estimate of $\hat{K}$ is positive only because both coefficients are incorrect in sign. In the quadratic specifications, 11 of 24 states produce an estimate of $\hat{K}$ that is statistically less than X_T, although one (Ohio) is based on both coefficients being incorrect in sign. For the U.S., only Arkansas produces an imaginary number estimate of $\hat{K}$, but this occurs for six countries (Burma, Ecuador, India, Mexico, Oman and Syria). In the quadratic specification 36 of 44 countries produce an estimate of $\hat{K}$ which is less than observed cumulative production, X_T, in 2008, or is an imaginary number. Thus, the HDPO model fails to produce reasonable out-of-sample estimates of K in 37 of 48 U.S. states and in 76 of 88 countries.

In the linear specification, 9 of 24 U.S. states and 23 of 44 countries reject the null of HDPO Hypothesis 3, and in the quadratic specification, 18 of 24 U.S. states and 18 of 44 countries reject the null. HDPO Hypothesis 4 is also soundly rejected: 19 of 24 states and 33 of 44 countries reject the null that of $K_{Q/X} = K_Q$, and only two states (Arkansas and Pennsylvania) and one country (Congo) fail to reject the null hypothesis that the estimated coefficients are equal in the linear and quadratic specifications, but both states reject that $\gamma_0 = 0$.

In the linear specification, none of the 24 states or 44 countries fails to reject one or more of the HDPO Hypotheses, and in the quadratic specification, only Illinois and New Zealand fail to reject at least one of the HDPO Hypotheses. Thus, the disaggregated data offer even less support for the HDPO model than does the aggregated data reported in Table 1.

3.3 Crude Oil Production from 15 U.S. Oil Fields

The concept of a peak in production arose from observations on the time paths of field production, where fields like Spindletop in Texas were so over drilled that production began to decline after the second year. Thus, I now turn in Table 5 to an analysis of 15 U.S. fields which combine production data from 1913-1952 published in Zimmermann (1957) with data from 1979-1998 on U.S. fields over 100 million barrels in size from the *Oil and Gas Journal*.[19] Production data between 1913 and 1952 is used to estimate the HDPO model and to estimate $\hat{K}$, and the latter is compared with observed cumulative output through 1998. This test is in the spirit of Nehring (2006a,b,c) and Maugeri (2009), who each examined production from specific fields and basins in Texas and California.

The estimates of the HDPO parameters in the linear specification reveal that in only 6 of the 15 fields do both regression parameter estimates satisfy HDPO Hypothesis 1, although all point estimates are of the HDPO expected sign. In the quadratic specification, only 3 of 15 fields satisfy all three requirements of HDPO Hypothesis 1 (i.e., that $\gamma_0 = 0$, $\gamma_X > 0$ and $\gamma_{X^2} < 0$), and for the Haynesville field the signs on the slope coefficients are statistically significant and incorrect. Thus, HDPO Hypothesis 1 is rejected in 21 of 30 specifications. HDPO Hypothesis 2 is rejected by 14 fields in the linear specification and by 13 fields in the quadratic specification, with only the Homer field in Louisiana producing an estimate of $\hat{K}$ that is greater than observed cumulative production through 1998. In addition, HDPO Hypothesis 3 is rejected in 11 of the linear specifications and in 12 of the quadratic specifications, including the Homer field. Thus, not one of the fields fails to reject one or more of HDPO Hypotheses 1-4. Therefore, the field level analysis of the HDPO model reveals the same problems as observed at higher aggregations: a forcing of a linear model onto inherently non-linear data.[20]

4 Analysis of the HDPO Model Using Discoveries Data

This section reports tests of the HDPO model hypotheses using discoveries data. I begin with discoveries data based on contemporary estimates of proved reserves at first for the U.S. and the world and then for a number of U.S. states and international countries, and then examine discoveries data on super-giant fields which uses 2009 estimates of reserves on each field rather than the estimates at the time of discovery.

Discoveries data from proved reserves are calculated as the change in cumulative discoveries, $D_t = C_t - C_{t-1}$, where cumuative discoveries are the sum of

[19] See the *Oil and Gas Journal*, "Forecast and Review" issue. This data was discontinued after 1998.

[20] These results also suggest that increases in production came from improvements to technology, since the median HDPO $\hat{K}$ from data 1913-1952 was about 30% of actual production through 1998.

cumulative production and proved reserves, $C_t = X_t + R_t$. Peak oil advocates typically ignore this data, preferring to discuss only the data on giant fields. They argue that OPEC countries have an incentive to overstate reserves since production quotas are based on reserves, and that the discoveries data from proved reserves is too noisy.[21] There are, however, at least two reasons to consider this data. First, as the same level of ultimately recoverable reserves underlies both the discovery and the production processes in the HDPO model, it interesting to see if the discoveries data produces a similar estimate of ultimately recoverable reserves as does the production data and whether the same problems as those identified for production data arises when using discoveries data. Second, as both production and exploration decisions in every year are based upon current knowledge, this data allows one to estimate the production and discoveries equations simultaneously.

4.1 Crude Oil Discoveries from the U.S. and World

Table 6 reports the analysis of the HDPO model on U.S. and world discoveries. Panel *A* reports samples ending in 2010 while panel *B* reports samples beginning in 1900 for the U.S. and in 1948 for the world. In each case, a linear and quadratic specification is estimated.

As with the production data, only 4 of 14 linear specification samples reject HDPO Hypothesis 1. In the quadratic specifications, however, 9 of 14 samples, all 6 of the world samples and 3 of 8 U.S. samples, reject HDPO Hypothesis 1. Thus one or more components of HDPO Hypothesis 1 is rejected in nearly half the samples, including Deffeyes' preferred sample for world production, the 1983-2010 linear sample. For HDPO Hypothesis 2, in only two cases, the 1900-1930 U.S. linear specification (for which the estimate of $\hat{K}$ is negative) and the 1948-1960 world quadratic specification, is the estimated $\hat{K}$ value less than cumulative discoveries in 2010. HDPO Hypothesis 3 is rejected in only 6 of 28 samples, 2 by an estimate of $\hat{K}$ less than C_T and 4 by an estimate of $\hat{K}$ which is an imaginary number. For HDPO Hypothesis 4, Table 6 reports both the tests (*i*) and (*ii*) between the linear and quadratic discoveries specifications (as in Tables 1 and 2) and the tests (*iii*) between the production and discoveries specifications. The tests of equality between the linear and quadratic samples show that one or both of HDPO Hypotheses 4(*i*) and 4(*ii*) are rejected in all samples. But for the tests of HDPO Hypotheses 4(*iii*) between the estimates of K between the production and discoveries equations, the hypothesis is only rejected for all 12 world samples and for 3 U.S. quadratic samples for the U.S.. Overall, the discoveries data satisfy HDPO Hypotheses 1-4 in not a single U.S. or world sample.

Interestingly, the point estimate for the linear 1983-2010 world discoveries

[21] Hubbert (1982), for example, plots U.S. discoveries data with most of the variation removed by presenting only 11 year averages (Fig. 32, p. 92).

sample is over 630% of current estimated world cumulative discoveries (although it is not statistically significant). This estimate would imply ultimately recoverable reserves of over fifteen trillion barrels, a number that is more than five times larger than any published estimates (Salvador, 2005). One can see why Deffeyes does not wish report results of the HDPO model using these data.

4.2 Crude Oil Discoveries from 24 U.S. States and 44 Countries

Tables 7 and 8 report the analysis of the HDPO model using discoveries data calculated from proved reserves data on 24 U.S. states and 44 countries. As with the production data, the out-of-sample tests are conducted using predictions of ultimately recoverable reserves based on samples from 1948 to 1972.[22] This estimate of $\hat{K}$ is compared to cumulative discoveries at the end of 2010.

For the 24 U.S. states in the linear specification, the signs on the estimated coefficients are in agreement with HDPO Hypothesis 1 in all but Alaska and Ohio (where in both cases the point estimate is statistically insignificant), although one or more coefficients are statistically insignificant in 9 states. In the quadratic specification, however, only 4 states (Alabama, Louisiana, Nebraska and New Mexico) have the correct signs and significance levels in accordance with HDPO Hypothesis 1. One or both of HDPO Hypothesis 3 are rejected in 16 of 24 linear samples and in 21 of 24 quadratic samples. Only in Florida (in the linear specification) and Louisiana (in both the linear and quadratic specifications) do all of the HDPO Hypotheses 1-3 fail to be rejected. Yet, even Florida and Louisiana reject HDPO Hypothesis 4 that the quadratic and linear specifications are equal, and only for Alaska and Montana do none of HDPO Hypothesis 4 get rejected.

Of the 44 countries in Table 8, in the linear specification only 8 countries fail to reject HDPO Hypothesis 1. Indeed, in 24 of 44 countries, the slope coefficient β_C on cumulative discoveries is positive in sign, and in 6 countries (Bahrain, Brunei, Congo, Ecuador, Trinidad & Tobago and the United Kingdom), the positive coefficient is statistically different from zero. Furthermore, not a single country in either the linear or quadratic specifications fails to reject one or more of the HDPO hypotheses, and only Canada, Chile and Columbia fail to reject one or more of HDPO Hypothesis 4.

OPEC Incentives and Analysis of the HDPO Model on Discoveries Data

Campbell (1997), Deffeyes (2005) and Simmons (2006) argue that OPEC mem-

[22] While reserves data are available for the U.S. as a whole back to 1900, estimated proved reserve data by state are only available from 1948.

bers have increased their reserves estimates to increase their share of the OPEC production quota, which are in part based on reserves. Ten of the 12 current OPEC members[23] plus 2 previous members (Indonesia and Gabon) appear in Table 8. Thus, 12 of the 44 countries are or have been OPEC members while the remaining 32 countries have never been OPEC members.

In the linear specification, 8 countries fail to reject HDPO Hypothesis 1 and in the quadratic specification only 3 countries fail to reject HDPO Hypothesis 1. Four of these 11 cases are OPEC members. Thus, OPEC members behave according to the HDPO Hypothesis 1 37.5% of the time, while non-OPEC countries behave according to HDPO Hypothesis 1 only 12.5% of the time. In contrast, for HDPO Hypothesis 2 there are 28 rejections in the linear specification and 23 rejections in the quadratic specification. OPEC countries reject HDPO Hypothesis 2 in 16 of 24 specifications while non-OPEC countries reject HDPO Hypothesis 2 in 35 of 64 specifications. HDPO Hypothesis 3 is rejected in 13 of 24 OPEC specifications and in 21 of 64 non-OPEC specifications. Thus OPEC countries fail to reject HDPO Hypothesis 1 with greater incidence than non-OPEC countries, but OPEC countries reject both HDPO Hypotheses 2 and 3 with higher rates of incidence than non-OPEC countries. And as stated above, not a single country failed to reject at least one of the HDPO Hypotheses.

An alternative explanation for the great number of positive signs on β_C (23 of 44) and γ_{C^2} (21 of 44) for the international data relative to the U.S. states (only 1 positive β_C, Alaska, and 6 positive γ_{C^2}) is likely due to the maturity of the U.S. industry relative to elsewhere. Because many countries lag the U.S. in the intensity with which they have developed their petroleum resources, they behave much like the U.S. did during the first thirty years of the 20$^{\text{th}}$ Century, when discoveries were increasing at an increasing rate (see the 1900-1930 sample for the U.S. in Panel B of Table 2).

4.3 Discoveries of 'Super-Giant' Fields

Deffeyes (2005) and other peak oil advocates use evidence from the giant (fields with oil reserves over 100 million barrels) to fit the HDPO model to discoveries data. Table 9 reports the the analysis of the HDPO model on super-giant field (those greater than 500 million barrels of oil equivalent) discoveries, which is the closest to the giant field hits data used by Deffeyes that I could obtain. I analyse the data by region (U.S. and world), by type of field (oil, gas, and oil, gas and condensates fields combined), and by period, reporting samples from 1860-1925, 1860-1950, 1860-1975, and 1860-2009.

For the 24 U.S. samples for each type of field, the signs in the 12 linear specification are as predicted in HDPO Hypothesis 1, although one or both of

[23]United Arab Emirates does not appear in Table 8 as data sources varied in their treatment of these producers. Nigeria also is excluded for lack of sufficient data.

the regression coefficient estimates is statistically zero in 3 of 12 U.S. cases. In the U.S. quadratic samples, γ_0 is statistically different from zero in only one case (1860-1950 sample for all fields) and γ_C is statistically positive in all samples and γ_{C^2} is negative in all samples and statistically different from zero in all but the earliest oil and all fields samples.

For the world samples, the estimated parameter β_C in the linear specification is positive in sign in 5 of 12 cases (and statistically significant in the two early gas cases) and is statistically insignificant in 6 of 12 cases. In the quadratic specifications, there are two instances (both for gas fields) where the parameter γ_0 is statistically different from zero, and in the 1860-1925 samples, both γ_C and γ_{C^2} are insignificant for oil and all fields and are significant but of the opposite sign predicted by the HDPO model for natural gas fields.

HDPO Hypothesis 2 is rejected for the U.S. for all samples ending on or before 1950, for both the linear or quadratic specifications, and for all samples in the linear specification for natural gas. For the world, 9 of 12 samples ending on or before 1950 for both the linear and quadratic specifications reject HDPO Hypothesis 2, but none of the samples in either specification ending after 1950 result in a rejection of HDPO Hypothesis 2. HDPO Hypothesis 3 is rejected in the linear model only for the world in the natural gas samples, but in the quadratic model, it is rejected in 9 of 16 cases and occurs in each field type and for both the U.S. and the world. HDPO Hypotheses 4 can only be tested using the linear versus quadratic specifications, but either the equality of coefficients or equality of estimated K is rejected in half of the samples, with rejections occurring more often in samples using more recent data.

