Property Rights Out of Anarchy?
The Demsetz Hypothesis in a Game of Conflict*

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Abstract

The Demsetz hypothesis states that secure claims to property arise when the value of creating those rights is sufficiently high. This paper examines the conditions under which this holds in an anarchy equilibrium in which players may allocate labor to production, to conflict, or to the public good of secure claims to property protection. In a simultaneous choice Nash equilibrium, no secure claims to property are created. However, if players play a sequential choice game in which secure claims to property protection occurs in the first stage, then the strategic benefit of reducing others' subsequent conflict allocation causes secure claims to property to arise. Secure claims to property in a social contract are imperfect, but for sufficiently high productivity of resources, the social contract welfare dominates autocracy.

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1 Introduction

It has long been recognized that property rights are an essential ingredient to a well-functioning economy. What is less well understood, however, is how secure claims to property arise, especially when no state exists to provide them. Demsetz (1967) observed that the Montagnais bands in Quebec created secure claims to property for beaver ponds once fur trade with Europeans made the rent dissipation from an open access property rights regime sufficiently costly. He hypothesized that exogenous increases in the value of a resource leads to the establishment of secure claims to property for that resource. Important contributions to the evidence for the Demsetz hypothesis include Anderson and Hill (1975), who observed that grazing rights evolved informally in the American west, Umbeck (1977), who observed that miners in the California gold rush devised and enforced their own rules for protecting their mining claims, and Ostrom (1990), who discusses examples in which secure claims to property arose organically in Swiss alpine meadows, Spanish irrigation canals, and Japanese forests.

The Demsetz hypothesis, however, is not without criticism. First it requires that agents somehow overcome the collective action problem of creating secure claims to property, but how this is accomplished is not clearly spelled out (Banner 2002). Second, secure claims to property have often arisen through collective actions that are at odds to the interests of many of the parties using the resource. Examples of this include the enclosure of European fields (Banner 2002), the Homestead Act in the U.S. (Anderson and Hill 1975), and property rights created in fisheries in Turkey and Nova Scotia, Canada (Ostrom 1990, at pp. 144-146, 174-178), where many of those who had formerly used the resource were subsequently excluded and were not compensated for their loss. Third, in many of the examples in which secure claims to property arose through collective action a significant amount of conflict also existed. This was true in the grazing association examples (Anderson and Hill 1975) and in the Californian mining camps (Clay and Wright 2005). In each case, this seems to have been exacerbated by the influx of large numbers of potential competitors. Thus it is also possible that an increase in the value, or productivity, of a resource may result in an increase in conflict, over the use of the resource, rather than the establishment of well defined property rights.

This paper addresses each of these three issues. We examine a model in which agents have an endowment that can be used either for production or expropriation. Like Hafer (2006), we have two productive uses, subsistence consumption, which is secure, and production, which may be insecure. In the absence of secure claims to property, there exists an *anarchy equilibrium* (e.g., Skaperdas 1992, Hirshleifer 1995, and Grossman and Kim 1995).[1] To implement the conditions of

[1] Grossman and Kim (1995) trace this literature to Haavelmo (1954). This growing literature is surveyed in Garfinkel and Skaperdas (2007).

the Demsetz hypothesis, we vary a parameter which measures the productivity of the endowment devoted to insecure production relative to subsistence consumption. As resources devoted to insecure production become more valuable, we determine what conditions must hold in order that the collective action problem of creating secure claims to property is overcome.

Not surprisingly, we find that it is difficult for secure claims to property to arise organically as the solution to a collective action problem. First, for low values of the marginal product of insecure production, players will simply revert to subsistence consumption, as in Murphy, Shleifer, and Vishny (1993). Even when the marginal productivity of insecure production is high, however, when players *simultaneously* allocate their endowment between subsistence consumption, insecure production, conflict, and a contribution to the public good of property protection, the symmetric Nash equilibrium results in *zero* contribution to the public good of ensuring claims to production secure. This occurs because investing in conflict dominates contributing to the public good of enforcing secure claims to property. Allocations to conflict increase the proportion of the *total* production a player appropriates, while devoting resources to ensuring all claims to property are secure only increases the proportion of his own production he appropriates; it *reduces* the proportion of others' production he expropriates. Thus, in the simultaneous Nash equilibrium, claims to property remain insecure. Furthermore, this result is independent of the marginal productivity of resources used in insecure production.

There are, however, two ways in which positive social provision to secure claims to property might arise in a non-cooperative game environment, and both require a version of the Demsetz hypothesis to hold. In the first, one player – the autocrat – offers his potential "citizens" protection of their property in exchange for his right to tax them. Since players may avoid insecure claims to property by allocating their endowment to subsistence consumption, which is not subject to theft, if the marginal productivity of insecure production is too low, even autocracy cannot arise. But when the marginal product of the insecure production is sufficiently high, an autocrat can offer his citizens a contract that they will accept. Thus, a nascent state may arise when the value of creating secure claims to property exceeds its social costs.

A more organic form of the Demsetz hypothesis is also possible. In what we call a *social contract*, players play a two-stage game. In the first stage, players allocate part of their endowment to the public good of enforcing secure claims to property. Then, having observed the contributions by all players to the public good property protection, in the second stage players allocate the remainder of their endowment between subsistence consumption, conflict and a potentially insecure production. In the symmetric, subgame perfect Nash equilibrium to this game, it is possible that players willingly contribute to the public good of property rights protection. Each player does so because he recognizes that an increase in his own contribution to secure claims to property, in the first stage, results in a reduction in *all* players' allocations to conflict in the second stage. This strategic effect causes the marginal return from property rights protection to exceed that of conflict. Although players make no explicit agreement with one another, the contribution to the public good of secure property is credible. Thus such an agreement, were it to be reached, is dynamically consistent. Relative to the Nash equilibrium, less resources are diverted away from production in the subgame perfect equilibrium. In order for voluntary contributions to the security of property to occur, however, the marginal product of insecure production must be sufficiently high to compensate for the lost production. Hence, the Demsetz hypothesis holds. The minimum marginal productivity parameter under which a social contract arises, however, is increasing in the number of players because of the free-riding problem inherent in the public good provision.

An autocracy, all else equal, can create secure claims to property at a lower level of the marginal product of insecure production than can occur in the social contract equilibrium. This is because the autocrat, by the power of coercion behind his ability to raise taxes, can overcome the free-riding problem inherent in the social contract. Like Grossman (2002), we find that the autocrat provides *perfectly secure* property, in the sense that citizens allocate nothing to conflict. The autocrat, however, simply replaces conflict with taxation. Hence, all of the surplus above the Nash equilibrium is captured by the autocrat (*cf.*, Wintrobe 1990, Olson 1993, Grossman 2002, Hurwicz 2008). In contrast, conflict remains under a social contract, whenever the number of players is greater than two, so that claims to property are not perfectly secure. This occurs because as the number of players rises, the strategic effect of contributing to protecting claims to property diminishes. Hence, secure claims to property can arise at a lower level of the marginal productivity of insecure production under autocracy than in the social contract.

Several comparisons to the Demsetz hypothesis should be emphasized. First, secure claims to property are more likely to arise in smaller societies, *ceteris paribus*. In the Demsetz hypothesis, this is usually attributed to transactions costs rising as the size of the population rises. In our model, it occurs because there is a tension between the free-riding of public good provision and the strategic effect upon one's rivals' allocations to conflict; as the size of the population rises, the strategic effect diminishes while the free-riding effect increases. Second, unlike the Demsetz hypothesis, we offer an explicit mechanism whereby secure claims to property might arise. Committing part of their endowment to the public good, prior to allocating the balance between production and conflict, is credible because of the strategic effect. English King John in 1215 signed the *Magna Carta* granting rights to his nobility over their property because the nobility was roused enough to march en masse upon London to demand those rights. Such an act would not be successful if nearly all of the nobles in England had not simultaneously pre-committed to contributing to the public good of countering the King. Similarly, in the American west and in the California mining camps, secure claims to property arose only after the ranchers and miners formed organizations whose purpose was to devote resources to creating and enforcing the security of property.

This paper also contributes to the literature on conflict. While the security of claims to property is endogenous in all models of conflict, no papers have examined the creation of secure claims to property explicitly as the equilibrium result to a private provision of public goods problem. Our model emphasizes the strategic effect of contributing to the public good of property rights protection. Strategic effects in contest games in which players move sequentially has been studied by Dixit (1987), Baik and Shogren (1992), Leninger (1993), Grossman and Kim (1995), and Kolmar (2008). Dixit simply considered games in which asymmetric players moved in different sequences. Baik and Shogren and Leninger extended this to games in which the sequencing of moves was endogenously determined. Grossman and Kim (1995) and Kolmar (2008) considered games in which agents first chose defensive investments and then chose offensive investments. They each find that relative to the simultaneous move games, agents strategically over invest in defense against predation in order to reduce the amount of equilibrium predation. Hafer (2006), like us, considers a model in which agents have access to two productive processes, one of which requires a resource that is subject to thievery. In her model, there is asymmetric information about the marginal productivity of the insecure production, and players learn the true value of the productivity parameter to its owner by observing the intensity with which he defends his property. In none of these models, however, is defense of property a public good. Finally, Grossman (2002) considers a model in which an autocrat provides secure claims to property. Like us, he finds that the autocrat eliminates conflict. Grossman, however, considers neither the organic alternative to an autocrat that we model, nor how welfare varies between the autocratic and social contract equilibria as the value of production varies.

