

## Optimal Capital Accumulation in a Fishery: A Nonlinear Irreversible Investment Model

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A nonlinear two-state variable, two-control variable model of a fishery with irreversible investment and harvest capacity constraints is examined. The model relaxes assumptions of linearity in investment costs and variable harvest profits in an earlier model by C. W. Clark, F. H. Clarke, and G. R. Munro (*Econometrica*, 47, 25-47, 1979). In both the linear and nonlinear models, the optimal capital accumulation paths in new fisheries is characterized by a period in which the physical capital stock level exceeds its long-run sustainable equilibrium. However, unlike the linear model, periods of positive net (but declining gross) investment are optimal in the nonlinear model. This accords with observed capital accumulation paths from a number of fisheries. The paper also finds different effects in optimal harvest policy depending upon whether the linearity appears in the variable profits function or the investment cost function. © 1995 Academic Press, Inc.

### 1. INTRODUCTION

The problem of capital accumulation in a fishery has been studied by economists for over 2 decades. Its analytical difficulty lies in the interdependence of the biological and physical capital stocks. At any point in time, the rate of harvest is limited by the available physical capital stock. If the physical capital stock cannot be increased or decreased costlessly, the harvest capacity constraint introduces irreversibility in the investment decision [1]. It is only under greatly simplified assumptions that optimal investment paths for this system have been derived.

Smith [15, 16] was the first to consider the problem explicitly in terms of two capital stocks. However, his assumption that investment in the physical capital stock increases or decreases linearly in proportion to profits or losses in the industry has been criticized as *ad hoc* (e.g., [11]). An internally consistent, optimization-based approach was taken by Clark, Clarke, and Munro [5] (hereafter, denoted CCM). The solution to the linear-in-control-variables model developed by CCM partitions the state space for the physical and biological capital stocks into a "feedback" control system in which the appropriate harvest and investment controls depend entirely upon the state of the system. If initially the biological stock exceeds its steady-state level and the physical capital stock is inadequate to bring the biological stock to equilibrium, CCM showed it is optimal to make an immediate one-time investment in the physical capital stock followed by a period in which gross investment is turned "off" but the existing physical capital stock is fully utilized. The two capital stocks then decline over time, the physical capital stock due to depreciation, the biological stock because the harvest rate exceeds the biological growth. CCM argue that "[w]hile the study has been carried out on the basis of a specific model of the commercial fishing industry, we believe that

\*I thank the referees and editors for helpful comments. All remaining errors are mine alone.

the qualitative nature of our results will prove to be robust" (p. 46). McKelvey found their characterization of the optimal capital accumulation paths to hold for an open access fishery [11] and for a fishery with imperfect competition [12]. The model has also been extended to the case of uncertainty by Charles [3].

The problem with these models is that they do not accord with observed patterns of capital accumulation in fisheries. Clark ([4], Fig. 4.7, p. 117, citing data from Clark and Lamberson [6]) plots the rise and decline of the Antarctic baleen whaling fleet following the Second World War (cf. McKelvey, [12]). The striking feature is the difference between the predicted "optimal path" and the actual path. Initially, the biological stock was greater than its exploited steady-state level and the physical capital stock was approximately one-third of its eventual maximum. The CCM model predicts an immediate jump in the physical capital stock followed by a period of decline in both the physical and biological capital stocks. McKelvey's linear models for multiple-owner fisheries predict an initial investment pulse that is larger than CCM's "optimal" surge but is followed by the same period of zero gross investment. The actual data show that the fleet size grew relatively slowly, taking 15 years to reach its maximum level. Clark observes that the fleet rose to a higher level than predicted by the optimal model (16 factory vessels, compared with 13 in the optimal model), and attributes the difference to the fact that the fleet was open access rather than managed by a single operator. However, he makes no mention of the fact that the fleet took 15 years to reach its maximum rather than instantaneously as in the "optimal model."

The approach taken by CCM requires a number of strong assumptions. In particular, it is assumed that the marginal cost of investment is independent of the rate of investment and that marginal harvest profits are independent of the harvest rate. Thus large increases in the capital stock are proportionately no more costly than small increases, and output price and marginal harvest costs are independent of the rate of harvest. These assumptions are not likely to hold where industry input demand affects the costs of inputs (especially specialized capital inputs) and/or industry output affects the spot price of the harvested resource.

This paper investigates a nonlinear version of the CCM model, relaxing their assumptions that the investment cost function is linear in the investment rate and that the variable harvest profits function is linear in the harvest rate. McKelvey [11] has also relaxed these assumptions, but when investment costs are nonlinear he has only been able to characterize the final stages of the approach path.<sup>1</sup> The present paper attempts a more complete characterization of the entire optimal path from arbitrary beginning to the steady state. The inability of CCM's and McKelvey's linear models to predict the rate of capital accumulation is shown to be due to their assumption of linearity in the investment cost and variable profits functions.<sup>2</sup> However, in the present model, as in the linear model, a period in

<sup>1</sup>McKelvey [11] derived a set of equations similar to the nonlinear optimization problem considered in this paper. However, he states: "to actually carry through this formulation would be, in general, quite difficult: it is essentially the same 'problem of synthesis' that makes most optimal control problems difficult to solve analytically." He finds "[t]he problem is tractable for at least one special case, namely for the standard fisheries model ... and where investment costs are linear" (p. 293). The standard fisheries model he refers to is the model considered by CCM [5].

<sup>2</sup>An anonymous reviewer has pointed out that similar dynamics could be obtained by imposing an upper limit on the investment rate. Mathematically, such an assumption would clearly force the optimal path to approach more gradually than evidenced by CCM's approach path. However, such an

which the physical capital stock level is above its long-run sustainable equilibrium is found to be optimal. Unlike the linear model, a period of positive but declining gross investment, as is observed in capital accumulation data from developing fisheries, is also optimal. Several authors (e.g., [2], p. 24, [14], pp. 178–182) have suggested that convex investment costs might have this effect. However, the intuition for such conclusions is based on control models with a single state variable and a single control variable [13]. I show that such conclusions also hold in more complicated models with two state and two control variables.