When taken together all 24 U.S. samples reject one or more of the HDPO Hypotheses 1-4, and 23 of 24 of the world samples reject one or more of the HDPO Hypotheses 1-4. Thus, the results on the HDPO preferred data on discoveries rejects the HDPO Hypotheses with greater frequency than does the discoveries data using contemporary estimates of discoveries reported in Tables 6-8.

5 Peak Everything?

'Peak Everything?' is the title of a book by Heinberg (2007), which argues that for many resource commodities, consumption is about to peak. In this section, I consider the application of the HDPO model to resources other than oil. I begin with coal, which has been the subject of recent analysis in Heinberg and Fridley (2010), and then turn to a panel of 78 minerals, and finally to a panel of 21 agricultural goods. I also use the 78 minerals data, supplemented with data from coal and petroleum data, to briefly examine the case by optimists.

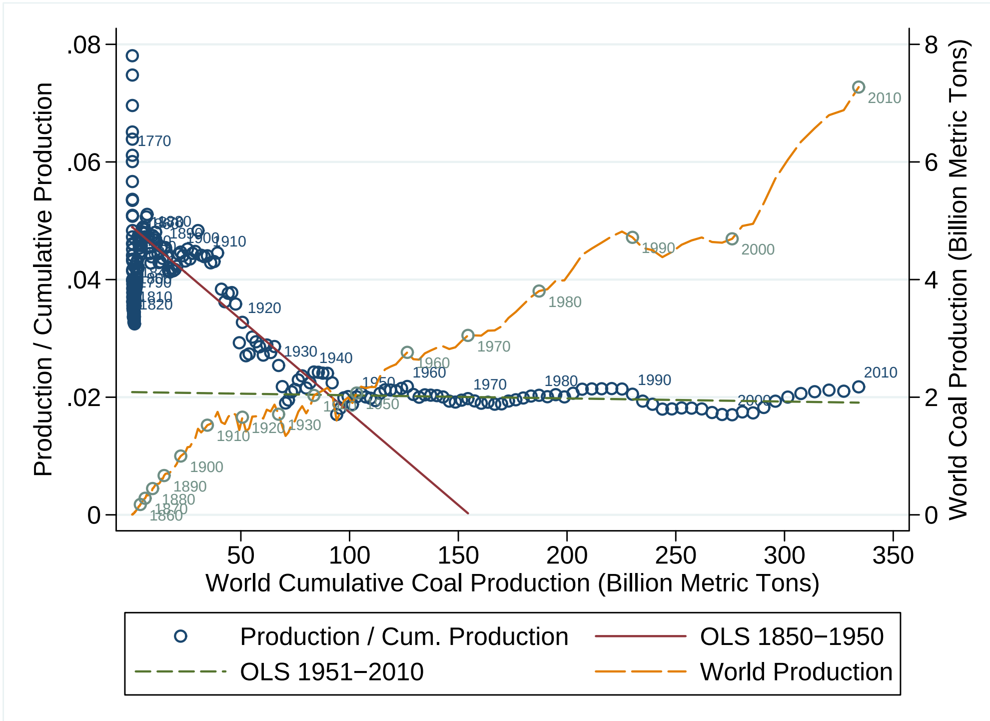


Figure 7: Hubbert-Deffeyes Peak Oil Applied to World Coal Production, 1751-2008

5.1 Coal Production Data

Given the obvious link between Jevon's (1866) concern about coal in Britain and the modern concerns of the peak oil literature, coal is a natural place to study the peak oil model. Coal has also been the subject of peak oil analysis by Hubbert (1956, Fig. 1) and, more recently, by Heinberg and Fridley (2010).

Fig. 7 plots the ratio of production-to-cumulative-production against cumulative production for world coal production from 1751.[24]As with the graphs for crude oil production in Fig. 4, the ratio Q_t/X_t is on average decreasing in cumulative production, X_t. As with oil, there also appears to be a 'kink' in the plot of Q_t/X_t against X_t occurring in approximately 1950.

To an observer like Hubbert (1956) looking at historical coal production data in 1950, at which point about 100 billion (10^9) metric tons of coal had been consumed, and using the HDPO methodology, it might appear that coal was nearing exhaustion. The downward sloping solid line is the predicted path of

[24]Coal data for 1981-2010 are from the 2010 *BP Statistical Review of World Energy*. Coal data from 1751-1980 are from Andres et al. (1999), from which I calculated coal production from "Emissions from solid fuel consumption" production using the average conversion of one metric ton of CO2 equaling 0.5 metric tons of coal for the world, 0.55 metric tons of coal for the U.S. and 1 metric ton of coal for the U.K. These factors are the average for the overlap of data from 1981-2006.

Q_t/X_t as a linear function of X_t based on a regression using data from 1850-1950, which, in keeping with the HDPO methodology, drops about half of the available data. This analysis indicates that exhaustion should occur at about 155 billion metric tons. Yet cumulative production in 2008 was more than twice this, at 300 billion metric tons. As can be seen in Fig. 7, cumulative world coal production has continued to rise well beyond the estimate based on 1850-1950 data. The ratio Q_t/X_t flattened out to around 2% of cumulative production, where it has remained. The second flatter dashed line is the regression using 1951-2010 data. It is has a much lower slope coefficient, which implies a much higher estimate of ultimately recoverable reserves ($\hat{K} = 3,900$ billion metric tons, which is more than twenty-five times larger than the estimate based on 1850-1950 data, and is about four and a half times larger than the World Energy Council (2010) estimate of 861 billion tons of recoverable coal reserves).

Comparing the USGS's estimate of 362 billion barrels of ultimately recoverable U.S. oil to his own estimate of 228 billion barrels, Deffeyes sarcastically remarked,

> "Either that straight line is going to make a sudden turn, or the USGS was counting on bringing in Iraq as the fifty-first state" (2005, p. 36).

What Fig. 7 shows is that the 'sudden turn' did occur for coal. The peak oil model applied to coal using data 1850-1950 produced an estimate of ultimate coal production that was passed only decade or two later. Hubbert did not mention this in his 1982 update.

Table 10 analyzes the HDPO model in greater detail for coal data, using three geographical regions (U.S., U.K., and the world), four sample periods 1751-1850, 1751-1900, 1751-1950, and 1751-2010, and a linear and quadratic specification of the HDPO. For the linear specification, HDPO Hypothesis 1 is rejected due to an insignificant slope coefficient, β_X, 2 of 4 times for the world, 0 of 4 times for the U.K., and 1 of 4 times for the U.S. In the quadratic specification, 3 of 4 samples for each region reject that γ_0 is zero, and in each of the 1741-1850 samples, the coefficient γ_{X^2} is positive and significant, in contrast to the HDPO model. Furthermore, HDPO Hypothesis 2 is rejected in all but the world 1751-2010 sample in the linear specification. In the quadratic specification, HDPO Hypothesis 2 is rejected in half of the samples in total, although not for any of the samples using the full data from 1751-2010. HDPO Hypothesis 3 is rejected in only one sample (world, 1751-2010) in the linear specification, and in 7 of 12 samples in the quadratic specification. Only the 1751-1900 world sample fails to reject one or more of HDPO Hypothesis 4. Finally, all 24 of the samples reject at least one of the HDPO Hypotheses.

I conclude by revisiting the British experience with coal. According to Maddison (2003), in the nine decades before coal peaked in Britain, from 1820 to 1913, British real per capita GDP rose at an annual rate of 1.14%, while in the nine decades after coal production peaked in Britain, British real per capita GDP rose by an annual rate of 1.63% from 1913 to 2003. Thus, in contrast to the predictions of Jevon, the British did much, much more after coal peaked than what they did before it peaked.

5.2 78 Minerals

Next, I apply the peak oil model to the production paths of 78 other mineral resources. Some of these resources differ from oil in that they are recyclable, and all, like coal, have substantially smaller direct world GDP shares than does crude oil. Nevertheless, the peak oil model has been applied to other resources in (e.g., Heinberg, 2007), so it is useful to see whether the problems identified for coal and petroleum also hold in this broader set of minerals.

Tables 11 and 12 report the analysis of the HDPO model on a set of seventy-eight minerals for the period 1900-2008.[25] The analysis uses data for the period 1900-1950 to estimate the HDPO model and to generate a predicted value of $\hat{K}$, which is then compared to the observed cumulative production through 2008. In addition, I also test HDPO Hypothesis 3.

For the linear specifications, HDPO Hypothesis 1 is rejected for only 17 of 78 minerals, and no instances occur where a sign is incorrect according to the HDPO model and is statistically significant. In the quadratic specifications, however, 58 of the 78 minerals reject HDPO Hypothesis 1 that γ_0 is zero, and there are several instances (asbestos, gallium, iron-oxides, mica flake, peat, rhenium, and talc) where the slope coefficient signs are incorrect according to the HDPO model and are statistically significant. Indeed, only bromine, cadmium, molybdenum, pumice, and strontium (5 of 78) have the HDPO expected signs and significance for all three parameters. HDPO Hypothesis 2 is rejected for all 78 minerals in the linear specification, and in the quadratic specification, $\hat{K}$ is statistically less than X_T in 38 of 78 cases and in and additional 25 cases $\hat{K}$ cannot be calculated because the point estimates yield an imaginary number. HDPO Hypothesis 3 is rejected in 62 of 78 cases in the linear specification and in 25 cases (plus one case, thallium, where the test could not be conducted due to multicollinearity) in the quadratic specification. Only 6 minerals fail to reject HDPO Hypothesis 4(i) regarding $K_{Q/X} = K_Q$, and only chromium and gravel industrial fail to reject HDPO Hypothesis 4(ii) that the estimated coefficients are not equal across the quadratic and linear specifications, and for gravel the intercept in the quadratic specification is statistically different from zero. Not a single mineral satisfies all of HDPO Hypotheses 1-4. This is evidence that the problem of non-linearity in the peak oil model is not confined to energy resources.

[25] Data for 76 non-fuel minerals is from Kelly and Matos (2007). Data for natural gas and uranium is from the 2009 *BP Statistical Review of Energy.* Natural gas production is in millions of cubic feet. Uranium production is in millions of pounds. Nuclear electricity production is converted into uranium production using the 1994-2006 average rate of 14.36 terra-Watt hours (10^{12} Watt hours) per million pounds of uranium calculated from *Energy Information Agency* data.

5.3 21 Agricultural Commodities

In this section, I show what happens when the HDPO model is applied to data for which the data generating process is undoubtedly different from the HDPO assumption. With agricultural production, there is no reason to suppose that cumulative production is bounded at all. In the linear HDPO model, an unbounded process is identified by a coefficient on cumulative production which is statistically equal to zero. The precedent for applying the HDPO model to such a commodity is Pauly et al. (2003), who have applied the HDPO model to fisheries, which, like agriculture production, are potentially sustainable, so there is no reason to expect that cumulative agricultural production to be bounded.

Table 13, applies the HDPO model to twenty-one agricultural commodities, using production data for the U.S. based on data in the *U.S. Historical Statistics*, supplemented by more recent data in the *Statistical Abstract of the United States* to bring the series complete through 2008. I use data between 1800 and 1950 to estimate the HDPO model and then I compare the predicted ultimate cumulative production with observed cumulative production through 2008, and to test whether a higher order terms on cumulative production should be included in the regressions.

The results are very similar to those in the previous tables. For HDPO Hypothesis 1, in the linear specification the coefficient on cumulative production is negative and statistically different from zero in all twenty-one commodities. This implies that the HDPO model identifies each of these commodities as having an upper bound on production. Since output of every one of these commodities except wool has been increasing, this casts doubt on the ability of the HDPO model to identify a sustainable process. The quadratic specification rejects that the intercept is zero for 19 of 21 commodities, and rejects the slope coefficients as having the HDPO predicted signs in 11 of 21 cases, including cattle for which the coefficient on C_t^2 is positive and significant. For HDPO Hypothesis 2, in the linear specification the data reject the hypothesis that $\hat{K} = X_T$ in favour of the alternative hypothesis that $\hat{K} < X_T$ in all 21 commodities; in the quadratic specification, however, this only occurs in 8 commodities. In addition, HDPO Hypothesis 3 is also rejected in 19 of 21 commodities in the linear specification and in 13 of 21 commodities in the quadratic specification. Finally, HDPO Hypotheses 4 are rejected for all 21 commodities. These results imply that the HDPO model is incapable of distinguishing between processes for which cumulative production is fundamentally unbounded and those for which it is bounded.

In short, the non-linearity identified in the petroleum data appear to be a much more universal truth than is the HDPO assumption that the ratio of production-to-cumulative-production is linearly decreasing in cumulative production.