2 Model Assumptions

We now present a model of conflict in which enforcement of secure claims to property is a public good. Suppose there are $N \geq 2$ identical players, indexed $i = 1, \ldots, N$. Each player has an endowment of ω units of labor and one unit of land. First, labor may be used to consume from the land directly: c_i units of the endowment consumed in this fashion yields c_i units of payoff. We call c_i "subsistence" consumption, as it corresponds to the minimum level of equilibrium payoff. Second, labor may be used to produce a consumable good, "corn". An investment of k_i units of labor into corn production produces Ak_i units of payoff, where A is the marginal product of labor used in corn production. We shall vary the parameter A, which measures how valuable corn production is relative to subsistence consumption, to examine the Demsetz hypothesis.

What distinguishes corn production from subsistence consumption is that, unlike subsistence consumption, corn production can be expropriated by others. While both subsistence gatherings and corn production can be stored, in ancient times conquerers generally took only the surplus above the subsistence level, leaving the conquered on the land to be plucked again another year.[2] Given subsistence consumption has a normalized marginal productivity of unity, if $A \leq 1$, insecure claims to property for corn are not of economic importance.

Let p_{ii} denote the proportion of i's own corn production i appropriates, and let p_{ij} denote the proportion of j's corn production expropriated by i. Thus $p_{ii} + \sum_{j \neq i}^{N} p_{ji} = 1$, for all $i = 1, \ldots, N$. Then because corn production can be expropriated while subsistence consumption cannot implies the following:

Assumption 1. *Each player's payoff is the sum of what he appropriates from his own corn production and what he expropriates from the corn production of the other players, plus his subsistence consumption:*

$$u_i = p_{ii} A k_i + \sum_{j \neq i}^{N} p_{ij} A k_j + c_i, \qquad i = 1, \ldots, N. \tag{1}$$

This follows the guns-or-butter model of Garfinkel and Skaperdas (2007). However, we depart slightly from Garfinkel and Skaperdas with the presence of subsistence consumption, which follows Murphy, Shleifer and Vishney (1993). An important implication of this specification is, for values of $A < 1$, no conflict occurs. Furthermore, when A is high enough to induce no subsistence consumption, the equilibrium allocations to production and conflict do not vary with A.

A player may also allocate his endowment towards two goods that affect the security of property. The tool of conflict, x_i, is the amount of labor utilized protecting one's own property and expropriating the property of others.[3] An increase in x_i increases both the share of i's own corn production that i appropriates and the share of the others' corn production that i expropriates. But conflict is not the only tool that affects the security of property.

Finally, players may also privately provide the public good of enforcing secure claims to property, y_i, which we call "security", by supplying labor in the defense of their own and others' corn production.[4] An increase in the public good of security increases the share of player i's own corn that i appropriates and reduces the share of others' corn that i expropriates. Thus while

[2] In Ostrom's Spanish irrigation example, farmers could lose their access to water, but not the land. In Umbeck's California gold rush example, however, miners could lose the whole claim.

[3] Grossman and Kim (1995) and others have split the investment in conflict into defensive (e.g., locks) and offensive (lock picks) investments. However, if we think of conflict as investment in weapons, then weapons work both as offensive and defensive investments. When offensive and defensive tools of conflict are differentiated, it can be shown that their sum equals the value of x_i in the simultaneous choice Nash equilibrium (furthermore, in the sequential choice game, the second stage first-order-necessary-conditions require solving a system of four third order polynomials, which is not analytically tractable).

[4] We include in security all aspects of protection of secure claims to property including prevention, enforcement,

an increase in i's conflict increases both his appropriation and his expropriation, an increase in i's contribution to security has an asymmetric effect on appropriation and expropriation. Conflict makes the aggressor better off and others worse off (and so generates a negative externality); security is a public good that makes the provider both better off (by making their own production secure) and worse off (by making their thievery less effective).

Indexing the public good of enforcing secure claims to property as G, a positive value of G ensures that the contest success function is asymmetric in the sense that one's appropriation of one's own output in a symmetric equilibrium in a society of size N is greater than $\frac{1}{N}$ and that one's expropriation of the other players output is less than $\frac{1}{N}$:

Assumption 2. *The proportion of player i's corn production that player i appropriates be given by*

$$p_{ii} = \frac{G + x_i}{G + X}, \qquad i = 1, \dots, N, \tag{2}$$

where $X \equiv \sum_{j=1}^{N} x_j$. *Under this technology of conflict, the proportion of player j's corn production that player i expropriates is*

$$p_{ij} = \frac{x_i}{G + X}, \qquad i \neq j, i = 1, \dots, N. \tag{3}$$

It is natural to think of the security of claims to property in terms of p_{ii}. Only when $p_{ii} = 1$, claims to property are perfectly secure. Thus, claims to property are perfectly secure if, and only if, conflict is driven to zero: i.e., if $x_1 = x_2 = \cdots = x_N = 0$.

In writing p_{ii} and p_{ij} in this manner, we are making several assumptions relative to the literature. First, compared to Hirschleifer's specification (with a "decisiveness parameter" set equal to one), where $p_{ii} = x_i/X$, we see that G increases the security of claims to one's own property and decreases one's claims to other's property.[5] Second, compared to the specification in Grossman and Kim (1995), which can be written as $p_{ii} = \phi x_i/(\phi x_i + X_{-i})$, where ϕ measures the effectiveness of defensive weapons versus offensive weapons and where $X_{-i} = \sum_{j \neq i}^{N} x_j$, we see that an increase in G plays a similar role to an increase in ϕ. If, however, we were to use Grossman and Kim's specification, writing $\phi = G$, then the denominators of p_{ii} and p_{ij} each differ, which causes the derivatives of p_{ii} and p_{ij} with respect to G and x_i to be too complicated for closed form solutions.[6] The specification we use assumes the denominators of p_{ii} and p_{ij} to be equal, which restricts the

dispute resolution, and sanctions. Miners in the California gold rush acted both as makers and enforcers of rules regarding mining claims, as did farmers in Spanish irrigation districts, and fishermen in the Nova Scotia cod fishery.

[5] Hirshleifer, following Tullock (1980), lets the proportion of one's own endowment that one appropriates be $p_{ii} = (x_i/X)^m$, where $m > 0$ is the decisiveness parameter. We follow Grossman and Kim (1995), who also set $m = 1$.

[6] The first-order necessary conditions have denominators containing different terms that are each squared, resulting in fourth order polynomials for G and x_i.

first-order-necessary-conditions to be tractable.[7]

The public good, G, of enforcing secure claims to property has three components. First, we assume that there exists a natural asymmetry that favors the owner over the expropriator. This is indexed by the parameter $\theta > 0$, which could occur because of a barrier such as a mountain or a river that divides one's property from others, or simply due to the same evolutionary pressures that require the fox fail more often than the hare. The effect of $\theta > 0$ is that as $X \to 0$, $p_{ii} \to 1$ and $p_{ij} \to 0$. This natural advantage to protecting one's own production, however, is limited:

Assumption 3. $\omega(N-1) > \theta > 0$.

In addition, the public good of enhancing the security of claims to property may be provided either privately or by the state. Private security, $Y \equiv \sum_{i=1}^{N} y_i$, is provided by a private contribution from from each individual. Public security, S, is paid for through a tax, t, imposed by the state. The total contribution to enforcing the security of claims to property is $G = \theta + Y + S$. Thus, natural, private and state provided security are assumed to be perfect substitutes.

From Assumption 2, the rates at which the appropriation and expropriation parameters change as x_i and y_i increase are given by

$$\frac{\partial p_{ii}}{\partial Y} = \frac{\partial p_{ii}}{\partial x_i} = \frac{X_{-i}}{\left(G+X\right)^2} > 0, \qquad \frac{\partial p_{ij}}{\partial Y} = \frac{-x_i}{\left(G+X\right)^2} < 0, \qquad \text{and} \qquad \frac{\partial p_{ij}}{\partial x_i} = \frac{G + X_{-i}}{\left(G+X\right)^2} > 0. \tag{4}$$

Thus the security of one's claim to one's own production in (2) is increasing in G, while one's claim to other's production in (3) is decreasing in G. In contrast, both (2) and (3) are increasing in x_i. Note also that $\partial^2 p_{ii}/\partial x_i^2 < 0$ and $\partial^2 p_{ij}/\partial x_i^2 < 0$, so the marginal productivity of conflict is strictly decreasing. Similarly, $\partial^2 p_{ii}/\partial y_i^2 < 0$, while $\partial^2 p_{ij}/\partial y_i^2 > 0$, so the marginal productivity of security of property is strictly decreasing as well.

