Imperfect Competition, Nominal Wage Rigidity and the Business Cycle*

Zuzana Janko†

University of Calgary

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Abstract

We introduce nominal wage contracts into a competitive and a noncompetitive labor market structure. We find these models to have similar business cycle properties, but we argue that the imperfectly competitive market structure is more appropriate for nominal wage contract analyses. We introduce imperfect competition to the labor market by assuming that households have market power and consequently choose nominal wage contracts as part of their maximization problem, while in a competitive structure nominal wage contracts are introduced via an exogenous rule. The latter formulation lacks microeconomic foundation that makes the model inappropriate for the analysis of nominal wage contracts.

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† Department of Economics, University of Calgary, 2500 University Drive, N.W., Calgary, Alberta, Canada T2N 1N4, Phone: (403) 220-6101, Fax (403) 282-5262, E-mail: zjanko@ucalgary.ca.

1. Introduction

In their 1989 paper, Cooley and Hansen (1989) introduce money into a standard RBC formulation to investigate the business cycle properties of nominal variables. They find that their model poorly matches the business cycle properties of nominal variables due to the lack of a transmission mechanism for monetary shocks.[1] In the subsequent literature, nominal rigidities are seen as one way of introducing a propagation mechanism for money supply shocks. For example, following the work of Gray (1976) and Fisher (1977), Cho (1993), Cho and Cooley (1995), Cooley and Hansen (1995), Cho, Cooley, and Phaneuf (1997) incorporate nominal wage contracts into Hansen's indivisible-labor economy. Although, monetary shocks are propagated in the economy, the approach through which they are is seen by many as add-hoc since nominal wage contracts are set exogenously. Moreover, nominal wages are not chosen as part of an agents' optimization problem, which is a clear lack of microeconomic foundation within these models. In this chapter, we build on Cho and Cooley's (1995) model by introducing imperfect competition whereby households choose nominal wages in advance as part of their maximization problem. This allows us to construct a richer and more realistic story for nominal wage setting. We find that despite the add-hoc nominal wage setting structure, the quantitative results of Cho and Cooley (1995) are in line with our imperfectly competitive model.

As in Erceg (1997), Huang, Liu, and Phaneuf (2000), Christiano, Eichenbaum and Evans (2001), we introduce imperfect competition into the Cho-Cooley model. Specifically, we assume that households have market power in the labor market, and

[1] Moreover, they find that on the real side their model behaves as a standard Real Business Cycle model, such as the model of Kydland and Prescott (1982), Long and Plosser (1983) and Hansen (1985).

consequently households choose nominal wages as part of their decision-making. This implies that in the labor market, each household is a monopolistic supplier of a differentiated labor input[2]. Subsequently, an intermediary, such as an employment agency, uses Dixit-Stiglitz technology to transform individual labor services into an aggregate labor input that is sold to the final-goods producers. The final-goods firm produces output in a perfectly competitive environment.

In addition, we introduce nominal wage rigidity into the model by assuming that households choose nominal wages in advance with a markup over the wage prevailing under perfect competition. Hence, at the beginning of each period, households choose nominal wage for a time that is several periods ahead in the future, which is equal to a constant markup over the expected value of marginal rate of substitution between consumption and leisure. In contrast to Cho and Cooley (1995) where nominal wages are chosen based on a rule, with market power households choose nominal wages in advance as part of their maximization problem.

We compare the results of a Basic model, a formulation analogous to the Cho-Cooley model, with the Imperfectly Competitive (IC) model. As in Cho and Cooley (1995), we find that the volatilities of real variables are unrealistically high in both the Basic and the IC models with nominal wage rigidity, as compared to what we observe in the U.S. data. In the Basic model, households cede to the firm the right to determine labor hours once the wage is adopted. This allows the firm to adjust to unexpected technology and money supply shocks. In the IC model, the intermediary demands labor taking nominal wage as given. Hence, the intermediary is able to adjust to technology

[2] Griffin (1996) provides microeconomic evidence of this feature, reporting that the estimated elasticity of substitution between labor skills is greater than one in U.S. firms.

and money supply shocks as well. In both models, the adjustments to unexpected shocks lead to high volatility in hours, which in turn transfers to high fluctuations in output and productivity. The differences in volatilities across models are only marginally different.

Furthermore, we find that labor productivity is countercyclical in both models. With nominal wage rigidity, both models exhibit a propagation mechanism that raises labor hours much more than output. A positive monetary shock causes an increase in the price level, which in turn decreases the marginal product of labor, thus increasing first labor and then output. Again, both models generate almost identical results. In sum, we find that the results on the real side of the economy are very similar between the Basic model and the Imperfectly Competitive model with nominal wage rigidity.