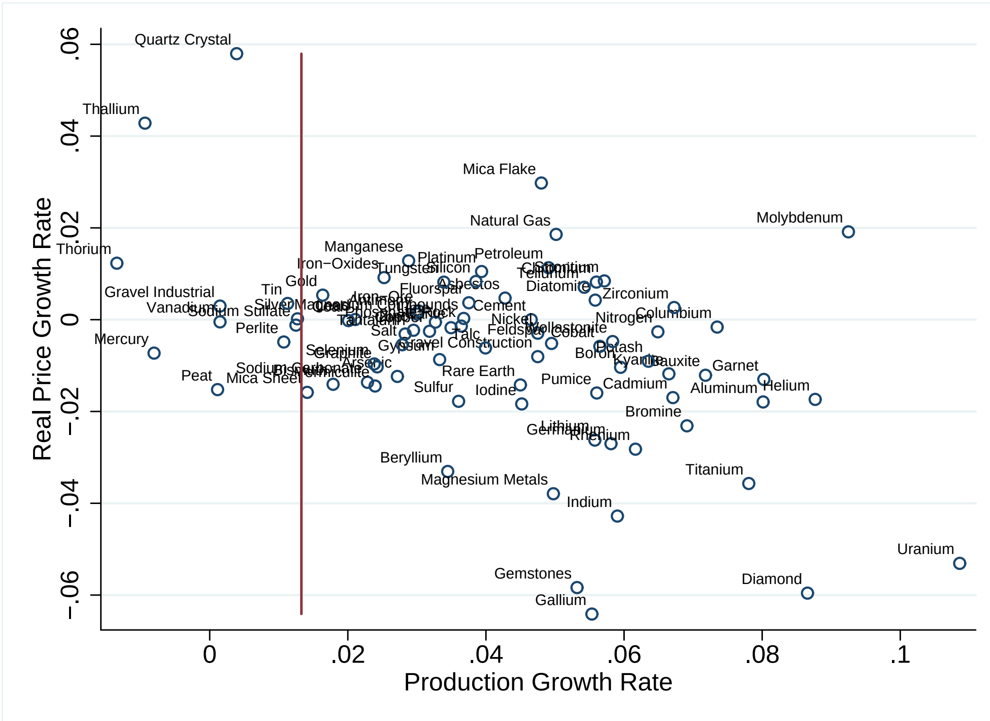


Figure 8: Growth in Real Prices and Quantities, 1900-2007

5.4 The Optimists' Case

I conclude this section by briefly considering why the HDPO model consistently underestimates potential production. Figure 8 shows the average annual percentage growth rate in real prices and in output over the period 1900-2008 for the 78 mineral commodities from Table 11 plus crude oil and coal production. Two important features stand out in the data presented in Fig. 8. First, real prices declined in 51 of 80 minerals (62%).[26] Second, per capita production grew in 62 of 80 (78%) minerals, as is indicated in Fig. 8 by observations to the right of the vertical line, which corresponds to Maddison's estimate of world population annual growth 1950-2003, $n = 1.35\%$. Indeed, in 48 of the 80 minerals (60%), real prices fell while per capita consumption increased.

The only way to get rising output with falling prices is for the supply curve to be shifting faster than the demand curve. This effect is generally associated with technological change. Technological change explains why all of the estimates of ultimately recoverable reserves, including those of peak oil analysts, have been increasing over time (Salvador, 2005). Maugeri (2009) provides a specific example: for the Kern River field in California, after producing 278 million barrels since

[26]This result has been shown for earlier periods by Barnett and Morse (1963) and Slade (1982), among others.

discovery in 1899, estimates in 1942 were that 54 million barrels of oil remained; yet by 1988, an additional 736 million barrels were produced and estimates of remaining reserves were raised to 970 million barrels; and today, after producing over 2 billion barrels, estimates are that 627 million barrels remain recoverable. Nehring (2006c) found similar results for the Permian Basin.

6 Conclusions

The Hubbert-Deffeyes peak oil model predicts that world petroleum production is about to peak and that once this occurs, the pressures of rising prices and declining production will have profound and serious consequences on the world economy. While most observers believe that production must eventually peak—either because reserves are physically exhausted or because a substitute makes them economically unviable—the peak oil model claims that this peak is both imminent and inevitable. Emotionally, the argument is powerful: we have squandered an endowment millions of years in the making in a brief century and a half. But unlike geological methods of estimating potential oil reserves, the HDPO model relies entirely upon using the historical paths of production and discoveries to predict ultimately recoverable reserves.

In this paper I have investigate the empirical robustness of these methods. I argue that the HDPO model attempts to force a linear relationship onto data which are inherently non-linear. The problems this causes manifest themselves in two ways. First, Hubbert (1956) and Deffeyes (2005) are forced to argue that historical data of sufficient time in the past is irrelevant for predicting ultimately recoverable reserves ('it happened too early'). They are forced to do this because estimates of future production based on data sufficiently far into the past produces estimates of ultimately recoverable reserves that are less than observed cumulative production, offering an obvious refutation of their hypothesis. Second, Campbell (1997), Deffeyes (2005) and Simmons (2006) are forced to argue that estimates of proved reserves among OPEC countries are overstated and unreliable. They must argue this in order to defend estimates of ultimately recoverable reserves which are smaller than some estimates of current proved reserves.

I find, however, that the non-linearities in the data undo peak oil model attempts to measure ultimately recoverable reserves. This non-linearity causes a plot of the ratio of production-to-cumulative-production against cumulative production to continually become flatter as cumulative production rises. Thus, HDPO estimates of ultimately recoverable reserves made using data more than 30 years into the past are incapable of predicting ultimately recoverable reserves even as large as currently observed cumulative production or discoveries. Furthermore, the HDPO specification is shown to produce statistically different results using mathematically equivalent linear and quadratic specifications of the model, and estimates using production and discoveries data yield statistically different predictions about ultimately recoverable reserves. These problems with the HDPO model are shown to occur for U.S. and world petroleum production and discoveries, and for discoveries of super-giant fields, as well as for less aggregated production and discoveries data at U.S. states, international countries, and at individual fields levels. Furthermore, these problems are shown to also occur for data on production from other resources, such as coal and a set of 78 minerals. Finally, I show that the HDPO model is incapable of identifying a data for which no logical bound exists on cumulative production, by applying the model to 21 agricultural commodities, where I find that the HDPO model predicts imminent collapse of production of these commodities.

While it is clear that oil production might eventually peak, the evidence I present in this paper suggests that the Hubbert-Deffeyes peak oil model has little, if anything, to say about either the timing of that eventuality or of the magnitude of ultimately recoverable reserves.

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Table 1: HDPO Model for United States and World Crude Oil Production, 1859-2010

	United States, $X_T = 202$ Billion Barrels					World, $X_T = 1,161$ Billion Barrels				
Linear HDPO Model	1859-2010	1900-2010	1931-2010	1958-2010	1983-2010	1859-2010	1900-2010	1931-2010	1958-2010	1983-2010
β_0	0.105	0.079	0.064	0.057	0.054	0.104	0.076	0.068	0.075	0.046
	(0.011)**	(0.004)**	(0.001)**	(0.001)**	(0.003)**	(0.011)**	(0.003)**	(0.003)**	(0.004)**	(0.003)**
β_X	-0.597	-0.403	-0.296	-0.251	-0.231	-0.096	-0.054	-0.043	-0.052	-0.019
	(0.086)**	(0.030)**	(0.010)**	(0.008)**	(0.019)**	(0.018)**	(0.006)**	(0.004)**	(0.006)**	(0.004)**
N	147	111	80	53	27	147	111	80	53	27
$K_{Q/X}/X_T = 100\%$, $\chi^2(1)$	*87.1	97	**107.4	**113.4	**115.8	93.5	*120.2	**136.4	*123.3	**211
$\beta_{X^2} = 0, \chi^2(1)$	**13.5	**39.1	**27.6	1.2	**162	**11.2	*4.4	0	*6.9	**86.5
Quadratic HDPO Model	1859-2010	1900-2010	1931-2010	1958-2010	1983-2010	1859-2010	1900-2010	1931-2010	1958-2010	1983-2010
γ_0	0.097	0.188	0.324	0.561	12.238	0.495	0.824	1.674	7.315	23.192
	(0.029)**	(0.046)**	(0.081)**	(0.351)	(1.175)**	(0.188)**	(0.326)*	(0.759)*	(2.558)**	(3.349)**
γ_X	56.894	54.807	51.811	47.392	-92.915	58.61	57.132	53.488	33.143	-15.803
	(1.579)**	(1.809)**	(2.497)**	(5.887)**	(14.319)**	(6.374)**	(6.581)**	(7.206)**	(9.412)**	(8.653)
γ_{X^2}	-250.485	-241.432	-228.635	-211.136	204.854	-32.299	-31.089	-28.159	-12.994	20.508
	(9.674)**	(10.208)**	(12.605)**	(23.063)**	(43.295)**	(7.250)**	(7.350)**	(7.616)**	(7.690)	(5.244)**
N	147	111	80	53	27	147	111	80	53	27
$K_Q/X_T = 100\%$, $\chi^2(1)$	**113.4	**113.9	**115.3	**116.8	[a]	**157	**159.5	**166.2	237.3	[a]
$\gamma_{X^3} = 0$, $\chi^2(1)$	**26.2	**15.9	*6.9	**9.3	2.4	**64.5	**64.3	**64.2	**29.2	0
Cross-Equation Tests	1859-2010	1900-2010	1931-2010	1958-2010	1983-2010	1859-2010	1900-2010	1931-2010	1958-2010	1983-2010
$K_{Q/X} = K_Q$, $\chi^2(1)$	**261.5	**1482	**6873.4	**14422	**1.5E+11	**103.5	**548.4	**624.3	**840.8	**3.3E+7
$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$, $\chi^2(2)$	**10650	**6019	**2550	**2530	**1496	**3713	**2342	**1331	**360	**316

Notes: The dependent variable is Q_t/X_t, where cumulative production, X_t, is measured at January 1 of each year (10^9 barrels) and production, Q_t, is production from January 1 to December 31 of each year (10^9 barrels). N is the number of observations. The estimator is Newey-West; standard errors (in parentheses) are corrected for fifth-order autocorrelation and for hetoeroskedasticity. '$K_{Q/X}/X_T = 100\%$' and '$K_Q/X_T = 100\%$' report the ratio between the HDPO linear prediction of ultimately recoverable reserves, $K_{Q/X}$ or K_Q, to 2010 cumulative production, $X_T = X_{2010}$, multiplied by 100 so that $K_{Q/X}$ and K_Q are expressed as a percentage of X_T; the asterisks report the results of the null hypothesis that $K_{Q/X} = X_T$ and $K_Q = X_T$; these tests are a Wald test, distributed as a $\chi^2(1)$. '$\beta_{X^2} = 0$' reports the test of the hypothesis that when included the variable X_t^2 is statistically insignificant in the linear HDPO model and $\gamma_{X^3} = 0$ reports the test of the hypothesis that when included the variable X_t^3 is statistically insignificant in the quadratic HDPO model; these tests are a Wald test, distributed as a $\chi^2(1)$. $K_{Q/X} = K_Q$ tests whether the estimated value of K from the linear and quadratic models are equal; these tests are a Wald test, distributed as a $\chi^2(1)$. $\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$ tests whether the estimated coefficients in the linear and quadratic model are equal; these tests are a Wald test, distributed as a $\chi^2(2)$. [a] Estimate of K_Q is an imaginary number. [b] Test of $\gamma_{X^3} = 0$ could not be conducted as X^3 was dropped due to multicollinearity. Statistical significance: ** $p < 0.01$, * $p < 0.05$.

Table 2: Out-of-Sample HDPO Predictions of Ultimately Recoverable Reserves for U.S. and World, 1859-2010

A. Samples Beginning in 1859

	United States, $X_T = 202$ Billion Barrels					World, $X_T = 1,161$ Billion Barrels				
Linear HDPO Model	1859-1899	1859-1930	1859-1960	1859-1990	1859-2010	1859-1899	1859-1930	1859-1960	1859-1990	1859-2010
β_0	0.189	0.132	0.118	0.109	0.105	0.191	0.145	0.123	0.108	0.104
	(0.014)**	(0.017)**	(0.013)**	(0.011)**	(0.011)**	(0.014)**	(0.015)**	(0.013)**	(0.011)**	(0.011)**
β_X	-175.758	-6.708	-1.7	-0.759	-0.597	-88.495	-5.964	-0.923	-0.15	-0.096
	(30.615)**	(2.948)*	(0.414)**	(0.132)**	(0.086)**	(18.062)**	(1.967)**	(0.287)**	(0.032)**	(0.018)**
N	36	67	97	127	147	36	67	97	127	147
$K_{Q/X}/X_T = 100\%$, $\chi^2(1)$	**0.5	**9.9	**34.2	**70.8	*87.1	**0.2	**2.1	**11.5	**62	93.5
$\beta_{X^2} = 0$, $\chi^2(1)$	**9.7	*6.4	**8	**11.5	**13.5	*6.1	**11.9	**14.9	*5.2	**11.2
Quadratic HDPO Model	1859-1899	1859-1930	1859-1960	1859-1990	1859-2010	1859-1899	1859-1930	1859-1960	1859-1990	1859-2010
γ_0	0.003	-0.004	0.03	0.073	0.091	0.002	0.01	0.076	-0.255	0.402
	(0.001)*	(0.005)	(0.012)*	(0.024)**	(0.027)**	(0.001)*	(0.005)	(0.025)**	(0.120)*	(0.176)*
γ_X	120.686	97.642	72.638	60.354	57.703	141.649	83.174	58.259	88.023	61.583
	(13.000)**	(8.077)**	(2.379)**	(1.681)**	(1.464)**	(6.606)**	(6.056)**	(3.760)**	(4.872)**	(6.643)**
γ_{X^2}	-64651	-1036.923	-502.126	-279.499	-256.71	-37150	-79.68	63.507	-87.631	-36.688
	(16309)**	(808.725)	(46.422)**	(12.401)**	(8.661)**	(4977)**	(375.5)	(36.573)	(8.830)**	(8.063)**
N	36	67	97	127	145	36	67	97	127	145
$K_Q/X_T = 100\%$, $\chi^2(1)$	**1	46.5	**71.8	**107.4	**111.9	**0.3	89.9	**-0.1	**86.3	*145.1
$\gamma_{X^3} = 0$, $\chi^2(1)$	[b]	**15.3	0.8	**7.4	**26.2	[b]	*5.1	0.7	0.8	**64.5

