# Damned if you do, damned if you don't: an option value approach to evaluating the subsidy of the CPR mainline 

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#### Abstract

We revisit the 'excess subsidy' granted to the CPR for the first transcontinental railway in 1881. Previous studies have evaluated the subsidy from an ex post perspective, and concluded that it was 'too large.' We argue that the subsidy should be evaluated from an ex ante perspective, which explicitly accounts for uncertainty. Using an option value approach, which takes account of the risk and irreversibility of the investment, we show that it is likely that the subsidy was not excessive at all, but rather was required to compensate the CPR for forgoing the option to construct the line at the privately optimal time in the future.

On a tort quoi qu'on fasse: une approche en termes de valeur d'option à l'évaluation de la subvention du gouvernement pour la ligne de chemin de fer du Canadien Pacifique. Les auteurs ré-examinent la question de la 'subvention excédentaire' qu'on aurait donnée au Canadien Pacifique pour mettre en place le premier chemin de fer transcontinental en 1881. Des études antérieures ont évalué cette subvention dans une perspective ex post et sont arrivées à la conclusion qu'elle avait été 'trop grande.' Les auteurs suggèrent que l'évaluation doit se faire dans une perspective ex ante qui tienne compte explicitement de l'incertitude. En utilisant une approche en termes de valeur d'option, qui prend en compte le risque et l'irréversibilité de l'investissement, on montre qu'il est probable que la subvention n'a pas du tout été excessive, mais qu'elle était requise pour compenser le Canadian Pacifique pour son abandon de l'option de construire la ligne de chemin de fer à un moment optimal dans l'avenir du point de vue du secteur privé.


## I. INTRODUCTION

Governments have a long tradition of subsidizing, and sometimes constructing, infrastructure mega-projects like canals (early nineteenth century), railways (late nineteenth and twentieth centuries), and highways, tunnels, and fixed-link bridges

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(of the present). The rationale for government intervention in such projects is that the investments generate sizeable social returns but have private returns too low to attract private investors.

No less well established is the tradition among economists of analysing government intervention in these projects, particularly those thought to have had an important impact on national economic development. There has been a tendency to address both the positive and the normative aspects of government intervention in these projects from an ex post perspective, ignoring the high degree of risk associated with these projects at the time the decisions are made. While useful for positive analysis, the ex post perspective is of limited use for addressing issues of a normative nature.

A case in point is Canada's first transcontinental rail line. Built by the Canadian Pacific Railway Company (CPR) over the period from 1881 to 1885 , the line is perhaps Canada's most famous example of large-scale government intervention. ${ }^{1}$ Like many North American transcontinental railways, the CPR mainline was 'built ahead of demand' in the sense that Canadian prairie settlement did not begin in earnest until after 1896. Private interests were understandably leery of a Canadian transcontinental railway project. The prairies were practically unsettled at the time, and even when they did begin to fill up, wheat farming was an extremely volatile enterprise (Norrie 1974). George $(1968,741)$ recognized the inherent uncertainty of the transcontinental project, pointing out that 'private enterprise never seriously considered building the CPR without financial assistance of the government' because 'anticipated profits were too small in view of the considerable risks attached to the project.'

George $(1968,741)$ argued that an all-Canadian transcontinental railway 'was considered to be necessary politically and "profitable" in the sense that it would promote the general economic progress of the country.' Thus, to entice a private-sector entity to construct the line, the government in 1880 granted a subsidy consisting of in-kind transfers (track lines previously built by the government and transferred to the CPR), cash, and land.

Based upon ex post calculations of the line's realized rate of return, George (1968) has argued that a subsidy was necessary for the construction of the CPR mainline to be privately profitable. He also makes the normative assertion that the ex post size of the subsidy was grossly excessive given the realized rate of return to the CPR: he calculated that the ex post 'excess subsidy' could have been as high as $\$ 61$ million ( 1885 dollars), which amounted to about 40 per cent of the capital costs of the line (in present value terms). Although subsequent studies have challenged the magnitude of George's calculations, they concur that, in retrospect, the subsidy provided by the Canadian government was too high. Mercer (1982), for example, placed the value of the ex post 'excess subsidy' at around $\$ 20$ million (1900 dollars), about 13 per cent of the capital cost.