Each player simultaneously maximizes his payoff by choosing how he allocates his after-tax labor endowment, $\omega - t$, across the four possible choices: corn production, private provision of security, subsistence consumption, and conflict:

$$k_i = \omega - t - x_i - y_i - c_i, \qquad i = 1, \ldots, N. \tag{5}$$

Thus k_i is the residual from the choices of c_i, x_i, y_i and tax on the labor endowment, t.

[7] See Dixit (1987), Hilman and Riley (1989), Hirshleifer (1991, 2000), Baik and Shogren (1992), Skaperdas (1992, 1996), and Grossman and Kim (1995, *n.* 6 at p. 1279) for detailed discussions of alternative specifications for the technology of conflict. Mueller (2003, at p. 379) provides a rent-seeking contest function analogous to the conflict technology we use.

As we have assumed that each player's endowment is identical, we restrict our attention to symmetric equilibria. However, all of the results presented can be derived with few alterations if we were to allow the endowments to differ.[8]

3 The Simultaneous Choice Nash Equilibrium

Our objective in this section is to see how well claims to property are protected absent a state and to characterize the Nash equilibrium in terms of the productivity parameter, A, the number of competitors, N, and the security of property parameter, θ. Since no state exists, we set $S = t = 0$.

In the Nash equilibrium, each player simultaneously chooses x_i, c_i, and y_i to maximize u_i, taking the other players' actions as given. The first-order-necessary-conditions for player i include (5) and the following:

$$\frac{\partial u_i}{\partial y_i} = \frac{\partial p_{ii}}{\partial Y} A k_i + \sum_{j \neq i}^{N} \frac{\partial p_{ij}}{\partial Y} A k_j - A p_{ii} \leq 0, i = 1, \ldots, N, \tag{6}$$

$$\frac{\partial u_i}{\partial c_i} = 1 - A p_{ii} \leq 0, \qquad i = 1, \ldots, N, \tag{7}$$

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial p_{ii}}{\partial x_i} A k_i + \sum_{j \neq i}^{N} \frac{\partial p_{ij}}{\partial x_i} A k_j - A p_{ii} \leq 0, i = 1, \ldots, N. \tag{8}$$

Each unit of the endowment allocated to any of subsistence consumption, conflict or security has an opportunity cost in foregone appropriated corn production of Ap_{ii}. From (7), the marginal benefit from an increase in subsistence consumption is simply the direct increase in payoff from the subsistence consumption. Since (4) implies that $\frac{\partial p_{ii}}{\partial Y} = \frac{\partial p_{ii}}{\partial x_i}$, the only difference between (6) and (8) is in how expropriation is affected by an increase in security and in conflict. From (8, an increase in expenditures on conflict by i increases both i's appropriation and expropriation shares, while in (6) an increase in expenditures on security by i only increases i's appropriation, while it decreases i's expropriation. Hence, conflict dominates security as a strategy, no matter what others do. This results in the following:

Proposition 1. *In the Nash equilibrium to the conflict game, each individual contributes zero to the public good of secure claims to property protection.*

Proof. In the symmetric Nash equilibrium, (4) implies $\partial p_{ii}/\partial Y = -(N-1)\partial p_{ij}/\partial Y$. Therefore,

[8] When endowments differ, players with lower endowments devote a larger portion of their endowment to conflict in the Nash equilibrium (Hirshleifer 1991); an autocrat taxes those with a larger endowment at a higher rate; under a social contract, those with larger endowments devote a larger share of their resources towards security; and in each, players with sufficiently low endowments may devote all of their resources to conflict.

(6) can be written as

$$\frac{\partial u_i}{\partial y_i} = -p_{ii} A < 0, \qquad i = 1, \ldots, N.$$

Thus in the symmetric Nash equilibrium, each player sets $y_i = 0$. □

It is well known that free-riding effects cause private contributions to public goods to be less than the social optimum, since agents ignore the positive externality they impose upon others. Here, the free riding effect is magnified by the presence of the individually superior investment in conflict. Therefore, Proposition 1 implies that insecure property rights remain insecure in a game of conflict. Furthermore, as this result holds for all $A > 0$, the Demsetz hypothesis does not hold in the simultaneous choice Nash equilibrium. There is also nothing in this result that depends upon the imposition of symmetry in the strategies of players. A similar result can be shown to hold when conflict is separated into defensive and offensive functions as well, since, unlike an allocation to security, an allocation to defense increases the amount one appropriates of one's own production without affecting the amount one expropriates from others. Indeed, this result depends only upon the assumption that an increase in private conflict expenditure increases both the share expropriated and appropriated, while an increase in the public good of enforcement of security of property increases the appropriation share while decreasing the expropriation share.

Given that $y^{NE} = 0$, the symmetric Nash equilibrium conditions for the choice of conflict, x, and subsistence consumption, c, given by (8) and (7), respectively, can be written, using (5), as

$$\frac{\partial u_i}{\partial c_i} = \frac{1}{\theta + X}\big[(\theta + X) - A(\theta + x)\big] \leq 0, \qquad i = 1, \ldots, N. \tag{7'}$$

$$\frac{\partial u_i}{\partial x_i} = \frac{A}{\theta + X}\big[(\omega - x - c)(N-1) - (\theta + x)\big] \leq 0, \qquad i = 1, \ldots, N, \tag{8'}$$

The next result shows that $k^{NE} < \omega$:

Proposition 2. *Under Assumption 3, in the symmetric Nash equilibrium, no player devotes his entire endowment to production.*

Proof. Suppose not. Suppose that players devote their entire endowment to production, then $x^{NE} = c^{NE} = 0$. However, $x^{NE} = c^{NE} = 0$ implies that (8′) can be written as

$$\frac{\partial u_i}{\partial x_i} = \frac{A}{\theta}\big[(N-1)\omega - \theta\big],$$

which is positive by Assumption 3. This contradicts $x^{NE} = 0$. □

This result occurs because setting $c = x = 0$ results in positive net marginal payoff to x_i by (8′), and, if $A < 1$, to c_i by (7′).

Given Propositions 1 and 2, there are three types of equilibria that may arise. Equilibria where $x^{NE} > 0$, $k^{NE} > 0$ and $c^{NE} = 0$ are called the *conflict Nash equilibrium* (CNE), since conflict and production are both positive in this equilibrium. Equilibria where $x^{NE} = k^{NE} = 0$ and $c^{NE} = \omega$ are called the *subsistence Nash equilibrium* (SNE), as there is neither conflict nor production in this equilibrium. Finally, equilibria in which $k^{NE} > 0$, $x^{NE} \geq 0$ and $c^{NE} > 0$ are called the *conflict-subsistence Nash equilibrium* (CSNE), as there is simultaneously conflict, production and subsistence consumption in these equilibria.

First, consider the conflict Nash equilibrium. In this equilibrium, $\frac{\partial u_i}{\partial c_i} < 0$, $x^{NE} > 0$ and $k^{NE} > 0$. Therefore, from (7′), the marginal product of labor allocated to corn production must satisfy $A \geq \bar{A} \equiv \frac{N\omega}{\omega+\theta}$. If this condition holds, then (8′), (5), and (1) imply that

$$y^{NE} = 0, \qquad x^{NE} = \frac{\omega(N-1)-\theta}{N}, \qquad k^{NE} = \frac{\omega+\theta}{N}, \qquad u^{NE} = \frac{A(\omega+\theta)}{N},$$
$$p_{ii}^{NE} = \frac{\omega+\theta}{N\omega}, \qquad p_{ij}^{NE} = \frac{(N-1)\omega-\theta}{N(N-1)\omega}, \qquad \text{for } A \geq \bar{A} \equiv \frac{N\omega}{\omega+\theta}. \quad (9)$$

By Assumption 3, the level of conflict in the CNE is positive for all $N \geq 2$. Hence, property is less than perfectly secure in the CNE. Also, the minimum value of the marginal product of corn production such that the CNE occurs, $\bar{A}$, is increasing in N and ω, and decreasing in θ.

Next, consider the subsistence Nash equilibrium. In this equilibrium, $x^{NE} = k^{NE} = 0$, so that $u^{NE} = c^{NE} = \omega$. Since $x^{NE} = 0$ implies that $p_{ii}^{NE} = 1$, (7′) implies that $c^{NE} > 0$ only if $A < 1$. Therefore,

$$y^{NE} = x^{NE} = k^{NE} = p_{ij}^{NE} = 0, \qquad p_{ii}^{NE} = 1, \qquad c^{NE} = u^{NE} = \omega,$$
$$\text{for } A < 1. \quad (10)$$

Security of property is irrelevant in the SNE since everyone devotes all of their endowment to subsistence consumption, which cannot be stolen.