B. Samples Beginning in 1900

	United States, $X_T = 202$ Billion Barrels				World, $X_T = 1,161$ Billion Barrels			
Linear HDPO Model	1900-1930	1900-1960	1900-1990	1900-2010	1900-1930	1900-1960	1900-1990	1900-2010
β_0	0.087	0.088	0.082	0.079	0.087	0.082	0.077	0.076
	(0.004)**	(0.003)**	(0.004)**	(0.004)**	(0.004)**	(0.003)**	(0.003)**	(0.003)**
β_X	0.242	-0.87	-0.479	-0.403	-0.285	-0.265	-0.066	-0.054
	(0.686)	(0.097)**	(0.043)**	(0.030)**	(0.309)	(0.081)**	(0.009)**	(0.006)**
N	31	61	91	111	31	61	91	111
$K_{Q/X}/X_T = 100\%$, $\chi^2(1)$	-178.7	**50	**84.2	97	*26.4	**26.7	101.3	*120.2
$\beta_{X^2} = 0$, $\chi^2(1)$	2.1	**8.6	**33.3	**39.1	0.6	**27.1	0	*4.4
Quadratic HDPO Model	1900-1930	1900-1960	1900-1990	1900-2010	1900-1930	1900-1960	1900-1990	1900-2010
γ_0	-0.039	0.077	0.148	0.177	-0.008	0.177	-0.485	0.672
	(0.022)	(0.030)*	(0.042)**	(0.043)**	(0.032)	(0.058)**	(0.219)*	(0.306)*
γ_X	112.033	68.986	58.011	55.696	87.307	53.76	90.061	60.288
	(13.391)**	(2.447)**	(2.009)**	(1.682)**	(11.573)**	(4.253)**	(5.192)**	(6.936)**
γ_{X^2}	-2112	-449.18	-266.021	-247.821	-268.1	99.67	-90.668	-35.565
	(1152)	(42.968)**	(13.345)**	(9.172)**	(621.606)	(38.735)*	(9.236)**	(8.259)**
N	31	61	91	109	31	61	91	109
$K_Q/X_T = 100\%$, $\chi^2(1)$	**26.2	**76.7	**109.4	**112.9	28.1	**-0.3	**85.1	*146.9
$\gamma_{X^3} = 0$, $\chi^2(1)$	**10.7	0.1	3.5	**15.9	**8.8	0	0.4	**64.3
Cross-Equation Tests	1900-1930	1900-1960	1900-1990	1900-2010	1900-1930	1900-1960	1900-1990	1900-2010
$K_{Q/X} = K_Q$, $\chi^2(1)$	**15.7	**61.7	**776.7	**1482.6	1.3	**54.5	**144.5	**103.5
$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$, $\chi^2(2)$	**4017	**10669	**7447	**6019	**1616	**8393	**6638	**3713

Notes: See the notes to Table 1. [a] Estimate of K_Q is an imaginary number. [b] Test of $\gamma_{X^3} = 0$ could not be conducted as X^3 was dropped due to multicollinearity. Statistical significance: ** $p < 0.01$, * $p < 0.05$.

Table 3: HDPO Model for Crude Oil Production in 24 U.S. States, 1859-1972

			Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
			Coefficients		Hypothesis Tests, $\chi^2(1)$		Coefficients			Hypothesis Tests, $\chi^2(1)$		$K_{Q/X} = K_Q$	$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$
State	X_T	N	β_0	β_X	$K_{Q/X}/X_T = 100\%$	$\beta_{X^2} = 0$	γ_0	γ_X	γ_{X^2}	$K_Q/X_T = 100\%$	$\gamma_{X^3} = 0$	$\chi^2(1)$	$\chi^2(2)$
Alabama	0.68	28	*0.718	-7.346	**14.3	2.4	*0.81	**199.5	**-1211.9	**24.7	**171.9	**9.4	**233.1
Alaska	16.43	13	*2.026	-6.686	**1.8	3.2	-0.76	**622.3	**-1113.8	**3.4	0	3.7	**210.9
Arkansas	1.80	51	0.42	-0.418	**55.8	**11	**35.305	-23.2	11.7	[a]	*5.9	[a]	2.5
California	27.68	96	**0.275	**-0.024	**41.8	**11.6	**26.613	**59	**-2.6	**82.6	**7.4	**83.3	**1108.8
Colorado	2.01	85	*0.178	-0.167	**53.2	0	-1.309	**196.7	**-183.7	**53.1	**36.1	1.3	**1277.7
Florida	0.60	46	*0.546	-1.157	**78.3	0.4	-0.122	**394	**-769	**85	**17	*6.3	**631.2
Illinois	3.63	67	1.157	-0.601	**53.1	1.5	-0.212	**88.6	**-25.3	96.5	0.2	*5.4	**64
Indiana	0.56	83	0.432	-1.644	**47	*4.9	1.491	26.2	-8.3	571.9	**27.6	**25.9	**53.3
Kansas	6.25	83	**0.426	**-0.136	**50.1	*6	3.878	**84.5	**-15.5	**88	0.6	**26.8	**1797.9
Kentucky	0.78	87	**0.252	**-0.578	**55.8	*6.1	0.655	**65.9	**-71.2	119.6	**10.1	**37.3	**367.2
Louisiana	29.97	70	*0.246	-0.024	**34.5	2	3.222	**85.9	-0.8	372.9	*4.3	*4.8	**8200
Michigan	1.27	47	2.313	-5.435	**33.6	2.5	*7.39	**59.2	**-92.5	**58.9	**14.3	**21.8	**23.4
Mississippi	2.40	33	3.955	-4.599	**35.9	2.3	**18.402	**64.3	**-23.3	126.2	0.6	**11.8	**118.7
Montana	1.67	56	*0.38	-0.725	**31.5	*4.1	-0.166	**100.3	**-62.8	95.8	**11	**12.7	**829.3
Nebraska	0.50	33	8.227	-35.125	**46.5	1.3	1.131	**277.7	**-831.3	**67.1	**122.6	1.9	**236.6
New Mexico	5.52	48	0.794	-0.423	**34.1	2.5	*8.311	**94.1	**-19.1	90.7	2.4	*6.1	**1447
North Dakota	1.61	21	8.129	-31.525	**16	2.8	**6.355	**189.2	**-422.1	**29.7	**42.8	*6.6	**283.5
Ohio	1.12	96	**0.527	**-0.902	**52.2	**8.1	*8.171	22	-39.1	*73.2	**92.6	**34.9	**10.6
Oklahoma	14.84	71	1.897	-0.271	**47.1	3.4	**65.897	**36.1	*-2.3	117.1	**11.2	**20.2	**90.6
Pennsylvania	1.37	113	8.257	-8.936	**67.4	1.2	**8.787	*21.1	**-18.9	105	3.1	**17.5	4.2
Texas	63.54	76	1.942	-0.086	**35.7	1.3	**58.536	**78.8	**-1.4	88	**45.4	2.1	**761.7
Utah	1.33	35	0.952	-2.827	**25.2	1	2.704	**263.3	**-564.4	**35.7	**27.7	2	**90
West Virginia	0.58	96	**0.316	**-0.76	**71.3	*5.3	3.51	**57.6	**-131.2	**84.6	**37.9	**211.4	**106
Wyoming	7.05	78	**0.369	*-0.156	**33.6	*5.4	-0.371	**87.2	**-12.3	100.6	**8.2	**12.2	**1714.8

Notes: See the notes to Table 1. Standard errors are not reported. $X_T = X_{2008}$ is in billions (10^9) of barrels. [a] Estimate of K_Q is an imaginary number.

Table 4: HDPO Model for Crude Oil Production in 44 Countries, 1859-1972

			Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
			Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{Q/X} = K_Q$	$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$
Country	X_T	N	β_{q0}	β_X	$K_{Q/X}/X_T = 1$	$\beta_{X^2} = 0$	γ_0	γ_X	γ_{X^2}	$K_Q/X_T = 1$	$\gamma_{X^3} = 0$	$\chi^2(1)$	$\chi^2(2)$
Algeria[a]	18.5	58	*0.452	-0.122	**20	0	8.2	**338.7	**-75.9	**24.2	**11.6	1.5	**1133.8
Angola[a]	7.2	13	**0.89	-6.213	**2	*4.4	-0.7	**498.1	-639.7	**10.7	**8.8	*3.9	**119.2
Argentina	9.9	64	**0.339	*-0.24	**14.3	**7.1	1.6	**72.3	4.6	-0.2	**7.5	**21.7	**1706.2
Australia	6.8	8	**1.14	**-2.401	**6.9	0.4	-3.9	**1500	**-4248	**5.1	2.9	2.3	**288
Austria	0.8	37	*0.701	*-1.997	**43.1	*6.2	**3.1	**128.4	**-225.1	**72.9	**13.1	**44.7	**112
Bahrain	1.0	39	1.248	-4.044	**31.2	3.2	**4.1	**54.6	-6.7	837.5	0.3	**16.8	**403
Bolivia	0.5	42	**0.245	-1.673	**27.2	2.2	0	**152.5	-159.3	177.3	0	*5	**241.3
Brazil	10.3	32	**0.86	*-1.805	**4.6	2.5	2.7	**238.7	**-241.3	**9.7	1.6	*4.4	**528.1
Brunei	3.6	59	**0.268	-0.278	**26.5	3.2	-0.3	*114.4	-48	65.3	0.3	**8.4	**32.1
Burma (Myanmar)	0.4	37	0.332	-3.315	**25.7	**22.9	**9.4	**-201	**1440	[c]	0.3	[c]	**37.4
Canada	26.3	110	*0.183	-0.02	34.8	0	0.8	**160.1	**-13	**46.8	**16.3	0.1	**6429
Chile	0.4	22	**0.514	**-3.299	**35.4	**8.6	**1.5	**229.7	**-1016	**52.7	**49.8	**53.7	**390.5
Colombia	6.7	51	*0.665	-0.629	**15.8	*5.8	*7.7	**56.6	-9.7	89	0	**23.3	**288.5
Congo (Brazzaville)	2.1	12	1.502	-22.547	**3.2	0.9	-5.3	4249	-67460	**3	3.7	0.2	3.7
Denmark	2.0	[b]15	**0.876	*-4.956	**8.9	3.5	**1.3	**365.6	**-864.8	**21.4	**8.1	*6.2	**696.3
Ecuador[a]	4.2	55	**0.306	**-3.441	**2.1	**17.6	0.9	16.1	278.6	[c]	3.4	[c]	**10.9
Egypt	10.0	61	0.465	-0.77	**6.1	1.7	0.2	**87.8	22.8	**0	2.6	1.7	**495.4
France	0.9	54	**0.156	-0.291	61.2	0.2	**-1.2	**255.6	**-723.6	**39.8	0.6	1.7	**3121.9
Gabon	3.4	13	**0.474	*-1.429	**9.8	1.3	0.9	**380.4	**-807	**14	0.8	*5.4	**306.5
Germany	2.1	92	**0.179	-0.17	*50.7	0.8	**-1.6	**162.2	**-111.8	**69.2	*5.4	1.2	**5932.7
India	7.4	83	**0.254	*-0.673	**5.1	**13.8	*3.6	-19.8	**136.9	[c]	0	[c]	**60.6
Indonesia[a]	23.0	79	**0.184	-0.053	**15.1	*5.3	**8.5	**28.8	**11.4	**-1.5	0.4	**17.3	**1697.2
Iran[a]	61.3	59	**0.225	-0.018	**21	*4.1	-1	**78.6	**4.4	**0	**8.3	*5.4	**2925
Iraq[a]	30.7	45	**0.414	*-0.064	**21.1	*4.2	6.1	**163.4	**-12.2	**43.7	*4.2	**8	**1685.4
Italy	1.1	107	**0.118	-0.086	121.1	3	-0.1	**281.6	*-1846	**13.4	**15.3	0.2	**175.7
Japan	0.4	97	**0.232	**-1.909	**31.2	**8.8	*0.7	15	70.2	**-15.5	*6.2	**4.7E+9	**269.8
Kuwait[a]	35.2	26	*0.826	*-0.084	**27.8	*6.1	**131.3	**132.2	**-4.8	*80.3	**22.2	**30.3	**777
Libya[a]	26.1	11	*2.888	*-0.596	**18.5	*4.7	**97.9	**522	**-63.5	**32.2	0	**7	**478.1
Malaysia	6.9	[b]26	2.477	-1.67	**21.5	1.6	6	**324.6	**-115.9	**40.8	0.3	2.6	**20.1
Mexico	37.6	71	**0.678	**-0.222	**8.1	**10.9	36.9	6.4	3.9	[c]	1.9	[c]	**50.9
Netherlands	1.0	29	2.269	-12.647	**18.2	*4.2	**1.5	**183.7	**-573.9	**33.3	3.5	**14.4	**1246.1
New Zealand	0.3	[b]54	*0.266	-1.549	51.2	0	0.2	**195.8	-512.7	113.9	0.1	0.4	**1040.4
Norway	21.6	[b]17	**0.754	**-0.286	**12.2	**14.3	**57.1	**141.3	2.7	**-1.9	**28.2	**25.8	**348.3
Oman	8.5	[b]22	*0.929	-0.461	**23.7	*5.8	**89.6	7.3	*16.5	[c]	1.1	[c]	**183.2
Pakistan	0.6	25	**0.456	*-4.371	**16.5	0.8	-2.6	*587.4	*-5121.1	**17.4	2.9	**13	**40.5
Peru	2.5	76	**0.255	**-0.41	**24.9	**6.7	**2.6	**49.9	**-29.5	*69.7	**7.9	**50.7	**563.9
Qatar[a]	9.0	23	2.134	-2.072	**11.5	2.8	**21.6	**102.2	-5.9	196.8	*3.9	**8.7	**588.8
Saudi Arabia[a]	112.9	36	*1.141	-0.122	**8.3	3.3	75.7	**85.1	**3.3	**-0.8	**16	**9.7	**1565.8
Syria	4.6	[b]18	**0.526	*-0.564	**20.2	**10	**57	*-41.9	**78.2	[c]	**8	[c]	**31
Trinidad & Tobago	3.4	63	**0.328	*-0.364	**26.4	*4	*2.2	**76.9	**-31.9	71.5	1.7	**18.7	**279.9
Tunisia	1.4	[b]23	*0.934	-1.623	**42.5	*5.9	**18.3	**91.4	**-91.1	*86.7	0.2	**42	**70.9
Turkey	1.0	24	*0.942	-6.795	**14.5	2.1	-0.3	**386.6	**-1418	**28.3	1.3	2.2	**1448.3
United Kingdom	23.8	53	0.471	-35.414	**0.1	1.5	0.1	**97.7	**-4179	**0.1	0.1	**13.5	**59.9
Venezuela[a]	59.9	55	**0.47	*-0.024	**32.8	**7.6	24.3	**113.9	**-2.5	**75.4	0	**20.9	**6663.9

Notes: See the notes to Table 1. Standard errors are not reported. $X_T = X_{2008}$ is in billions (10^9) of barrels. [a] Current or former OPEC member. [b] HDPO estimation uses data through 1990. [c] Estimate of K_Q is an imaginary number.