The policy concerns from models of irreversible investment pertain to the optimal time to impose moratoriums on either harvesting or investment. This paper shows that the source of the nonlinearity affects the policy conclusions considerably. When the variable harvest profits function is linear, a moratorium on harvesting (but not investment) is optimal for larger stock levels than in a nonlinear model. When the investment cost function is linear, positive net investment is optimal only when the harvest capacity constraint is binding. Thus a moratorium on investment in times of excess capacity is optimal. This paper shows that when the system is being controlled to maximize social benefits and investment costs are convex in the rate of investment, an absolute moratorium on gross investment will not be optimal once investment begins. This is in sharp contrast to the linear model of CCM where investment is allowed to push the capital stock to a level high enough to reduce the biological stock to below its maximum sustainable yield stock level and is then turned off.

The paper is organized as follows. Section 2 develops the model and examines the steady state. Section 3 examines the out-of-equilibrium dynamics. Section 4 compares the effects of restrictive assumptions made in CCM and in this model. Section 5 compares McKelvey's [11] model of open access to the model of Sections 2 and 3. Section 6 concludes the paper with a discussion of the results.

## 2. THE MODEL

The optimization problem is to choose the investment and harvest rates to maximize  $V$ , the infinite stream of discounted variable harvest profits  $u(h)$  less investment costs  $c(I)$  per unit time,

$$V = \int_0^{\infty} e^{-rt} [u(h) - c(I)] dt. \quad (1)$$

Variable harvest profits depend only upon the harvest rate  $h$  and investment costs depend only upon the investment rate  $I$ . Both  $I$  and  $h$  can vary over time, though time notation is suppressed wherever possible. In CCM, the variable profits function also depended upon the biological stock. The implications of restricting variable profits to a function only of  $h$  are explored in Section 4. The variable harvest profits function is strictly increasing and concave, with  $u(0) = 0$ , and  $0 < u'(0) < \infty$ .<sup>3</sup> The concavity of  $u$  could be either because marginal harvesting

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assumption would be *ad hoc*. The assumption that larger rates of investment are possible, but at higher marginal costs, does not suffer from this complaint.

<sup>3</sup>The assumption that  $u'(h) > 0$  for all  $h$  avoids the problems of saturation in consumption that might be encountered when  $u'(h)$  can be negative for extremely large levels of harvest (e.g., [7], [10]).

costs rise as output increases or because demand for the output is less than perfectly elastic. The investment cost function is increasing and strictly convex, implying that supply of the capital inputs is less than perfectly elastic. In addition,  $c(0) = 0$ , and  $0 < c'(0) < \infty$ . It is also assumed that the discount rate,  $0 < r < 1$ , is constant.

The harvest and investment rates chosen to maximize  $V$  must satisfy equations of motion for the biological and physical capital stocks and initial conditions on the physical and capital stocks as well as nonnegativity constraints and a harvest capacity constraint. The equation of motion for the biological stock is Smith's [15] differential equation for growth of an exploited biological resource  $x$ ,

$$dx/dt \equiv x' = g(x) - h, \quad (2)$$

where  $g(x)$  is a positive-valued, strictly concave function on the domain  $(0, X)$ , with  $g(0) = g(X) = 0$ . The physical capital stock  $k$  moves according to

$$dk/dt \equiv k' = I - \gamma k, \quad (3)$$

where  $0 < \gamma < 1$  is the (constant) rate at which the capital stock depreciates each instant of time.

The harvest rate and the investment rate are subject to nonnegativity constraints, i.e.,

$$I \geq 0, \quad \text{and } h \geq 0, \quad (4)$$

and the harvest rate is limited by the productivity of the available physical capital stock. The harvest capacity constraint is

$$h \leq f(k), \quad (5)$$

where the harvest capacity constraint is given by the production function  $f(k)$ , which is an increasing, and by the concave function with  $f(0) = 0$  and  $0 < f'(0) < \infty$ . Unlike CCM, I am assuming that the biological capital stock does *not* affect the harvest capacity. They assumed that  $h \leq f(k)x$ . The implications of this assumption are explored in Section 4.<sup>4</sup>

The current-value Hamiltonian for this optimization problem may be stated as

$$H = u(h) - c(I) + \lambda[g(x) - h] + \mu[I - \gamma k],$$

where  $\lambda$  and  $\mu$  are the current value costate variables for the biological and physical capital stocks, respectively. Because of the constraints on the control variables, the problem is stated as a Lagrangian

$$L = H + \rho_h h + \rho_I I + \rho_f[f(k) - h],$$

where the  $\rho_i (i = I, h, f)$  are Lagrange multipliers for the nonnegativity constraints on the harvest and investment rates, and the harvest capacity constraint, respectively. The system of equations describing the solution to the optimization problem

<sup>4</sup>An implicit assumption hereafter is that  $h < x$ , that is, extinction is not optimal.

for (1) subject to (2–5), are (2), (3), and

$$\partial L/\partial h = u'(h) - \lambda + \rho_h - \rho_f = 0, \quad (6)$$

$$\partial L/\partial I = -c'(I) + \mu + rI = 0, \quad (7)$$

$$x' = r\lambda - \partial L/\partial x = [r - g'(x)]\lambda, \quad (8)$$

$$\mu' = r\mu - \partial L/\partial k = [r + \gamma]\mu - f'(k)\rho_f, \quad (9)$$

$$I \geq 0, \quad \rho_I \geq 0, \quad \rho_I I = 0, \quad (10)$$

$$h \geq 0, \quad \rho_h \geq 0, \quad \rho_h h = 0, \quad (11)$$

$$f(k) \geq h, \quad \rho_f \geq 0, \quad [f(k) - h]\rho_f = 0. \quad (12)$$

Consider the steady-state equilibrium for this problem (assuming one exists). The solution requires that the shadow value of both the biological and physical capital stocks to be positive in a steady-state equilibrium. It is found by solving a recursive system of equations