[^2]Given the uncertainty associated with the CPR mainline project, the distinction between the notions of ex post and ex ante efficiency is crucial. The concept of ex post efficiency is largely vacuous in this context, since it provides little guidance for policy makers. To see this, consider the counterfactual that the Canadian 'wheat boom' had been a 'wheat bust' and the CPR line never turned a profit. Using an ex post approach to evaluate the merits of the project, we would conclude that the Canadian government wasted a small fortune to subsidize an inherently doomed project. ${ }^{2}$ In general, an ex ante decision, based on a distribution of possible outcomes, will always be condemned as wasteful if the type of ex post approach commonly employed is used, unless the government hits an ex post 'bulls eye,' where their expectations and the ultimate realization of the state of the world exactly coincide. By ignoring the fundamental uncertainty that existed at the time, the ex post approach effectively assumes away the possibility that the CPR could have been caught holding the bag had the state of nature not emerged as it did.

George himself recognized the limitations of the ex post approach ${ }^{3}$ : 'Of course, a complete answer to the question whether the CPR was excessively subsidized would require an examination of the ex ante situation governing the bargain made between the company and the government in the autumn of 1880. Such an examination, however, does not appear feasible on the basis of available information and data' (1968, 762).

Despite the widespread recognition of the limitations of ex post efficiency, some economists have attached normative significance to the concept. Mercer (1982, 93), for example, argued that it is clear that 'the Canadian Pacific System received (ex post) more subsidy than necessary on grounds of efficiency.' Commenting on a similar 'excess subsidy' for a U.S. transcontinental, he claimed that 'society would have been better served by a substantially smaller subsidy’ (Mercer 1982, 88). Indeed, Mercer also analysed U.S. transcontinentals for which he discovered subsidies that were necessary but inadequate ex post - this despite the fact that the construction of the lines proceeded in any event! While he did point out that this lack of ex post profitability 'does not necessarily reflect a mistake in policy ex ante' (Mercer 1982, 87), he none the less concludes that in these cases 'the subsidy policy ... was not optimal, because insufficient subsidy was provided’ (Mercer 1982, 85).

To our knowledge there has been no formal attempt to examine the role of the railway subsidies from an ex ante perspective. Recent approaches to investment analysis, which focus on the timing of irreversible investments in an uncertain environment, seem appropriate for analysing railway construction (see McDonald and Siegel 1985, 1986; Brennan and Schwartz 1985; Dixit 1991; Pindyck 1992; Dixit and Pindyck 1994). ${ }^{4}$ Railway lines are an example of a 'once-and-for-all'

[^3]irreversible investment under uncertainty. Building the line now precludes doing so in the future, and once built, the line has few (if any) alternatives uses. In a risky environment, building a railway line is much like exercising a financial option: an exercise price, equal to the capital cost of the line, is incurred to secure an asset with an uncertain future return. This interpretation of a railway line as an irreversible investment involving a 'real option' gives rise to some important insights. In particular, it suggests an additional opportunity cost of undertaking the investment associated with forgoing the option to build the line in the future. As such, there is a benefit to delaying construction of the line to see how the uncertainty unfolds. Ex post analysis of rates of return obviously does not take account of this additional opportunity cost.

In this paper we employ a real options approach to revisit the issue of government subsidization of the CPR transcontinental. While we agree that given the data limitations it is virtually impossible to conduct a true analysis of the CPR's ex ante investment decision, we can explore the implications of uncertainty for evaluating the role of the government subsidy. Our hope is to shed some new light on the efficiency of government intervention in infrastructure projects by emphasizing the important role that option values can play in the timing of once-and-for-all investment decisions in an uncertain environment.

We conclude that given the uncertainty over both the investment's future returns and the credibility of the government's policy stance at the time, the private option value associated with delaying the construction of the transcontinental may have been substantial. Thus, in the face of this uncertainty, the subsidy provided by the government may not have been excessive at all, but rather was compensation to the CPR for exercising its investment option early.

## II. A MODEL OF RAILWAY INVESTMENT UNDER UNCERTAINTY

We closely follow the approach described in Dixit and Pindyck (1994), who base their discussion on McDonald and Siegel (1986). We briefly summarize the essential aspects of the methodology and then show how it can be applied to analysing the 'excess subsidy' provided to the CPR.

In this section, in order to illustrate the methodology, we treat the grant of the CPR charter as conveying to its owners the monopoly right to build the line immediately or at any point in the future. The subsidy is then treated as an incentive to induce immediate construction. Historically, however, the CPR charter, the subsidy, and the commitment to begin immediate construction all were part of a single policy package. In the next section we demonstrate that the methodology easily can be generalized and reinterpreted to account for this.