Table 5: HDPO Model for 15 U.S. Oil Fields, 1913-1952

					Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
					Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{Q/X} = K_Q$	$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$
Field	State	First Year	X_T	N	β_{q0}	β_X	$K_{Q/X}/X_T = 1$ (%)	$\beta_{X^2} = 0, \chi2(1)$	β_{q0}	β_X	β_{X^2}	$K_Q/X_T = 1$ (%)	$\beta_{X^3} = 0, \chi2(1)$	$\chi^2(1)$	$\chi^2(2)$
Anahuac	TX	1935	288.2	18	1.46	-0.017	**30.6	3	**1384	**285.9	**-2.214	**46.4	1.5	**53.9	**9.5
Conroe	TX	1932	742.4	21	1.75	-0.008	**31.3	*5.4	**10981.2	71	-0.202	**62.9	*4.5	3.6	**15.4
Cushing	OK	1913	508.6	40	**1.23	**-0.004	**58.6	**32.1	**29646.9	-79	-0.011	**70.3	**8.4	**211.2	**7927
Goldsmith	TX	1935	799.3	18	*2.35	-0.023	**12.6	**7.4	**3534.6	*91.7	0.082	**-5	0.2	**70	**2135
Hawkins	TX	1940	865.2	13	309.32	-2.997	**11.9	*4.3	**5800.2	**200.1	-0.972	**26.8	**44.5	1.8	*6.2
Haynesville	LA	1921	177.6	32	1.77	-0.024	**42.1	**16.4	**11663.1	**-268.5	**1.828	-563.1	0	**123	**19316
Healdton	OK	1914	348.7	39	**0.91	*-0.006	**47.5	**29.1	**11563.3	-39.6	-0.054	**64.1	*6.3	**105.8	**13303
Hendricks	TX	1926	256.4	27	*231.03	*-1.176	**76.6	**10.1	*23812.9	98.1	-0.977	**83.5	**35.9	**70.9	0.5
Homer	LA	1919	101.4	34	**5.23	**-0.045	**114.7	**10.2	**18673.8	47.9	-1.648	**120.2	**22.6	**39.2	0.2
Keystone	TX	1935	325.7	18	*1.07	-0.012	**28	2.1	496.2	**406.7	**-3.001	**42	*5.6	**73.9	*5.5
Slaughter	TX	1937	1177.9	16	*3.43	*-0.024	**12	*5.7	4049.7	**344.4	**-1.733	**17.8	**45.8	**26.1	**11.9
Smackover	AR	1922	574.4	31	*5.5	*-0.015	**62.9	**9.2	**35541.4	31.4	-0.287	**71.5	**39.3	**106.9	0.5
Tom O'Connor	TX	1934	787.7	19	2.7	-0.02	**16.9	3.3	**1911.2	**297.6	**-1.411	**27.6	**13.8	**87.6	**7.1
Wasson	TX	1937	1964.1	16	2.5	-0.013	**9.6	*5	**4942.6	**270.8	**-0.891	**16.3	**17.1	**52.9	**13.3
Webster	TX	1937	596.2	16	2.43	-0.018	**23.1	3.2	3161.8	**365.9	**-1.897	**33.7	**75	**34.4	**7.3

Notes: See the notes to Table 1. Standard errors are not reported. $X_T = X_{1998}$ is in millions (10^6) of barrels.

Table 6: HDPO Model for U.S. and World Discoveries, Measured as Net Additions to Proved Reserves, 1900-2010

A. Samples Ending in 2010	United States, $C_{2010} = 219$ Billion Barrels				World, $C_{2010} = 2,483$ Billion Barrels		
Linear HDPO Model	1900-2010	1931-2010	1958-2010	1983-2010	1948-2010	1958-2010	1983-2010
β_0	0.062	0.053	0.048	0.048	0.08	0.059	0.033
	(0.004)**	(0.007)**	(0.004)**	(0.009)**	(0.014)**	(0.014)**	(0.023)
β_C	-0.267	-0.216	-0.186	-0.188	-0.029	-0.016	-0.002
	(0.025)**	(0.043)**	(0.021)**	(0.045)**	(0.009)**	(0.007)*	(0.010)
N	110	80	53	27	62	53	27
$K_{D/C}/C_T = 100\%$, $\chi^2(1)$	105.5	112.3	**116.9	*116.9	112	144.8	631.5
$\beta_{C^2} = 0, \chi^2(1)$	2.3	0	0	3.3	*6.8	2.7	0.4
Quadratic HDPO Model	1900-2010	1931-2010	1958-2010	1983-2010	1948-2010	1958-2010	1983-2010
γ_0	0.226	0.339	0.301	30.021	23.109	31.686	126.637
	(0.154)	(0.657)	(2.091)	(17.532)	(9.784)*	(20.727)	(144.968)
γ_C	48.927	46.304	43.607	-264.624	5.394	-8.04	-113.401
	(6.045)**	(13.147)**	(33.037)	(184.731)	(26.189)	(38.649)	(173.476)
γ_{C^2}	-199.52	-188.81	-173.559	620.552	5.757	10.188	38.283
	(27.123)**	(50.328)**	(112.725)	(482.969)	(10.404)	(13.717)	(48.687)
N	110	80	53	27	62	53	27
$K_D/C_T = 100\%$, $\chi^2(1)$	**114.2	**115.1	117.8	**194.5	[a]	[a]	[a]
$\gamma_{C^3} = 0, \chi^2(1)$	2.1	2.4	0.6	2.6	1	0.7	0.3
Cross-Equation Tests	1900-2010	1931-2010	1958-2010	1983-2010	1948-2010	1958-2010	1983-2010
$K_{D/C} = K_D$, $\chi^2(1)$	**251.1	**218.9	**207.9	*5.3	[a]	[a]	[a]
$\beta_0 = \gamma_C$ & $\beta_C = \gamma_{C^2}$, $\chi^2(2)$	**116.2	**42.5	**65	*7.6	**14.1	5.6	1.7
$K_Q = K_D$, $\chi^2(1)$	0.2	0.3	0	0	[a]	[a]	[a]
$K_{Q/X} = K_{D/C}$, $\chi^2(1)$	*5.3	2.4	1.6	1.8	**9.3	3.6	0
B. Samples Begining in 1900	United States, $C_{2010} = 219$ Billion Barrels				World, $C_{2010} = 2,483$ Billion Barrels		
Linear HDPO Model	1900-1930	1900-1960	1900-1990	1900-2010	1948-1960	1948-1990	1948-2010
β_0	0.046	0.063	0.062	0.062	0.094	0.09	0.08
	(0.011)**	(0.005)**	(0.004)**	(0.004)**	(0.028)**	(0.018)**	(0.014)**
β_C	1.715	-0.301	-0.287	-0.267	0.003	-0.042	-0.029
	(0.917)	(0.073)**	(0.035)**	(0.025)**	(0.106)	(0.021)	(0.009)**
N	30	60	90	110	12	42	62
$K_{D/C}/C_T = 100\%$, $\chi^2(1)$	**-12.3	95.9	99.5	105.5	-1083.2	85.9	112
$\beta_{C^2} = 0, \chi^2(1)$	**14.7	0	0.7	2.3	**54.4	3.3	*6.8
Quadratic HDPO Model	1900-1930	1900-1960	1900-1990	1900-2010	1948-1960	1948-1990	1948-2010
γ_0	0.502	-0.027	0.146	0.219	-57.867	35.306	19.499
	(0.136)**	(0.108)	(0.133)	(0.154)	(14.466)**	(18.378)	(9.350)*
γ_C	-65.65	66.447	53.323	49.286	613.526	-57.76	16.577
	(27.283)*	(13.170)**	(5.965)**	(6.109)**	(125.962)**	(72.974)	(27.217)
γ_{C^2}	6417.802	-345.337	-227.083	-201.732	-1006.052	59.685	0.088
	(1024.266)**	(147.645)*	(31.970)**	(27.685)**	(235.975)**	(45.734)	(11.630)
N	30	60	90	108	12	42	60
$K_D/C_T = 100\%$, $\chi^2(1)$	-456621	87.7	108.7	**111.4	**19.9	[a]	-47.7
$\gamma_{C^3} = 0, \chi^2(1)$	3.5	0	0.6	2.1	4.7	**10.4	1
Cross-Equation Tests	1900-1930	1900-1960	1900-1990	1900-2010	1948-1960	1948-1990	1948-2010
$K_{D/C} = K_D$, $\chi^2(1)$	**2.8E+9	**9.4	**105	**251.1	0	[a]	**47.9
$\beta_0 = \gamma_C$ & $\beta_C = \gamma_{C^2}$, $\chi^2(2)$	**133.5	**176.5	**81.9	**116.2	**29.8	*6.6	**14.1
$K_Q = K_D$, $\chi^2(1)$	[b]	0.4	0	0.2	**60.1	[a]	**1.6E+9
$K_{Q/X} = K_{D/C}$, $\chi^2(1)$	0	2.5	*4.6	*5.3	0	2.3	**9.3

Notes: The dependent variable is D_t/C_t, where cumulative discoveries, C_t, is measured at January 1 of each year (10^9 barrels) and discoveries, D_t, is discoveries from January 1 to December 31 of each year (10^9 barrels). Discoveries and cumulative discoveries are calculated from estimated proved reserves data, where cumulative discoveries are the sum of cumulative production and proved reserves, $C_t = X_t + R_t$, and discoveries are the change in cumulative discoveries, $D_t = C_t - C_{t-1}$. N is the number of observations. The estimator is Newey-West with five lags; standard errors (in parentheses) are corrected for first-order autocorrelation and for hetoeroskedasticity. '$K_D/C_T = 100\%$' and '$K_{D/C}/C_T = 100\%$' report the ratio between the HDPO linear prediction of ultimately recoverable reserves, K_D or $K_{D/C}$, to 2010 cumulative discoveries, $C_T = C_{2010}$, multiplied by 100 so that the ratio is expressed as a percentage of C_T; the asterisks report the results of the null hypothesis that $K_D = C_T$ or $K_{D/C} = C_T$; these tests are a Wald test, distributed as a $\chi^2(1)$. '$\beta_{C^2} = 0$' reports the test of the hypothesis that when included the variable C_t^2 is statistically insignificant in the linear HDPO model and $\gamma_{C^3} = 0$ reports the test of the hypothesis that when included the variable C_t^3 is statistically insignificant in the quadratic HDPO model; these tests are a Wald test, distributed as a $\chi^2(1)$. [a] Quadratic estimate of K_D is an imaginary number. [b] Test could not be conducted. Statistical significance: ** $p < 0.01$, * $p < 0.05$.

