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Resource and Energy Economics 30 (2008) 428–454

RESOURCE and ENERGY ECONOMICS

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An 'oil'igopoly theory of exploration

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Received 13 September 2006; received in revised form 10 October 2007; accepted 11 October 2007
Available online 22 October 2007

Abstract

This paper develops a theory of 'oil'igopoly exploration of an exhaustible resource. Strategic exploration and production are jointly derived in a three period subgame perfect equilibrium. While the 'oil'igopoly theory of exploration shares many features with non-strategic models of exploration and production, there is one important difference. The 'oil'igopoly theory of exploration predicts that firms who exhaust their proved reserves before they can convert their unproved reserves into proved reserves have an incentive to over-explore, relative to the Nash equilibrium level of exploration. A simple empirical prediction is that firms holding smaller proved reserves should be observed doing more exploration. This prediction is consistent with country-level production and reserve data in the post-World War II era.

JEL classification: Q30; L71; C73

Keywords: Oligopoly; Exploration; Exhaustible resources; Dynamic consistency

1. Introduction

The theory of 'oil'igopoly, developed by Salant (1976) and extended by Loury (1986) and Polasky (1992), has the simple yet elegant prediction that firms holding larger proved reserves tend to produce quantities which are larger in absolute size but smaller as a proportion of their reserves. Polasky found support for this prediction using data on proved reserves and production

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0928-7655/$ – see front matter
doi:10.1016/j.reseneeco.2007.10.001

in a cross-section of oil producing nations. However, there are two aspects of the theory of 'oil'igopoly that limit its general appeal.

The first of these limitations is that the results are derived using the Nash equilibrium concept rather than the subgame perfection equilibrium concept.[2] It is well known that the Nash equilibrium to dynamic games is not generally dynamically consistent. When the resource stocks are commonly owned, Levhari and Mirman (1980) and Reinganam and Stokey (1985) found substantial differences between the time paths of production in the Nash and subgame perfect equilibriums. However, Eswaran and Lewis (1985) showed that when firms possess well defined property rights, the Nash equilibrium differs only slightly from the subgame perfect Nash equilibrium.[3]

The theory of 'oil'igopoly also ignores exploration. When exploration is added to the game, it is no longer clear that the Nash equilibrium will yield results that are qualitatively similar to the dynamically consistent subgame perfect equilibrium.[4] The reason for this difference can be seen if one views exploration as the costly process of moving reserves from the "unproved" to the "proved" state, then exploration may have strategic implications.[5] This happens because once exploration occurs, the exploration costs become sunk. As exploration costs are on the order of hundreds of thousands of dollars for a well drilled on land to millions of dollars for a well drilled at sea, sinking the exploration cost results in a substantial lowering of the marginal cost of production.[6] By lowering its marginal costs of future production, a firm has a credible threat to its rivals that it will produce a larger quantity in the next period.[7] This threat induces one's rivals to tilt their production profile towards the present, which raises the present value of future production to the firm.

This paper examines a model of 'oil'igopoly exploration and production using subgame perfection as the equilibrium concept. Our first objective is to ascertain the conditions under which firms strategically use exploration to affect the behaviour of their rivals. Given that an

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[2] Salant (1981, 1982), Lewis and Schmalensee (1980) and Ulph and Folie (1980) have also used Nash strategies to model the world oil market. See Mason and Polasky (2005) and Benchekroun et al. (2006) for recent extensions to the Nash model. See Gilbert (1978), Newbery (1981), Groot et al. (1992, 2003) for Stackelberg cartel-fringe models. Karp (1984), Maskin and Newbery (1990), Karp and Newbery (1993) consider Stackelberg models in which governments extract rents from exhaustible resource industries over time. These models also focus on the difference between open loop (Nash) and dynamically consistent (subgame perfect) equilibria.

[3] For example, if demand is linear, $p = \alpha - \beta Q$, and costs are quadratic, $c(q) = (\gamma/2)q^2$, then the solutions to the two period problem are independent of proved reserves, R_1 and R_2, which means that the Nash and subgame perfect equilibria in a game of more than two periods coincide exactly so long as the firms produce the same number of periods. Polasky (1992, no. 1, p. 217), citing Stiglitz (1976) claims that when marginal costs are constant and demand is iso-elastic, the Nash and subgame perfect equilibria also coincide. In the asymmetric game considered by Eswaran and Lewis (1985, Table 1, p. 466), the larger firm's output varied between −4.4 to +1.6%. The smaller firm's output varied from −1.1 to −4.4% in the first three periods, but by almost 30% in the final period in which it operated.

[4] Competitive models of exploration appear in Pindyck (1978), Arrow and Chang (1982), and Swierzbinski and Mendelsohn (1989).

[5] Proved reserves are those reserves for which exploration has already demonstrated the existence of an economically viable deposit. Unproved reserves are those reserves that the geologic indicators suggest exist, but which have not yet been discovered, or transformed into proved reserves, through exploration.

[6] Average drilling costs in the United States were approximately seven hundred thousand dollars for an onshore well and over twelve million dollars for an offshore well in 2002. *Source*: *Basic Petroleum Databook*, American Petroleum Institute, 2006.

[7] The strategic advantage conveyed by exploration is similar to that obtained from an increase in plant capacity, or R&D research to lower production costs in the industrial organization literature (e.g., Dixit, 1980, 1986; Fudenberg and Tirole, 1984; Bulow et al., 1985). The literature on strategic investments is surveyed in Tirole (1990, pp. 314–336).

exhaustible resource market exhausts all stocks in the final period of the game, there can be no strategic effects in a two period game. Hence, we solve for the subgame perfect Nash equilibrium in a three period game in which firms compete not only in the output market, but also in the process of exploration. Thus, our second objective is to examine how strategic exploration affects strategic production, and visa versa. Thirdly, the subgame perfect equilibrium is compared with the Nash equilibrium.

Not surprisingly, we find that the strategic 'oil'igopolistic exploration model shares a number of features with non-strategic models. For example, firms extract from proved reserves until those reserves are exhausted before extracting from unproved reserves (e.g., Hartwick, 1978). When possible, firms also equate the present value of marginal discovery costs over time. Thus, independent of strategic effects, firms with larger unproved reserves do more exploration in each period. In addition, when possible, firms equate the present value of marginal extraction profits over time. This means that if a firm has sufficient proved reserves relative to unproved reserves that it continues to have proved reserves up to the last period in which it produces, then it equates both the present value of marginal discovery costs and marginal extraction profits in each period in which it produces.

The difference between the strategic and non-strategic models only becomes apparent for those firms who exhaust their proved reserves before they have converted their unproved reserves into proved reserves. It is for these firms that the "when possible" qualification exits in the previous paragraph. These firms face declining resource quality along the extraction path as they switch from proved reserves to higher cost unproved reserves. Thus the present value of marginal extraction profits and the marginal discovery costs declines over time. It is for these firms that we find a strategic incentive to over-explore relative to the Nash equilibrium. However, this incentive only occurs in period one in a three period game, and furthermore, it only occurs for the subset of firms who exhaust their proved reserves before they have converted their unproved reserves into proved reserves. For these firms, numerical simulations suggest that this effect can be on the order of a between a seven to 10% increase in exploration and an increase in 10–20% in the quantity of proved reserves taken into period two. In contrast, firms holding proved reserves in sufficient quantity to produce from these reserves in the next period already have a credible commitment device to signal to rivals that they will produce a larger quantity in subsequent periods. For them, the benefit of holding larger quantities of proved reserves is small. By way of example, Saudi Arabia's Aramco holds proved reserves that will last between 70 and 80 years at its current production levels—it is not surprising, then, that they have done little exploration.[8]

Finally, as in the theory of 'oil'igopoly production, we find that, all else constant, firms holding larger proved reserves extract a larger quantity but a smaller proportion of their reserves in each period prior to exhaustion. However, the *equilibrium* first period output of firms is affected by two offsetting forces. The strategic effect causes firms to under-produce relative to the Nash equilibrium. But if all firms are under producing, the marginal profits of first period production increase, which causes the firm to increase its first period production. The equilibrium result is the sum of these two effects. Thus, we provide an explanation for why the differences between the Nash and subgame perfect solutions are quite small in models in which only production occurs.

[8] In the post-World war period, there have been only about 2000 wells drilled in the Gulf region, compared to over two million drilled in the United States (see "Really Big Oil," *The Economist*, 10 August 2006).

An important limitation to the theory of 'oil'igopoly production is that firms face no uncertainty over their reserve holdings. We too maintain this assumption in our theory of 'oil'igopoly exploration. Implicitly, this means that unproved reserves are known in quantity, but are costly to develop into proved reserves. We also assume that firms face no competition for their unproved reserves. Thus, there is no "race" to capture these reserves.[9] An important implication of these assumptions is that we can abstract from informational issues associated with exploration.[10]

This paper is closest in spirit to Bulow and Geanakopolis (1983) and Hartwick and Sadorsky (1990). Those papers were also interested in the strategic effects from exploration from higher-cost stocks due to exploration's role as a commitment device. However, those papers make an important assumption that limits the generality of their results. They only consider two period models. Since if firms were to exhaust all reserves in the second period there can be no strategic effect from exploration, these authors effectively leave some reserves unexploited.[11] Thus in each of these models there exists a subgame in which the excess reserves are exploited, but the effect of this subgame on the remainder of the game is ignored. In contrast, we solve for entire production and exploration path, including the endgame in which reserves are completely exhausted. Thus, we offer a complete characterization of the dynamics of the game.

The remainder of the paper is organized as follows. Section 2 presents the equilibrium beginning when there is only one period left before all oil is exhausted, and then characterizes the equilibrium when two periods remain before exhaustion. Section 3 derives the main results regarding strategic exploration by moving back three periods from exhaustion and asking how firms behave at that point, given the effects on rivals' subsequent behaviour. Section 4 presents numerical simulations of the equilibria. Section 5 presents a simple empirical model of exploration as a function of beginning reserves using country-level data over the post-World War II era. Section 6 concludes.

2. 'Oil'igopoly exploration and production

The game we consider is restricted to three periods. Implicitly, this means there is a restriction on the sum of reserves held by all firms. In this section, we describe the behaviour of firms in the last two periods in the game. In Section 3, we show how these results affect strategic exploration.

2.1. Notation and assumptions

At the beginning of each period t, let n_t firms hold either proved, unproved, or both types of reserves. Proved reserves held by the ith firm at the beginning of period t are denoted as R_{it}, and

[9] This assumption follows from evidence that most of the significant players in the world oil market are state-owned firms, which face little or no competition for access to the resource stocks within their own countries. Sixteen of the top twenty oil firms by reserve holdings are state owned firms. See "Really Big Oil," *The Economist*, 10 August 2006).

[10] See Mason (1986), Isaac (1987), Polasky (1996), and Hendricks and Porter (1996) for models of information transmission in exploration. These models implicitly assume that mineral rights are not secure. These models have focused on whether there is too little or too much exploration from an information gathering perspective and whether the timing of exploration has strategic information effects.