$$x_r = g'^{-1}(r) \quad (\text{from } x' = 0) \quad (13)$$

$$h_r = g(x_r) \quad (\text{from } x' = 0) \quad (14)$$

$$k_r = f^{-1}(h_r) \quad (\text{from } \mu > 0) \quad (15)$$

$$I_r = \gamma k_r \quad (\text{from } k' = 0) \quad (16)$$

$$\mu_r = c'(I_r) \quad (\text{from (7)}) \quad (17)$$

$$\rho_f = [r + \gamma]\mu_r/f'(k_r) \quad (\text{from } \mu' = 0) \quad (18)$$

$$\lambda_r = u'(f(k_r)) - \rho_f \quad (\text{from (6)}) \quad (19)$$

$$\rho_h = \rho_I = 0 \quad (\text{from (14) and (17)}). \quad (20)$$

Upon rearrangement, (19) with (18) substituted in for  $\rho_f$  shows that at the steady-state equilibrium,

$$u'(f(k_r))f'(k_r) = \lambda_r f'(k_r) + [r + \gamma]\mu_r. \quad (21)$$

The term on the left-hand side of (21) is the value of the marginal product of the physical capital stock. In the steady-state equilibrium this is equated to the cost of an additional unit of harvest, which is the sum of the marginal value of the reduction in the biological capital stock and interest and depreciation cost of the capital.

### 3. OUT-OF-EQUILIBRIUM BEHAVIOR

In this section, the optimal path to the steady-state equilibrium is characterized for various initial conditions. As in CCM, there are six possible combinations of binding and slack constraints on the investment and harvest rates to consider. The technique used is similar to methods used by Kamien and Schwartz [8, 9] for models involving multiple state and control variables. Phase diagrams in the  $\lambda$ - $x$

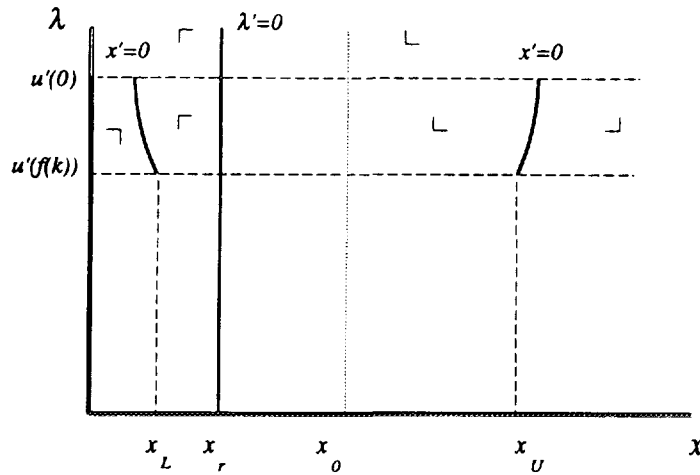


FIG. 1. The  $\lambda$ - $x$  phase plane when  $k^* < k_r$ .

space, showing the dynamics of the biological population, and in the  $\mu$ - $k$  space, showing the dynamics of the physical capital stock, are derived using standard methods. The optimal capital accumulation paths are characterized through a set of lemmas and theorems which make use of the phase diagrams in Figs. 1-5. Proofs to Theorems 1-3 are given in the Appendix. Hereafter,  $h^*$ ,  $I^*$ ,  $x^*$ , and  $k^*$  will refer to the values of the respective variables along the optimal path.

The dynamics in the  $\lambda$ - $x$  space when  $0 < h^* < f(k)$  are depicted in Figs. 1, 2, and 3. The region where  $h^* = 0$  is bound from below by  $\lambda = u'(0)$ , and the region where  $h^* = f(k)$  is bound from above by  $\lambda = u'(f(k))$ . The values of  $x$  such that  $g(x) = f(k)$  are denoted  $x_L(k)$  and  $x_U(k)$  (there are two since  $g$  is single-peaked). The maximum sustainable yield stock level is denoted  $x_0$  (i.e.,  $g'(x_0) = 0$ ). In Fig. 1, the  $h^* = f(k)$  boundary, given by  $\lambda = u'(f(k))$ , is above the steady-state

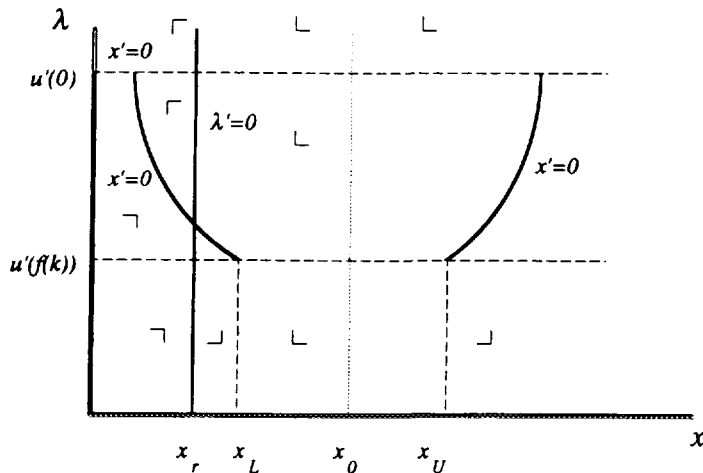


FIG. 2. The  $\lambda$ - $x$  phase plane when  $k_r < k^* < k_0$ .

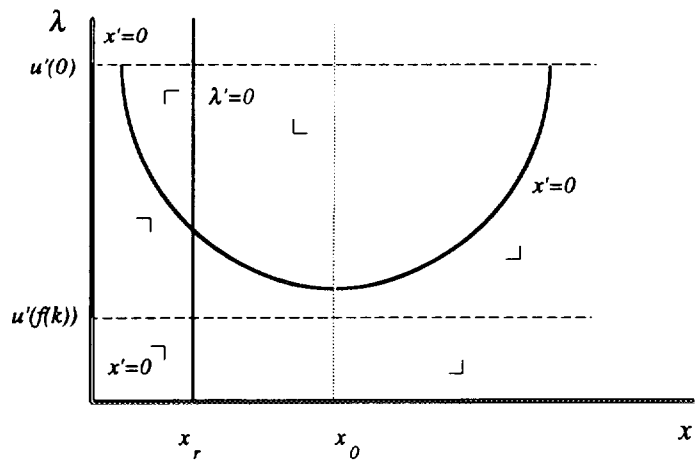


FIG. 3. The  $\lambda$ - $x$  phase plane when  $k_0 < k^*$ .