Table 7: HDPO Model for Crude Oil Discoveries (Net Additions to Proved Reserves) in 24 U.S. States, 1948-1972

			Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests			
			Coefficients		Hypothesis Tests, $\chi^2(1)$		Coefficients			Hypothesis Tests, $\chi^2(1)$		$K_{D/C} = K_D$	$\beta_0 = \gamma_C$ & $\beta_C = \gamma_{C^2}$	$K_{Q/X} = K_{D/C}$	$K_Q = K_D$
State	C_T	N	β_0	β_C	$K_{D/C}/C_T = 100\%$	$\beta_{C^2} = 0$	γ_0	γ_C	γ_{C^2}	$K_D/C_T = 100\%$	$\gamma_{C^3} = 0$	$\chi^2(1)$	$\chi^2(2)$	$\chi^2(1)$	$\chi^2(1)$
Alabama	0.72	25	**0.227	**-1.088	**29	0.3	-1.067	**259.4	**-1239.4	**28.5	*6.2	**23.5	**19.5	3.5	1.5
Alaska	19.94	9	**0.249	0.006	-199.3	0	-1114.467	3461	-291.4	**57.9	**43.3	0.1	4.1	0.1	1.1
Arkansas	1.83	25	**0.127	**-0.085	**81.4	0.1	-9.174	142.4	-91.4	**81.2	*5.6	**10.8	**25	**8.2	[b]
California	30.39	25	**0.072	**-0.003	**76.2	1	943.806	-54.6	1	[a]	**10.4	[a]	**11.6	0	[a]
Colorado	2.29	25	0.058	-0.02	127.1	*6.3	*-120.691	**395.3	*-222.2	**60.5	**7.8	1.2	0.6	0.5	**31
Florida	0.61	19	**0.333	*-0.587	93.6	0.1	32.389	143.9	-341.8	96.3	0	**15.1	*8.8	0.7	0
Illinois	3.68	25	**0.153	**-0.049	**85	3.1	*271.743	-75.9	-2.1	**89.1	1.2	**40.3	**64.9	**17.1	0.1
Indiana	0.58	25	**0.091	**-0.17	93.4	**9.6	**-35.354	**309.3	**-488.5	**84.1	2.9	**43	**15.2	0.5	3
Kansas	6.50	25	**0.135	**-0.027	**77.4	2.6	221.158	8.7	-9.6	**81.3	0.1	**104.6	**87.4	**10.6	0.2
Kentucky	0.80	25	*0.075	-0.087	107.8	**7.3	**-100.914	**556.4	**-614.6	**81.9	**7	**14.4	4.5	1.7	**6.9
Louisiana	30.35	25	**0.09	**-0.003	107.4	0.2	-12.052	**94.4	*-3	102.8	0	**46	**42.7	2.3	0.1
Michigan	1.32	25	**0.051	-0.047	82.7	0.7	25.414	-55.2	60.3	[a]	0	[a]	1.6	0.4	[a]
Mississippi	2.65	25	**0.104	-0.05	78.5	2.4	84.31	-78.8	38.6	[a]	*5.5	[a]	2.9	2	[a]
Montana	1.99	25	*0.087	-0.054	80.3	0.3	2.724	83.2	-55.5	77.1	1.5	0.9	0.5	0.4	0
Nebraska	0.51	25	**0.281	**-0.76	**72.4	1	0.424	**296.5	**-823.3	**70.7	0.3	**83.4	**64	0.6	0.4
New Mexico	6.17	25	**0.099	**-0.022	**71.5	*4.8	-129.444	**233.9	**-53.1	**60.8	0	**47.7	6	2.3	1.9
North Dakota	2.19	21	**0.539	**-0.973	**25.3	2	26.739	238.9	-491.6	**26.5	0	**77.5	**19.8	0	0.1
Ohio	1.16	25	-0.018	0.042	**36.8	**13	**-349.979	**912	**-567.2	**84.3	1.4	0.4	4.3	0.1	2.5
Oklahoma	15.42	25	**0.087	**-0.007	**85.5	*5.3	*1610.813	-265.5	12.2	[a]	1	[a]	**12.8	**22.4	[a]
Pennsylvania	1.39	25	0.049	-0.036	99.5	0.9	731.415	-1118.7	429.7	[a]	**0.1	[a]	0.4	0.2	[a]
Texas	68.09	25	**0.098	**-0.002	**78.4	**24	**4322.756	**-153.8	*1.7	[a]	0.1	[a]	**9.5	0.3	[a]
Utah	1.62	23	**0.229	**-0.31	**45.7	0	12.759	77.2	-61.2	87	**45.8	*6	2.5	3.5	**4.6E+9
West Virginia	0.61	25	0.044	-0.075	97.1	1.1	-265.894	1089.4	-1099.7	**91.7	0.8	0	1	2.3	**7.2E+9
Wyoming	7.61	25	**0.108	**-0.02	**69.2	0	131.622	7.8	-3.3	100.6	**12.4	**10.2	2.7	*5.5	0

Notes: See the notes to Table 5. Standard errors are not reported. $C_T = C_{2008}$ is in billions (10^9) of barrels. [a] Estimate of K_D is an imaginary number. [b] Estimate of K_Q is an imaginary number (see Table 3).

Table 8: HDPO Model for Crude Oil Discoveries (Net Additions to Proved Reserves) in 44 Countries, 1948-1972

			Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests			
			Coefficients		Hypothesis Tests, $\chi^2(1)$		Coefficients			Hypothesis Tests, $\chi^2(1)$		$K_{D/C} = K_D$	$\beta_0 = \gamma_C$ & $\beta_C = \gamma_{C^2}$	$K_{Q/X} = K_{D/C}$	$K_Q = K_D$
Country	C_T	N	β_{d0}	β_C	$K_{D/C}/C_T = 1$	$\beta_{C^2} = 0$	γ_0	γ_C	γ_{C^2}	$K_D/C_T = 1$	$\gamma_{C^3} = 0$	$\chi^2(1)$	$\chi^2(2)$	$\chi^2(1)$	$\chi^2(1)$
Algeria[a]	30.7	18	0.232	-0.008	91.9	**8.7	2785.6	-895.4	**43.8	*54.1	**39	0.2	**15.1	0.5	**16.5
Angola[a]	16.3	13	0.114	0.177	**-4	0.2	16.2	-80.6	426.8	[c]	1.4	[c]	**9.3	0.2	[c]
Argentina	12.5	20	0.097	-0.009	88	0.1	278.2	-154	31.6	[c]	1.6	[c]	2.2	1.7	[c]
Australia	8.3	17	0.048	0.092	**-6.2	0.1	60.2	-208.2	176.7	8	0.7	0.1	4.8	1.7	[e]
Austria	0.9	17	0.081	-0.122	77.4	**38.4	**274.5	**-1326	**1479	**66.2	0.2	0.5	1.5	0	**3.6E+9
Bahrain	1.1	20	*-0.104	**0.261	**35.9	1.6	-169.1	433.1	-110.6	312.3	**10.5	0.2	**10.7	0.1	**67.7
Bolivia	1.0	20	-0.039	0.23	*17	0.1	23.2	-246.2	507	35.6	0.2	0.3	2.9	0	[e]
Brazil	22.5	20	0.091	0.03	*-13.6	0	33.9	-118.1	174.2	[c]	0.2	[c]	3.8	0.1	[c]
Brunei	4.7	20	-0.034	*0.065	**11.1	0.2	-19.3	-11.3	59.1	**14.2	**7.9	0.7	*6.4	0.7	0.1
Burma (Myanmar)	0.4	20	-0.279	1.84	**34.5	2	-160.6	1983	-5871	**46.2	0.4	0.2	1.5	**24.9	[d]
Canada	204.9	20	**0.125	-0.005	**12.2	0	214.5	62.8	-1.5	22	0.7	0.4	0.5	0.6	0
Chile	0.6	20	0.081	-0.035	389.3	*4.8	**-26.1	**723.1	**-2094	**51.5	*5.6	0	3.2	0.1	3.5
Colombia	8.2	20	**0.076	-0.009	103.6	2.3	-296.6	386.8	-79.4	**47.8	0.6	1	0.8	0.3	1.1
Congo (Brazzaville)	3.7	[b]13	0.072	**0.274	**-7.2	**799.5	**-12.1	**1011	*-8.2	*3378	**5.7	1.1	**7568120	0.9	0.2
Denmark	3.2	17	-1.134	2.357	**15.1	3.5	-105.4	223.7	252.5	**10.7	**18.3	1.3	**28.5	*5.5	**33.4
Ecuador[a]	8.7	20	0.063	**0.145	**-5	**11.5	**-25.5	**320.3	**91.9	**0.9	**20	3.5	**13730.4	1.1	[d]
Egypt	13.7	20	0.059	0.021	**-20.7	0	82.3	-52.9	40.8	[c]	3.6	[c]	5.2	0.1	[c]
France	1.0	20	**0.286	**-0.655	**43.8	2.6	-1.6	**352.2	**-831.8	**42.1	1.5	**48.1	**16.5	1.7	0.1
Gabon	5.4	13	*0.107	0.114	**-17.4	*6.3	**-78.7	**611.5	**-395.9	**26	0.2	0.6	**22.1	0.8	**11.3
Germany	2.4	20	**0.165	*-0.091	**74	0.9	34.4	74.7	-42.1	88.1	0.3	**20.2	4.3	0.5	0
India	13.0	20	0.031	0.022	*-11.1	0.5	85.7	-184.5	122.1	[c]	0	[c]	1.7	0	[c d]
Indonesia[a]	27.3	20	**0.195	*-0.012	**61.9	1.3	-746.2	499.4	-31.4	**52.1	**30.8	3.7	0.9	**47	**246984
Iran[a]	199.7	20	0.055	0	-252.2	0	4968	-195.2	2.8	[c]	*4.7	[c]	4.2	0	[c]
Iraq[a]	145.7	20	0.139	-0.003	**36.1	**7.1	**7845	**-553.4	**11.1	[c]	0.8	[c]	2.2	1.9	[c]
Italy	1.5	20	**0.436	**-1.058	**26.7	3.6	4.5	**499.1	**-1277	**25.9	0.7	**41.8	*8.6	**30.7	0.3
Japan	0.4	20	-0.015	0.226	15.4	2.5	-238.1	2965	-8835	**46.6	**8.5	0	1.1	0.2	**29.2
Kuwait[a]	136.7	20	**0.308	**-0.004	**59.5	1	-2043	440.7	-5.3	**57.6	**39.3	**20.2	**11.7	**108	1.6
Libya[a]	67.6	13	**0.469	**-0.012	**59.7	0.5	-89.8	441.1	-10.5	**62	0.2	**15.9	3.6	**12.1	0
Malaysia	10.9	[b]13	0.016	0.008	-18.6	1.5	-766.9	436.2	-47.4	*62.8	0.1	**3732.9	0.8	0.1	*4.9
Mexico	49.2	20	-0.055	0.014	**7.9	*6.3	*4331	*-1369	**107.3	**14.2	0.5	0.4	3.6	**11.1	[d]
Netherlands	1.1	20	**0.151	*-0.214	*65	3.7	-33.4	**454.6	*-708.9	**51.3	1.1	**10.9	2	3.1	2
New Zealand	0.4	[b]25	-0.102	0.865	30	3	-2.1	-437.4	2576	**44.6	*4	0.1	3	0.1	**21.5
Norway	28.4	[b]18	-0.527	0.061	**30.3	3.7	-2963	666	-23.1	82.2	**9.9	**180557	3.6	**11	**28.5
Oman	14.0	[b]24	-0.236	0.053	**32	0	-5183	2206	-204.3	**52.5	1	1.1	2.3	**7.1	[d]
Pakistan	0.9	20	**0.075	0.069	-118.1	**7.7	*-21.9	**790.6	**-4032	**17.6	3.5	0	3.9	0	**15.7
Peru	2.9	20	0.014	0.015	-31.7	0.5	365.6	-792.4	443.1	[c]	**8.6	[c]	3.3	0.5	[c]
Qatar[a]	24.2	20	0.039	0.011	**-14.6	*4.8	*855.9	*-515.3	**82.5	[c]	*4.6	[c]	**11.7	0.3	[c]
Saudi Arabia[a]	377.2	20	0.112	0	445.7	**26.1	**14406	**-363.3	**2.9	[c]	1.2	[c]	*6.4	0	[c]
Syria	7.1	[b]31	0.026	0.004	-90.5	**11	-48	-215.3	*69.2	**46.6	0.5	0	4.9	3.1	[d]
Trinidad & Tobago	4.1	20	-0.054	*0.092	**14.4	**39.8	**510.5	**-857.6	**370	[c]	**17.4	[c]	**159.8	*6.3	[c]
Tunisia	1.8	[b]24	**0.273	-0.095	163.3	**11.8	**478	*-790.9	*259.5	126.4	**8.5	**1.6E+6	0.9	2.5	**53.1
Turkey	1.3	20	*0.13	-0.046	223.8	0.6	72.8	-764.9	951	**55.1	**33	0.1	*7.2	0.1	**94.1
United Kingdom	27.4	20	*0.075	**0.173	**-1.6	**1595.8	**-16.1	**1041	**-48	**79.2	**48.8	1.8	**606744	1.9	**72.8
Venezuela[a]	146.9	20	**0.142	**-0.003	**32.2	2	-2464	339.2	*-6.5	**29.3	**16.7	**16.6	*7.3	0.4	3.8

Notes: See the notes to Table 5. Standard errors are not reported. $C_T = C_{2008}$ is in billions (10^9) of barrels. [a] Current or former OPEC member. [b] HDPO estimation uses data through 1990. [c] Estimate of K_D is an imaginary number. [d] Estimate of K_Q is an imaginary number (see Table 4). [e] Test Statistic could not be calculated.