When $\theta > 0$, there also exists an intermediate case in which $1 \leq A < \bar{A}$.[9] The CSNE is characterized by $c^{NE} > 0$, $x^{NE} \geq 0$ and $k^{NE} > 0$. These are the simultaneous solutions to (8′), (7′) and (5):

$$y^{NE} = 0, \qquad x^{NE} = \frac{\theta(A-1)}{N-A}, \qquad c^{NE} = \omega - \frac{\theta A}{N-A}, \qquad k^{NE} = \frac{\theta}{N-A},$$
$$u^{NE} = \omega, \qquad p_{ii}^{NE} = \frac{1}{A}, \qquad p_{ij}^{NE} = \frac{A-1}{A(N-1)}, \qquad \text{for } 1 \leq A < \bar{A}. \quad (11)$$

[9]When $\theta = 0$, the condition under which $c > 0$ when $x = 0$ is found by applying L'Hopital's Rule to (7′), which yields $A < N$. Similarly, when $\theta = 0$, the condition under which $c = 0$ given that $x > 0$ is that $A \geq \bar{A} = N$.

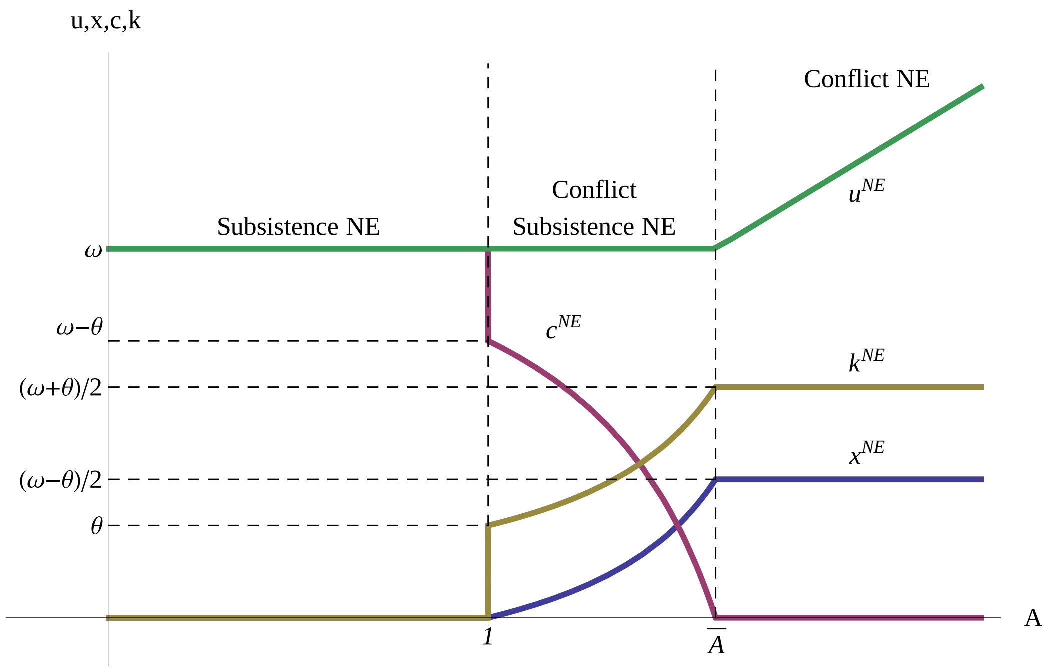

Figure 1: The Nash Equilibrium with $N = 2$ Players ($\theta = \frac{1}{4}\omega$).

Two properties of the CSNE are noteworthy. First, when $\theta = 0$, $x^{NE} = k^{NE} = y^{NE} = 0$ and $c^{NE} = \omega$. Thus, $\theta > 0$ is necessary for the CSNE to exist. Second, in the CSNE, $u^{NE} = \omega$, which is the same level of payoff obtained in the SNE. The marginal payoff of subsistence consumption is equal to unity. Since subsistence consumption coexists with production and conflict in the CSNE, the marginal product of conflict and production are also unity.

Figure 1 shows how the equilibrium choices and equilibrium payoff vary for a range of values of A, when $N = 2$. The SNE occurs for $A < 1$. The CSNE begins at $A = 1$, where k^{NE} jumps from zero to θ and c^{NE} drops to $\omega - \theta$. As A increases in the CSNE, c^{NE} decreases towards zero and k^{NE} and x^{NE} each increase. In the CNE, $A \geq \bar{A}$, $c^{NE} = 0$ and k^{NE} and x^{NE} are each constant, hence the payoff is increasing in A. The lower boundary of the CNE, $\bar{A}$, is increasing in N and decreasing in θ.

Diamond (1997) provides a stark example of how the marginal product of labor allocated to corn production determines the nature of the Nash equilibrium. Around 1000AD, Polynesians settled both New Zealand and the Chatham Islands, some 500 miles southeast of New Zealand. The rich environment of New Zealand contrasted sharply with the cold Chatham Islands climate which was unsuited to Polynesian agriculture. While the Maori grew to a rich but warlike society, the Moriori society reverted to an unstructured hunter-gatherer society. In 1835, upon learning of their existence, 900 Maori sailed to the Chatham Islands where they encountered some 2000 Moriori, whom the Maori declared to be their slaves. The Moriori, who "had a tradition of resolving disputes peacefully," decided to share their resources with the Maori, but before an offer could be made the Maori attacked. A Moriori survivor described the ensuing slaughter: "[The Maori] commenced to kill us like sheep... [We] were terrified, fled to the bush, concealed ourselves in holes underground and in any place to escape our enemies. It was of no avail; we were discovered and killed—men, women, and children indiscriminately" (*id.* at p. 53). Hence the Moriori, who had existed for over 800 years in a SNE, were ill suited for surviving in the CNE to which the Maori had become accustomed.

4 Autocratically Provided Property Rights

Now we consider the case where one of the citizens becomes an autocrat who offers to create secure claims to property by providing state-sponsored security of size S to supplement the existing natural secure claims to property, θ, in exchange for the right to impose a tax of t on the labor endowment of each player.

Bloch (1960) describes the method of taxation of a lord upon his subject during the Medieval period in Europe:

> "The powerful individual who forced his weaker neighbor to submit to him was apt to require the surrender of his property as well as his person. The lesser men, therefore, in offering themselves to the chief, also offered their lands. The lord, once the bond of personal subordination had been sealed, restored to his new dependent the property thus temporarily surrendered, but subject now to his superior right, expressed by the various obligations imposed upon it." (Bloch 1960, at p. 171)

The tax used by William the Conquerer and his successors was a tax on the labor and land endowments, not upon the output, per se. A tax on the endowment does not subject the autocrat to the moral hazard problem that occurs when output is taxed.

The autocrat uses his power of taxation to expropriate wealth for his own consumption. This corresponds to the "tinpot" form of dictatorship in Wintrobe (1990).[10] Since the autocrat keeps any surplus tax revenues above the costs of supplying the security for himself, he chooses the level of security to provide that maximizes the surplus he is able to grab from his citizens. Since even a benevolent autocrat provides a level of S just high enough that citizens allocate nothing to conflict

[10] Mueller (2003, at p. 409) gives examples of several such autocrats. The Roman emperor Nero composed and sang in public, bribed his way to winning in Olympic games, and was alleged to have played his lyre while Rome burned. French autocrat Louis XIV and English autocrat Henry VIII are other examples of autocrats whose consumption (houses and wives, respectively) was deemed extravagant. In modern times, Imelda Marcos, wife of a Philippines dictator, became famous for her 3,000 pairs of shoes.

(*cf.* Grossman 2002), thereby monopolizing the use of force, it is easy for a despotic autocrat to use this force for his own benefit.[11] The problem of despotism, however, is more serious than an inequitable distribution of wealth: the net surplus created under an autocrat is invariant to A, since the autocrat faces a participation constraint that his citizens require at least as much payoff as they gain in the simultaneous choice Nash equilibrium.

In the simultaneous choice Nash equilibrium, players take the tax t and the autocrat's choice of S as given when choosing how to allocate their after-tax endowment between corn production, conflict, subsistence consumption and private provision of the public good of secure claims to property protection. Therefore, in the symmetric simultaneous choice Nash equilibrium, the first-order-necessary conditions for the choices of y, c, and x, respectively, satisfy

$$\frac{\partial u_i}{\partial y_i} = -A\left[\frac{G+x}{G+X}\right] \leq 0, \qquad i = 1, \ldots, N, \tag{12}$$

$$\frac{\partial u_i}{\partial c_i} = 1 - A\left[\frac{G+x}{G+X}\right] \leq 0, \qquad i = 1, \ldots, N, \tag{13}$$

$$\frac{\partial u_i}{\partial x_i} = \frac{A\left[(\omega - x - c - y - t)(N-1) - (G+x)\right]}{G} \leq 0,\ i = 1, \ldots, N. \tag{14}$$

As in the case where no autocrat exists, the first-order condition (12) for y_i is negative for all feasible values of y, x, and c. Hence:

Proposition 3. *In the symmetric Nash equilibrium under an autocrat, each player sets* $y^* = 0$.