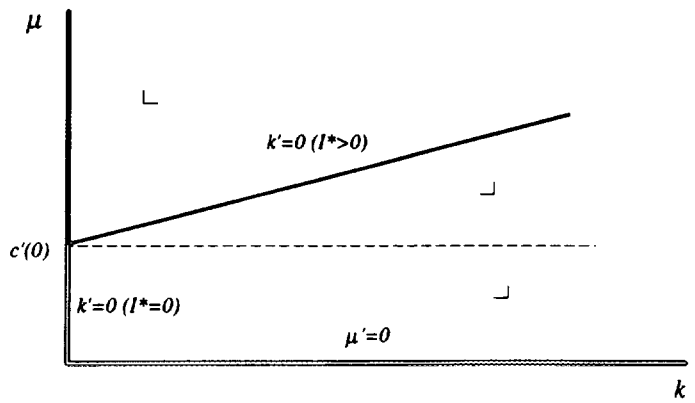


FIG. 4. The  $\mu$ - $k$  phase plane when  $h^* < f(k)$ .

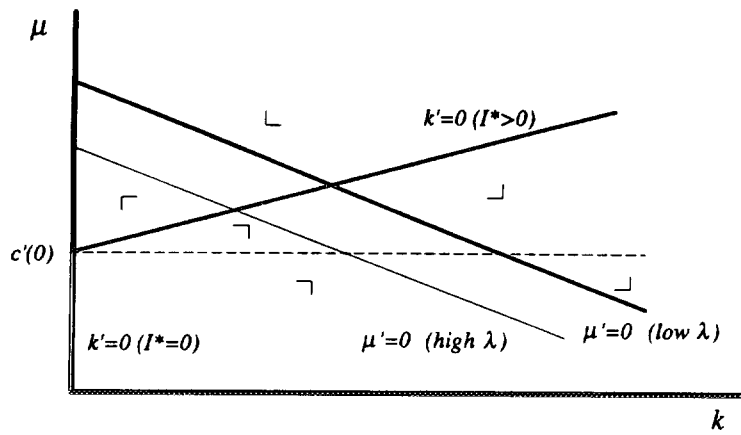


FIG. 5. The  $\mu$ - $k$  phase plane when  $h^* = f(k)$ .

equilibrium  $(x_r, \lambda_r)$ , formed by the intersection between the  $x' = 0$  locus and the  $\lambda' = 0$  locus. In Fig. 2, the physical capital stock is greater than the steady-state level, but less than the amount necessary to harvest the maximum sustainable yield,  $h_0$ , where  $h_0 = g(x_0)$ . In Fig. 3, the physical capital stock is greater than is necessary to harvest even the maximum sustainable yield. Thus the upper bound where  $h^* = f(k)$  is below the entire  $x' = 0$  locus.

The  $\mu$ - $k$  phase diagram for the case where  $h^* < f(k)$  is depicted in Fig. 4. The  $\mu' = 0$  locus is the  $k$ -axis ( $\mu = 0$ ). The  $k' = 0$  locus is the  $\mu$ -axis from the origin up to the point  $\mu = c'(0)$  (the point where investment is just choked off); thereafter, it has a positive slope. Below the  $k' = 0$  locus,  $\mu$  is increasing and  $k$  is decreasing. Above the  $k' = 0$  locus, both  $\mu$  and  $k$  are increasing. It is only for  $\mu > c'(0)$  that  $I^* > 0$ . The  $\mu$ - $k$  dynamics when  $h^* = f(k)$  are depicted in Fig. 5. The  $\mu' = 0$  locus is downward sloping and extends across the  $c'(0)$  locus. Above the  $\mu' = 0$  locus,  $\mu$  is increasing, and below it,  $\mu$  is decreasing. Increases in  $\lambda$  cause the  $\mu' = 0$  locus to shift in toward the origin. The  $k' = 0$  locus is the  $\mu$ -axis from the origin up to the point  $c'(0)$ , and is positively sloped thereafter. Everywhere above the  $k' = 0$  locus,  $k' < 0$ . The primary difference between the  $\mu$ - $k$  phase diagram in Fig. 4, where  $h^* < f(k)$ , and that in Fig. 5, where  $h^* = f(k)$ , concerns the  $\mu' = 0$  locus. When  $h^* < f(k)$ , the  $\mu' = 0$  locus is the  $k$ -axis; when  $h^* = f(k)$ , the  $\mu' = 0$  locus has a positive intercept, is downward sloping, and shifts downward as  $\lambda$  increases.

Let us now characterize the optimal capital accumulation paths.

**LEMMA 1.** *During the final stage in approaching the steady-state equilibrium,  $h^* = f(k)$  (see [11]).*

At the steady-state equilibrium, it must be that  $x = x_r$  and  $k = k_r$ . From Lemma 1, this means that  $x_L \rightarrow x_r$  near the steady-state equilibrium (see Fig. 2). Let us now show that if  $x(t) < x_r$ , then *both* capital stocks will be increasing in the final approach to the steady-state equilibrium, and if  $x(t) > x_r$ , then both capital stocks will be decreasing in the final approach to the steady-state equilibrium.

**THEOREM 1.** *In the final approach to the steady-state equilibrium, if  $x(t) < x_r$ , then  $k(t) < k_r$ , and if  $x(t) > x_r$ , then  $k(t) > k_r$ .*

Lemma 1 and Theorem 1 both pertain to the interval of time just before reaching the steady-state equilibrium. The next theorem pertains to the entire approach path.