Table 9: HDPO Model for U.S. and World Discoveries of Fields 100 Million Barrels of Oil Equivalent or Larger, 1860-2009

A. Giant Oil Field Discoveries

				Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
				Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{D/C} = K_D$	$\beta_0 = \gamma_C$ & $\beta_C = \gamma_{C^2}$
Region	Sample	C_T [a]	N	β_{d0}	β_C	$K_{D/C}/C_T = 1$ (%)	$\beta_{C^2} = 0, \chi^2(1)$	γ_0	γ_C	γ_{C^2}	$K_D/C_T = 1$	$\gamma_{C^3} = 0$	$\chi^2(2)$	$\chi^2(2)$
United States	1860-1925	87.52	54	***113.5	**-5.8	**22.4	0.95	70.2	*68.3	-2.5	**32.1	3.64	2.7	0.1
	1860-1950	87.52	79	***93.9	***-1.5	*71.1	0.1	-176.3	***127.1	***-2.1	**66.8	**6.93	3.3	1.3
	1860-1975	87.52	104	***90.9	***-1.3	81.9	0.62	84.7	***65.9	***-0.8	91.6	0.73	**10.3	0.3
	1860-2009	87.52	138	***88.5	***-1.1	90.2	1.02	134.6	**56.4	**-0.7	97.4	1.08	**26.9	1.1
World	1860-1925	1403.35	57	*144.5	-4.5	**2.3	1.22	159.1	58.8	-1.1	**3.9	*5.45	*6.1	0.4
	1860-1950	1403.35	82	***98.7	-0.1	52	0.04	-545.8	***132.9	**-0.3	**36.6	0	0.4	0.4
	1860-1975	1403.35	107	***95.3	*-0.1	93.7	0.03	-391.7	***96.9	***-0.1	95.2	3.05	2.7	0
	1860-2009	1403.35	141	***94.8	***-0.1	98.5	0.04	-151.1	***91.6	***-0.1	99.7	0.07	**14.2	0

B. Giant Gas Field Discoveries

				Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
				Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{D/C} = K_D$	$\beta_0 = \gamma_C$ & $\beta_C = \gamma_{C^2}$
Region	Sample	C_T [b]	N	β_{d0}	β_C	$K_{D/C}/C_T = 1$ (%)	$\beta_{C^2} = 0, \chi^2(1)$	γ_0	γ_C	γ_{C^2}	$K_D/C_T = 1$	$\gamma_{C^3} = 0$	$\chi^2(2)$	$\chi^2(2)$
United States	1860-1925	0.25	41	206.9	-9709.9	**8.4	2.24	-0.1	**419.7	**-26152	**6.3	0.23	0.3	0
	1860-1950	0.25	66	**225.9	*-1656.3	**53.9	0.24	0.3	***205.9	***-1551.9	**53	0.56	2	0.1
	1860-1975	0.25	91	**214.5	*-1194.8	**70.9	0.63	0.7	***70.2	***-331	87.6	1.36	*6.2	2.7
	1860-2009	0.25	125	**201.9	**-910.2	*87.6	1.42	0.8	***56	***-229.9	101.5	0.63	**13.6	4.2
World	1860-1925	6.71	57	-5.9	*1273.1	**0.1	2.22	***12.1	***-1104.2	***23116.6	**0.5	**30.02	0.2	0.5
	1860-1950	6.71	82	*41.6	*137.5	**-4.5	**7.04	-1.1	**133.7	-139.2	**14.2	0.15	0.7	1.4
	1860-1975	6.71	107	***55.5	11.9	-69.8	**7.75	*-13	***188.3	**-34.1	81.2	*5.72	0.6	**11.8
	1860-2009	6.71	141	***61.6	*-6.9	134	*5.76	-2.8	***127.4	***-19.6	96.5	3.42	3.6	*6.7

C. Giant Oil/Gas/Condensate Field Discoveries

				Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
				Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{D/C} = K_D$	$\beta_0 = \gamma_C$ & $\beta_C = \gamma_{C^2}$
Region	Sample	C_T [c]	N	β_{d0}	β_C	$K_{D/C}/C_T = 1$ (%)	$\beta_{C^2} = 0, \chi^2(1)$	γ_0	γ_C	γ_{C^2}	$K_D/C_T = 1$	$\gamma_{C^3} = 0$	$\chi^2(2)$	$\chi^2(2)$
United States	1860-1925	130.13	54	***112	*-4.7	**18.3	0.68	28.9	**89.1	-3.1	**22.7	**8.26	2	0.1
	1860-1950	130.13	79	***97.4	***-1.04	*71.5	0.11	*-305.6	***159.3	***-1.9	**61.3	*3.92	3	2.8
	1860-1975	130.13	104	***94.8	***-0.9	82.3	0.22	152	***69.4	***-0.6	90.2	**-1	**10.3	0.5
	1860-2009	130.13	138	***92.3	***-0.8	91.7	0.83	224.1	***57.7	***-0.5	98.8	1.64	**27.7	1.7
World	1860-1925	2644.20	57	**69.3	-1.2	**2.2	0.27	439.2	17.8	0	284.6	**11.38	1.9	1.7
	1860-1950	2644.20	82	***67.1	0.016	-157	2.04	-705.5	***130.5	**-0.2	**25.5	0	0	0.9
	1860-1975	2644.20	107	***69.8	-0.014	188.2	0.94	-1105.9	***95.5	***-0.68	103.3	**7.19	0.5	1.4
	1860-2009	2644.20	141	***72	***-0.025	109.1	1.55	-712.6	***93.1	***-0.03	99.4	0.06	**14.5	1.7

Notes: See the notes to Table 5. The data are the size of fields as measured in 2009. [a] $C_T = C_{2009}$ billion (10^9) barrels of Oil. [b] $C_T = C_{2009}$ trillion (10^{12}) cubic feet. [c] $C_T = C_{2009}$ billion (10^9) barrels of oil equivalent.

Table 10: HDPO Model for U.S., U.K., and World Coal Production, 1751-2010

				Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
				Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{Q/X} = K_Q$	$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$
Region	Sample	X_T	N	β_{q0}	β_X	$K_{Q/X}/X_T = 1$ (%)	$\beta_{X^3} = 0$	γ_{q0}	γ_X	γ_{X^2}	$K_Q/X_T = 1$ (%)	$\gamma_{X^3} = 0$	$\chi^2(1)$	$\chi^2(2)$
World	1751-1850	307.5	99	***0.101	-0.049	**0.7	3.58	***0.004	***0.024	***0.0074	**-0.1	*3.89	**17.4	**24.1
	1751-1900		149	***0.069	-0.002	**10.3	1.78	**-0.004	***0.048	***-0.0003	62	0.72	2.9	4.7
	1751-1950		199	***0.063	**-0.001	**37.3	1.15	0.006	***0.047	***-0.0003	**52	**13.64	**9.2	5
	1751-2010		255	***0.058	***-0.0002	90.8	*5.26	***0.119	***0.023	***-0.00002	**401.5	**7.24	**19.9	**44.1
United Kingdom	1751-1850	15.3	99	***0.108	*-0.139	**5	3.77	***0.002	***0.021	***0.0099	**-0.8	3.38	**30.4	**27.9
	1751-1900		149	***0.072	*-0.018	**26.8	2.84	0	***0.038	***-0.0029	84.2	*5.03	**11.2	**11.5
	1751-1950		199	***0.063	**-0.007	**60.2	2.73	*0.001	***0.035	***-0.0024	95.2	**11.85	**21.5	**13.8
	1751-2010		255	***0.059	***-0.004	*86.6	3.2	***0.003	***0.031	***-0.0018	**110.6	1.53	**52.4	**20.1
United States	1751-1850	73.6	50	***0.173	-1.43	**0.2	2.85	0	***0.097	**0.2911	**0	1.89	3	2.3
	1751-1900		100	***0.132	**-0.023	**7.7	2.53	**0.001	***0.08	***-0.004	**27.1	2.88	*6.4	**85
	1751-1950		150	***0.116	***-0.004	**41.2	2.75	***0.015	***0.063	***-0.0014	**59.5	**24.69	**29.9	**130.2
	1751-2010		206	***0.105	***-0.002	**75.2	1.63	***0.079	***0.022	**-0.0002	*203.5	**30.03	**103	**174.8

Notes: See the notes to Table 1. Standard errors are not reported. $X_T = X_{2006}$ is measured in billions of metric tons.

Table 11: HDPO Model for 78 Mineral Commodities, 1900-1950 (Aluminum to Magnesium Metals)

			Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
			Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{Q/X} = K_Q$	$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$
Mineral	X_T	N	β_0	β_X	$K_{Q/X}/X_T = 1$ (%)	$\beta_{X^2} = 0$	γ_0	γ_X	γ_{X^2}	$K_Q/X_T = 1$ (%)	$\gamma_{X^2} = 0$	$\chi^2(1)$	$\chi^2(2)$
Aluminum	8.72E+08	50	**0.228	*-0.106	**2.1	3.13	-0.044	**1.458	-0.391	**3.7	0.47	**11.4	**163.5
Antimony	6.04E+06	50	**0.258	**-0.015	**17.8	**8.03	**0.016	0.002	-0.00002	90.7	0.7	**27.6	**65.3
Arsenic	3.61E+06	46	**0.25	**-0.007	**34	**8.02	**0.008	**0.003	**0	**50.3	*3.91	**63.9	**77
Asbestos	1.90E+08	50	**0.244	*-0.04	**6.1	*4.79	**0.083	**0.066	*0.009	**-1.6	**29.05	**22.4	**14.3
Barite	3.13E+08	32	*0.377	-0.063	**6	*4.57	**0.326	**0.165	-0.003	49.2	0.18	**32.4	*7.3
Bauxite	4.90E+09	50	**0.217	-0.095	**2.3	2.33	-0.181	**6.959	-1.547	**4.5	0.35	**8	**136.5
Beryllium	1.63E+04	15	**0.714	**-0.097	**7.3	**19.92	*0.00005	0.00001	0.0000007	[z]	**72.08	**68.5	**109.2
Bismuth	2.40E+05	23	**0.512	*-0.058	**8.8	**14.45	**0.001	-0.00002	0.00001	[z]	0.16	**8.6E+5	**35.4
Boron	1.19E+08	13	**0.554	**-0.755	**0.7	**9.74	**0.03	**0.249	**-0.268	**1	0.01	**76.9	**11.5
Bromine	1.80E+07	50	**0.186	-0.032	**5.8	0.35	-0.001	**0.046	**-0.013	**3.5	**6.85	0.7	**30.3
Cadmium	1.03E+06	50	**0.246	**-0.027	**9.2	1.06	0.0001	**0.002	**-0.00015	**11.9	**10.54	**15.1	**119
Cement	5.86E+10	24	**0.406	*-0.161	**2.5	**9.29	**71.174	-16.889	10.211	[z]	2.32	**9.7E+6	*8
Chromium	1.76E+08	50	*0.21	-0.043	**4.9	1.97	0.001	**0.18	-0.011	**16.1	0.08	*5.9	2.2
Cobalt	1.88E+06	49	0.286	-0.081	**3.5	2.48	0.00014	*0.001	0.000001	**-0.1	0.48	*6.4	**11.3
Columbium	8.08E+05	16	**0.613	**-0.047	**13	**10.94	**0.004	*0.001	-0.000002	322.8	**10.33	**15.2	**43.6
Copper	5.26E+08	50	**0.226	*-0.021	**10.6	*5.16	**0.571	**0.208	-0.005	**40.7	0.97	**51.8	**17.5
Industrial Diamond	4.06E+03	12	**0.646	**-1.087	**0.6	**17.39	**0.000002	**-0.000002	**0.000003	[z]	0	**2.2E+8	**46.3
Diatomite	9.28E+07	16	**0.61	-0.279	**2.2	1.98	0.007	0.274	0.027	**0	**14.59	1.6	3.1
Feldspar	3.20E+08	42	*0.239	-0.071	**3.4	*4.52	**0.169	0.045	**0.023	[z]	0.11	**2.9E+6	**18.6
Fluorspar	2.39E+08	37	**0.241	-0.044	**5.5	*5.71	0.105	*0.174	-0.009	**19.2	*4.57	**34.3	**10.5
Gallium	1.69E+03	12	**0.412	*-0.028	**14.5	**15.78	**0.000013	-0.0000007	**0.0000002	[z]	**20.47	**211000	**94.1
Garnet	7.38E+06	13	**0.454	**-0.369	**1.2	*4.49	0.002	0.014	-0.007	**2.2	**35.24	**23.3	**46.8
Gemstones	3.64E+02	10	**0.638	**-0.304	**2.1	**7.28	**0.0000004	*0.0000003	0.0000001	[z]	0.79	**10.4	**41.2
Germanium	3.63E+02	25	**0.395	**-0.038	**10.5	*6.1	**0.0000007	**0.0000004	**-0.00000003	**16.7	0.05	**45.5	**40.4
Gold	1.34E+05	50	**0.248	*-0.012	**21.2	**7.43	**0.00049	0.00002	-0.0000001	194.9	3.15	2	**50.5
Graphite	4.08E+07	50	**0.227	*-0.017	**13.2	*6.56	**0.083	0.01	-0.000225	50.5	0.62	**16.5	**47.4
Gravel Industrial	2.70E+09	10	**0.701	**-0.018	**39.8	**21.42	**116.77	-0.416	0.01	[z]	**15.04	**64706	3.8
Gypsum	4.69E+09	26	**0.408	*-0.089	**4.6	**14.93	**13.311	**-4.932	**1.12	[z]	2.77	**3.8E+6	**108.7
Helium	7.85E+05	15	**0.507	**-1.014	**0.5	1.38	0.00005	**0.004	*-0.008	**0.5	**31.17	**7.3	**32.7
Indium	6.89E+03	13	**0.548	**-0.068	**8.1	**15.58	**0.00006	-0.000003	0	2260.5	**24.21	**18.7	**66.1
Iodine	6.53E+05	20	**0.424	**-0.02	**20.8	*5.74	**0.002	**0.001	**-0.000026	**39.4	0.06	**34.1	**46.1
Iron-Ore	5.71E+10	46	*0.268	*-0.028	**9.5	*6.53	**119.525	1.367	0.626	[z]	0.01	**6.8E+5	**36.9
Iron-Oxides	1.15E+07	14	**0.595	**-0.021	**28.7	**17.81	**0.308	**-0.007	**0.00019	[z]	0.01	**1.1E+5	**47.3
Kyanite	1.49E+07	22	**0.583	*-0.25	**2.3	**6.96	*0.009	0.005	0.004	[z]	1.33	**5.1E+6	**28.4
Lead	2.17E+08	33	*0.368	*-0.025	**14.9	*6.32	**0.964	0.044	-0.001	60	0.32	**26.5	**15.3
Lime	6.16E+09	17	**0.52	**-0.022	**23.3	**8.88	**70.331	**3.047	**-0.044	87.6	0.11	**89.2	**297.8
Lithium	7.54E+06	25	0.332	-0.265	**1.3	*5.54	*0.004	-0.004	0.006	[z]	0.01	**15.3	**16.8
Magnesium Compounds	5.98E+08	37	**0.217	*-0.033	**6.6	3.17	0.184	**0.627	**-0.056	**11.6	2.21	**24.9	**61.8
Magnesium Metals	1.75E+07	13	**0.843	**-0.165	**5.1	0.02	0.004	**0.147	**-0.029	**5.1	3.47	**101.9	**62.7

Notes: See the notes to Table 1. Standard errors are not reported. Production is measured in metric tons, except in the quadratic specification, where production is measured in millions of metric tons. $X_T = X_{2008}$ is cumulative production in 2008. In the regressions, cumulative production is normalized so that $X_T = 1$. [a] Sample is not continuous. [b] Uses data to 1980. [c] Uses data to 1985. [d] Uses data to 1990. [e] Uses data to 1995. [f] Estimate of K_Q is an imaginary number. [g] An estimate of γ_{X^3} could not be calculated due to multicollinearity between X^3, X^2 and X.