[11] In Hartwick and Sadorsky, firms in the first period choose both the level of exploration and production, but in the second period firms only produce from their remaining proved reserves—they do no further exploration. In Bulow and Geanakopolis, firms in each period extract from lower cost reserves and from a higher cost backstop technology. Depletion of reserves raises the future marginal costs of extraction from those reserves. However, the lower cost reserves are not exhausted in their model. In each of these models, current production affects subsequent profits, but it cannot affect subsequent behaviour.

unproved reserves are denoted as S_{it}. Thus initial reserves held by firm i are denoted as R_{i1} and S_{i1}, respectively. We let $\boldsymbol{R}_t = \{R_{1t}, R_{2t}, \ldots, R_{n_t t}\}$ and $\boldsymbol{S}_t = \{S_{1t}, S_{2t}, \ldots, S_{n_t t}\}$ denote the vectors of stocks held at the beginning of period t by all n_t firms active in that period; $R_t = \sum_{i=1}^{n_t} R_{it}$ and $S_t = \sum_{i=1}^{n_t} S_{it}$ denote the stocks held at the beginning of period t by all n_t firms; and $R_{-it} = \sum_{j\neq i}^{n_t} R_{jt}$ and $S_{-it} \sum_{j\neq i}^{n_t} S_{jt}$ denote the sum of reserves held by all firms other than firm i at the beginning of period t. We assume that the stocks of proved and unproved reserves for all firms are common knowledge.

Since the game ends in three periods, all firms exhaust their reserves by period three (i.e., $n_4 = 0$). For a given allocation of reserves of each type, the number of firms exhausting each type is endogenous. However, rather than deriving the equilibrium number of firms that exhaust in each period, without loss of generality, we fix the number of firms exhausting in each period, i.e., the $\{n_{t+1} - n_t\}_{t=1,2,3}$, and derive the conditions on the reserve holdings that have to be satisfied in equilibrium in order for this number of firms to rationally exhaust in each period.

In each period, each firm chooses a level of output, q_{it}, and a level of reserve additions, w_{it}, $t = 1, 2, 3$, $i = 1, \ldots, n_t$. The model is deterministic, so each unit of exploration yields a fixed quantity of reserve additions. Given the production and reserve additions choices made by firm i in period t, the stocks of proved and unproved reserves evolve according to

$$R_{it+1} = R_{it} + w_{it} - q_{it}, \quad i = 1, \ldots, n_t, t = 1, 2, 3, \tag{1}$$

$$S_{it+1} = S_{it} - w_{it}, \quad i = 1, \ldots, n_t, t = 1, 2, 3. \tag{2}$$

The price at time t is denoted by $P_t = P(Q_t)$, where $Q_t = \sum_{i=1}^{n_t} q_{it}$, and where the demand function, $P(Q_t)$, is positive valued, finite, and decreasing in aggregate output. The extraction and discovery costs are denoted by $c_i(q_{it})$ and $d_i(w_{it})$, respectively. These have the properties that $c_i'(q_{it}) > 0$ and $c_i''(q_{it}) \geq 0$ for all $q_{it} \geq 0$, and $d_i'(0) = 0$, but $d_i'(w_{it}) > 0$ and $d_i''(w_{it}) > 0$ for all $w_{it} > 0$. For simplicity, we assume that the cost of extraction and discovery are independent of reserves.[12]

As we wish to restrict the model to three periods so that we can use backwards induction, we impose further restrictions on demand and cost functions:

Assumption A.1. $c_i'(0) + d_i'(0) < P(0) < \infty$.

The first inequality ensures that firms wish to ultimately exhaust their reserves, as the marginal revenue exceeds the cost of extraction for the last unit of production. The second inequality implies that the resource is not essential to production in the economy. This assumption ensures that exhaustion occurs in finite time.

In addition, we make two regularity assumptions which, when taken together, ensure that the best-response functions are stable. This is necessary in order for strategic effects to take place. These assumptions are

Assumption A.2. $P'(Q_t) + q_{it}P''(Q_t) < 0$, for all $q_{it} > 0$ and for all $Q_t > 0$.

Assumption A.3. $c_i''(q_{it}) - P'(Q_t) > 0$, for all $q_{it} > 0$ and for all $Q_t > 0$.

[12] Thus the only grade differential in the stocks is the difference between proved and unproved reserves. See Swierzbinski and Mendelsohn (1989), *inter alia*, for a model of grade differentials under competitive extraction and exploration.

Assumption A.2 implies that firm i's marginal profit is lowered by an increase in the output of any other firm. This implies that the goods are strategic substitutes and occurs because the choice variable is the output (Bulow et al., 1985). Assumption A.3 implies that the demand function intersects the marginal cost function from above. Taken together, Assumptions A.2 and A.3 imply that second order conditions are satisfied for each firm, and these assumptions together are also sufficient conditions to yield the existence of a unique and stable equilibrium (Vives, 1999, Theorem 2.7).

We also assume that firm i's total profits, not just marginal profits, are decreasing in the output level of other firms (*cf.* Tirole, 1990, p. 326). In a single period game, these automatically hold, since the effect on firm i's profits of an increase in the output of other firms, Q_{-it}, is simply $P'(Q_t)q_{it} < 0$. However, when we move to the second period in a game in which all firms that produce in period three exhaust their stock by the third period, then the assumption that profits are decreasing in the second period output of other firms is

Assumption A.4. $q_{i2}P'(Q_2) - \beta q_{i3}P'(Q_3) < 0$, for $q_{i2} > q_{i3}$ and $Q_2 > Q_3$, $i = 1, \ldots, n_2$.

Finally, one dollar earned or spent one period in the future is discounted at the common rate $\beta \in (0, 1)$.

Now, we turn to the analysis of the game, which we begin in period three, the final period in which any firms produce.

2.2. The period three equilibrium

In the final period of the game, there are n_3 firms each holding reserves $R_{i3} \geq 0$ and $S_{i3} \geq 0$ (with $R_{i3} \pm S_{i3} > 0$). The problem faced by firm i at the beginning of period three is to choose q_{i3} and w_{i3}, taking the actions of the other firms fixed, to maximize

$$\textbf{P3} \quad V_{i3} = P(Q_3)q_{i3} - c_i(q_{i3}) - d_i(w_{i3}), \quad i = 1, \ldots, n_3.$$

subject to the constraints that (Kuhn–Tucker multipliers in parentheses)

$$R_{i3} + w_{i3} - q_{i3} \geq 0, \qquad (\lambda_{i3}) \quad i = 1, \ldots, n_3, \tag{3}$$

$$S_{i3} - w_{i3} \geq 0, \qquad (\mu_{i3}) \quad i = 1, \ldots, n_3, \tag{4}$$

Given that all firms are assumed to exhaust by period three, the constraints (3) and (4) are each binding, so that in equilibrium, $w_{i3} = S_{i3}$ and $q_{i3} = R_{i3} + S_{i3}$. Then the first-order necessary conditions for each firm can be written as

$$\begin{aligned} \frac{\partial V_{i3}}{\partial q_{i3}} &= P(R_3 + S_3) + (R_{i3} + S_{i3})P\prime(R_3 + S_3) \\ &\quad - c_i'(R_{i3} + S_{i3}) - \lambda_{i3} = 0, \quad i = 1, \ldots, n_3, \end{aligned} \tag{5}$$

$$\frac{\partial V_{i3}}{\partial w_{i3}} = d_i'(S_{i3}) + \lambda_{i3} - \mu_{i3} = 0, \quad i = 1, \ldots, n_3. \tag{6}$$

Condition (5) says that each firm equates marginal revenue with marginal extraction costs plus scarcity rent. Condition (6) implies that marginal discovery costs are equated with the net scarcity rent. Note that Assumption A.2 implies that λ_i is decreasing in the aggregate reserves held by all other firms at the beginning of period three.

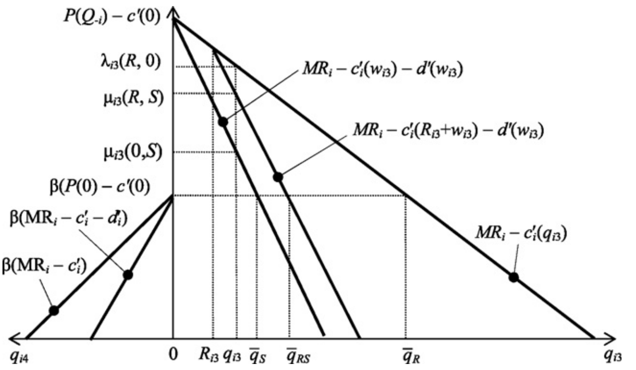

Fig. 1. Rational exhaustion in period three by firm i. *Note*: The left-hand-side horizontal axis is not drawn to scale. $MR_i = P(Q_t) + q_{it}P'(Q_t)$.

Next, we turn to the condition which is both necessary and sufficient for all firms to exhaust in period three. Suppose that firm i enters period three with positive values of both stocks. In order for firm i to exhaust in period three, the marginal profits from period three must exceed those from waiting another period, taking the actions of all other firms as fixed. The discounted marginal profits to such firm who waits until period four are $\beta[P(0) - c_i'(0) - d_i'(0)]$, since all other firms are assumed to have exhausted in period three. Combing (5) and (6) by eliminating λ_{i3} and comparing the marginal profits in period three and four yields the condition that must hold if firm i is to exhaust in period three, given that he holds both types of reserves:

$$\begin{aligned} &P(R_3 + S_3) + (R_{i3} + S_{i3})P'(R_3 + S_3) - c_i'(R_{i3} + S_{i3}) - d_i'(S_{i3}) \\ &= \mu_{i3} > \beta[P(0) - c_i'(0)], \quad \text{if } R_{i3}, S_{i3} > 0, \; i = 1, \ldots, n_3. \end{aligned} \tag{7}$$

For a firm that holds only unproved reserves the condition (7) is unchanged, except that $R_{i3} = 0$. But the condition for a firm that holds only proved reserves would not contain the $d_i'(S_{i3})$ term, and μ_{i3} would be replaced by λ_{i3} (see (5)):

$$\begin{aligned} &P(R_3 + S_3) + R_{i3}P'(R_3 + S_3) - c_i'(R_{i3}) \\ &= \lambda_{i3} > \beta[P(0) - c_i'(0)], \quad \text{if } R_{i3} > 0, S_{i3} = 0, i = 1, \ldots, n_3. \end{aligned} \tag{8}$$

These conditions say that in order to be satisfied by exhausting in period three, the rents earned by firm i in period three must be greater than the present value of the rents earned by waiting one period, taking as given the actions of all other firms. The Nash equilibrium is that all firms exhaust if, and only if, the inequalities in (7) and (8) holds for all n_3 firms. The equilibrium condition for a particular firm is illustrated graphically in Fig. 1.

Under Assumptions A.2 and A.3, the marginal profits to the firm in each period are downward sloping functions. The solid lines in Fig. 1 are the marginal profits for a firm holding only proved reserves. The dashed lines are the marginal profits for a firm who holds unproved reserves, with the lower dashed curve in the right-hand-side panel corresponding to the case where $R_{i3} = 0$ and the upper dashed curve corresponding to the case where $R_{i3} > 0$.[13]

[13] For brevity, we shall use time subscripts for the prices and costs when the functional arguments are suppressed. Thus, $c_{it} \equiv c_i(q_{it}), c_{it}' \equiv c_i'(q_{it})$, and $c_{it}'' \equiv c_i''(q_{it})$; and similarly for $d_i(w_{it})$ and $P(Q_t)$.