**THEOREM 2.** *Once the investment rate becomes positive, it will never go back to zero.*

Theorem 2 shows that the nonlinear model behaves very differently than the linear models. CCM and McKelvey argued that if the biological capital stock is above  $x_r$ , it is optimal to immediately increase the physical capital stock to a level greater than the steady-state level and then turn investment off. The biological capital stock is then reduced to the steady-state equilibrium using the existing capital stock, which is allowed to depreciate. Theorem 2 shows that if investment costs are convex, it will never be optimal to turn investment off once it has begun.



Let us now characterize the investment path for the case where the initial biological stock is above  $x_r$  and the initial physical capital stock is small enough that it is optimal to invest a positive quantity at  $t = 0$ . Then,

**THEOREM 3.** *If the physical capital stock is insufficient to bring the biological capital stock to the steady-state equilibrium, then once investment begins, the investment rate will be positive, finite, and decreasing.*

Theorem 3 shows that once investment begins the gross investment rate will remain positive during the remainder of the optimal accumulation path. Combining this result with Theorem 2 shows that if the physical capital stock is initially too low to bring the biological capital stock to the steady-state equilibrium, then gross investment in the fishery will begin promptly, but will never stop. Net investment may be negative, but gross investment will asymptotically approach its steady-state level from above.

#### 4. EFFECTS OF RESTRICTIVE ASSUMPTIONS

The model just presented is not a complete generalization of the CCM model. Consider a general model that encompasses both the CCM model and the nonlinear model studied in Sections 2 and 3. The model, using the notation from Section 2 and stated in Lagrangian form, is

$$L = \pi(x)u(h) - c(I) + \lambda[g(x) - h] + \mu[I - \gamma k] + \rho_I I + \rho_h h + \rho_f[f(k)s(x) - h]. \quad (22)$$

In (22), the variable harvest profits function (the entire first term) depends upon the biological capital stock through the function  $\pi(x)$ . The harvest capacity constraint (the last term) depends upon the biological capital stock through the function  $s(x)$ . The functions  $\pi(x)$  and  $s(x)$  appear in CCM, but not in the nonlinear model. As in CCM,  $x$  appears in the profits function because it appears in the production function.<sup>5</sup> The difference between the CCM model and the model analyzed above is thus that CCM restrict  $u(h)$  and  $c(I)$  to be linear, while I restrict  $s(x) = 1$  (implying  $p(x) = 1$ ) in this paper.

It would be interesting to analyze (22). Unfortunately, I have neither been able to complete such an analysis so far as characterization of the optimal paths is concerned, nor, when  $s(x)$  is allowed to vary with  $x$ , have I been able to analyze

<sup>5</sup>CCM [5] assumed that instantaneous variable harvesting profits are given by  $Ph - wE$ , where  $P$  is the output price and  $w$  is the marginal cost of effort,  $E$ . Harvest is related to the level of effort through the production function  $h = qxE \equiv s(x)E \leq s(x)f(k)$ , i.e.,  $s(x) = qx$ , where  $q$  is a positive constant. The harvest production function constraint (4) is given by the inequality above. It states that effort is restricted to being no more than the existing physical capital stock,  $k$ . Solving the first equality above for effort and substituting the result into the harvest profit function shows that

$$\pi(x)u(h) = [P - w/s(x)]h.$$

Thus CCM's harvest profit function is nonlinear in  $x$ , but linear in  $h$ . The profit function depends upon  $x$  because the production function depends upon  $x$ . If any of the terms  $Ph$ ,  $wE$ , or  $s(x)E$  were nonlinear in  $h$  or  $E$ , then the variable profits function would be nonlinear in  $h$ .

the out-of-equilibrium behavior of the solution to (22) with either just  $u(h)$  or  $c(I)$  nonlinear. However, it is possible to show the incremental effect of imposing the assumptions of one model on top of the assumptions already made in the other model. Therefore, consider what happens to the CCM model when  $s(x) = 1$ , or what happens in the nonlinear model analyzed in Sections 2 and 3 when either  $u(h)$  or  $c(I)$  is restricted to be linear.

A. Restricting  $s(x) = 1$  in the CCM Model

CCM obtain *two* biological equilibria for their model, corresponding to the cases where the harvest capacity constraint is and is not binding, with only the former sustainable over the long run. Assume, as do CCM, that  $u(h) = h$  and  $c(I) = cI$  in (22). When the harvest capacity constraint is slack, the equilibrium biological stock can be shown to be the solution to (see [5, pp. 29–30] for these derivations)

$$[\pi'(x)/\pi(x)]g(x) + g'(x) = r. \tag{23}$$

Equation (23) is a “modified golden rule” [5, p. 29]. The left-hand side of (23) represents the biological stock’s own rate of return, which must equal the return on other capital assets,  $r$ .

When the harvest capacity constraint is binding, the corresponding equilibrium level of the biological stock can be shown to be the solution to

$$[\pi'(x)g(x) - \rho_f s'(x)f(k)] / [\pi(x) - \rho_f] + g'(x) = r, \tag{24}$$

where  $\rho_f = c[r + \gamma]/s(x)f'(k)$ . The terms involving  $\rho_f$ , which do not appear in (23), appear in (24) because when the capital stock is fully utilized the replacement cost of capital is positive. In (23), the harvest capacity constraint is not binding, so additional capital can be utilized at no additional cost. It is now apparent why the nonlinear model studied in Sections 2 and 3 only has one equilibrium. If  $\pi(x) = s(x) = 1$ , then  $s'(x) = \pi'(x) = 0$ , so the first terms in (23) and (24) vanish, making the solutions identical to one another and to the biological steady-state obtained in (13).