[^4]Let $V(t)$ denote the expected present value (EPV) of the income from the mainline at time $t$,

$$
\begin{equation*}
V(t)=\int_{t}^{\infty} \pi(s) e^{-\rho(s-t)} d s \tag{1}
\end{equation*}
$$

where $\pi(t)$ is income in year $t$ and $\rho$ is a discount rate.
We assume that $V$ evolves stochastically over time, following geometric Brownian motion,

$$
\begin{equation*}
\frac{d V(t)}{V(t)}=\sigma d W(t) \tag{2}
\end{equation*}
$$

where $\sigma$ is the standard deviation in the expected growth rate in $V$ (the expected growth rate is assumed to be zero) and $d W(t)$ the increment in a standard Weiner process. ${ }^{5}$ Equation (2) can be thought of as the continuous time analog of a geometric random walk. ${ }^{6,7}$

Consider the owners of the CPR charter at time 0 (1881), deciding whether to build the mainline at that time or at some point in the future. Let $F(V(t))$ denote the value of the investment opportunity at any time $t$, expressed as a function of the EPV of the line if it was constructed at that time. Thus, at the current time 0 , the CPR charter can be viewed as an asset worth $F(V(0)$ ), which gives the charter holders the right, but not the obligation, to build a line worth $V(t)(t>0)$ at any time in the future. The current EPV of the line, $V(0)$, is known, but the EPV in the future, $V(t)$, is not. Moreover, the charter holders know the equation of motion that governs $V(t)$ (equation (2)). The charter holders 'exercise' the investment option by incurring capital costs of $I$.

The standard 'textbook' approach to investment project evaluation would prescribe building the line as soon as its EPV, $V(t)$, equalled or exceeded its capital cost, $I$. When the investment is irreversible, as is clearly the case with a transcontinental rail line, this approach does not lead to the optimal timing of investment, since it ignores the fact that the firm has other investment opportunities not captured in the standard opportunity cost used to discount the expected cash flows. Namely, an irreversible investment of this type can be thought of as 'competing with itself over time' - the firm must choose among the non-mutually exclusive alternatives of building now or at any time in the future. This requires the explicit

[^5]recognition of the opportunity cost of exercising the investment option. The firm should thus seek a decision rule that maximizes the current value of the investment opportunity, $F(V)$.

The value of the investment opportunity is maximized by determining a critical value for $V, V^{*}$, sometimes called the 'trigger value,' such that the investment is made as soon as $V$ equals, or exceeds, $V^{*}$. As McDonald and Siegel (1986) show, the investment option, $F(V)$, must satisfy the following partial differential equation: ${ }^{8}$

$$
\begin{equation*}
\frac{1}{2} \sigma^{2} V^{2} F^{\prime \prime}(V)-\rho F(V)=0 \tag{3}
\end{equation*}
$$

and three boundary conditions:

$$
\begin{align*}
& F(0)=0  \tag{4}\\
& V^{*}-F\left(V^{*}\right)=I  \tag{5}\\
& 1-F^{\prime}\left(V^{*}\right)=0 \tag{6}
\end{align*}
$$

Equation (4) follows from the fact that $V$ follows a geometric random walk, and therefore if $V$ reaches zero, it stays there - the option to invest then has no value. Equation (5) is the 'value matching' condition. The left-hand side, $V^{*}-F\left(V^{*}\right)$, represents the fact that when the firm undertakes the investment, it receives an asset worth $V^{*}$, the EPV of the completed line at that time, but it gives up an asset worth $F\left(V^{*}\right)$, the value of the option to build the line at some point in the future. This condition thus reflects the idea that there is an additional opportunity cost associated with making an irreversible investment - giving up other opportunity to invest in the future. At the trigger value $V^{*}$, the net gain from this asset exchange is just equal to the cost of the investment, $I$. Equation (6) is the 'smooth pasting' condition. It follows from the fact that $V^{*}$ is chosen optimally so as to maximize the value of the investment opportunity.

Using equation (4), the solution to the partial differential equation (3) is

$$
\begin{equation*}
F[V(0)]=A V(0)^{\beta_{1}} \tag{7}
\end{equation*}
$$

where $A$ is a constant to be determined below, and

$$
\begin{equation*}
\beta_{1}=\frac{1}{2}+\sqrt{\frac{1}{4}+2 \frac{\rho}{\sigma^{2}}}>1 \tag{8}
\end{equation*}
$$

[^6]is the positive root of the quadratic equation,
\[

$$
\begin{equation*}
\frac{1}{2} \sigma^{2} \beta(\beta-1)-\rho=0 \tag{9}
\end{equation*}
$$

\]

From the smooth pasting and value matching conditions, the solutions for $V^{*}$ and $A$ are

$$
\begin{align*}
& V^{*}=\frac{\beta_{1}}{\beta_{1}-1} I  \tag{10}\\
& A=\frac{V^{*}-I}{V^{* \beta_{1}}} \tag{11}
\end{align*}
$$