Therefore, when an autocrat exists, *only* the autocrat provides secure claims to property. Nevertheless, relative to the Nash equilibrium, there is no crowding out of private provision, as equilibrium contributions to the public good of property protection was already zero by Proposition 1.

Recall that players can ensure payoff equal to ω by consuming some of the endowment directly as subsistence consumption. Since there is no need for secure claims to property in a subsistence economy, the autocrat cannot improve welfare. Thus the interesting outcomes occur when A is large enough that players do not wish to devote any of their labor endowment to subsistence consumption. Absent an autocrat, this occurs when $A \geq \bar{A}$. From (13), (14), and (5), for any feasible values of S and t, when $A \geq \bar{A}$ the Nash equilibrium level of investment in conflict and corn production and the corresponding symmetric equilibrium payoff satisfy the following:

$$x^*(S,t) = \frac{(N-1)(\omega - t) - \theta - S}{N}, \qquad k^*(S,t) = \frac{\omega + \theta + S - t}{N}$$
$$\text{and } u^*(t,S) = \frac{A(\omega + \theta + S - t)}{N}. \tag{15}$$

[11] If citizens can emigrate or revolt, the autocrat's ability to extract surplus is limited. These options, however, are ignored in the model.

Comparing (15) with (9), these are identical to the CNE values with ω replaced by the after-tax endowment, $\omega - t$, and with $G = \theta$ replaced by $G = \theta + S$.

The surplus, R_K, the autocrat earns is the difference between his tax revenues and his costs of providing state-sponsored security:[12]

$$R_K = Nt - S. \tag{16}$$

The autocrat sets the tax rate, t_K^* such that his citizens are indifferent between the simultaneous choice Nash equilibrium outcome – since that is the best that they can do in his absence – and the equilibrium outcome in which the autocrat taxes them at rate t and provides security equal to S.[13] Thus, the surplus gain is entirely captured by the autocrat, which means that R_K measures the welfare gain under an autocrat. There are two cases to consider, depending upon whether $A \geq \bar{A}$ or $A < \bar{A}$.

When $A \geq \bar{A}$, the equilibrium payoffs absent an autocrat are the payoffs in the CNE. From, (9), that payoff is $u^{NE} = \frac{A}{N}(\omega + \theta)$. When each citizen takes S and t as given when choosing x_i and k_i, the payoff with an autocrat is given by (15). Thus the participation constraint faced by an autocrat is

$$u_K^*(t, S) = \frac{A(\omega + \theta + S - t)}{N} \geq \frac{A}{N}(\omega + \theta) \equiv u^{NE}, \qquad \text{for } A \geq \bar{A}. \tag{17}$$

Taking the tax rate as given and solving for the level of security that just makes each player indifferent between having an autocrat who provides security of claims to property of S and doing without an autocrat and experiencing the CNE payoff yields $S_K^*(t) = t$. Substituting this into the surplus function (16) yields $R_K^*(t) = (N-1)t$. The surplus the autocrat earns is strictly increasing in t. From (15), the t_K^* that maximizes the autocrat's surplus is the value of t_K^* such that $x^*(t_K^*) = 0$. An autocrat, therefore, provides the level of security just high enough to prevent any conflict between his citizens. Therefore, when $A > \bar{A}$, under an autocrat, the equilibrium satisfies

$$y_K^* = c_K^* = x_K^* = 0, \qquad k_K^* = \frac{\omega + \theta}{N}, \qquad t_K^* = S_K^* = \frac{(N-1)\omega - \theta}{N},$$
$$\text{and } R_K^* = \frac{(N-1)[(N-1)\omega - \theta]}{N}, \qquad \text{for } A \geq \bar{A}. \tag{18}$$

The values y_K^*, c_K^*, and k_K^* are identical to the CNE allocations to private provision of security, subsistence consumption, and corn production, respectively, given in (9). Furthermore, the tax

[12] Implicit in this calculation is the assumption that N is large enough that we might neglect the opportunity cost of the king's labor.

[13] Bloch (1960, at p. 146) notes that the vassal was often referred to in medieval times as the "man of mouth and hands," to his lord. The reference to 'mouth' implied that the lord was responsible for providing the vassal with the means to provide for himself. We take this to imply that the autocrat cannot reduce welfare beyond the Nash equilibrium welfare.

charged by the autocrat, t_K^*, is identical to the CNE allocation to conflict, x^{NE}, and is independent of A. By Assumption 3, the autocrat improves welfare since $\Delta S_K^* = R_K^* > 0$, for all $A \geq \bar{A}$ and $N \geq 2$. The autocrat is unable to induce his citizens to increase their investment in corn relative to the CNE, howeverm as he has extracted through his taxes exactly the same amount of labor his citizens invested in conflict in the CNE. Hence, the autocrat simply replaces conflict between individual citizens with exploitation by the autocrat. As t_K^* is independent of A, the surplus gain under the autocrat is independent of A.

An increase in θ forces the autocrat to decrease the tax rate and provision of secure claims to property, which means that more is available for production and less is available for expropriation by the autocrat. Thus in a country with good natural protection, the autocrat will thereby prosper less than in a country in which the landscape makes it more difficult to protect one's own property.

When $A < \bar{A}$, the payoff each citizen earns in the CSNE absent an autocrat is $u^{NE} = \omega$. Solving for the size of state-sponsored security, $S_K^*(t)$, that equates the left side of (17) to payoff in the CSNE yields

$$S_K^*(t) = \frac{N\omega}{A} - \omega - \theta + t.$$

Therefore, the autocrat's surplus is

$$R_K^*(t) = (N-1)t - \frac{N\omega}{A} + \omega + \theta.$$

Again, R_K^* is strictly increasing in t, which implies that t_K^* is chosen to set $x_K^* = 0$. Thus, an autocrat always drives conflict to zero.[14] From (13), $x_K^* = 0$ implies that $c_K^* = 0$ for all $A > 1$. Therefore, the autocrat chooses

$$t_K^* = \frac{(A-1)\omega}{A}, \text{ and } S_K^* = \frac{(N-1)\omega}{A} - \theta, \text{ for } A_K \equiv \frac{(2N-1)\omega}{N\omega + \theta} \leq A < \bar{A}, \tag{19}$$

where A_K is the minimum marginal product of labor allocated to corn production such that an autocrat may exist. Hence,

$$y_K^* = c_K^* = x_K^* = 0, \qquad k_K^* = \frac{\omega}{A}, \qquad \text{and } R_K^* = \frac{\omega[N(A-2)+1] + \theta A}{A}, \qquad \text{for } A_K \leq A < \bar{A}. \tag{20}$$

At $A = A_K$, $R_K^* = 0$. Since A_K is decreasing in θ, an increase in θ increases the range of values

[14] Describing the order which William the Conquerer brought to England during his reign of 1066-1087AD, it was remarked that "It was such than any man, who was himself aught, might travel over the kingdom with bosom full of gold unmolested; and no man durst kill another however great the injury he might have received from him" [*sic*] Barrington (1900 at p. 57).

$\{A, N\}$ such that an autocrat can improve welfare relative to the Nash equilibrium. Varying θ reveals that A_K is bounded between $\frac{2N-1}{N}$ and $\frac{2N-1}{N+1}$, which never rises above $A = 2$.

Comparing (11) with (20), the the labor allocated to corn production under the autocrat, k_K^*, is greater than the corresponding CSNE level.[15] This is because there is no subsistence consumption with an autocrat.

When $A_K \leq A < \bar{A}$, (20) implies k_K^* is independent of θ, since an increase in θ is exactly offset by a decrease in S_K^* by (19). Thus, an increase in θ is fully expropriated by the autocrat when $A_K \leq A < \bar{A}$, since the payoff in the Nash equilibrium is independent of θ. This is the opposite of what happens when $A \geq \bar{A}$, where the Nash equilibrium payoff is increasing in θ. Hence, the participation constraint forces the autocrat to leave that surplus with his citizens.

5 The Sequential Choice Social Contract Equilibrium

Now consider a sequential choice game, in which private provision to the public good of enforcing secure claims to property occurs in the first stage, and the remaining endowment is allocated between subsistence consumption, production and conflict in the second stage. Assume that there is no state, so that $S = t = 0$.

We call this game a *social contract*, since players commit to the allocation of y_i prior to making each of the other choices. While there is no explicit agreement made between the parties, the equilibrium allocations to security are derived using subgame perfection, so the implicit agreement is credible. As there is no need for secure claims to property if subsistence consumption is chosen in the second stage, we suppose that $A \geq A_{SC}$, where A_{SC} is the minimum value of A such that $c_1 = c_2 = \cdots = c_N = 0$ in the second stage. In solving the second stage of the game, we let $y_1, y_2, \ldots, y_N$ take arbitrary non-negative values. The allocation of labor between x_i and k_i then depends upon the values of $y_1, y_2, \ldots, y_N$ from the first stage decisions.