On the nominal side, the Basic model and the IC model without nominal wage rigidity both generate weak correlations between output and nominal variables. Nominal wage rigidity, acting as a propagation mechanism for monetary shocks, improves upon these results in both labor structures. Specifically, the correlations between output and inflation and nominal interest rate more closely mimic the U.S. data. Furthermore, a model without nominal wage rigidity, irrespective of the labor market structure, best matches the standard deviation of nominal interest rate. Finally, none of the models considered here are able to mimic the low volatility of inflation found in the data.

The remainder of the paper is organized as follows. In section 2, we set up the Basic model and define the recursive competitive equilibrium in this economy. In section 3, we build the Imperfectly Competitive model and describe the recursive competitive equilibrium when households have market power. Section 4 gives a brief discussion of the solution method used to solve the dynamic programming problem. The model is

calibrated in section 5. We present our simulation results, which include standard deviations and cross-correlations for the Basic model and the Imperfectly Competitive model in section 6. Section 7 concludes the chapter.

2. Basic Model

The economy consists of firms, households, and the government. Firms produce output using a Cobb-Douglas production function subject to a technology shock. In this economy, households and firms enter into a contract in advance by agreeing to a nominal wage where expected marginal rate of substitution between consumption and leisure is set equal to the expected value of marginal product of labor. Once nominal wages are set households cede to the firm the right to determine labor hours in the future. Hence, at the time the nominal wage is adopted, firms choose labor hours by setting real wage equal to marginal product of labor, taking nominal wage as given. In section 3, we give a detail analysis of how actual labor hours are determined.

Households are composed of identical infinitely-lived agents who gain utility from leisure and two types of consumption goods, a credit good and a cash good. The cash good is a result of introducing money into the economy via a cash-in-advance (CIA) constraint. Finally, the government injects money into the economy through a lump-sum transfer to households.

2.1 Firms

The production function representing the technology in this economy is a standard Cobb-Douglas production formulation:

(1) $$Y_t = e^{z_t} K_t^{\theta} H_t^{1-\theta}, \qquad 0 < \theta < 1,$$

where Y, K, and H are aggregate output, capital stock and labor hours, respectively.[3] The parameter θ represents capital's share in total income. The production function exhibits constant returns-to-scale, thus it can be assumed, without loss of generality, that there is only one competitive firm. This production function is subject to a technology shock z_t, such that changes in z_t cause shifts in the production function, altering factor productivity. As in the real business cycle literature, we let z_t evolve according to the following law of motion:

(2) $$z_{t+1} = \gamma z_t + \varepsilon_{t+1}, \qquad 0<\gamma<1,$$

where ε is normally distributed with mean zero and a standard deviation of σ_ε.

The profits of the firm are given by $Y_t - \frac{W_t^c}{P_t} H_t - r_t K_t$, where W_t^c is the nominal contract wage, P_t is the price level, $\frac{W_t^c}{P_t}$ is the real wage rate, and r_t is the real rental rate. The objective of the firm is to maximize profits subject to (1). The first-order conditions of the firm's maximization problem yield the rental rate and the wage rate as follows:

(3) $$r_t = \theta e^{z_t} \left(\frac{H_t}{K_t} \right)^{1-\theta},$$

(4) $$\frac{W_t^c}{P_t} = (1-\theta) e^{z_t} \left(\frac{K_t}{H_t} \right)^{\theta}.$$

Notice that the firm takes nominal wage W_t^c as given.

2.2 Households

[3] The convention in this chapter is to use capital letters to represent aggregate variables, while lower-case letters are used to denote the variables of the household.

The economy consists of a unit measure of identical infinitely-lived agents, who maximize the following utility function:

(5) $$U = E_t \sum_{t=0}^{\infty} \beta^t [\alpha \log c_{1t} + (1-\alpha) \log c_{2t} - \psi h_t],$$

where $\beta \in (0,1)$, $\alpha \in (0,1)$, and $\psi > 0$. β is the discount factor, α determines the relative importance of consumption of the cash good, c_{1t}, to the consumption of the credit good, c_{2t}. Finally, ψ is the preference parameter on hours worked h_t. Households are endowed with one unit of time, which they can allocate towards leisure or labor.

In this economy, households purchase two types of consumption goods. One is a credit good acquired through capital and labor income. The other is a cash good that can only be attained through cash. Consequently, the representative household is required to hold money in the beginning of period t in order to purchase the cash good, c_{1t}. The currency holdings of the representative household is given by

(6) $$m_t + T_t,$$

which can be used to purchase cash goods. Households obtain currency through last-period's money holdings, m_t, and a lump-sum government money transfer, T_t.[4] The resulting cash-in-advance constraint is given by[5]

(7) $$P_t c_{1t} \leq m_t + T_t.$$

In addition to the CIA constraint, the representative household must satisfy the budget constraint. The total real income that a household has available to carry out transactions is income earned from labor and capital, and real currency holdings as

[4] $T_t = M_{t+1} - M_t$, where M_t is the money supply in period t.