If firm i has cumulative reserves equal to q_{i3} in Fig. 1, then the marginal profits to the firm are λ_{i3} if $q_{i3} = R_{i3} > 0$ and $S_{i3} = 0$; the marginal profits are $\mu_{i3}(R, S)$ if $q_{i3} = R_{i3} + S_{i3}$ where both R_{i3} and S_{i3} are positive; and the marginal profits are $\mu_{i3}(S)$ if $q_{i3} = S_{i3} > 0$ and $R_{i3} = 0$. At q_{i3}, each of these values is larger than $\beta[P(0) - c_i'(0)]$, the present value of marginal profits in period four. The quantities $\bar{q}_{\mathrm{S}}$, $\bar{q}_{\mathrm{RS}}$ and $\bar{q}_{\mathrm{R}}$ are thus the maximum total reserves that can be held by a firm holding only unproved reserves, holding both proved and unproved reserves, or holding only proved reserves, respectively, given the holdings of all other firms, such that firm i will rationally exhaust in period three. If the appropriate condition holds for all n_3 firms, then period three is the equilibrium time of exhaustion. Note that $\bar{q}_{\mathrm{S}} < \bar{q}_{\mathrm{RS}} < \bar{q}_{\mathrm{R}}$. This occurs because unproved reserves face the additional cost of discovery, $d_i'(S_{i3})$, relative to proved reserves. We can see that a firm carrying only proved reserves into period three can carry more reserves, and still want to exhaust in the third period, than a firm carrying only unproved reserves or a firm carrying both types of reserves, all else constant. Note also that the marginal profit curve for a firm holding both types of stocks in positive quantities is kinked at R_{i3}. This reflects the change in costs once the firm runs out of proved reserves.

The key point to note from Fig. 1 is that so long as the reserves for all firms satisfy the limits, small changes in the reserve holdings of other firms do not affect the *behaviour* of the firm i.[14] Rather, a small change in other firms' reserves affects firm i's *profits*, but it does not affect q_{i3} or w_{i3}.[15] This implies that the subgame perfect equilibrium and the Nash equilibrium for the subgame beginning at period two are identical. An important implication is that, without loss of generality, we may solve for the equilibrium beginning in period two using the Nash solution, since the Nash and subgame perfect solutions are identical.

2.3. Period two-subgame perfect equilibrium

We turn now to the problem faced by firm i in period two, given that it holds reserves R_{i2} and S_{i2}, where $R_{i2} + S_{i2} > 0$, but either R_{i2} or S_{i2} could be zero. We continue to assume that some firms may exhaust in period two, but no firms exhaust beyond period three. Thus, q_{i3} is less than the appropriate $\bar{q}$ in Fig. 1.

Firm i's problem in period two is to choose exploration and production $\{q_{i2}, q_{i3}, w_{i2}, w_{i3}\}$, taking the choices of all other firms as fixed, to maximize[16]

$$\mathbf{P2} \quad V_{i2} = P(Q_2)q_{i2} - c_i(q_{i2}) - d_i(w_{i2}) + \beta[P(Q_3)q_{i3} - c_i(q_{i3}) - d_i(w_{i3})],$$

subject to the following constraints (Kuhn–Tucker multipliers in parentheses)[17]:

$$R_{i2} + w_{i2} + w_{i3} - q_{i2} - q_{i3} \geq 0, \quad (\lambda_{i2}) \quad i = 1, \ldots, n_2, \tag{9}$$

[14] If, by carrying additional reserves into period three, firm j were to cause firm i to alter when firm i exhausted, there would be a change in the *behavior* of firm i. In Fig. 1, this corresponds to lowering the marginal profit function sufficiently in period three so that $R_{i3} + S_{i3} > \bar{q}_{\mathrm{R,S}}$. We do not consider this strategic effect.

[15] The effect of an increase in reserves held by other firms on the marginal profits of firm i is given by $\partial\lambda_i/\partial Q_{-i3} = \partial\mu_i/\partial Q_{-i3} = P'(R_3 + S_3) + (R_{i3} + S_{i3})P''(R_3 + S_3) < 0$ by A.2. The effect on profits is also negative: $\partial V_{i3}/\partial Q_{-i3} = (R_{i3} + S_{i3})P'(R_3 + S_3) < 0$.

[16] Since firm i's reserves are exhausted in period three, we could write this as a backwards induction: $\mathbf{P2}'$ $V_{i2} = P(Q_2)q_{i2} - c_{i2}(q_{i2}) - d_{i2}(w_{i2}) + \beta V_{i3}^*(\boldsymbol{R}_3, \boldsymbol{S}_3)$, where $V_{i3}^*(\boldsymbol{R}_3, \boldsymbol{S}_3)$ is the solution to problem **P3**. However, since firm j cannot affect firm i's period three production, problems $\mathbf{P2}'$ and **P2** are equivalent.

[17] The multipliers λ_{i2} and μ_{i2} are the present value of the resource stocks R_{it} and S_{it} in *period two*. The values of λ_{i3} and μ_{i3} in Section 2, which were written as the present value in period three, equal $\lambda_{i3} = \lambda_{i2}/\beta$ and $\mu_{i3} = \mu_{i2}/\beta$ in period two, respectively.

$$S_{i2} - w_{i2} - w_{i3} \geq 0, \quad (\mu_{i2}) \quad i = 1, \ldots, n_2, \tag{10}$$

$$S_{i2} - w_{i2} \geq 0, \quad (\theta_{i2}) \quad i = 1, \ldots, n_2. \tag{11}$$

$$R_{i2} + w_{i2} - q_{i2} \geq 0, \quad (\phi_{i2}) \quad i = 1, \ldots, n_2. \tag{12}$$

Constraint (9) is a feasibility constraint on production due to the exhaustible nature of the resource. Constraint (10) is a feasibility constraint on exploration. Constraints (11) and (12) ensure that in period two exploration and extractions, respectively, are each feasible. Both (9) and (10) are binding, given that exhaustion occurs by period three.

The first-order necessary conditions for maximization of **P2** include (9)–(12) and

$$\frac{\partial V_{i2}}{\partial q_{i2}} = P(Q_2) + P'(Q_2)q_{i2} - c_i'(q_{i2}) - \lambda_{i2} - \phi_{i2} \leq 0, \quad i = 1, \ldots, n_2, \tag{13}$$

$$\frac{\partial V_{i2}}{\partial q_{i3}} = \beta[P(Q_3) + P'(Q_3)q_{i3} - c_i'(q_{i3})] - \lambda_{i2} \leq 0, \quad i = 1, \ldots, n_3, \tag{14}$$

$$\frac{\partial V_{i2}}{\partial w_{i2}} = -d_i'(w_{i2}) + \lambda_{i2} - \mu_{i2} + \phi_{i2} - \theta_{i2} \leq 0, \qquad i = 1, \ldots, n_2, \tag{15}$$

$$\frac{\partial V_{i2}}{\partial w_{i3}} = -\beta d_i'(w_{i3}) + \lambda_{i2} - \mu_{i2} \leq 0, \quad i = 1, \ldots, n_3. \tag{16}$$

Each of these holds as an equality when the choice variable is non-negative. The marginal value of the proved reserves, R_{i2}, to firm i at the beginning of period two is $\lambda_{i2} + \phi_{i2}$ and the marginal value of unproved reserves to firm i at the beginning of period two is μ_{i2}. The conditions (13) and (14) have the usual interpretation that the marginal profit from extraction in each period is equal to the marginal value of the remaining resource stock. Eqs. (15) and (16) reveal that a similar dynamic is at work with unproved reserves. We show in Appendix A (Proposition A.1) that constraint (11) only binds for firms that exhaust all reserves in period two. Thus, it is ignored in what follows.

Given that a firm with positive stocks will produce at least to period two, we may use (14) to eliminate the shadow value of proved reserves, λ_{i2}, from (13) to yield an intertemporal arbitrage rule in terms of marginal profits from production:

$$\pi_{i2}'(q_{i2}|Q_{-i2}) - \beta\pi_{i3}'(q_{i3}) - \phi_{i2} \geq 0, \quad i = 1, \ldots, n_2, \tag{17}$$

where $\pi_{it}'(q_{it}|Q_{-it}) \equiv P(Q_t) + P'(Q_t)q_{it} - c_i'(q_{it})$ is the marginal *extraction* profit to firm i in period t, taking discovery costs as sunk and holding the output of all other firms, Q_{-it}, constant. When production is positive in both periods, then (17) holds as a strict equality. If in addition, firm i takes a positive quantity of proved reserves into period three, so that ϕ_i is zero, then (17) implies that the marginal profits from production are equal in present value, which is Hotelling's rule for an oligopolist (e.g., Salant, 1976; Loury, 1986, or Polasky, 1992). In contrast, when $\phi_i > 0$ and production is positive in both periods, then the Hotelling condition (17) reflects the increase in extraction costs due to having to extract only from unproved reserves in period three. Thus, $\pi_{i2}' > \beta\pi_{i3}'$. Finally, when firm i chooses not to produce in period three, (17) implies that $\pi_{i2}' > \beta\pi_{i3}'$, which is the condition under which the firm does better by exhausting in period two than by taking some reserves into period three.

A similar expression can be obtained for marginal discovery costs, Using (15) and (16) to eliminate $\lambda_{i2} - \mu_{i2}$ yields the intertemporal optimization condition for reserve additions:

$$d'_i(w_{it}) - \beta d'_i(S_{i2} - w_{it}) - \phi_{i2} \le 0, \quad i = 1, \ldots, n_2. \tag{18}$$

The inequality in (18) becomes an equality if exploration occurs in both periods two and three. When production is positive in both periods and firm i has sufficient proved reserves to take some proved reserves into period three, (18) implies that the present value of marginal discovery costs are equated over time. In contrast, if production is positive in both periods but firm i has insufficient proved reserves to take proved reserves into period three, (18) implies that the present value of marginal discovery costs is higher in period two than in period three. Finally, if firm i exhausts in period two, then $d'_i(S_{i2}) < \phi_{i2}$, which implies that the marginal cost of the last unit discovered is less than the value of that unit to production.

An important implication of (18) is that each firm will produce from the higher cost unproved reserves in the final period in which it operates. This occurs because unproved reserves have higher marginal extraction costs, $c'_i + d'_i$, than proved reserves, which only have marginal cost c'_i. Firms have an incentive to produce from the lowest cost reserves first (Hartwick, 1978). Note that Hartwick and Sadorsky (1990) did not allow firms to explore in the final period of their two period game, but (18) shows that if the firm is given the choice of exploring in the last period, he will do so. Given that there are no strategic effects between periods two and three, this effect entirely characterizes firm's behaviour in the final two periods of the game.

Finally, note that if production is positive in both periods then (17) and (18) imply that the present value of marginal profits are equated:

$$\pi'_{i2}(q_{i2}|Q_{-i2}) - d'_i(w_{i2}) = \beta[\pi'_{i3}(q_{i3}|Q_{-i3}) - d'_i(S_{i2} - w_{i2})], \quad i = 1, \ldots, n_2. \tag{19}$$

In what follows it is convenient to let $w_{i2}(S_{i2})$ denote the equilibrium level of exploration in period two when firm i takes a positive quantity of proved reserves into period three, i.e., when $\phi_{i2} = 0$. Then (18) implies that $w_{i2}(S_{i2})$ is implicitly given by

$$d'_i(w_{i2}(S_{i2})) = \beta d'_i(S_{i2} - w_{i2}(S_{i2})), \quad \text{for } S_{i2} > 0, i = 1, \ldots, n_2. \tag{20}$$

It follows from (20) that $0 < w'_{i2}(S_{i2}) < 1$.[18] Note also that $w_{i2}(0) = 0$, since the quantity of unproved reserves is known with certainty.