If the restrictions  $\pi(x) = s(x) = 1$  are imposed on (22) in addition to CCM’s linearity assumptions for  $u(h)$  and  $c(I)$ , the dynamics of this system are as depicted in Fig. 6. The difference between Fig. 6 and the more general linear-in-control-variables model considered by CCM (see Fig. 1 in [5, p. 31]) is that the restrictions  $\pi(x) = s(x) = 1$  result in a *single* steady-state biological stock, occurring at  $x_r$ . As in the more general linear-in-control-variables model, there exist two “switching curves,”  $\sigma_1$  to the left of  $x_r$ , and  $\sigma_2$ , to the right of  $x_r$ . To the left and above  $\sigma_1$ , both investment and harvest are optimally zero. Below and to the right of  $\sigma_2$ , investment is infinite. Elsewhere, investment is zero but harvest is positive and the harvest capacity constraint is binding. If an initial investment pulse occurs, it is followed by a period where investment is turned off. Investment is not turned on again until both capital stocks are at the steady-state equilibrium. This implies:

LEMMA 2. *The instantaneous jump in the physical capital stock in CCM is independent of whether the biological capital stock affects harvest profit or the harvest production function (see [5]).*

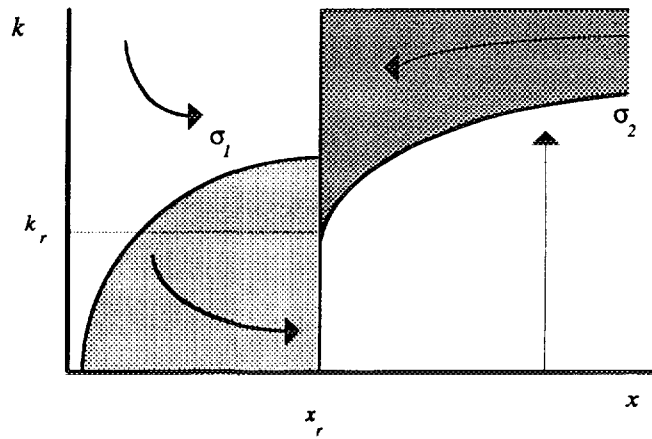


FIG. 6. Feedback controls when costs are independent of  $x$ .

*B. Linearity Restrictions on  $c(I)$*

Now, consider the effect of CCM's linearity assumption in the investment cost function when the remainder of the model maintains the assumption that  $\pi(x) = s(x) = 1$ . When  $c(I)$  is linear, the maximum principle equation (7) for investment becomes

$$c = \mu + \rho_I. \tag{25}$$

Equation (25) implies a singular solution for investment. Independent of whether harvest profit is linear in  $h$  or whether  $x$  appears in either the harvest profit or production function, the following is true:

**THEOREM 4.** *When the model is linear in investment costs, if  $h^* < f(k)$ , then  $I^* = 0$ .*

This is an important difference between the linear investment cost and nonlinear investment cost models. In the nonlinear model it may be optimal to invest in physical capital even though the capital is not currently being used. The reason, of course, is that to forego a unit of investment for a moment can raise investment costs due to the larger rate of investment. In a linear model, the cost of investment is independent of the rate of investment, so it does not pay to accumulate capital in anticipation of future requirements.

Theorem 4 does *not* mean that  $h^* = f(k)$  implies  $I^* > 0$ . The next theorem characterizes the optimal path for the case where  $x \geq x_r$ .

**THEOREM 5.** *If investment costs are linear, then (i)  $h^* > 0$  when  $x \geq x_r$ , (ii)  $k' \leq 0$  when  $I^*$  is singular, and (iii) an instantaneous jump in the physical capital stock, if it occurs, will occur at time  $t = 0$ .*

*C. Linearity Restrictions on  $u(h)$*

Consider the case where harvest profits are linear in the harvest rate, e.g.,  $u(h) = \pi h$ , where  $\pi$  is a positive constant. The maximum principle equation for

the harvest rate (6) becomes

$$\pi - \lambda = \rho_f - \rho_h. \tag{26}$$

Equation (26) implies that the harvest rate will be singular. It is easy to show the following:

**THEOREM 6.** *When the model is linear in the harvest rate but nonlinear in the investment rate, (i) if the harvest variable is singular, then the biological capital stock is at the steady-state equilibrium level  $x = x_r$ , (ii) if  $x < x_r$ , then  $h^* = 0$ , and (iii) if  $x > x_r$ , then  $h^* = f(k)$ .*

Theorem 6 implies that when the harvest rate is linear, the selection of the investment rate drives most of the dynamics. The question is whether or not the investment rate will follow the pattern suggested by CCM, where if  $x > x_r$  and the physical capital stock is inadequate to reduce the stock to  $x_r$ , then an initial investment pulse is followed by a period of no gross investment. For  $x > x_r$ , Theorem 6 implies that the harvest capacity constraint will be everywhere binding along the optimal path. The model considered in Sections 2 and 3 for the case where  $h^* = f(k)$  and  $I^* > 0$  is identical to the present problem except for one term in the costate equation for the physical capital stock. If the harvest profit function is linear in  $h$ , the costate equation (9) becomes  $\mu' = \mu[r + \gamma] - [\pi - \lambda]f'(k)$ . The only difference is that here  $\pi$  replaces  $u'(f(k))$ . This difference does not affect the nature of the dynamics. In each case, the  $\mu' = 0$  locus is downward sloping and an increase in the value of  $\lambda$  causes the  $\mu' = 0$  locus to shift downward (see Fig. 5). Thus,

**THEOREM 7.** *When the harvest profit function is linear in the harvest rate, a nonlinear investment cost function causes the physical capital stock to be built up with the gross investment rate remaining positive once investment starts, and with net investment declining along the optimal path.*

### 5. ECONOMIC RATIONALIZATION OF OPEN ACCESS

McKelvey [11] also developed a model of open access under conditions of irreversible investment. Under open access, a number of things change in the model considered in Sections 2 and 3. Open access implies  $\lambda$  equals zero. Let the variable harvest function be specified more fully as

$$u(h) = \int_0^h p(s) ds - w(h),$$

where  $p(h)$  is the demand function ( $p' < 0$ ) and  $w(h)$  is the variable harvest cost function ( $w' > 0$ ). Note that  $u'(h) = p(h) - w'(h)$ . Under open access, the zero profits condition is that  $p(h) - w'(h)/h = 0$  when the harvest capacity constraint is not binding. In the event that each unit of harvest is taxed at rate  $\tau$  (or there exists an efficient market for harvest quotas), McKelvey shows that under open access, the value of an additional unit of capital is