Taking $y_1, y_2, \ldots, y_N$ as given, but holding $c_1 = c_2 = \cdots = c_N = 0$, from (8), the first-order-necessary-conditions for the choice of $x_1, x_2, \ldots, x_N$ satisfy

$$\frac{\partial u_i}{\partial x_i} = \frac{A}{(\theta + Y + X)^2}\Big[X_{-i}(\omega - x_i - y_i) + \sum_{j \neq i}(\theta + Y + X_{-i})(\omega - x_j - y_j) \qquad (21)$$
$$- (\theta + Y + x_i)(\theta + Y + X)\Big] = 0, \qquad i = 1, \ldots, N.$$

[15]From (20), $k_K^* = \frac{\omega}{A} > \frac{\theta}{N-A} = k^{NE}$ in the subsistence-conflict Nash equilibrium (see (11)). Rearranging this inequality yields $A < \bar{A}$, which must hold in the CSNE. Similarly, $A < \bar{A}$ implies that $t_K^* = \frac{(A-1)(\omega)}{A} > \frac{\theta(A-1)}{N-A} = x^{NE}$.

Solving the joint system of (21) for $x_i(Y, y_i)$ yields

$$x_i^*(Y, y_i) = \frac{(N-1)(N\omega - Y)^2 - N^2(\theta + Y)(\omega - y_i)}{N^2(N\omega - Y)}, \qquad i = 1, \ldots, N. \tag{22}$$

Observe that $x_i^*(Y, y_i)$ is decreasing in Y_{-i}:

$$\frac{\partial x_i^*}{\partial Y_{-i}} = \frac{-(N-1)(N\omega - Y)^2 - N^2(\omega - y_i)(N\omega - \theta)}{N^2(N\omega - Y)^2} < 0. \tag{23}$$

Furthermore, u_i is decreasing in X_{-i}:

$$\frac{\partial u_i}{\partial X_{-i}} = -(N-1)Ap_{ij} + A(\omega - x_i - y_i)\frac{\partial p_{ii}}{\partial X_{-i}} + \sum_{j \neq i}^{N} A(\omega - x_j - y_j)\frac{\partial p_{ij}}{\partial X_{-i}} < 0, \tag{24}$$

since $\frac{\partial p_{ii}}{\partial X_{-i}} < 0$ and $\frac{\partial p_{ij}}{\partial X_{-i}} < 0$.

Thus, the choice of y_i satisfies

$$\frac{du_i}{dy_i} = \frac{\partial u_i}{\partial y_i} + \sum_{j \neq i}^{N} \frac{\partial u_i}{\partial x_i}\frac{\partial x_j^*}{\partial Y_{-j}} \leq 0, \qquad y_i \geq 0, \qquad \text{and} \qquad \frac{du_i}{dy_i} y_i = 0, \tag{25}$$

where $\frac{\partial u_i}{\partial y_i}$ is given by (6). In the social contract there is a strategic effect associated with property rights protection, given by the second expression in (25), which is positive since an increase in y_i causes x_j to decrease for *all* j, which increases the payoff to i. This suggests that, unlike in the simultaneous choice Nash equilibrium, it may be possible for some $\{A, N\}$ values, that the sequential choice subgame perfect Nash equilibrium contributions to the public good of enforcing the security of claims to property may be positive.

The following proposition characterizes the condition under which $y^{SC} > 0$, and how the subgame perfect Nash equilibrium allocations to conflict, security of property, and production, as well as the resulting payoff and security of property, are related to the parameters.

Proposition 4. *For all $A \geq A_{SC} \equiv \frac{N(N+1)\omega}{2(N\omega+\theta)}$, the symmetric social contract subgame perfect equilibrium (SCSPE) satisfies*

$$c^{SC} = 0, \qquad x^{SC} = \frac{(N\omega - \theta)(N-2)}{(N+1)N}, \qquad y^{SC} = \frac{\omega - \theta}{N+1}, \qquad k^{SC} = \frac{2(N\omega + \theta)}{N(N+1)},$$
$$p_{ii}^{SC} = \frac{2}{N}, \qquad p_{ij}^{SC} = \frac{N-2}{N(N-1)}, \qquad \text{and} \qquad u^{SC} = \frac{2A(N\omega + \theta)}{N(N+1)}; \tag{26}$$

for $1 \leq A < A_{SC}$, the SCSPE is identical to the CSNE; and for $A < 1$ the SCSPE is identical to *the SNE.*

Proof. See the Appendix. □

When $N = 2$, Proposition 4 shows that conflict is completely eliminated, $x^{SC} = 0$, so that secure claims to property are perfectly enforced, i.e., $p_{ii}^{SC} = 1$ when $N = 2$. Furthermore, the expenditure on private provision of security is less than the CNE expenditures on conflict: $y^{SC} = \frac{\omega-\theta}{3} < \frac{\omega-\theta}{2} = x^{NE}$. Thus, the resources spent on security of property are less than the resources spent on conflict in the Nash equilibrium. This means more is left over for production in the SCSPE, i.e., $k^{SC} = \frac{2(N\omega+\theta)}{N(N+1)} > \frac{\omega+\theta}{N} = k^{NE}$. Thus, the payoff is also higher in the SCSPE than in the CNE. These results are all due to the strategic effect investing in security has upon one's rival.[16]

When $N > 2$, the SCSPE does not fully eliminate conflict, since $x^{SC} > 0$ for $N > 2$. The effect any player can have upon influencing the behavior of the balance of the population diminishes as N grows. Indeed, the limit as $N \to \infty$ is that $k^{SC} = y^{SC} = u^{SC} = 0$ and $x^{SC} = \omega$, which is identical to the limit as $N \to \infty$ to the CNE. This occurs because the aggregate provision to security, $Y^{SC} = \frac{N(\omega-\theta)}{N+1}$, is increasing in N, but is bounded from above by $Y^{SC} \leq \omega - \theta$. In contrast, total conflict expenditure is increasing approximately linearly in N, so is unbounded as $N \to \infty$. Thus, the proportion of the dwindling corn production that one appropriates is diminishing towards zero as N increases.

In order for the social contract to arise as an equilibrium arrangement, distinct from the simultaneous choice Nash equilibrium, the marginal product of labor allocated to corn must be sufficiently large, $A > A_{SC} = \frac{N(N+1)\omega}{2(N\omega+\theta)}$. This expression is increasing in N.[17] Indeed, A_{SC} is unbounded as $N \to \infty$. Nevertheless, because $\omega > \theta$ by Assumption 1, A_{SC} is less than $\bar{A}$ for all N. Hence the social contract is possible even for $\{A, N\}$ values such that the payoff in the simultaneous choice Nash equilibrium is equal to the subsistence level.

The surplus created under a social contract is the difference between the payoffs under the Nash

[16] In Dixit (1987), Baik and Shogren (1992), Leninger (1993), Grossman and Kim (1995), and Kolmar (2008), the strategic effect of an increase in the allocation to security of property reduces the subsequent allocation to conflict. Baik and Shogren (1992) and Leninger (1993) consider games in which players simultaneously choose in which period to move in a contest game. The interesting results of those papers occur when players are asymmetric. Here, all players are symmetric, but each player has multiple uses for his endowment.

[17] With around 150,000 people poring into California during the gold rush, gold mining claims remained insecure. According to Clay and Wright (2005), "every two or three claims supported at least one lawyer" (at p. 170). Thus in large societies, conflict dominates the SCSPE, just as it does the Nash equilibrium.

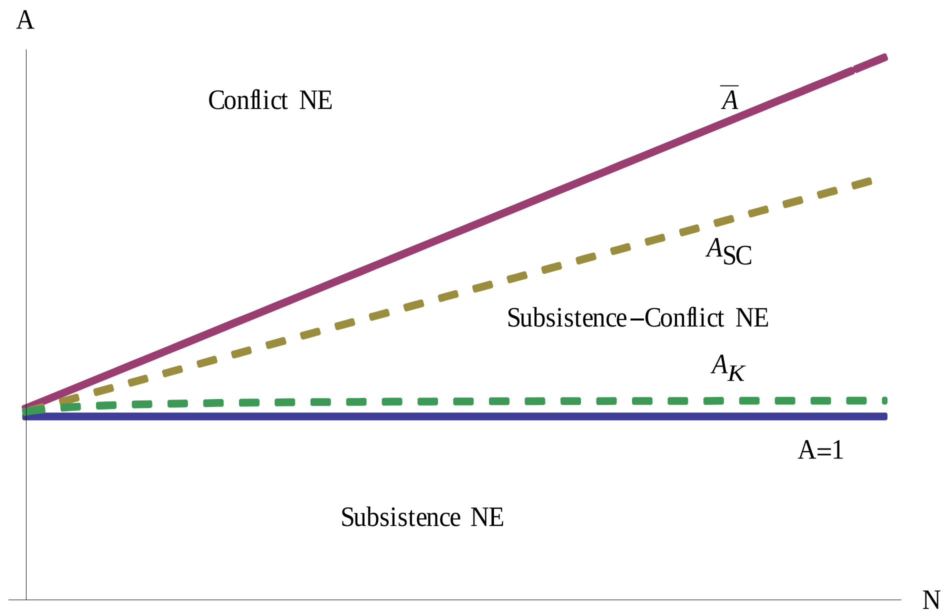

Figure 2: The Conflict Equilibrium Boundaries.