While (18) eliminates all equilibria with zero exploration in any period in which production is positive, there remain three possible outcomes for a firm that produces in one or both of the two remaining periods. These outcomes depend upon whether or not the firm brings proved reserves in the last period, i.e., whether constraint (12) binds when production occurs in period three, and upon whether or not the firm produces in period three:

Case A: Producer i explores and extracts in periods two and three and brings positive quantities of both types of reserves into period three, i.e., the constraint (12) does not bind. In this case,

$$\begin{aligned} &w^*_{i2} = w_{i2}(S_{i2}), \quad q^*_{i2} \le R_{i2} + w_{i2}(S_{i2}), \qquad w^*_{i3} = S_{i2} - w_{i2}(S_{i2}), \quad \text{and} \\ &q^*_{i3} = R_{i2} + S_{i2} - q^*_{i2}. \end{aligned} \tag{21}$$

[18] Writing $d''_{it} \equiv d''_i(w_{it}), w'_{i2}(S_{i2}) = \beta d''_{i3}/(d''_{i2} + \beta d''_{i3})$.

Case B: Firm i explores and extracts in both periods two and three but exhausts his proved reserves in period two. Thus the constraint (12) binds. In this case,

$$S_{i2} > w_{i2}^* > w_{i2}(S_{i2}), \quad q_{i2}^* = R_{i2} + w_{i2}^*, \quad \text{and} \quad q_{i3}^* = w_{i3}^* = R_{i2} + S_{i2} - q_{i2}^* \tag{22}$$

Case C: Producer i exhausts all remaining reserves in period two. In this case,

$$w_{i2}^* = S_{i2}, \quad q_{i2}^* = R_{i2} + S_{i2}, \quad \text{and} \quad q_{i3}^* = w_{i3}^* = 0 \tag{23}$$

Let us begin by characterizing when a firm is in case C. Taking as given the actions of all other firms, firm i rationally exhausts in period two if, and only if

$$\pi_{i2}'(R_{i2} + S_{i2}|Q_{-i2}) - d_i'(S_{i2}) \geq \beta\pi_{i3}'(0|Q_{-i3}), \quad i = 1, \ldots, n_2, \tag{24}$$

where we have used the assumption that $d_i'(0) = 0$ in writing (24). In $\{R_{i2}, S_{i2}\}$ space (see Fig. 2), the boundary between cases B and C corresponds to the loci where (24) holds as an equality. The slope of this boundary is

$$\left.\frac{dR_{i2}}{dS_{i2}}\right|_{BC} = \frac{\pi_{i2}'' - d_{i2}''}{-\pi_{i2}''} < 0, \quad i = 1, \ldots, n_2. \tag{25}$$

For stocks below this locus, the firm's best-response to all other firm's output choices is to exhaust in period two.

Conversely, a necessary condition for firm i's best-response to be to produce in period three is that the inequality in (24) is reversed:

$$\pi_{i2}'(R_{i2} + S_{i2}|Q_{-i2}) - d_i'(S_{i2}) < \beta\pi_{i3}'(0|Q_{-i3}), \quad i = 1, \ldots, n_2. \tag{26}$$

While (26) guarantees that the firm will wish to produce into period three, an additional condition is required to ensure that the proved reserves taken into period three are positive. This condition is

$$\pi_{i2}'(R_{i2} + w_{i2}(S_{i2})|Q_{-i2}) \leq \beta\pi_{i3}'(S_{i2} - w_{i2}(S_{i2})|Q_{-i3}), \quad i = 1, \ldots, n_2. \tag{27}$$

When (27) holds as a strict inequality, firm i's marginal profits if he exhausts his proved reserves in period two are less than the discounted marginal profits of consuming the balance of his reserves in period three. Thus, firm i's best-response is to take positive proved reserves into period three.

When condition (27) is reversed, firm i's best-response is to exhaust proved reserves in period two, i.e., a necessary condition for exhaustion of proved reserves in period two is

$$\pi_{i2}'(R_{i2} + w_{i2}(S_{i2})|Q_{-i2}) > \beta\pi_{i3}'(S_{i2} - w_{i2}(S_{i2})|Q_{-i3}), \quad i = 1, \ldots, n_2. \tag{28}$$

To distinguish between case A (where a positive quantity of proved reserves exists at the end of period two) and case B (where proved reserves are exhausted in period two) note that firm i just exhausts his proved reserves when $q_{i2}^* = R_{i2} + w_{i2}(S_{i2})$. Since (19) holds exactly when production is positive in both periods, the loci splitting $\{R_{i2}, S_{i2}\}$ space into cases A and B satisfies (27) with equality.

The first thing to notice is that when $S_{i2} = 0$, the locus dividing cases A and B, given implicitly when (27) holds as an equality, has the same intercept as the locus dividing cases B and C, given implicitly when (24) holds as an equality.[19] Second, the loci dividing cases A and B is above the loci dividing cases B and C in Fig. 2. To see this notice that the slope of the loci dividing case A

[19] To see this, recall that $d_{it}'(0) = 0$.

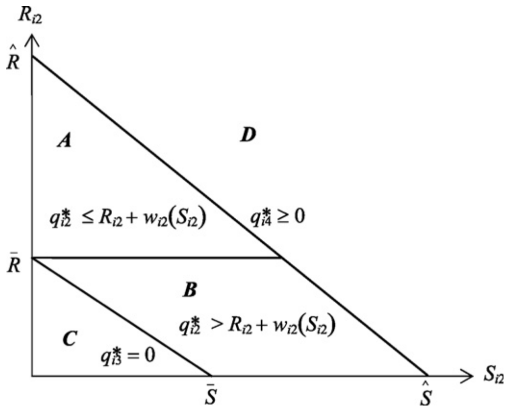

Fig. 2. Equilibrium exploration and exhaustion for firm *i*. *Note*: In area *C*, firm *i* has insufficient reserves of each type to continue producing in to three. In area *B* firm *i* has insufficient proved reserves to carry positive quantity of proved reserves into period three. In area *A*, firm *i* has sufficient proved reserves to carry proved reserves into period three. In area *D*, firm *i* has sufficient reserves of each type to continue producing into period four. The reserves and actions of all other firms are held constant. See Proposition A.2 in Appendix A for the definitions of $\hat{R}, \bar{R}, \hat{S}$, and $\bar{S}$.

from case *B* is

$$\left.\frac{\partial R_{i2}}{\partial S_{i2}}\right|_{\mathrm{AB}} = \frac{w'_{i2}\pi''_{i2} - (1 - w'_{i2})\beta\pi''_{i3}}{-\pi''_{i2}}, \quad i = 1, \ldots, n_2, \tag{29}$$

where $w'_{i2}(S_{i2}) = \beta d''_{i3}/(d''_{i2} + \beta d''_{i3})$. Comparing (29) and (25) reveals that $\partial R_{i2}/\partial S_{i2}|_{\mathrm{AB}} > \partial R_{i2}/\partial S_{i2}|_{\mathrm{BC}}$. Thus, in Fig. 2, we have drawn the AB boundary above the BC boundary.[20] Since each firm is assumed to exhaust by the third period, no firm holds reserves in the area *D* in Fig. 2. Proposition A.2 in Appendix A derives the properties of the boundaries between regions *A* and *D* and between regions *B* and *D* and proves that regions *A* and *B* exist.

We conclude this section by proving that for a given set of reserves ($\boldsymbol{R}_2, \boldsymbol{S}_2$) such that all firms exhaust by period three, a Nash equilibrium among the set of active firms exists and is unique.

Proposition 1. *Under* Assumptions A.1–A.4, *there exists a unique Nash equilibrium beginning in period two.*

Proof. See the mathematical Appendix A. □

2.4. Properties of the second period value function

The one thing that cannot be seen in Fig. 2 is the relationship between the best-response production and exploration levels of firm *i* and the reserves held by other firms. However, this is the key to understanding the strategic behaviour. As this is equivalent to examining how firm *i*'s reserves affect the behaviour of other firms, we examine this question from this perspective.

[20] It can be shown that the AB boundary has a slope of zero when (i) demand is linear, (ii) marginal extraction costs are constant, and (iii) marginal exploration costs are linear. This is the case shown in Fig. 2.

We begin by showing the effect on firm i's stream of future profits beginning in period two of an increase in R_{i2} and S_{i2} when that firm produces in both periods two and three. The stream of profits for a firm producing in both periods two and three can be written as

$$V_{i2}^*(\mathbf{R}_2, \mathbf{S}_2) = P(Q_2)q_{i2}^* - c_i(q_{i2}^*) - d_i(w_{i2}^*) + \beta[P(Q_3)q_{i3}^* - c_i(q_{i3}^*) - d_i(w_{i3}^*)], \tag{30}$$

where $q_{i2}^*, q_{i3}^*, w_{i2}^*$, and w_{i3}^* are given by (21)–(23).

By the envelope theorem, differentiating firm i's second period profits with respect to firm i's own proved reserves, R_{i2}, yields

$$\frac{\mathrm{d}V_{i2}^*}{\mathrm{d}R_{i2}} = \frac{\partial V_{i2}^*}{\partial R_{i2}} + \frac{\partial V_{i2}^*}{\partial Q_{-i2}} \sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial R_{i2}}, \quad i = 1, \ldots, n_2, \tag{31}$$

The direct effect of an increase in firm i's proved reserves on firm i's stream of profits beginning in period two is given by $\partial V_{i2}^*/\partial R_{i2}$. In cases A and B, where production occurs in both periods, the direct effect is $\partial V_{i2}^*/\partial R_{i2} = \beta\pi_{i3}' + d_{i2}' - \beta d_{i3}'$, which is positive in sign. In case A, where the firm has sufficient proved reserves to produce from them in period three, (20) implies $d_{i2}' - \beta d_{i3}' = 0$. In case B, where firm i exhausts his proved reserves in period two, (18) implies that $d_{i2}' - \beta d_{i3}' > 0$. A firm who exhausts all of his proved and unproved reserves in period two (case C) has $\partial V_{i2}^*/\partial R_{i2} = \pi_{i2}' > 0$.

The indirect effect of an increase in proved reserves is comprised of two effects. If firm i produces in both periods two and three (cases A and B), the effect that an increase in second period production by other firms upon firm i's stream of profits is given by $\partial V_{i2}^*/\partial Q_{-i2} = q_{i2}^* P_2' - \beta q_{i3}^* P_3'$, which is negative in sign by Assumption A.4. In contrast, if firm i exhausts in period two (case C), then $\partial V_{i2}^*/\partial Q_{-i2} = q_{i2}^* P_2'$. Again this effect is negative in sign.

The second component to the indirect effect is the sign of $\sum_{j \neq i} \partial q_{j2}^*/\partial R_{i2}$. This effect is similar to that of changes in unproved reserves, which we discuss first.

Differentiating firm i's second period profits with respect to firm i's own unproved reserve holdings, S_{i2}, yields

$$\frac{\mathrm{d}V_{i2}^*}{\mathrm{d}S_{i2}} = \frac{\partial V_{i2}^*}{\partial S_{i2}} + \frac{\partial V_{i2}^*}{\partial Q_{-i2}} \sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial S_{i2}}, \quad i = 1, \ldots, n_2. \tag{32}$$

The direct effect of an increase in holdings of unproved reserves is $\partial V_{i2}^*/\partial S_{i2} = \beta(\pi_{i3}' - d_{i3}') > 0$ for both cases A and B, where firm i produces into period three, and $\partial V_{i2}^*/\partial S_{i2} = \pi_{i2}' - d_{i2}' > 0$ if firm i exhausts all stocks in period two (case C).