Equation (10) gives the optimal decision rule: invest as soon as $V$ equals $V^{*}$. Note how the decision rule differs from the more familiar expected net present value rule. In the absence of either risk or irreversibility, or ignoring the option value associated with the investment opportunity, the trigger value would be $V^{*}=I$ : invest as soon as the EPV of the income from the completed mainline equals its cost. The presence of the term $\beta_{1} /\left(\beta_{1}-1\right)$ in equation (10), which is greater than one, means that the holder of the railway charter optimally should delay construction until the EPV of the completed line exceeds its cost by a discrete amount before undertaking the investment. This additional amount is required to cover the opportunity cost of exercising the investment option.

To see how the investment option affects the interpretation of the notion of an 'excess subsidy' for the CPR, consider how a subsidy alters the decision rule. If, in the absence of the subsidy, the current (1881) EPV of the rail line is less than the trigger value - that is, $V(0)<V^{*}$ - then a private entity would not be interested in building the line at this time. To entice private interests to build the line in 1881, the government must offer a subsidy. ${ }^{9}$ Depending upon the way the subsidy is structured, it could increase the EPV of the project by augmenting the income stream, and/or it could lower the trigger value, for example, by reducing the capital cost. ${ }^{10}$ The subsidy package offered to the CPR contained elements of both: transferring previously laid track to the CPR increasing their revenue stream, while the cash subsidies and land grants can be viewed as lowering their capital costs. ${ }^{11}$

[^7]Let $R(0)$ denote the current EPV of the income augmenting element of the government's subsidy package, $C(0)$ the present value of the cash grants, and $L(0)$ the present value of the land grants. Private interests would then be willing to construct the mainline currently if $V(0)+R(0) \geqq V^{*}-C(0)-L(0)$, where $V^{*}$ is given above in equation (10). The subsidy is excessive if the relationship holds with strict inequality, in which case the government paid more than necessary to entice the private sector to undertake the project. In this case, the size of the ex ante excess subsidy is

$$
\begin{equation*}
\mathrm{ES}(0)=V(0)+R(0)+C(0)+L(0)-V^{*} \tag{12}
\end{equation*}
$$

To determine whether or not the CPR subsidy was excessive, we need to know the ex ante values of the terms on the right-hand size of equation (12) in 1881; as George (1968) points out, this is difficult. To our knowledge, 'hard' numbers reflecting the expectations of the investors at the time do not exist. Another important parameter for which we have no information is the variance of the EPV, $\sigma^{2}$, which helps to determine $V^{*}$.

Our approach is to follow George (1968) and Mercer (1982) and treat the subsequent realization of the 1881 present value of the project as the ex ante expected present value. This presumes that the CPR's expectations regarding the value of the completed project were exactly fulfilled. We then determine the level of uncertainty, as measured by the standard deviation, which implies an ex ante excess subsidy of zero in this case - that is, the standard deviation which suggests that the level of the subsidy was just sufficient to compensate the CPR for forgoing their option to build the mainline at the privately optimal time in the future.

The calculations are presented in table 1. The data are from Mercer (1982). To compute the 1881 present values, we use Mercer's calculated discount rate of 6.75 per cent, which reflects the real average rate of return on the U.S. stock exchange around that time. It is interesting to note that this discount rate therefore contains a systematic market risk premium. Thus, although Mercer conducts ex post analysis based upon profit realizations, he discounts them by a risk-adjusted discount rate but ignores the opportunity cost associated with exercising the option to delay. Like Mercer, we compute the present value over a ten-year period, with a 'terminal adjustment' in 1900 to account for the value of the company at that time. ${ }^{12}$

If the opportunity cost of exercising the investment option is ignored by setting $\sigma=0$, which is tantamount to following the standard ex post approach, then the 'excess subsidy' is about $\$ 19$ million. ${ }^{13}$ If, on the other hand, $\sigma=0.044$, the ex ante excess subsidy is zero. This is a very low level of uncertainty indeed, as the long-run standard deviation on the 'market portfolio' of stocks traded on North

[^8]TABLE 1
1881 expected present values (thousands 1900 \$)

| Income $^{a} V(0)+R(0)$ | 113,318 |
| :--- | ---: |
| Subsidies |  |
| $\quad$ Cash $C(0)$ | 21,114 |
| Land $L(0)$ | 33,730 |
| $\quad$ Total | 54,844 |
| Capital cost $I$ | 149,213 |
| Trigger value $V^{*}$ |  |
| $\sigma=0$ | 149,213 |
| $\sigma=0.044$ | 168,163 |
| Excess subsidy ES(0) |  |
| $\sigma=0$ | 18,948 |
| $\sigma=0.044$ | 0 |

NOTES
Assumes a discount rate of 6.75 per cent. The raw data are taken from Mercer (1982).
$a$ Includes income from the rail lines built by the government and transferred to the CPR.