[5] Stockman (1981) considers a CIA constraint on investment as well.

described above. This income is used to invest, and acquire purchases of the two consumption goods as well as real money balances that are carried over to next period for future transactions. The budget constraint for the households is

$$c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{P_t} \leq \frac{W_t^c}{P_t} h_t + r_t k_t + \frac{m_t}{P_t} + \frac{T_t}{P_t}, \tag{8}$$

where i_t is investment.

Output produced in this economy is either consumed or invested in physical capital. Capital evolves according to the following law of motion

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{9}$$

where $\delta \in (0,1)$ represents the capital depreciation rate.

The first-order conditions of the household's problem are given by

$$\frac{1}{c_{2t}} = \beta \underset{t}{E}\left[\frac{1}{c_{2t+1}}\left(\theta e^{z_{t+1}} K_t^{\theta-1} H_t^{1-\theta} + 1 - \delta\right)\right], \tag{10}$$

$$\frac{1}{P_t c_{2t}} = \left(\frac{\alpha}{1-\alpha}\right)\beta \underset{t}{E}\left(\frac{1}{P_{t+1} c_{2t+1}}\right), \tag{11}$$

$$\frac{W_t^c}{P_t} = \psi c_{2t}, \tag{12}$$

where (10) is analogous to the standard Euler equation for intertemporal consumption choices and (11) is the equation determining money demand.[6] Equation (12) is an intra-temporal condition that equates the real wage to the household's marginal rate of substitution between consumption of the credit good and leisure.

2.3 Government

[6] In a model with money the consumption choices (in equation (10)) are not total consumption but instead the consumption of the credit good only.

By introducing money into the model, the government takes up the role of printing money and injecting it into the economy. The government budget constraint is

(13) $$G_t + \frac{T_t}{P_t} = \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t},$$

where G_t is government expenditure, and M_t is aggregate money supply. Since in this model, the impact of changes in government expenditure is not of interest, the assumption is made that G_t is constant over time. Without loss of generality, the assumption is extended to G_t being equal to zero in every time period. As a result, money enters the economy by directly financing the lump-sum transfer.

The government issues money according to the following rule:

(14) $$M_{t+1} = e^{\mu_t} M_t,$$

where the money growth rate is $e^{\mu_t} - 1$ in period t. Hence, μ_t is a random variable governing the growth rate of money supply, and is assumed to evolve according to the autoregressive process

(15) $$\mu_{t+1} = \eta\mu_t + \xi_{t+1},$$

where η falls in (0,1). To ensure that the cash-in-advance constraint (7) is binding, μ_t must be positive in every period.[7] This can be assured by assuming that ξ is log-normally distributed with mean $(1-\eta)\bar{\mu}$ and standard deviation σ_ξ, where $\bar{\mu}$ represents the average growth rate of money.

2.4 Nominal Wage Contracts

[7] If μ_t were negative, this would imply that the government transfer is negative, and equation (7) may not be binding in some periods.

In this section, we set up a nominal wage contract and use it to derive labor hours. This contract follows Cho and Cooley (1995) whereby at the beginning of *each* period, agents agree to a contract for a time that is j periods ahead in the future.[8] In the case of j=2, households and firms set up a contract this period for two quarters from now. Thus, at time t-1, households and firms agree to a nominal wage for the period t+1; at t, they agree to a wage for period t+2; at t+1, they agree to a wage for t+3, and this process repeats over time. A graphical representation of j=2 is given in Figure 1 below.

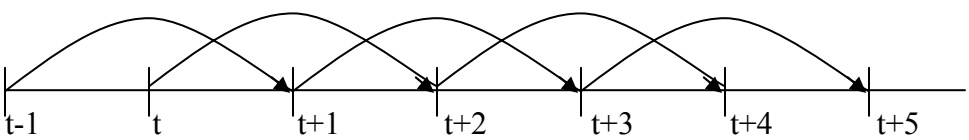

FIGURE 1
A two-period Nominal Wage Contract

Households and firms enter into a contract in advance by agreeing to a nominal wage where the expected marginal rate of substitution between consumption of credit good and leisure equals to the expected value of marginal product of labor. In this way, households cede to the firm the right to determine labor hours. At the time the nominal wage is adopted, firms choose hours by setting real wage equal to marginal product of labor, taking the nominal wage as given. As a result, households supply the amount of labor demanded by the firm.

The wage rate agreed upon at time t-j is W_t^c and is a function of $\Omega_{t-j} = (z_{t-j}, \mu_{t-j}, K_{t-j+1})$. After the money growth shock and the technology shock are revealed at time t, households choose their optimal level of consumption, investment and money balances as a function of $z_t, \mu_t, k_t, K_t, m_t$ and W_t^c.