The effect that an increase in second period production by other firms upon firm i's stream of profits is $\partial V_{i2}^*/\partial Q_{-i2} = q_{i2}^* P_2' - \beta q_{i3}^* P_3'$, which is negative by Assumption A.4, when firm i produces in period three (cases A and B), and $\partial V_{i2}^*/\partial Q_{-i2} = q_{i2}^* P_2'$, which is negative, in case C, where firm i exhausts all stocks in period two.

Now let us turn to the terms involving the derivatives $\partial q_{j2}^*/\partial R_{i2}$ and $\partial q_{j2}^*/\partial S_{i2}$. It is through these terms that the strategic effect, if one exists, occurs. Following Tirole (1990, p. 326), we may use the chain rule to write these sums as

$$\sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial R_{i2}} = \left(\frac{\partial q_{i2}^*}{\partial R_{i2}}\right) \sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial q_{i2}} \quad \text{and} \quad \sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial S_{i2}} = \left(\frac{\partial q_{i2}^*}{\partial S_{i2}}\right) \sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial q_{i2}}, \quad i = 1, \ldots, n_2. \tag{33}$$

The sign of these expressions depend on the slopes of the best-response functions of all other firms to firm i's output level. Given Assumptions A.2–A.4, the goods are strategic substitutes, so that the slopes of the best-response functions are negative. However, we need these summations to be negative in *net* given the interactions among the set of all other firms.[21] A result due to Dixit (1986) shows that this is so.

Lemma 1. (Dixit, 1986) *Under* Assumptions A.1–A.4, *the summation* $\sum_{j\neq i}\partial q_{j2}^*/\partial q_{i2}$ *in* (33) *is negative for all firms who produce in period two and three* (*cases A and B*), *and is zero for firms who end production in period two* (*case C*).

Proof. See the mathematical Appendix A. □

This result shows that any result that causes firm i to increase output in period two *tilts* the production profile of other firms by lowering production by these firms in period two and raising it in period three. Thus, the strategic effect in an exhaustible resource model, if it exists, alters the *timing* of production rather than the level of production.[22]

Next, we show the effect of own proved and unproved reserve holdings on the second period output of each type of firm.

Proposition 2. Under Assumptions A.1–A.4:

(a) $\partial q_{i2}^*/\partial R_{i2} > 0$ and $\partial q_{i2}^*/\partial S_{i2} > 0$, $i = 1, \ldots, n_2$,
(b) $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2}$, $i = 1, \ldots, n_2$, when (i) $d_i''(w_{it}) = 0$, or (ii) firm i has sufficient proved reserves to take positive proved reserves into period three (case A), or (iii) firm i exhausts all reserves in period two (case C),
(c) $\partial q_{i2}^*/\partial R_{i2} > \partial q_{i2}^*/\partial S_{i2}$, $i = 1, \ldots, n_2$, when (i) $d_i''(w_{it}) > 0$, or (ii) firm i has insufficient proved reserves to take positive proved reserves into period three, but still produces in period three (case B).

Proof. See the mathematical Appendix A. □

Results (b) and (c) of Proposition 2 show that firms who hold insufficient proved reserves to extend production from those reserves into period three but sufficient total reserves to produce in period three (case B) are unlike both firms with sufficient proved reserves to produce in both period two and three (case A) and firms with insufficient total reserves to produce into period three (case C). Results (b) and (c) also show that $d_i''(w_{it}) > 0$ is necessary to obtain any effect on second period production from first period exploration, since any reserves found through first period exploration reduce unproved reserves, S_{i2}, at the same rate as proved reserves, R_{i2}, are increased.

In the 'oil'igopoly theory of production, Polasky (1992) showed that $0 < \partial q_{i2}^*/\partial R_{i2} < 1$. Result (a) of Proposition 2 extends the lower inequality of this result to the case where firms hold two

[21] The best response functions $q_{i2}^* = \rho_i(Q_{-i2})$ describe how firm i responds to changes in the output of all other firms. To see how all other firms simultaneously respond to a change in firm i's output, we need to solve the system of equations $H_{-i}\boldsymbol{dq_{-i2}} = bdq_{i2}$ to obtain $\boldsymbol{dq}_{-i2} = H_{-i}^{-1}bdq_{i2}$, where $\boldsymbol{dq}_{-i2} = \{dq_{12}, \ldots, dq_{i-12}, dq_{i+12}, \ldots, dq_{n2}\}$ is the vector of dq_{j2} for $j \neq i$, H_{-i} is the Jacobian matrix for the first-order conditions for all firms other than firm i with diagonal elements a_j equal to the second order conditions on q_{j2} and off diagonal elements b_j in row j (where these are defined in the text below), and $b = \{b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_{n2}\}$ is the vector of the cross-effects on marginal profits.

[22] Inspection of (A.19) in the appendix reveals that $\sum_{j\neq i}\partial q_{j2}^*/\partial q_{i2} = -(n_2 - 1)/n_2$ when $c_{it}'' = P_t'' = 0$. This implies that the limit as $n_2 \to \infty$ of $\sum_{j\neq i}\partial q_{j2}^*/\partial q_{i2}$ equals -1. The term $b_j/(a_j - b_j)$ in (A.19) is greater than one in value if $P_t'' = 0$ or if $c_{it}'' = 0$ and demand is inelastic. Then it can be shown that $0 > \sum_{j\neq i}\partial q_{j2}^*/\partial q_{i2} > -1$.

types of reserves. However, it remains to be seen whether or not we can also extend the result that $\partial q_{i2}^{*}/\partial R_{i2} < 1$ to the case where proved and unproved reserves exist for firms who produce in both periods two and three. The next result provides a sufficient condition for this to be true:

Corollary 2 (*Corollary to Proposition 2*).
Under Assumptions A.1–A.4, for firms that produce in both periods two and three (cases A and B), a sufficient condition for $\partial q_{i2}^{*}/\partial R_{i2} < 1$ *is that*

$$\frac{d_{i2}'' + \beta d_{i3}'' - (P_2' + q_{i2}^{*}P_2'' - c_{i2}'')}{d_{i2}'' + \beta d_{i3}'' - P_2' - \beta(P_3' + q_{i3}^{*}P_3'')} < \sum_{j \neq i} \frac{P_2' + q_{j2}^{*}P_2'' + \beta(P_3' + q_{j3}^{*}P_3'')}{P_2' - c_{j2}'' + \beta(P_3' - c_{j3}'')} \tag{34}$$

Proof. See the mathematical Appendix A. □

The condition (34) is much stronger than the conditions needed to prove that $\partial q_{i2}^{*}/\partial R_{i2} < 1$ when there is only one type of stock. However, when $P_t'' = c_{it}'' = d_{it}'' = 0$, (34) collapses to $2/(N-1) < 1 + \beta$, which holds for *all* $N \geq 3$. Thus, like the theory of 'oil'igopoly, this model also has the property that output is increasing at a decreasing rate in proved reserves.[23]

3. Strategic exploration and extraction

Now we are prepared to ask the central question of this paper. In this section we derive the strategic effects from exploration and from production, and show which types of firms will alter their behaviour relative to the Nash equilibrium based on those incentives.

3.1. Strategic exploration and production

The problem faced by firm i in period one is to choose output q_{i1} and exploration w_{i1} to maximize

$$\textbf{P1} \quad \max_{\{q_{i1} w_{i1}\}} V_{i1} = P(Q_1)q_{i1} - c_i(q_{i1}) - d_i(w_{i1}) + \beta V_{i2}^{*}(\mathbf{R}_2, \mathbf{S}_2), \quad i = 1, \ldots, n_1,$$

where the value function $V_{i2}^{*}(\mathbf{R}_2, \mathbf{S}_2)$ is given by (30). Firm i's choices are subject to the constraints:

$$w_{i1} \leq S_{i1}, \quad (\theta_{i1}) \quad i = 1, \ldots, n_1, \tag{35}$$

$$q_{i1} \leq R_{i1} + w_{i1}, \quad (\phi_{i1}) \quad i = 1, \ldots, n_1, \tag{36}$$

which are analogous to the constraints (11) and (12).

[23] A similar condition can be obtained for the effect of an increase in unproved reserves. Indeed, when (12) is not binding, we know that the effects are identical, and when (12) is binding, we need only substitute $f_i = e_i + d_{it}''$ for e_i in the condition (A.23).

Ignoring the cases where the constraints (35) and (36) hold, we may use (31)–(33) to write the solution to **P1** for a firm who produces in periods two and three as

$$\frac{\partial V_{i1}}{\partial q_{i1}} = \pi'_{i1}(q^*_{i1}, Q_{-i1}) - \beta\left[\frac{\partial V^*_{i2}}{\partial R_{i2}}\right] - \beta\left(\frac{\partial q^*_{i2}}{\partial R_{i2}}\right)\left[\frac{\partial V^*_{i2}}{\partial Q_{-i2}}\sum_{j\neq i}\frac{\partial q^*_{j2}}{\partial q_{i2}}\right] = 0, \quad i = 1, \ldots, n_1, \tag{37}$$

$$\frac{\partial V_{i1}}{\partial w_{i1}} = -d'_i(w^*_{i1}) + \beta\left[\frac{\partial V^*_{i2}}{\partial R_{i2}} - \frac{V^*_{i2}}{\partial S_{i2}}\right] + \beta\left(\frac{\partial q^*_{i2}}{\partial R_{i2}} - \frac{\partial q^*_{i2}}{\partial S_{i2}}\right)\left[\frac{\partial V^*_{i2}}{Q_{-i2}}\sum_{j\neq i}\frac{\partial q^*_{j2}}{\partial q_{i2}}\right] = 0, \tag{38}$$

$$i = 1, \ldots n_1.$$

In the Nash equilibrium, the third terms on the right-hand-side of (37) and (38) are zero for all firms, $i = 1, \ldots, n_1$. In contrast, in the subgame perfect Nash equilibrium, these terms are non-zero for some types of firms. This means that a comparison of the Nash equilibrium and the subgame perfect Nash equilibrium needs to distinguish between how the best-response functions (37) and (38) change across the two equilibria and how changes in the behaviour of the other firms, i.e., changes in Q_{-i1}, shifts the best-response functions.

In the Nash equilibrium, (37) implies that firm i equates the marginal profits from extraction today with the present value of lost marginal profits beginning in period two. Similarly, the Nash condition in (38) implies that the firm equates the present value of net benefit of exploration to second period profits with marginal exploration costs in the first period.

The strategic effects appear in the third terms of (37) and (38). Lemma 1 and Assumption A.4 imply that the terms $\sum_{j\neq i}\partial q^*_{j2}/\partial q_{i2}$ and $\partial V^*_{i2}/\partial Q_{-i2}$ are each negative in sign, and Proposition 2, part (a), implies that $\partial q^*_{i2}/\partial R_{i2} > 0$. Together, these imply that the strategic effect from production raises the costs of current production. Therefore,

Proposition 3. *Holding the output of all other firms constant, under* Assumptions A.1–A.4, *the best-response first period production level for any firm who holds positive quantities of reserves in period two is lower in the subgame perfect Nash equilibrium than in the Nash equilibrium.*

This result occurs because Lemma 1 and Assumption A.4 imply that the terms $\sum_{j\neq i}\partial q^*_{j2}/\partial q_{i2}$ and $\partial V^*_{i2}/\partial Q_{-i2}$ are each negative in sign, and Proposition 2, part (a), implies that $\partial q^*_{i2}/\partial R_{i2} > 0$ for all types of firms who take positive level of some type of reserves into period two. In Fig. 2, this is shown as the movement from q^{NE}_{i1} to q'_{i1}.