American stock exchanges is about 0.20 . As such, if we presume that the expected returns on the mainline coincided with the returns ultimately realized, only a very small level of uncertainty is required to eliminate the so-called excess subsidy.

It seems to us that it is unlikely that the perceived volatility of the mainline was this low. If the standard deviation was higher, then given the level of subsidies, the EPV of the income from the line would have to have been higher for the CPR to agree to build the line in 1881 . For example, if $\sigma=0.20$, the 1881 EPV of the line would have to have been at least $\$ 200$ million for the CPR to agree to build the line at that time (this includes the income from the government tracks transferred to the CPR). Compare this figure with the 1881 realized present value of $\$ 113.3$ million, and the present value of the construction expenditures, $\$ 149.2$ million.

Another way to look at the issue is to note that for $\sigma=0.044$, which, recall, is an extremely low level of volatility, the trigger value is about $\$ 168$ million, which exceeds the capital cost of the line by almost 13 per cent. At a higher level of uncertainty, for example, $\sigma=0.20$, which is still conservative, the trigger value is about $\$ 255$ million, 71 per cent higher than the cost of the investment. This suggests that even at very low levels of uncertainty the option value associated with the ability to delay the investment is significant; at higher, but still modest, levels of uncertainty the option value is huge! The government would have to compensate the holders of the CPR charter for forgoing this value.

It is possible to calculate all of the combinations of volatility and EPV that yield a zero excess subsidy. The locus of these zero excess subsidy points is illustrated in the top curve of figure 1 (ignore the other two curves for now). Points that lie
on the curve imply a zero excess subsidy; points above the curve involve an excess subsidy; while at points below the curve the CPR would not have been willing to undertake the investment in 1881. For example, if, as discussed above, $\sigma=0.20$ and the EPV of the line was about $\$ 200$ million, then the level of subsidy granted to the CPR was just enough to convince the CPR to build the line when it did and was not at all excessive. If, on the other hand, at that same standard deviation the EPV of the line was $\$ 250$ million, then the government would have granted an ex ante excess subsidy of $\$ 50$ million - that is, paid a subsidy of $\$ 50$ million more than it had to in order to entice the CPR to build the line by 1885. If this was the case, then one could conclude that the subsidy was, to borrow Mercer's words, 'more subsidy than necessary on grounds of efficiency' (1982, 93), and that 'society would have been better served by a substantially smaller subsidy' (ibid., 87).

## III. A GENERALIZATION AND REINTERPRETATION ${ }^{14}$

As mentioned above, it may not appear to be appropriate to model the government of Canada as having first given the CPR the legal option to develop the line and then determining the level of subsidy required to encourage an early exercise of this option. Rather, it may be more appropriate to view the government as having offered the CPR a 'take it or leave it' package which involved commencing construction 'now or never.' With such a 'now or never' offer there would be no private option value, since the CPR syndicate would not have the ability to delay the project. The opportunity cost of exercising the investment option would then be zero; the trigger value would be $V^{*}=I$, and the analysis of the 'excess subsidy' undertaken in Section is would not be appropriate.

The existence of a private option value hinges on the ability of the CPR to delay construction, and therefore on the credibility of a 'now or never' subsidy package offered by the government. If such an offer was completely credible, by declining the offer, the CPR owners would lose the opportunity to invest in the future. In this case, as long as the EPV exceeded the cost of the project (i.e., $V>I$ ), even by an infinitesimal amount, the CPR would accept the offer. On the other hand, if a 'now or never' offer was not at all credible, then the CPR syndicate had the option of declining the policy package and returning to the government at a later date with an offer to construct the transcontinental, either when conditions were more favourable or when the subsidy package was enriched. In this case the CPR syndicate had a de facto option to delay construction.