Table 12: HDPO Model for 78 Mineral Commodities, 1900-1950 (Manganese to Zirconium)

			Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
			Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{Q/X} = K_Q$	$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$
Mineral	X_{2008}	N	β_0	β_X	$K_{Q/X}/X_T = 1$ (%)	$\beta_{X^2} = 0$	γ_0	γ_X	γ_{X^2}	$K_Q/X_T = 1$ (%)	$\gamma_{X^2} = 0$	$\chi^2(1)$	$\chi^2(2)$
Manganese	5.22E+08	50	**0.192	*-0.02	**9.6	**8.26	**0.413	**0.236	-0.007	**33.4	1.44	**55.5	**32.8
Mercury	5.46E+05	50	*0.234	*-0.008	**29.7	*6.29	**0.003	0.000045	0.000001	[z]	2.29	0.2	**47.2
Mica Flake	1.21E+07	33	*0.364	-0.063	**5.8	**9.89	**0.023	-0.003	**0.001	[z]	3.05	**1.4E+6	**29.6
Mica Sheet	7.75E+05	44	**0.245	*-0.009	**27.9	*5.56	**0.002	**0.0005	-0.000003	177.7	0.2	**29.5	**45.2
Molybdenum	5.62E+06	50	*0.324	-0.063	**5.1	0.13	-0.0005	**0.018	**-0.003	**5.3	1.73	2.8	**21.2
Natural Gas	2.97E+09	31	**0.312	*-0.014	**22.6	*4.15	**5.423	**3.128	**-0.054	**60	0	**25.1	**1936.1
Nickel	4.79E+07	50	**0.206	-0.038	**5.5	3.65	0.002	**0.043	-0.003	**15	**11.89	-999	**21.4
Nitrogen	3.76E+09	14	**0.63	*-0.284	**2.2	**8.47	**2.988	**4.457	-0.169	**27.1	1.41	**43.6	**514.5
Peat	7.91E+09	15	**0.504	*-0.116	**4.3	**14.73	**25.606	-7.999	**2.936	[z]	1.21	**2.3E+6	*8.5
Perlite	5.61E+07	14	**0.589	**-0.017	**35.6	**12.9	**1.297	**0.027	**-0.0004	97.2	0.1	**56802	**46.4
Phosphate Rock	6.70E+09	50	**0.215	*-0.044	**4.9	*6.11	**4.304	0.948	0.241	[z]	*6.28	**2.1E+6	**171.3
Platinum	1.29E+04	50	*0.228	-0.087	**2.6	*4.73	**0.000006	-0.000001	0.000002	[z]	**29.85	**4.1E+6	**21.1
Potash	1.29E+09	31	*0.697	-0.191	**3.6	*5.68	**0.834	**1.125	**-0.155	**7.9	0.15	**37.7	**13.7
Pumice	7.47E+08	8	**1	**-1.193	**0.8	0.43	-0.016	**6.724	-6.014	**1.1	0.52	0.7	**25.4
Quartz Crystal	2.29E+04	20	**1.19	**-0.035	**34	**12.85	**0.001	-0.00008	0.000002	[z]	**29.66	**1.5E+5	**27.4
Rare Earth	2.46E+06	50	**0.379	**-0.171	**2.2	**32.87	**0.003	-0.001	0.0004	[z]	0.31	**5.0E+7	**74.6
Rhenium	9.15E+02	7	**0.741	**-0.147	**5.1	**18.84	**0.000007	**-0.000002	**0.000001	[z]	0.07	**1.8E+6	**36.9
Salt	1.02E+10	44	**0.238	*-0.026	**9.1	*5.59	**12.735	**3.323	-0.041	85.6	*6.49	**72.6	**774.3
Selenium	7.94E+04	12	**0.705	**-0.114	**6.2	**18.27	**0.0003	*0.0001	*-0.00002	**9.2	*4.46	**86	**74.2
Silicon	1.32E+08	16	**0.501	**-0.026	**19.5	**12.73	**1.167	**0.062	0.001	*-30	0	**1.4E+5	**39.7
Silver	1.03E+06	50	**0.238	*-0.009	**25.4	*6.1	**0.005	**0.0003	**-0.00001	**48.5	1.99	**91.1	**50.7
Sodium Carbonate	1.23E+09	12	**0.681	**-0.035	**19.5	**13.09	**14.152	**1.275	**-0.029	**53.2	2.7	**43.9	**32.2
Sodium Sulfate	1.74E+08	8	**0.805	**-0.04	**20.2	**17.48	**3.722	**0.115	-0.003	63.5	1.96	**1.7E+5	**38.9
Strontium	9.82E+06	29	**0.192	*-0.01	**19.8	0.04	-0.003	**0.021	*-0.001	**17.9	0.13	*3.9	**61.7
Sulfur	2.76E+09	50	0.224	-0.037	**6	2.09	1.174	**1.293	-0.023	57.2	0.09	**15	**217.4
Talc	3.74E+08	46	**0.225	-0.049	**4.6	*5.04	**0.158	*0.108	**0.021	[z]	0.43	**1.4E+6	**21
Tantalum	2.37E+04	11	**0.645	**-0.037	**17.6	**39.24	**0.00041	-0.00001	0.000001	[z]	**29.86	**4.0E+5	**81.7
Tellurium	7.33E+03	20	**0.765	**-0.102	**7.5	*5.1	0.00001	*0.00002	-0.000002	**9.9	*5.98	**45.5	**54.6
Thallium	4.00E+02	10	**0.675	**-0.02	**34.4	**13.39	**0.00001	**0.0000002	0	132.7	[w]	**29.2	**55.6
Thorium	6.20E+05	17	**0.344	**-0.009	**37.3	**12.37	**0.008	0.0003	0	[z]	0.31	**9.5	**104.2
Tin	1.91E+07	45	**0.263	*-0.01	**26.4	*6.32	**0.093	**0.006	*-0.00015	**53.1	0.19	**62.9	**45.4
Titanium	2.80E+08	25	**0.447	*-0.203	**2.2	*4.18	*0.046	**0.409	-0.022	**18.6	0.14	**15.1	*8.5
Tungsten	2.89E+06	45	**0.252	*-0.021	**11.8	*5.61	0.001	*0.002	0	**27.7	*5.7	**27.6	**43.2
Uranium	1.92E+06	15	**0.566	*-0.085	**6.7	3.01	**0.00043	**0.007	**-0.001	**11.4	0.39	**8.2	**54.6
Vanadium	1.39E+06	33	**0.242	*-0.071	**3.4	**7.18	0.001	0.001	0	**9.9	2.69	**31.6	**51.9
Vermiculite	2.52E+07	32	*0.349	-0.01	**33.7	*5.51	**0.158	**0.012	0	235.9	**18.67	**54.5	**30
Wollastonite	1.22E+07	21	**0.385	*-0.044	**8.7	*6.37	**0.032	0.005	0	[z]	0.15	**28.6	**34.1
Zinc	4.07E+08	50	**0.233	*-0.023	**10.3	*5.86	**0.607	*0.079	0.001	-9.3	0.01	**15.8	**16.1
Zirconium	3.31E+07	6	**1.03	**-2.362	**0.4	**50.26	**0.017	**0.066	**-0.12	**0.7	**7.4	**137.6	**100.8

Notes: See the notes to Table 1. Standard errors are not reported. Production is measured in metric tons, except in the quadratic specification, where production is measured in millions of metric tons. $X_T = X_{2008}$ is cumulative production in 2008. In the regressions, cumulative production is normalized so that $X_T = 1$. [a] Sample is not continuous. [b] Uses data to 1980. [c] Uses data to 1985. [d] Uses data to 1990. [e] Uses data to 1995. [f] Estimate of K_Q is an imaginary number. [g] An estimate of γ_{X^3} could not be calculated due to multicollinearity between X^3, X^2 and X.

Table 13: HDPO Model for U.S. Agricultural Commodities, 1790-1950

					Linear HDPO Model				Quadratic HDPO Model					Cross-Equation Tests	
					Coefficients		Hypothesis Tests		Coefficients			Hypothesis Tests		$K_{Q/X} = K_Q$	$\beta_0 = \gamma_X$ & $\beta_X = \gamma_{X^2}$
Commodity	1st Year of Production	X_T	Units	N	β_0	β_X	$K_{Q/X}/X_T$	$\beta_{X^2} = 0$	γ_0	γ_X	γ_{X^2}	K_Q/X_T	$\gamma_{X^2} = 0$	$\chi^2(1)$	$\chi^2(2)$
Alfalfa	1919	5.15	10^9 Tons	31	**0.326	*-0.479	**13.2	**7.71	**0.0203	0.011	0.015	[a]	0.09	**335.1	**127.9
Barley	1866	34.90	10^9 Bushels	84	**0.158	*-0.016	**28	3.79	**0.0372	**0.037	**-0.001	82.8	0.48	**36.5	**353.6
Cattle	1900	2.64	10^9 Animals Slaughtered	50	**0.252	*-0.446	**21.4	**6.75	**0.0136	-0.007	**0.023	[a]	2.41	**621.4	**106.1
Corn	1866	554.85	10^9 Bushels	84	**0.179	*-0.001	**25.4	*4.88	**1.1962	**0.025	**-0.0001	**48.9	**24.88	**85.4	**97.1
Cotton	1790	1.75	10^9 Bales	160	**0.137	**-0.2	**39.2	*5.52	0.0003	**0.043	**-0.034	**71.7	0.55	**36.9	**1922.9
Dry Beans	1909	1.71	10^9 hundred weight	41	**0.241	*-0.639	**22.2	*5.42	**0.0053	**0.04	*-0.029	88.7	2.71	**52.8	**209.5
Flaxseed	1866	2.35	10^9 Bushels	84	**0.188	**-0.189	**42.3	**7.66	**0.0092	0.006	0.01	[a]	**18.06	**369	**48.1
Hay	1866	13.25	10^9 Tons	84	**0.151	*-0.035	**32.3	*3.98	**0.0244	**0.03	**-0.003	**76.5	**8.5	**65.7	**654.6
Hogs	1900	8.31	10^9 Animals Slaughtered	50	**0.235	*-0.091	**31	*5.89	**0.0512	*0.011	-0.001	[a]	0.63	**138.2	**44.2
Oats	1866	112.08	10^9 Bushels	84	**0.159	*-0.003	**53.3	*4.75	**0.343	**0.033	**-0.0003	105.5	**15.52	**73	**271.1
Peanuts	1909	223.75	10^9 Pounds	41	**0.247	*-0.007	**16	*6.34	**0.3477	**0.051	-0.0001	165	**17.91	**31.8	**305.4
Potatoes	1866	34.59	10^9 hundred weight	84	**0.151	*-0.013	**32.5	*4.39	**0.0741	**0.026	**-0.001	*80	**16.57	**77.5	**485.3
Rice	1895	8.13	10^9 hundred weight	55	**0.211	**-0.285	**9.1	**0	**0.0068	**0.03	0.004	**-2.9	**14.22	**10	**477.1
Rye	1866	3.95	10^9 Bushels	84	**0.147	*-0.064	**58.2	3.41	*0.0088	**0.048	**-0.015	*85.9	*4.95	**55.3	**84
Sheep	1900	1.41	10^9 Animals Slaughtered	50	**0.239	*-0.337	**50.3	*5.46	**0.0122	**0.02	-0.01	177.6	**8.2	**53.6	**60.9
Sorghum	1929	35.94	10^9 Bushels	21	**0.366	*-0.218	**4.7	*6.23	**0.0439	0.046	0.018	[a]	0.91	**27.8	**33.7
Soybeans	1924	94.47	10^9 Bushels	26	**0.36	**-0.14	**2.7	2.39	0.0064	**0.235	**-0.056	**4.5	**15.75	**14.1	**598
Sweet Potatoes	1868	3.20	10^9 hundred weight	82	**0.153	*-0.081	**59	*4.84	**0.0134	**0.029	**-0.009	**69	*4.61	**111.8	**267.9
Tobacco	1866	182.09	10^9 Pounds	84	**0.141	*-0.002	**36.5	*5.4	**0.3859	**0.024	-0.00007	196.7	**18.03	**61.9	**544.7
Wheat	1866	161.50	10^9 Bushels	84	**0.177	*-0.004	**25.8	*4.39	**0.3424	**0.018	-0.0001	119.3	**15.1	**59.8	**229.1
Wool	1869	30.07	10^9 Pounds	81	**0.174	*-0.01	**57.2	*5.95	**0.2009	*0.012	-0.00033	162.7	0.01	**111.1	**46.7

Notes: See the notes to Table 1. Standard errors are not reported. $X_T = X_{2008}$ is in the units given in the table. [a] Estimate of K_Q is an imaginary number.