While Proposition 3 tells us that the best-response to a given level of output by other firms is lower in the subgame perfect Nash equilibrium, it does not follow that the *equilibrium* first period production levels are always smaller in the subgame perfect Nash equilibrium relative to the Nash equilibrium. There are two reasons for this. First, suppose that all other firms reduce first period output. Then by Assumption A.4, the marginal profit of first period production, π'_{i1}, rises. In Fig. 3, this is shown as a shift upwards in the π'_{i1} curve. While as drawn, $q^{\mathrm{SP}}_{i1} < q^{\mathrm{NE}}_{i1}$, it is clear that if the shift in the π'_{i1} curve dominates the shift in the opportunity cost, it is possible that $q^{\mathrm{SP}}_{i1} \geq q^{\mathrm{NE}}_{i1}$. The other possibility is that Q_{-i1} increases. In this case, π'_{i1} shifts down rather than up. Thus if all other firms on average increase their first period output relative to the Nash equilibrium, firm i will decrease its first period output relative to its Nash equilibrium level.

Note that the two offsetting equilibrium effects identified in Fig. 3 explain why it is possible to get similar results between the Nash and subgame perfect cases in a model in which only production from a fixed stock occurs. These offsetting results also suggest a reason why asymmetric models

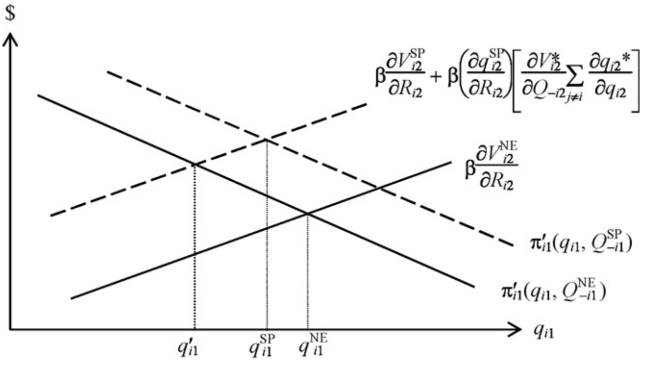

Fig. 3. Comparison of subgame perfect and nash equilibrium first period production.

often yield differences between the two concepts. When producers are quite different from one another, both the shift of the marginal profit from first period production due to $\partial\pi_{i1}'/\partial Q_{-i1}$ and the slopes of the best-response curves, $\partial q_{j2}^{*}/\partial q_{i2}$, can differ substantially across producers.

Next, let us consider the strategic effect on the choice of first period exploration, given in (38).

Proposition 4. *Under* Assumptions A.1–A.4, *the only type of firms who have a strategic incentive to over explore relative to the Nash equilibrium are those firms who have sufficient reserves to produce in period three, but have insufficient proved reserves to take positive proved reserves into period three (i.e., case B). Firms who have sufficient proved reserves to produce from these reserves in period three (case A) or insufficient total reserves to produce beyond period two (case C), faces the same incentives for exploration as occur in the Nash equilibrium.*

The reason for this result is that firms whose reserves place them in case A, where they take positive proved reserves to period three, have $\partial q_{i2}^{*}/\partial R_{i2} = \partial q_{i2}^{*}/\partial S_{i2}$ from Proposition 2, part (b), so the strategic effect disappears for those types of firms. Similarly, those firms whose reserves place them in case C, so that they exhaust both types of reserves in period two, do not affect the output of other firms, i.e., $\sum_{j\neq i}\partial q_{j2}^{*}/\partial q_{i2}$.

Two important differences between the strategic effect on exploration and the strategic effect on production can be seen by comparing Fig. 4 to Fig. 3. First, in Fig. 4, in the Nash equilibrium, $d_i'(w_{i1})$, the opportunity cost of exploring in period one, is equated with the net value of exploration, $\beta(\partial V_{i2}^{*}/\partial R_{i2} - \partial V_{i2}^{*}/\partial S_{i2})$. This is also the subgame perfect Nash equilibrium for firms in case A and case C. However, for firms in case B, the strategic effect shifts up the marginal benefit of exploration. Thus, the first important difference is simply that there is *no* strategic effect for firms in cases A and C. The second important difference is that for case B firms, for whom there is a change in the best-response function, there is only one curve that shifts in Fig. 4. This is unlike the first period production equilibrium, which shifts both the marginal benefit and marginal cost of first period production. Thus, the strategic effect on these types of firms is an unambiguous increase in the *equilibrium* level of exploration in period one.

3.2. Characterization of period one exploration

Next, we show that if the constraint (36) is binding, so that firm i's proved reserves in period two are driven to zero, then firm i will not have positive proved reserves at the end of any subsequent period.

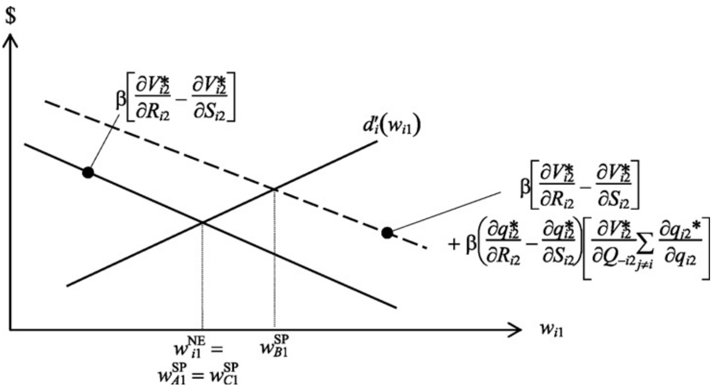

Fig. 4. Comparison of subgame perfect and nash equilibrium first period exploration.

Proposition 5. *Under Assumptions A.1–A.4, if a firm extracts all of its proved reserves in period one, he will not subsequently hold positive quantities of proved reserves.*

Proof. See the mathematical Appendix A. □

These propositions eliminate all but three possible combinations of exploration activities for the n_3 firms that produce in all three periods.[24] We conclude that so long as initial unproved reserves are positive, the firm explores in every subsequent period. Furthermore, for each period in which firm i takes some proved reserves into the next period, it extracts only from the lower cost proved reserves. Lastly, if proved reserves are exhausted prior to unproved reserves, then the firm will not rebuild these proved reserves in any subsequent period.

4. Numerical simulations

To examine the magnitudes of the effects that we have identified, we report two numerical simulations.

Table 1 reports the results of a numerical simulation of the Nash and subgame perfect Nash equilibria. In the example, demand is given by $P_t = 100 - Q_t$, marginal extraction costs are zero, exploration costs are $d(w) = w^2$, and the common discount factor is $\beta = 1/2$. There are three firms in the example, corresponding to the three types of firms (cases A, B, and C) who take positive reserves into period two. Firm A (case A) begins with proved reserves $R_A = 50$ and unproved reserves $S_A = 10$. In equilibrium this firm takes proved reserves into period three. Firm B (case B), begins with proved reserves of $R_B = 20$ and unproved reserves $S_B = 15$. This firm produces for three periods, but exhausts his proved reserves in period 2. Firm C (case C) begins with proved reserves $R_C = 15$ and unproved reserves $S_C = 5$. This firm takes proved reserves into period 2, but exhausts all reserves in period 2.

The first thing to notice from Table 1 is that only firm B strategically over explores in period one. This is what was predicted by Proposition 4. The magnitude of the increase in firm B's

[24] There are also m_2 firms which only hold sufficient reserves to produce in period two (those who take reserves equivalent to the area C in Fig. 2), and there are m_1 firms whose reserve holdings are insufficient to even produce in period two. These firms' choices cannot be influenced by the actions of the remaining firms, but those who take reserves into period two can affect the behavior of firms who continue to produce into period three.

Table 1
A three-firm asymmetric 'oil'igopoly example

Firm	Period	Nash equilibrium			Subgame perfect Nash equilibrium			Percent difference ($x^{SP}/x^{NE} - 1$)		
		1	2	3	1	2	3	1	2	3
A	q_{it}	24.4	23.8	11.8	24.7	23.5	11.8	+1.3	−1.4	0
B	q_{it}	18.4	11.7	4.9	18.1	12.0	4.9	−1.5	+2.4	0
C	q_{it}	15.0	5.0		14.6	5.4		−2.4	+7.1	
A	w_{it}	1.4	2.9	5.7	1.4	2.9	5.7	0	0	0
B	w_{it}	3.4	6.7	4.9	3.7	6.4	4.9	+9.0	−4.5	0
C	w_{it}	1.7	3.3		1.7	3.3		0	0	
A	R_{it-1}	50	28.4	6.1	50	26.7	6.1		−1.1	0
B	R_{it-1}	20	5.0	0	20	5.6	0		+11.8	0
C	R_{it-1}	15	1.7	0	15	2.0	0		+21.5	
A	S_{it-1}	10	8.6	5.7	10	5.7	5.7		0	0
B	S_{it-1}	15	11.6	4.9	15	11.3	4.9		−2.6	0
C	S_{it-1}	5	3.3	0	5	3.3	0		0	
Aggregates	P_t	42.2	59.5	83.3	42.6	59.2	83.3	+0.7	−0.5	0
	Q_t	57.8	40.5	16.7	57.4	40.8	16.7	−0.6	+0.8	0
	W_t	6.5	12.9	10.6	6.7	12.6	10.6	+4.7	−2.4	0
	R_t	85	39.1	6.1	85	34.3	6.1		−12.3	0

Note: Sums may not add due to rounding. Percentage differences are calculated on exact quantities.

exploration is an increase of 9% relative to the Nash equilibrium. The second thing to notice is that the ambiguity of the strategic effect on equilibrium production also is evident. While firm *B* and firm *C* both under-produce relative to the Nash equilibrium, firm *A* produces a larger quantity relative to the Nash equilibrium. Because firm *B* both under produces and over explores, he enters period two with 11.8% higher proved reserves in the subgame perfect equilibrium. Firm *C* also enters with higher proved reserves (by 21.5%), but this is entirely due to restricting production in the subgame perfect Nash equilibrium. Firm *A*, on the other hand, enters period two with 1.1% lower reserves as a result of increasing production in period one in response to the decrease in production by firms *B* and *C*. Aggregate exploration is 4.7% higher in period one relative to the Nash equilibrium, and aggregate proved reserves are 12.3% lower in period one relative to the Nash equilibrium. The change in aggregate price and quantity is less than 1% in each period.

Table 2 presents a simulation in which there are three identical firms, each of which is a type-*B* firm, so that each firm exhausts proved reserves in period two but continues to produce from unproved reserves in period three. The demand and cost parameters are the same as in Table 1. However, to get exhaustion in the proper periods, we change the reserve holding so that each firm owns $R_i = 15$ and $S_i = 20$.

This example also has small changes in first period exploration (an increase of 4.5%), but this, combined with the small reduction in first period output results in a large (20%) increase in proved reserves at the beginning of period two.

While these simulations are only suggestive, they suggest that the magnitudes may be economically meaningful.