equilibrium and the social contract:

$$\Delta S_{SC}^{*} = N(u^{SC} - u^{NE}) = \begin{cases} \frac{(N-1)A(\omega-\theta)}{N(N+1)} & \text{if } A \geq \bar{A} \\ \frac{\omega N[2A-(N+1)]+2A\theta}{N+1} & \text{if } A_{SC} \leq A < \bar{A} \end{cases} \tag{27}$$

When $A \geq \bar{A}$, the welfare gain from a social contract is positive since $\omega > \theta$ by Assumption 1. It is also positive for all values of $A_{SC} < A < \bar{A}$. For $A < \bar{A}$, a player can always guarantee payoff equal to ω by reverting to subsistence consumption. Thus, $u^{SC} \geq \omega$. However, A_{SC} is the value of A such that $u^{SC} = \omega$. Thus, we have proved:

Proposition 5. *For all values of $A > A_{SC}$, such that the sequential choice social contract subgame perfect Nash equilibrium differs from the simultaneous choice Nash equilibrium, welfare is higher under the social contract than under the Nash equilibrium.*

This occurs because conflict is reduced and production is increased in the SCSPE relative to the CNE.

An example of how the dissipation of rents is controlled in the sequential game is in how rent-dissipation was controlled on oil fields under the rule of capture.[18] One of the consequences

[18]See Hotte (2001) for an example of how conflict games have been used to analyze common property problems.

of the rule of capture in oil fields was that surface owners, recognizing the migratory nature of oil, typically drilled the first wells at the corners of their property. The common law, however, developed an ingenious response to this problem. Producers, as agents for the surface rights owner, were required to drill an offset well adjacent to any well drilled on adjoining properties. While this might at first glance appear to be doubly wasteful, it had the opposite effect. Since most wells were drilled on the boundaries of surface properties, the effect of the offset wells was that most of the oil recovered by a particular producer was from his own property. An example of how effective this was in preserving rents is given by the 71,000 acre Slaughter field discovered in west Texas in 1936, which had over a hundred surface rights owners. An attempt to coordinate production on the whole field failed, but twenty-eight cooperative sub-units were created. By 1975, 427 wells had been drilled along the boundaries of the sub-units at a total cost of 156 million dollars (Libecap and Wiggins (1985). Boyce and Nostbakken (2011) estimated the field generated over five billion 2007 dollars in net-of-drilling-cost revenues for its owners between 1936 and 2008. Thus, the 427 wells drilled on the subunit boundaries dissipated less than three percent of the rents. By prohibiting migration of oil across sub-unit boundaries, these wells provided a public good which prevented even worse dissipation of rents had the oil been allowed to migrate. While not conflict in the sense that property was destroyed through violence, the common property rent-dissipation had the same potential as violence to destroy wealth. But by investing in boundary wells, the field's owners found a way to enhance the security of their claims to property.

6 When Does the Social Contract Welfare Dominate an Autocrat?

Now we can compare the equilibrium welfare under a social contract with that in which secure claims to property are provided by an autocrat.

The minimum value of the marginal product of labor allocated to corn production such that an autocrat can arise is A_K. This is increasing in N at a decreasing rate, and the limit as $N \to \infty$ is $A_K = 2$. Likewise, the minimum level of A under which a social contract can arise, A_{SC}, is also increasing in N, but as $N \to \infty$, A_{SC} is unbounded. This suggests that when N is large, secure claims to property can arise under an autocrat at lower values of A than under the social contract. This is made exact by the following:

Proposition 6. *For $N = 2$, $A_{SC} = A_K$, but $A_{SC} > A_K$ when $N > 2$.*

Proof. We saw above that an autocrat is able to create property rights only if $A \geq A_K = \frac{\omega(2N-1)}{N\omega+\theta}$.

For all $N \geq 2$,

$$\begin{aligned} 0 &\leq (N-2)(N-1) \\ 0 &\leq N^2 - 3N + 2 \\ 2(2N-1) &\leq N(N+1) \\ A_K = \frac{(2N-1)\omega}{N\omega + \theta} &\leq \frac{N(N+1)\omega}{2(N\omega+\theta)} = A_{SC}. \end{aligned}$$

When $N = 2$, the minimum value of the marginal product of labor allocated to corn production under which a social contract can exist is the same as when an autocrat can exist. However, for $N > 2$, the inequalities hold strictly. □

This result is shown in Figure 2, which shows the boundaries between the three Nash equilibria cases as solid lines $A = 1$ and $A = \bar{A}$, and the lower bounds where the autocrat and a social contract can create property rights as the dashed lines A_K and A_{SC}. Both A_K and A_{SC} occur in the conflict-subsistence Nash equilibrium region, which implies that both can create property rights at $\{A, N\}$ values below the $\bar{A}$ locus, where it becomes profitable in the Nash equilibrium to forego subsistence consumption. The A_K locus for $N > 2$ lies below the A_{SC} locus. For large N, players under a social contract have difficulty creating secure claims to property because the free-riding problem overwhelms the strategic incentive. In contrast, an autocrat is able to coerce his citizens to pay taxes to support the provision of secure claims to property, so long as the participation constraint (17) is satisfied. Hence, the autocrat is able to create property rights at much lower levels of marginal productivity of insecure production than is possible with a the social contract.

For all values of A between A_K and A_{SC}, the social contract is dominated by an autocrat because the autocrat is able to create surplus relative to the Nash equilibrium, while the social contract can do no better than the Nash equilibrium. However, for $A \geq A_{SC}$, the welfare gain under the social contract is linearly increasing in A (see (27)), while the welfare gain to an autocrat is independent of A (see (18)). Therefore, we may state the following:

Proposition 7. *For $A \leq A_{SC}$, an autocrat creates greater welfare than the social contract, but for A sufficiently high, the social contract welfare dominates an autocrat.*

Proof. When $A = A_{SC}$, the welfare gain to the social contract relative to the Nash equilibrium is zero by (27). However, under an autocrat the welfare gain relative to the Nash equilibrium is

$$R_K^*(A_{SC}) = \frac{(N-2)(N-1)(N\omega+\theta)}{N(N+1)}.$$

This is positive for all $N > 2$ and equal to zero for $N = 2$. For $A \geq \bar{A}$, the surplus gain relative to the Nash equilibrium from the social contract is linearly increasing in A by (27). In contrast, the surplus gain relative to the Nash equilibrium from the autocrat is given by (18), which is positive, but independent of A. Thus, for A sufficiently large, $\Delta S_{SC}(A) > R_K^*(A)$. $\square$

Both the autocrat (explicitly) and the social contract (implicitly) face the constraint that payoff of the citizens must be at least as large as the Nash equilibrium payoff. Between A_K and A_{SC}, only an autocrat can create surplus relative to the simultaneous choice Nash equilibrium because of the free-riding problem. While an autocrat's surplus is increasing in A when $A_K \leq A < \bar{A}$, for $A \geq \bar{A}$ the net surplus the autocrat creates, $R_K^* = (N-1)x^{NE}$, is independent of A. Hence, this bounds the social gain under an autocrat relative to the Nash equilibrium. In contrast, under a social contract, the welfare gain relative to the Nash equilibrium is linearly increasing in A for all $A \geq A_{SC}$. Thus, for some A sufficiently larger than A_{SC}, the social contract dominates the autocrat.

To illustrate this, suppose that $A \geq \bar{A}$. Then the surplus created under a social contract exceeds the surplus created under an autocrat if $A \geq A^* \geq \bar{A}$, where

$$A^* = \frac{[(N-1)\omega - \theta](N+1)}{\omega - \theta}. \tag{28}$$

Thus a social contract is able to do better than an autocrat so long as the marginal product of labor allocated to corn production is sufficiently large. However, as A^* is increasing at an increasing rate in N, increasing N while holding A constant implies that an autocrat does better than a social contract for large societies. This is because the autocrat is able to overcome the free-riding problem that overwhelms the social contract equilibrium as N grows large.

Figure 3 illustrates the regions in which each form of protection of claims to property may arise.[19] Below the locus A_K, neither a social contract nor an autocrat may arise. Thus, there is no social investment in secure claims to property. The Nash equilibrium (CNE) involves only subsistence consumption when $A < 1$, and entails investments in corn production and conflict, in addition to subsistence consumption, when $1 \leq A < A_K$. In the region where $A_K \leq A < A_{SC}$, a welfare gain is only possible through an autocrat, but no welfare gain through a social contract is possible. In the region $A_{SC} \leq A < A^*$, secure claims to property may be provided by either a social contract or an autocrat, but the welfare gain under an autocrat is higher. When $A \geq A^*$, secure claims to property may still be provided by either a social contract or an autocrat, but the welfare gain under a social contract is larger. This result is consistent with the model of Acemoglu et al. (2000), which implies the rich extend the franchise to avoid a revolt by committing to future wealth

[19] Fig. 3 is not drawn to scale. The locus A^* is scaled by a factor of 1/2 so that the area between A_K and A_{SC} may be distinguished.