Of course, the issue of credibility is not as clear cut as this. It may be more appropriate to view the CPR as having assessed the probability that it would lose the investment opportunity if it refused to accept a package that involved immediate construction. We model this idea by representing the EPV of the project to the

[^9]$C P R$ as following a mixed Brownian/Poisson (jump) process. Thus, equation (1) is replaced with
\[

$$
\begin{equation*}
d V(t)=\sigma V(t) d W(t)-V(t) d q(t) \tag{13}
\end{equation*}
$$

\]

where $d q(t)=1$ is the increment of a Poisson process with a (constant) intensity parameter, or mean arrival rate, $\lambda .{ }^{15}$

The idea behind equation (13) is that $V$ evolves continuously and stochastically over time, but at each instant in time there is a probability $\lambda d t$ that an 'event' occurs which will cause $V$ to 'jump' by a discrete amount, $-V(t)$; in which case the EPV of the unbuilt line to the CPR collapses to zero, and stays there. The 'event' in this case is the elimination of the CPR syndicate from further consideration for the transcontinental project. This could occur, for example, if the government decided to build the rail line on its own or an agreement was struck with a competing group to build the line. The Poisson intensity parameter reflects the likelihood of either of these events occurring, and thus it can be thought of as representing the credibility of the 'now or never' aspect of the subsidy package. ${ }^{16}$

The Poisson intensity parameter can be used to compute the 'probability of ruin' over a specified period. For example, for a given $\lambda$ the probability that the CPR would for some reason lose the investment opportunity over the following year is Prob $(1)=1-e^{-\lambda}$. The case where $\operatorname{Prob}(1)=0(\lambda=0)$ represents a situation where a 'now or never' offer had no credibility - as would be the case if government construction was not a viable alternative nor did there exist a competing group that was both interested in and capable of constructing the line. In this case, the CPR owners would rationally decline a 'now or never' package that did not compensate them for forgoing the option to build the line at a later time, even though the EPV of the project exceeded its cost. This, of course, was the situation analysed in section II. The case where $\operatorname{Prob}(1)=1(\lambda=\infty)$ represents a situation where a 'take it or leave it' offer was credible in the extreme, either because the government had the ability to 'go it alone' or because a viable competitor was waiting in the wings. In either case, the value of the investment opportunity to the CPR would collapse to zero with certainty unless they accepted the 'now or never' terms. In this case, there would be no ability to delay and therefore no option value. Although they did not put it in these terms, this case is essentially that analysed by both George and Mercer.

In all likelihood, neither of these limiting cases provides an accurate description of the actual state of affairs in 1881. A viable competing interest to the CPR syndicate could have emerged, but such an emergence was far from certain. Since the early 1870s numerous syndicates had offered to build a transcontinental railway. Most of these syndicates, however, ultimately withdrew, were declared unsuitable

15 It is assumed that $d q$ and $d W$ are independent.
16 Dixit and Pindyck $(1994,167)$ suggest using a similar approach as a 'reduced form' way to model a situation in which a company faces the potential of increased competition from new entrants or owing to a technological innovation from existing rivals.
by the government, or, as was the case of the Sir Hugh Allan syndicate, collapsed owing to a political scandal. The owners of the Grand Trunk Railway twice rejected the government's offer of the transcontinental charter on the basis of the government's demands for an all-Canadian route. The Liberal Mackenzie government actually started construction in the 1870s. After total government expenditure on railway construction from 1871 to 1879 amounted to $\$ 14.3$ million, general agreement emerged that a transcontinental railway could be constructed more efficiently and more cheaply by a private company. ${ }^{17}$

Thus, neither the emergence of a viable competing interest in the immediate future nor the government's 'going it alone' could be considered a certainty if the syndicate refused a package involving immediate construction. As such, some value of $0 \leqq \operatorname{Prob}(1) \leqq 1(0 \leqq \lambda \leqq \infty)$ was most likely the case. In any event, we can incorporate the impact that a positive probability of ruin had on the investment decision in general and on the value of an 'excess subsidy' granted to the CPR in particular.

When the EPV of the project is governed by the mixed Brownian/jump process described in equation (13), the partial differential equation (3) must be modified by replacing $\rho$ with $(\rho+\lambda) .{ }^{18}$ The same substitution takes place in the determination of $\beta_{1}$ (equation (8)), which in turn enters the equation that determines the trigger value $V^{*}$ (equation (10)). As such, introducing a positive $\lambda$ has an impact similar to that of an increase in the discount rate.

On top of the usual trade-off associated with an irreversible investment decision under uncertainty, a positive probability of ruin introduces an additional trade-off arising from the possibility that the investment opportunity may disappear altogether if the CPR delayed accepting the package. It is important to note that this does not mean that the option to delay disappears completely, but rather that it becomes less valuable. To see this, note from inspection of equations (8) and (10) that as $\lambda$ (or $\rho$ ) increases, $\beta_{1}$ increases and $V^{*}$ decreases. Thus, as the probability that the CPR may lose the investment opportunity in the future increases (i.e., the government's 'now or never' offer becomes more credible), the trigger value that determines the optimal timing of the investment declines. Only in the limit, when the likelihood that the CPR will lose the project unless it accepts the offer approaches certainty, does the trigger value equal the cost of the investment $\left(V^{*}=I\right)$, and the standard 'textbook' decision rule becomes optimal.