Table 2
A three-firm symmetric 'oil'igopoly example

Period	Nash equilibrium			Subgame perfect Nash equilibrium			Percent difference ($x^{SP}/x^{NE} - 1$)		
	1	2	3	1	2	3	1	2	3
q_{it}	18.2	11.5	5.2	18.1	11.6	5.2	−0.61	+0.97	0
w_{it}	4.9	9.8	5.2	5.1	9.6	5.2	+4.51	−2.26	0
P_t	45.6	65.5	84.3	45.6	65.1	84.3	0	−0.51	0
R_{it-1}	15	1.67	0	15	2.0	0		+20.0	
S_{it-1}	20	15.1	5.2	20	14.9	5.2		−1.47	

Note: Sums may not add due to rounding. Percentage differences are calculated on exact quantities.

5. Empirical evidence of strategic exploration

In this section, we present evidence on the relationship between exploration and proved reserve holdings for the 99 countries for which data on oil reserves is available in the post-World War II era. Fig. 5 displays the reserves in 2002 on the vertical axis and the reserves in the first year for which data is available for each country on the horizontal axis. A data point above the 45° line is a country for which ending reserves are larger than beginning reserves. While this data does not indicate reserve additions, because we have not added production during the interval to the ending reserves data, it suggests that countries with smaller initial reserves must have added much more reserves than those with larger initial proved reserves. We interpret this evidence as support for the hypothesis that firms with smaller proved reserves are more likely, all else equal, to engage in strategic exploration.

To see whether the negative relationship between initial proved reserves and ending proved reserves observed in the data is statistically different from zero, we regress the rate of reserves

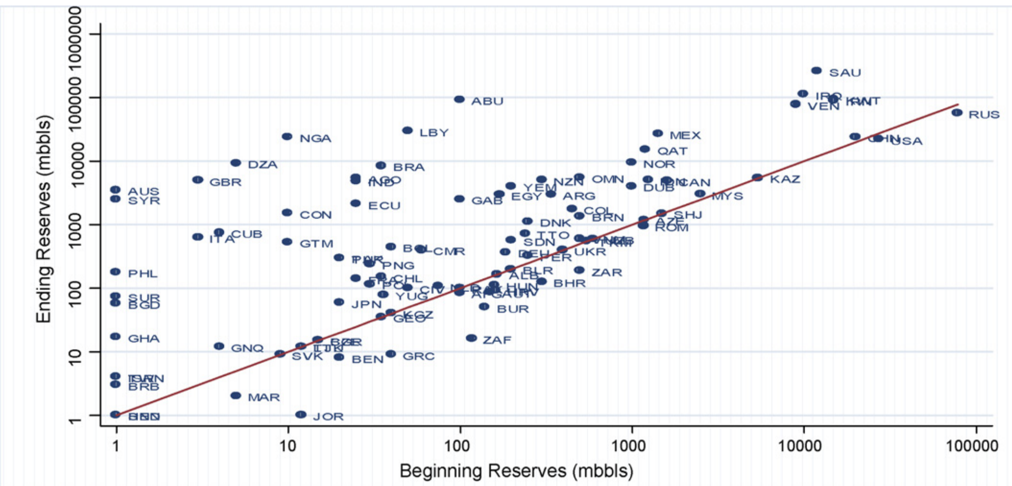

Fig. 5. Reserves growth and initial reserves, 1952–2002. *Note*: Reserves data is by country and is in log scale. The number of years between initial reserves and ending reserves differs across countries. Countries above the 45° line exhibit gross reserve growth. *Source*: *Oil & Gas Journal*.

growth on initial reserves using the data depicted in Fig. 5. The regression result is

$$\frac{\ln(R_{iT}) - \ln(R_{i0})}{T_i} = \underset{(0.51)}{4.09} - \underset{(0.24)}{1.11}\ln(R_{i0}), \quad \text{adjusted} \quad R^2 = 0.18, N = 99.$$

(Standard errors are in parentheses.) T_i is the number of years each country is observed in the data, and $\ln(R_{iT})$ and $\ln(R_{i0})$ are the natural log of ending and beginning reserves, respectively.

Fig. 5 and the regression results show that countries with smaller initial reserves tended to have higher rates of growth in their proved reserves over the period 1952–2002. While there is greater variation in the countries with smaller reserves, the percentage changes in reserves is highest for countries that initially had smaller reserves. This regression supports the hypothesis that smaller countries do indeed explore more relative to the larger countries: a 1% increase in initial reserves results in a 1.11% reduction in reserves growth.

While these results appear to support the hypothesis that producers who are more likely to exhaust their reserves are more likely to over explore, there are a number of caveats that need to be mentioned. First, we have not estimated the growth rate in actual net reserve additions, rather, we have estimated the growth rate in gross of production reserve additions. However, the results of Polasky (1992) suggest that smaller firms tend to produce a larger share of their reserves than larger firms, so the bias by excluding production should imply an even larger effect of an increase in proved reserves on reserve additions. Second, the producing countries are not producing firms. In addition, those countries that are associated with a single firm are often dominated by state-owned-firms, whose behaviour may or may not fit with profit maximization. However, it would be an interesting coincidence if these firms tended to behave as we suggest forward looking strategic firms would behave. Finally, there may be other reasons – political unrest, higher levels of risk, etc. (e.g., Bohn and Deacon, 2000) – that would cause firms in parts of the world with larger reserve bases to behave differently. We have not conditioned for this in our regressions. Nevertheless, our results, simple as they are, provide tantalizing evidence that the world oil market behaves as suggested by the 'oil'igopoly theory of exploration.

6. Conclusions

This paper has studied a three period model of 'oil'igopolistic exploration and production. We have solved for the dynamically consistent subgame perfect Nash equilibrium in a model in which firms compete both by production in the output market and by converting unproved reserves into proved reserves. The results of this model have been compared with those obtained using a full commitment Nash equilibrium.

The main question asked in this paper is whether exploration may lead firms to over explore relative to the Nash equilibrium. We do find that firms may over-explore, but we find that they do so only in certain circumstances. Two factors counter against firms using exploration as a strategic commitment device. One of these factors is that firms have an incentive to use lower cost reserves first. This means that any firm owning both proved and unproved reserves will always produce from the proved reserves first. Second, profit maximizing firms, whether behaving strategically or not, have an incentive to equate the present value of marginal discovery costs over time. This means that even in the Nash equilibrium, firms tend to explore in all periods in which they also produce. This incentive is only overturned when a firm happens to exhaust its proved reserves prior to having converted the last of its unproved reserves into proved reserves. It is these types of firms, and only these types of firms, that a strategic incentive exists to over explore relative to the Nash equilibrium. Interestingly, these firms tend to be firms who hold low quantities of proved reserves relative to their unproved reserves. This is consistent with international data on oil exploration, which suggests that countries that started with smaller proved reserves have tended to have higher reserve additions over time.

Our numerical simulations suggest that the effect of over-exploration, when combined with the incentive to under produce, may yield proved reserve results that are significantly different from those predicted by the Nash equilibrium.

Acknowledgements

We have benefited from comments on earlier drafts by Curtis Eaton, Jeffrey Church, Aidan Hollis, John Livernois, Scott Taylor, and Steve Salant, as well as from participants at the Canadian Resource and Environmental Economics (CREE) working group meetings in Montreal, October 2005 and the Alberta Industrial Organization Conference, October 2005. Two anonymous referees made suggestions that have improved the exposition and corrected our thinking on several points. All remaining errors are our own.

Appendix A

Proposition A.1. *Under* Assumptions A.1–A.4, *if firm i has a positive quantity of unproved reserves at the beginning of period two* (i.e., $S_{i2} > 0$) *and rationally exhausts in period two or three*, *then firm i explores in each period in which he produces*.

Proof. This result is proved by the following three steps. □

Lemma A.1. *With positive quantities of the unproved reserves and production in both periods two and three*, *if there is zero exploration in some period*, *it will be in period two, not period three*.

Proof. Suppose not. Suppose that $w_{i2} > 0$ and that $\lambda_i - \mu_i < \beta d_i'(0) = 0$. Then, $d_i'(S_{i2}) - \phi_i = \lambda_i - \mu_i$ and $w_{i3} = 0$. Then we obtain that

$$d_i'(Si_2) - \phi_i = \lambda_i - \mu_i < \beta d_i'(0) = 0. \tag{A.1}$$

Since the firm is assumed to extract in the third period, it is not possible that $w_{i2} = S_{i2} > 0$ and $\phi_i > 0$ both occur. Thus, let $\phi_i = 0$. Then this equation implies that $d_i'(S_{i2}) < 0$ which is a contradiction, since $d_i' \geq 0$. □

Lemma A.2. *If firm i extracts all of his proved reserves by period two, so that the constraint* (12) *binds, then he also explores in period two.*

Proof. Suppose not. Suppose that $\phi_i > 0$, that $q_{i2} = R_{i2}$, and $w_{i2} = 0$ that implies $d_i'(0) - \phi_i > \lambda_i - \mu_i$. Then $w_{i3} = S_{i2}$, and $d_i'(0) = \beta d_i'(S_{i2}) > \phi_i$. Since $d_i'(0) = 0$, we get that $-\beta d_i'(S_{i2}) > \phi_i > 0$ that contradicts. □

Lemma A.3. *If firm i produces in both periods and the constraint* (12) *does not bind, then firm i will have positive level of exploration level in period two*.

Proof. Suppose not. Suppose that $q_{i2} < R_{i2}$, that production occurs in both periods, and that $w_{i2} = 0$. Then $q_{i2} < R_{i2}$ implies that $\phi_i = 0$ and $d_i'(0) > \lambda_i - \mu_i$ implies that $w_{i2} = 0$. Since

$w_{i2} = 0$, $w_{i3} = S_{i2} > 0$. Therefore, $d_i'(0) > \beta d_i'(S_{i2})$. Since $d_i'(0) = 0, \beta d_i'(S_{i2}) < 0$ contradicts. □

The only other possibility is that firm i exhausts in period two but does not explore in period two. Assumption A.4 implies that each firm exhausts all stocks, so it cannot be equilibrium behaviour for firm i to shut down before exhausting his unproved reserves.

This completes the proof of Lemma A.1.

Proposition A.2. *Firm i rationally ends production in period three only if*

$$\tau_i(R_{i2}, S_{i2}) = \pi_{i3}'(q_{i3}^*, Q_{-i2}) - d_i'(w_{i3}^*) - \beta\pi_{i4}'(0,0) \geq 0.$$

Proof.

(i) Suppose that $q_{i2}^* < R_{i2} + w_{i2}^*$. Then $q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$, and $w_{i3}^* = S_{i2} - w_{i2}^*$. Thus, let

$$\tau_{iA}(R_{i2}, S_{i2}) \equiv \pi_{i3}'(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i2}) - d_i'(S_{i2} - w_{i2}^*) - \beta\pi_{i4}'(0,0). \tag{A.10}$$

When $S_{i2} = 0$, the value of $R_{i2} = \hat{R}$ such that $\tau_{iA}(\hat{R}, 0) = 0$ must satisfy $\pi_{i3}'(\hat{R} - q_{i3}^*, Q_{-i3}) = \beta\pi_{i4}'(0,0)$. It can be shown that $0 < \partial q_{i2}^*/\partial R_{i2} < 1$. Thus, $\hat{R} - q_{i2}^*$ lies between zero and $\hat{R}$. Let $\bar{R}$ solve $\pi_{i2}'(\bar{R}, Q_{-i2}) \equiv \beta\pi_{i3}'(0, Q_{-i3})$ which is the boundary for ending production in period two, when $S_{i2} = 0$. Since $\hat{R} - q_{i2}^*$ is strictly positive, it follows that $\hat{R} > \bar{R}$. Thus, in the region where $S_{i2} = 0$, there exists a set of values of R_{i2} such that firm i wishes to exhaust in period three. It can also be shown that along the locus of points where $\tau_{iA}(R_{i2}, S_{i2}) = 0$, that

$$\left.\frac{\mathrm{d}R_{i2}}{\mathrm{d}S_{i2}}\right|_{\tau_{iA}(R_{i2},S_{i2})=0} = -\frac{(1 - \partial q_{i2}^*/\partial S_{i2})\pi_{i3}' - (1 - \partial w_{i2}^*/\partial S_{i2})d_{i3}''}{(1 - \partial q_{i2}^*/\partial R_{i2})\pi_{i3}'} < 0, \tag{A.11}$$

Since $0 < \partial w_{i2}^*/\partial S_{i2} < 1$, and $0 < \partial q_{i2}^*/\partial S_{i2} = \partial q_{i2}^*/\partial R_{i2} < 1$.