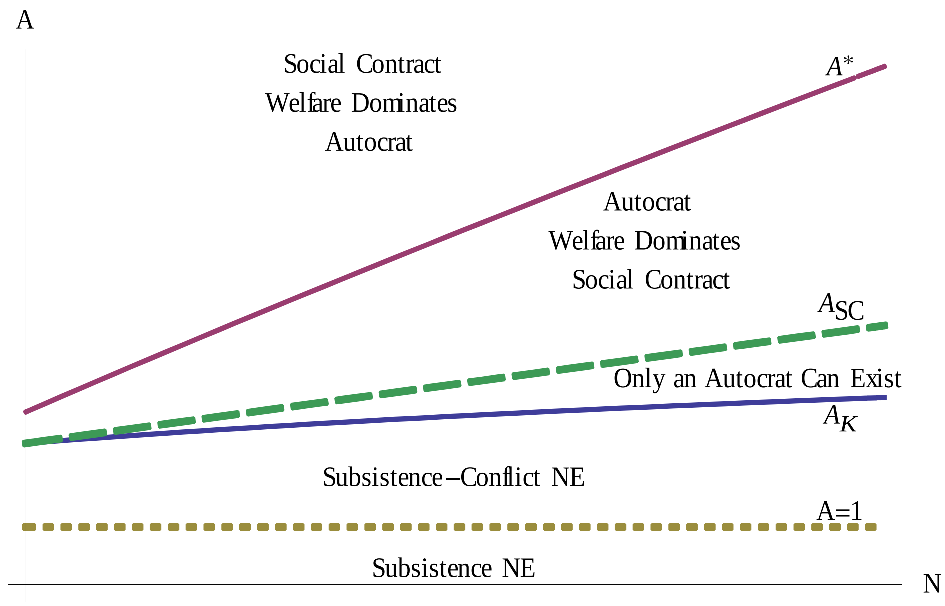

Figure 3: Welfare Maximizing Outcomes in A-N Space.

transfers. As A increases in the region $A_K \leq A \leq A^*$, inequality is on the rise as the autocratic reaps all the surplus gain. Once the inequality reaches A^*, a social contract arises (possibly through revolt) extending the franchise.

Assuming that all autocrats become tinpot dictators and that the welfare maximizing method of social organization is chosen, Figure 3 summarizes the equilibrium types of outcomes that can be sustained as a function of the size of the population and the marginal product of labor allocated to corn production. Holding N constant and raising A, such that $1 \leq A < A_K$, results in moving from a subsistence economy with no conflict, when $A < 1$, to a state in which conflict and production both occur but the payoff is unchanged. In the region $A_K \leq A < A^*$, an autocrat is able to offer a contract, which rational citizens are indifferent between accepting and not, that allows him to increase aggregate welfare, with the autocrat keeping the surplus he creates. In this region, there is no conflict. Above the A^* locus, however, the gains from insecure production are sufficiently high that aggregate welfare is improved by adopting a social contract, even though for $N > 2$ conflict is again positive.[20] Thus conflict is non-monotonically changing as A increases. Below $A = 1$, there is no conflict, and then conflict increases in the region between 1 and A_K. Then conflict is eliminated

[20]A social contract may also arise in the region $A_{SC} \leq A < A^*$, because citizens recognize that there is a surplus gain to themselves by redistributing some of the surplus of the autocrat among the citizens, but a rational autocrat who is able to return enough of the surplus to make his citizens indifferent between the social contract and the side-payments of the autocrat may successfully stay in power as autocracy welfare dominates the social contract in this region.

in the region of autocracy, but rises again once the social contract comes into effect.

The fragility of the social contract in terms of N is also evident in Figure 3. Suppose that $A > A_{SC}$, so that a social contract maximizes welfare. Then holding A constant, an increase in N makes it possible that an autocrat can increase aggregate welfare relative to an existing social contract. This could occur because a benevolent autocrat successfully argues (correctly) that he can increase welfare relative to the social contract because he can eliminate conflict, but then either he or his successor recognizes that he can capture that surplus for himself.

7 Conclusions

This paper has examined whether the Demsetz hypothesis, that secure claims to property arise when the value of creating them rises, holds in an anarchy equilibrium. We considered a game in which players may allocate their endowment across subsistence consumption, investment in insecure productive activities, conflict over the insecure production, and contributing to the public good of property protection. We evaluated how equilibrium behavior changes as the marginal product of labor allocated to insecure production, relative to subsistence consumption, and the number of players are varied.

Secure claims to property are not provided in a game in which players simultaneously choose to allocate their endowment among production, conflict, and the public good of secure claims to property, in the Nash equilibrium. This occurs because allocations to conflict raise both the proportion of a player's own production that he appropriates and the proportion of others' production that he expropriates. In contrast, contributions to secure claims to property only raise the proportion of his own production a player appropriates; it reduces the proportion of others' production he expropriates. Thus players prefer to allocate labor to conflict rather than the public good of property protection. As the marginal product of insecure production rises, all else equal, society becomes richer because the proportion of the endowment allocated to conflict is bounded from above. Larger societies, however, are characterized by higher levels of conflict, and hence players receive lower payoffs.

An autocrat who taxes the endowments of his citizens to provide secure claims to property, and who keeps all of the surplus above the Nash equilibrium level of payoff of his citizens, is able to provide secure claims to property, so long as the marginal product of labor allocated to insecure production is large enough to pay for the provision of secure claims to property. Indeed, such an autocrat creates perfectly secure claims to property, driving conflict to zero. When the value of the marginal product of labor allocated to insecure production is large, however, an autocrat merely replaces the Nash equilibrium level of conflict with expropriating taxes. As the level of conflict is bounded, so is the level of taxation. Thus the amount of surplus an autocrat can create relative to the Nash equilibrium is limited.

In a social contract, players first simultaneously allocate part of their endowment to protection of claims to property, and then allocate the remainder between conflict and insecure production. When there are only two players, the resulting subgame-perfect Nash equilibrium creates perfectly secure claims to property at the same cost as an autocrat. As the number of players rises, however, the minimum marginal product of labor allocated to insecure production necessary for a social contract to arise is always higher than that under which an autocrat can create secure claims to property. Furthermore, security of claims to property created in this fashion are imperfect, as players under-invest in property rights protection relative to the social optimum in an attempt to free-ride on others. Nevertheless, the social contract welfare dominates an autocrat when the marginal product of insecure production is sufficiently high. The level of the marginal product of insecure production, such that the social contract welfare dominates an autocrat, is increasing in the size of the population. Thus a social contract is most likely to arise in small populations with high productivity of investment.

The results of our model are consistent with historical examples. For example, the *Magna Carta* is the result of the English nobility replacing their despot with a social contract, presumably because it was welfare enhancing (as well as redistributive). However, as the mining camps of the California gold rush and the grazing associations of the American west demonstrate, the social contract is fragile. In both cases, a social contract evolved which provided secure claims to property. In both cases, however, the social contract involved a substantial amount of conflict and eventually collapsed. The deterioration of the social contract was a result of a substantial increase in the population, in the case of the mining camps, and a significant decline in the value of the livestock, in the case of the the grazing associations.

A Mathematical Appendix

A.1 Proof of Proposition 4

Proof. Substituting (22) into the payoff function yields, after some simplification, the value function in terms of y_i and Y:

$$u_i(Y, y_i) = \frac{A}{N^2}\left[\frac{N^2\omega(\omega+\theta) + N(N-2)\omega Y + Y^2 - N^2 y_i(Y+\theta)}{N\omega - Y}\right], \qquad i = 1, \ldots, N. \tag{29}$$

Each player in the public goods provision stage chooses y_i, taking the Y_{-i} as given. Hence, the first order condition in the choice of y_i is

$$\frac{\partial u_i}{\partial y_i} = \frac{A}{N^2(N\omega - Y)^2}\Big\{(N^2-1)Y^2 + NY[N\theta - (N-2)\omega] \\ - N^2[y_i(N\omega + \theta) - (N-1)(\omega - \theta)\omega]\Big\} = 0, \qquad i = 1, \ldots, N. \quad (30)$$

Imposing symmetry on (30), so that $y_1 = y_2 = \cdots = y_N \equiv y$, yields

$$\frac{\partial u_i}{\partial y_i} = \frac{A(N-1)[\omega - \theta - (N+1)y]}{N^2(\omega - y)} = 0.$$

Solving this for y yields the subgame perfect level of private provision to security, and substituting these results back into (2), (3), (7), and (22) yields the results in (26). $\square$

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