The impact of the credibility of the government's 'now or never' offer is illustrated in figures 1 and 2 . Figure 1 shows that the zero excess subsidy frontier shifts down as the probability of ruin increases. Figure 2 shows all of the combinations of Prob (1) and $\sigma$ that give rise to a zero excess subsidy if we again treat the realized present value of the project ( $\$ 113.3$ million) as the EPV. In both figures we see that, as discussed in section II, for $\operatorname{Prob}(1)=0(\lambda=0)$ a standard deviation

[^10]
$-\operatorname{Prob}(1)=0 \quad+\operatorname{Prob}(1)=9.5 \% \quad * \quad \operatorname{Prob}(1)=75.0 \%$

FIGURE 1 Zero excess subsidy surface EPV and $\sigma$
of 0.044 is required to eliminate the excess subsidy; for Prob $(1)=9.5$ per cent $(\lambda=0.10)$ the required standard deviation is about 0.067 ; while for Prob $(1)=$ 75 per cent $(\lambda=1.39)$ the required standard deviation is about 0.20 . Even if there is a 75 per cent probability of losing the investment opportunity over the next year, at $\sigma=0.20$ the amount of risk required to eliminate the excess subsidy is still relatively modest by historical standards. Moreover, the trigger value when Prob $(1)=75$ per cent and $\sigma=0.20$ is about 13 per cent higher than the capital cost of the project. Thus, even in the face of a very credible 'now or never' threat, the private benefit to waiting and the associated option value are still quite high. Even in this case the CPR would have to be compensated for constructing the line early, which could easily wipe out the 'excess subsidy.' More generally, as long as Prob (1) was less than $1(\lambda<\infty)$, which means that there was some positive probability that they would not lose the investment opportunity if they declined it currently, there was a benefit to the CPR of delaying, and the associated option value was positive. To entice the CPR into accepting the 'now or never' terms of the subsidy package, the CPR would have to be compensated for forgoing this option.


FIGURE 2 Zero excess subsidy surface Prob (1) and $\sigma$
IV. SUMMARYAND CONCLUSIONS

In this paper we revisit the issue of the subsidy granted to the CPR for its construction of the first Canadian transcontinental rail line over the period from 1881 to 1885 . Previous studies have investigated the issue from an ex post perspective and concluded that the government subsidy was excessive. The impression left by these studies is that the Canadian government's intervention was inefficient in the sense that the mainline could have been built with a substantially lower subsidy.

In this paper we question this widely accepted interpretation. We argue that the concept of ex post efficiency is a largely vacuous one, since it inevitably condemns the government to a judgment of inefficiency. If we are interested in evaluating the government subsidy from a normative perspective, we must adopt an ex ante notion of efficiency, which takes full account of the presence of uncertainty, and its interaction with the irreversible nature of the project.

The main point of the paper is a simple one. Given the uncertainty over the future EPV of the transcontinental line, and given the uncertainty regarding the credibility of a 'now or never' subsidy package offered by the government, the CPR had a de facto option to delay construction of the mainline. This gave rise to an additional opportunity cost associated with forgoing the option to build
the line in the future when more would be known about the potential returns. The CPR would have to be compensated for forgoing this opportunity and exercising its investment option early. Previous studies have not incorporated this fact in their calculations of the 'excess subsidy.'

While we are not able to undertake a true ex ante analysis of the investment decision, we show that the implications of this de facto option to delay are significant. Even if the probability that the CPR syndicate would lose the investment opportunity by delaying was quite high, only a relatively modest amount of uncertainty regarding the future value of the project was required to give rise to a fairly sizeable option value. As such, we think that it is quite likely that the portion of the subsidy granted to the CPR that traditionally has been viewed as excessive may well have been required to compensate the company for forgoing its investment option.

We think that the approach illustrated here is potentially quite important for evaluating government support of infrastructure projects and assessing their impact on economic growth and development. Although we examine the CPR subsidy in this paper, the approach can be used to examine government involvement in any infrastructure project. Based on our analysis, we would suggest that any attention devoted to examining the ex post efficiency of government involvement in such projects is largely misplaced. Of no small consequence, the danger of this approach is that policies based on ex post evaluations of project profitability may well obscure more relevant concerns, such as the design of mechanisms that ensure efficient subsidies ex ante.