(ii) Next, consider the case where $q_{i2}^* = R_{i2} + w_{i2}^*$. Then $w_{i3}^* = q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$. In this case, let

$$\tau_{iB}(R_{i2}, S_{i2}) \equiv \pi_{i3}'(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i3}) - d_i'(R_{i2} + S_{i2} - q_{i2}^*) - \beta\pi_{i4}'(0,0), \tag{A.12}$$

where q_{i2}^* solves $\pi_{i2}'(q_{i2}^*, Q_{-i3}) - d_i'(q_{i2}^* - R_{i2}) = \beta[\pi_{i3}'(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i3}) - d_i'(R_{i2} + S_{i2} - q_{i2}^*)]$. In this case, the $\tau_{iB}(R_{i2}, S_{i2})$ locus is again downward sloping:

$$\left.\frac{\mathrm{d}R_{i2}}{\mathrm{d}S_{i2}}\right|_{\tau_{iB}(R_{i2},S_{i2})=0} = -\frac{1 - \partial q_{i2}^*/\partial S_{i2}}{1 - \partial q_{i2}^*/\partial R_{i2}} < 0, \tag{A.13}$$

since $0 < \partial q_{i2}^*/\partial S_{i2} < \partial q_{i2}^*/\partial R_{i2} < 1$.

This implies that there exist values $\{R_{i2}, S_{i2}\}$ such that firm i wishes to produce in period three but not in period four. When $R_{i2} = 0$, the corresponding value of $S_{i2} = \hat{S}$ such that $\tau_{iB}(0, \hat{S}) = 0$ must satisfy

$$\pi_{i3}'(\hat{S} - q_{i2}^*, Q_{-i3}) - d_i'(\hat{S} - q_{i2}^*) = \beta\pi_{i4}'(0,0). \tag{A.14}$$

Finally, let $\bar{S}$ solve $\pi_{i2}'(\bar{S}, Q_{-i2}) \equiv \beta\pi_{i3}'(0, Q_{-i3})$ which is the boundary for ending production in period two, Since $0 < \partial q_{i2}^*/\partial S_{i2} < 1$, $\hat{S} - q_{i2}^*$ is strictly positive. This implies that in the region where $R_{i2} = 0$, that $\hat{S} > \bar{S}$, which completes the proof. □

Proof (*Proof of Proposition 1*).

(i) *Existence* (Vives, 1999, Theorem 2.7). To prove existence, it is necessary to prove that the best-reply functions are strongly decreasing in the output of the other firms. Assumptions A.2–A.4 ensure that the slope of the best-reply functions $\rho_{i2}(Q_{-i2})$ are strongly decreasing. For the case where production by firm i is positive in both periods, This can be seen by totally differentiating (19):

$$\rho'_{i2}(Q_{-i2}) = -\left(\frac{P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3)}{P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3) - (c''_{i2} - P'_2) - \beta(c''_{i3} - P'_3)}\right). \quad \text{(A.15)}$$

Both the numerator and the denominator of the term in brackets are negative, so the whole expression is negative. Therefore, under Assumptions A.2 and A.3, the best-response functions are strictly decreasing. Given this, Vives (1999, Theorem 2.7) implies that an equilibrium exists.

(ii) *Uniqueness*. To prove uniqueness, it is necessary to also show that the best-response map $\rho(\cdot) \equiv \{\rho_{12}(Q_{-1}), \ldots, \rho_{n_2 2}(Q_{-n_2})\}$ is a contraction. Vives (1999, Theorem 2.8) proves that if the slopes of the best-reply functions are strongly decreasing in the output of the other firms and greater than -1 in value, then a unique equilibrium exists. Note that Assumptions A.2 and A.3 imply that

$$\begin{aligned} 0 &> P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3) \\ &> P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3) - (c''_{i2} - P'_2) - \beta(c''_{i3} - P'_3) \end{aligned} \quad \text{(A.16)}$$

Dividing through by -1 times the right-hand-side reveals that $\rho'_{i2}(Q_{-i2}) > -1$. Thus, the condition on the best-response functions is met. This completes the proof. □

Proof (*Proof of Lemma 1*).

This proof follows Dixit (1986). Write the total differential of the jth firm's first-order condition on the choice of q^*_{j2} as

$$a_j\, dq^*_{j2} + b_j \sum_{k \neq i,j} dq^*_{k2} = -b_j\, dq_{i2}, \quad j \neq i, i = 1, \ldots n_2. \quad \text{(A.17)}$$

where, by A.2 and A.3, $a_j \equiv 2P'_2 + P''_2 q^*_{j2} - c''_{j2} + \beta(2P'_3 + P''_3 q^*_{j3} - c''_{j3}) < 0$ for a firm for whom (11) is not binding and $a_j \equiv 2P'_2 + P''_2 q^*_{j2} - c''_{j2} - d''_{j2} + \beta(2P'_3 + P''_3 q^*_{j3} - c''_{j3} - d''_{j3}) < 0$ for a firm for whom (11) is binding, and $b_j = P'_2 + P''_2 q^*_{j2} + \beta(P'_3 + P''_3 q^*_{j3}) < 0$ for all firms that continue to produce in period three. We can rewrite (A.17) as

$$dq^*_{j2} + \left(\frac{b_j}{a_j - b_j}\right) dQ_{-i2} = -\left(\frac{b_i}{a_j - b_j}\right) dq_{i2}, \quad i = 1, \ldots n_2. \quad \text{(A.18)}$$

Summing over all $j \neq i$ and solving for how the aggregate output by other firms changes as q_{i2} increases yields

$$\frac{dQ_{-i2}}{dq_{i2}} = -\sum_{j \neq i} \frac{b_j}{a_j - b_j}\left(1 + \sum_{j \neq i} \frac{b_j}{a_j - b_j}\right)^{-1}, \quad i = 1, \ldots, n_2. \quad \text{(A.19)}$$

Thus

$$\frac{\partial q_{j2}^*}{\partial q_{i2}} = -\left(\frac{b_j}{a_j - b_j}\right)\left(1 + \sum_{k \neq i} \frac{b_k}{a_k - b_k}\right)^{-1} < 0, \quad i = 1, \ldots, n_2, \tag{A.20}$$

since $b_j/(a_j - b_j) > 0$ for all j. This completes the pro of for those firms that produce into period three. For firms that end production in period two, $q_{j2}^* = R_{j2} + S_{j2}$. Thus, these firms do not respond at all to changes in q_{i2}. This completes the proof. □

Proof (*Proof of Proposition 2*).

(i) Write the total differential of a firm that produces in both periods two and three first-order condition in its own output q_{i2}^* as

$$a_i\, dq_{i2}^* + b_i \sum_{j \neq i} dq_{j2}^* = -\mathrm{e}_i\, dR_{i2} - f_i\, dS_{i2}, \quad i = 1, \ldots n_2, \tag{A.21}$$

where a_i and b_i are defined as above, and where $\mathrm{e}_i = f_i \equiv -\beta(2P_3' + P_3'' q_{i3}^* - c_{i3}'') > 0$ for firms for whom (12) is not binding, and for a firm for whom (12) binds $\mathrm{e}_i = d_{i2}'' - \beta(2P_3' + P_3'' q_{i3}^* - c_{i3}'' - d_{i3}'') > 0$ and $f_i = -\beta(2P_3' + P_3'' q_{i3}^* - c_{i3}'' - d_{i3}'') > 0$. For the case where R_{i2} changes, (A.21) implies

$$\frac{\partial q_{i2}^*}{\partial R_{i2}} = \frac{-\mathrm{e}_i}{\left(a_i + b_i \sum_{j \neq i} \partial q_{j2}^* / \partial q_{i2}\right)} = \left(\frac{\mathrm{e}_i}{-(a_i - b_i)}\right)\left(\frac{1 + \sum_{j \neq i} b_j / a_j - b_j}{a_i / a_i - b_i + \sum_{j \neq i} b_j / a_j - b_j}\right) \equiv \mathrm{e}_i \Gamma_i > 0, \qquad i = 1, \ldots, n_2 \tag{A.22}$$

where Γ_i is $-(a_i - b_i)^{-1}$ times the second expression in brackets in the second equality. Γ_i is positive since $a_i - b_i < 0$ and both $a_i/(a_i - b_i) > 0$ and $b_i/(a_i - b_i) > 0$ for all i. Thus, $\partial q_{i2}^*/\partial R_{i2} = \mathrm{e}_i \Gamma_i > 0$ and by an equivalent process, it can be shown that $\partial q_{i2}^*/\partial S_{i2} = f_i \Gamma_i > 0$. For firms that produce in only period two, $q_{i2}^* = R_{i2} + S_{i2}$, so that $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2} = 1$.

(ii) When the constraint (12) does not bind or $d_{i2}'' = 0$, $e_i = f_i$, so that $\partial q_{i2}^*/\partial R_{i2} - \partial q_{i2}^*/\partial S_{i2} = 0$.

(iii) When $d_{i2}'' > 0$, $\partial q_{i2}^*/\partial R_{i2} - \partial q_{i2}^*/\partial S_{i2} = d_{i2}'' \Gamma_i > 0$. This completes the proof. □

Proof (*Proof of Corollary to Proposition 2*).
The necessary condition for (A.22) to be less than one can be written as

$$\frac{\mathrm{e}_i - a_i}{\mathrm{e}_i - a_i + b_i} < \sum_{j \neq i} \frac{b_j}{a_j - b_j}, \quad i = 1, \ldots, n_2. \tag{A.23}$$

This expression can be rearranged to yield (34). This completes the proof. □

Proof (*Proof of Proposition 5*).
By assumption, $R_{i2} = 0$. Now, suppose that the conclusion does not follow. Then it must be that $q_{i2}^* < w_{i2}^*$, if the producer continues to produce to period three. (If the producer does not continue to produce in period three, then all reserves are exhausted in period two, which proves the proposition.) Thus $\phi_{i2} = 0$, since (12) is not binding. Since the feasibility constraint (9) must bind,

it requires that $q^*_{i3} > w^*_{i3}$. However, $\phi_{i2} = 0$ implies that w^*_{i2} solves (20), so that $w^*_{i3} > w^*_{i2}$. Therefore $q^*_{i2} < w^*_{i2} < w^*_{i3} < q^*_{i3}$. However, $\phi_{i2} = 0$ also implies that q^*_{i2} solves (19), so that $q^*_{i2} > q^*_{i3}$. This is a contradiction, which completes the proof. □

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