$$\mu(t) = \int_t^\infty \{ [f(k)/k] [p(h) - w'(h)/h - \tau]^+ e^{-(r+\gamma)(s-t)} \} ds, \tag{27}$$

where  $[p(h) - w(h)/h - \tau]^+ = \max(p(h) - w(h)/h - \tau, 0)$ . Differentiating (27) with respect to time yields

$$\mu' = (r + \gamma)\mu - [f(k)/k][p(h) - w(h)/h - \tau]^+. \quad (28)$$

If the capital market is competitive, then when investment is positive,  $\mu = c'(I)$ .<sup>6</sup> The optimal tax may now be derived. When the harvest capacity constraint is binding, we get from (28) and (9) that

$$\begin{aligned} p(h) &= [\mu(r + \gamma) - \mu'] / [f(k)/k] + w(h)/h + \tau \\ &= [\mu(r + \gamma) - \mu'] / [f(k)/k] + w'(h) + \lambda, \end{aligned}$$

from which the optimal tax is

$$\tau = \lambda + [w'(h) - w(h)/h] + [\mu(r + \gamma) - \mu'] \{ [1/f'(k)] - [k/f(k)] \}. \quad (29)$$

This shows clearly the three different parts that the tax must correct: the first term corrects for the intertemporal distortions due to the open access, the second term corrects for the averaging of the variable harvesting costs, and the third term corrects for the averaging of the productivity of capital. The optimal tax is nonnegative. The shadow price of the stock is nonnegative by assumption; McKelvey's assumption that  $w$  is convex ( $w' > 0$  and  $w'' > 0$ ) makes the second term nonnegative; and the concavity of  $f$  plus (9), which implies  $\mu(r + \gamma) - \mu' \geq 0$ , makes the third term nonnegative as well.

When the harvest constraint is not binding, the third term in (29) vanishes. The tax then need only correct for the neglect of the shadow value and the externality imposed through variable harvesting costs. When investment is optimally zero the tax must ensure that  $\mu < c'(0)$ . However, this is accomplished by sending the correct signal of the social cost of the additional harvest. From (27) investment will be zero if and only if the correct tax has been applied to force  $[f(k)/k][p(h) - w(h)/h - \tau]^+ = f'(k)\rho_f$ . In this case, a single tax on harvest is sufficient to adjust both harvest and investment to the optimal levels. This result is in contrast to McKelvey's result, with the difference due to his assumption that  $\mu = c(I)/I$ . Since the tax is nonnegative at all times, we conclude:

**LEMMA 3.** *Open access will result in excessive harvesting and overcapitalization relative to the optimal path.*

## 6. DISCUSSION AND CONCLUSIONS

The analysis of a nonlinear model of optimal capital accumulation in a fishery has markedly different conclusions about the shape of capital accumulation paths than predicted by Clark *et al.* [5]. They find that a large initial pulse in the physical capital stock is optimal if the biological stock is in excess of the steady-state

<sup>6</sup>McKelvey [11] assumed  $\mu = c(I)/I$ . However, this is incorrect so long as the market for capital is competitive.

equilibrium. This is followed by a period of zero gross investment in the fleet, resulting in a decrease in the physical capital stock due to depreciation. In contrast, in the nonlinear model it is never optimal to stop gross investment once it has started. The difference in the optimal capital accumulation paths is due to the linearity assumptions made by CCM with regards to the variable harvest profits and investment costs functions.

The nonlinear model finds a capital accumulation path that much more closely resembles that observed in actual fisheries. This is interpreted to mean that convex investment costs and/or concave industry variable harvest profits are important in the growth of new fisheries as well as in the decline of more mature fisheries. Not only does the data in Clark and Lamberson [6] for the Antarctic baleen whaling fleet in the post-Second-World-War era show that gross investment remained positive as long as the biological stock exceeded the steady-state equilibrium, but the rate of gross investment also declined over time, as predicted by the nonlinear model. Although no one would seriously argue that the whaling fleet was following an optimal path, it is plain that some nonlinearity was involved in the process of capital accumulation.

Even the nonlinear model, though, fails to account for the rapid decline in the capital stock of the Antarctic baleen whaling fleet following 1961. It is possible that such a decline is the result of poorly defined property rights, leading to excessive capitalization as predicted by McKelvey [11, 12]. It is also possible that what accounts for the decline is the dynamics observed by CCM, in which the biological steady-state equilibrium may be crossed once along the optimal path due to the effect of the biological stock on the production function.

This paper also shows that the nonlinear asymptotic approach path results common in one-state, one-control variable models appear to also hold in two-state, two-control variable models, with (at least for the model considered) the CCM provision that the physical capital stock exceed its steady-state level somewhere along the equilibrium path. In CCM, the temporary excessive capitalization was due in part to the linear investment cost function. In the nonlinear model studied in this paper, the temporary excessive capitalization is an artifact of the steady-state equilibrium occurring to the left of the maximum sustainable yield stock level. It is not clear whether this characteristic would survive in a model where the steady-state biological stock exceeded the maximum sustainable yield stock.

Finally, the analysis here shows that the source of the nonlinearity is important in guiding policy-makers with regards to imposing a moratorium on either harvesting or investment. If investment costs are nonlinear and harvest profits are linear, then a temporary moratorium on harvesting but not on investment may be optimal. If harvest profits are nonlinear but investment costs are linear, then a moratorium should be placed on investment only if the current capital stock is not fully utilized.