A case in points is George's conclusion that his ex post analysis 'calls attention to the government's failure to attach some provision for repayment, either partial or complete, of the principle of the subsidy once the profitability of the investment became apparent' (1968, 761). The methodology employed here emphasizes that, ex ante, such repayment provisions are of the nature of a contingent claim, like an option, and should be valued as such. Contingent repayment provisions would have lowered the ex ante value of the subsidy, possibly below the level required to compensate the company for forgoing its investment option. Moreover, the riskier the income stream the lower the value of the contingent subsidy to the firm. The up-front portion of the subsidy would have to be increased to compensate for the possibility that it may have to be paid back if the company is profitable. ${ }^{19}$ This sort of trade-off becomes clear only when one follows an explicitly ex ante approach that fully captures the important interactions between risk, irreversibility, and the ability to delay the investment.

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19 Although we do not do so here, it is fairly straightforward to model contingent repayment provision of this type.

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[^2]:    1 Government involvement in the CPR transcontinental was analysed in a series of papers in this Journal written over twenty years ago, by George $(1968,1975)$ and Mercer $(1973)$. Mercer incorporated the results of his 1973 paper in a book (1982), which examines government subsidies of numerous North American transcontinentals.

[^3]:    2 Or, somewhat paradoxically, that the subsidy was necessary but inadequate! See Mercer (1982) and the discussion below.
    3 For a more recent statement of the same point see Attack and Passell (1994, 441).
    4 Previous studies of the timing of railroad construction, such as Lewis and Robinson (1984) or Harley (1982), emphasize the importance of property rights in the branch lines to be built in the

[^4]:    future. They also focus on expectations of a growing revenue stream arising from future farm settlement (the argument is presented in a complete exposition by Harley and in more succinct mathematical form in Lewis and Robinson). These studies do not recognize the implications of the interaction between risk and irreversibility for the timing of railroad construction. This is a considerable weakness, considering the historical uncertainty about western settlement.

[^5]:    5 Lacking evidence to the contrary, we assume that the expected growth rate in $V(t)$ is zero. It is straightforward to modify the analysis to allow for geometric Brownian motion with drift.
    6 See Malliaris and Brock (1982) for a simple explanation of stochastic processes in an economic context. It is easy to show that if income from the rail line follows geometric Brownian motion, then so too does the EPV of that income stream.
    7 For simplicity we assume that $V$ is uncertain but that the capital costs of the project and the value of the subsidies are not. In reality, both the capital costs the value of the subsidies were stochastic - the latter because the subsidy consisted of land grants, the revenue from which was unknown, and the income from rail lines built by the government and transferred to the CPR. Although it is possible to allow the uncertainty associated with all of these things to be included in the model, doing so substantially complicates the analysis without changing our fundamental conclusions.

[^6]:    8 See also Pindyck (1991), Dixit (1992), and Dixit and Pindyck (1994). Intuitively, when it is optimal not to invest, capital market equilibrium implies that the expected return on the investment opportunity equals the expect capital gain, or $\rho F(V) d t=E(d f(V))$. Utilizing Ito's Lemma and the fact that $V$ follows a geometric random walk, this may be expanded to give equation (3). See Malliaris and Brock (1982) or Dixit and Pindyck (1994) for an explanation of Ito's Lemma in an economic context.

[^7]:    9 We do not address the question of whether or not the government was justified in wanting the project completed by 1885 . While this is obviously an extremely important aspect of any normative analysis of the transcontinental, our goal is more modest in this paper: given that the government wanted the line constructed at that time, did it pay too much, as the existing literature suggests?
    10 The trigger value could also be lowered by reducing the amount of risk faced by private investors, which would lower the value of the option term. Although the subsidy offered to the CPR did not contain this particular element, other infrastructure subsidies have done so. Examples of this approach include loan guarantees, and provisions whereby the government agrees to cover some portion of potential losses.
    11 As should become evident below, it is somewhat arbitrary whether a particular component of the subsidy package should be viewed as lowering capital costs or increasing expected revenues. The important thing is that it be depicted as one or the other, but not both; that is, it is important to avoid 'double counting.'

[^8]:    12 We use Mercer's terminal adjustment C, which is his 'preferred' estimate of the value of the CPR at that time, calculated using a stock adjustment method.
    13 This differs slightly from Mercer's (1982) measure of the ex post subsidy of $\$ 19.8$ million for reasons that we have not been able to determine.

[^9]:    14 We would like to thank an anonymous referee for crystalizing our thoughts on the issues addressed in this section.

[^10]:    17 Glazebrook (1964). This was the conclusion of Sandford Fleming, the engineer-in-chief for the government construction project.
    18 See Dixit and Pindyck (1994, chaps 3 and 4) for a discussion of how to modify Ito's Lemma for a mixed Brownian/jump process