## MATHEMATICAL APPENDIX

*Proof of Theorem 1.* We know from Lemma 1 that during the final stage of approach,  $h^* = f(k)$ . (i) Suppose  $x(t) > x_r$ . If  $k(t) < k_r$ , then the  $x_L$  locus (where  $x' = 0$ ) lies to the left of  $x_r$ . Then  $x' > 0$  for  $x > x_r$ . This causes  $x$  to move away from the steady-state equilibrium. However, if  $k(t) > k_r$ , then  $x_L > x_r$ . This means for  $x_r < x(t) < x_L$ , that  $x' < 0$ . If simultaneously,  $k' < 0$ , then  $x_L$

approaches  $x_r$ . Thus the region between  $x_r$  and  $x_L$  diminishes, vanishing in the limit when the steady-state equilibrium is reached. (ii) Suppose  $x(t) < x_r$ . If  $k(t) > k_r$ , then the  $x_L > x_r$ . Then  $x' < 0$  for  $x < x_r$ . Again, this causes  $x$  to move away from the steady-state equilibrium. However, if  $k(t) < k_r$ , then  $x_L < x_r$ . This means for  $x_L < x(t) < x_r$ , that  $x' > 0$ . If simultaneously  $k' > 0$ , then  $x_L$  approaches  $x_r$ . Thus the region between  $x_r$  and  $x_L$  diminishes, vanishing in the limit when the steady-state equilibrium is reached. ■

*Proof of Theorem 2.* For  $I^*$  to approach zero,  $\mu$  must approach  $c'(0)$  from above. As long as  $\mu > c'(0)$ , the investment rate will remain positive. (i) When  $h^* < f(k)$ ,  $\mu > c'(0)$ , and  $\mu' = [r + \gamma]\mu$ , which is positive. Thus for  $h^* < f(k)$ , if the investment rate is positive, it will never drop to zero. (ii) When  $h^* = f(k)$ ,  $I^* > 0$  implies  $\mu' = [r + \gamma]\mu - [u'(f(k)) - \lambda]f'(k)$ . Since the second term causes  $\mu'$  to decrease, the net effect appears ambiguous. However, the dynamics in  $\mu$ - $k$  space depend upon both the  $\mu' = 0$  locus and the  $k' = 0$  locus. From Fig. 5, the optimal path moves toward the intersection of the  $\mu' = 0$  locus and the  $k' = 0$  locus while  $h^* = f(k)$ . This intersection always occurs at values of  $\mu > c'(0)$ . Therefore, though  $\mu'$  may be negative, it never decreases to the point where  $I^* = 0$ .

*Proof of Theorem 3.* We suppose that the initial capital stock is small enough that the harvest capacity constraint  $h^* = f(k)$  is or becomes binding somewhere along the optimal path. If the initial biological stock is above  $x_0$ , then the amount of capital needed to push the biological capital stock down to the steady-state is greater than  $k_0$ . This means  $h^* = f(k)$  and  $I^* > 0$ , at least for some interval along the optimal path. Consider the dynamics when  $h^* = f(k)$  and  $I^* > 0$ . From Fig. 1, if  $x > x_U$ , then the optimal path will lead toward the intersection of the  $x_U$  locus and the  $\lambda' = 0$  locus. This means  $x$  is decreasing, and  $\lambda$  is increasing. From Fig. 5, if  $k$  is small (to the left of the intersection of the  $k' = 0$  and  $\mu' = 0$  loci), then  $\mu$  is decreasing, and  $k$  is increasing. Since  $I^* = c^{-1}(\mu)$ , we have that  $I^{*'} = c^{-1}(\mu)\mu'$ , which is negative. Thus the investment rate is positive, but declining, and  $h^* = f(k)$  initially. The case where  $x_r < x < x_U$  and where  $x_r < x_L < x < x_U$  are each similar, except that  $x'$  or  $\lambda'$  may be opposite sign. In any case, the dynamics in  $\mu$ - $k$  space are the same.

## REFERENCES

1. Arrow, K. J., Optimal capital policy with irreversible investment, in "Value, Capital and Growth" (J. N. Wolfe, Ed.), pp. 1-20. Edinburgh Univ. Press, Edinburgh (1986).
2. Bjorndal, T., Optimal management of North Sea herring, *J. Environ. Econom. Management*, **15**, 9-29 (1988).
3. Charles, A., Optimal fisheries investment under uncertainty, *Canad. J. Fish. Aquat. Sci.*, **40**, 2080-2091 (1983).
4. C. W. Clark, "Mathematical Bioeconomics: The Optimal Management of Renewable Resources," 2nd ed., Wiley, New York (1990).
5. C. W. Clark, F. H. Clarke, and G. R. Munro, The optimal exploitation of renewable resource stocks: Problems of irreversible investment, *Econometrica*, **47**, 25-47 (1979).
6. C. W. Clark and R. H. Lamberson, An economic history and analysis of pelagic whaling, *Marine Policy*, **6**, 103-20 (1982).
7. Hannesson, R., The effect of the discount rate on the optimal exploitation of renewable resources, *Marine Resour. Econom.*, **3**, 319-329 (1986).
8. M. I. Kamien and N. L. Schwartz, Optimal capital accumulation and durable goods production, *Z. Nationalokonom.*, **37**, 25-43 (1977).

9. M. I. Kamien and N. L. Schwartz, Optimal exhaustible resource depletion with endogenous technical change, *Rev. Econom. Stud.*, **45**, 179–196 (1978).
10. M. C. Kemp and N. V. Long, The management of a renewable resource: Monopoly versus competition, in “Exhaustible Resources, Optimality, and Trade” (M. C. Kemp and N. V. Long, Eds.), North-Holland, Amsterdam (1980).
11. R. McKelvey, Decentralized Regulation of a common property renewable resource industry with irreversible investment, *J. Environ. Econom. Management*, **12**, 287–307 (1985).
12. R. McKelvey, Fur seal and blue whale, in “The Bioeconomics of extinction: Lecture Notes in Biomathematics” (Y. Cohen, Ed.), Vol. 73, pp. 57–81. Springer-Verlag, Berlin (1986).
13. F. Milne, The adjustment cost problem with jumps in the state variable, in “Applications in Optimal Control Theory” (N. V. Long, N. J. Vousden, and S. J. Turnovsky, Eds.), North-Holland, Amsterdam (1977).
14. G. R. Munro, Mathematical economics and the evolution of modern fisheries economics, *Bull. Math. Bio.*, **53**, 163–184 (1992).
15. V. L. Smith, Economics of production from natural resources, *Amer. Econom. Rev.*, **58**, 409–431 (1968).
16. V. L. Smith, On models of commercial fishing, *J. Political Econom.*, **77**, 181–98 (1969).