[8] Notice that when j=0, the Basic model reduces to a Cooley and Hansen (1989) type of economy. We will consider this case when presenting our results.

In the contracting period t-j, the equilibrium nominal wage for time t is set equal to the expected value of the marginal product of labor, given the information set Ω_{t-j}. Hence, conditioning the firm's first-order condition with respect to labor hours, on the information set Ω_{t-j}, leads to the following general formulation for the nominal contract wage

$$W_t^c = E\left(e^{z_t} (1-\theta) \left(\frac{K_t}{H_t} \right)^{\theta} P_t \middle| \Omega_{t-j} \right). \tag{16}$$

To obtain the decision rule for actual labor hours, we take logs of equation (16)

$$\begin{aligned} \log W_t^c = {} & \log(1-\theta) + \theta E\left(\log K_t \middle| \Omega_{t-j}\right) - \theta E\left(\log H_t \middle| \Omega_{t-j}\right) + E\left(z_t \middle| \Omega_{t-j}\right) \\ & + E\left(\log P_t \middle| \Omega_{t-j}\right). \end{aligned} \tag{17}$$

At t, the firm taking the nominal wage as given, sets real wage equal to the marginal product of labor. Hence, we take logs of the firm's first-order condition with respect to labor hours, i.e. (4), and obtain

$$\log W_t^c = \log(1-\theta) + \theta(\log K_t) - \theta \log H_t + z_t + \log P_t. \tag{18}$$

Setting (17) equal to(18), we obtain the expression for $\log H_t$

$$\begin{aligned} \log H_t = {} & E\left(\log H_t \middle| \Omega_{t-j}\right) + \left[\log K_t - E\left(\log K_t \middle| \Omega_{t-j}\right)\right] \\ & + \frac{1}{\theta}\left[z_t - E\left(z_t \middle| \Omega_{t-j}\right)\right] + \frac{1}{\theta}\left[\log P_t - E\left(\log P_t \middle| \Omega_{t-j}\right)\right], \end{aligned} \tag{19}$$

The labor hours chosen by the firm via (19) are taken as given by the household.

2.5 Dynamic Programming

The problem solved by the household is to maximize the expected life-time utility (5), subject to (3) – (4), (7) –(15), where (7) and (8) are assumed to hold with equality, (18), (19), and the laws of motion for aggregate capital stock, $K_{t+1} = (1-\delta)K_t + I_t$. To ensure

that all variables converge to a steady state, m_t and P_t are eliminated from the problem by using $\hat{m}_t$ and $\hat{P}_t$, where $\hat{m}_t \equiv m_t/M_t$ and $\hat{P}_t \equiv P_t/M_{t+1}$.

The dynamic programming problem solved by the household is (primes denote next period values)

$$v\left(z,\mu,k,K,\hat{m},W^c\right) = \max_{\hat{m}'k'} \Big[\alpha \log c_1 + (1-\alpha)\log c_2 - \psi H^F + \beta E\left(\left(z',\mu',k',K',\hat{m}',W^{c\prime}\right)\middle| z,\mu,k,K,\hat{m},W^c\right)\Big] \tag{20}$$

subject to:

(i) $$c_2 + k' + \frac{\hat{m}'}{\hat{P}} = \frac{W^c}{\hat{P}} H^F + rk + (1-\delta)k,$$

(ii) $$c_1 = \frac{\hat{m} + e^{\mu} - 1}{e^{\mu}\hat{P}},$$

(iii) $$z' = \gamma z + \varepsilon',$$

(iv) $$\mu' = \eta\mu + \xi',$$

(vi) $$r = r\left(z,\mu,K,W^c\right),$$

(vii) $$H^F = H^F\left(z,\mu,K,W^c\right),$$

(viii) $$K' = K'\left(z,\mu,K,W^c\right) \text{ and } \hat{P} = \hat{P}\left(z,\mu,K,W^c\right),$$

where W^c is the nominal wage determined via the rule (16) and H^F are labors chosen by the firm via (19). Equation (i) is the resource constraint, and (ii) is the cash-in-advance constraint. Equations (iii) and (iv) are the laws of motion for the technology shock and the money growth shock, respectively. Finally, equation (vi) states the price of capital, (vii) gives the aggregate labor hours chosen by the firm and (vii) gives the perceived functional relationship between the aggregate state, and aggregate capital, and the price level.

A <u>recursive competitive equilibrium</u> consists of a set of decision rules for the household, $k' = k'(z,\mu,k,K,\hat{m},W^c)$ and $\hat{m}' = \hat{m}'(z,\mu,k,K,\hat{m},W^c)$; decision rules for the

firm, $H^F = H^F(z,\mu,K,W^c)$, $K^F = K^F(z,\mu,K,W^c)$;[9] the law of motion for the aggregate capital stock $K' = K'(z,\mu,K,W^c)$; pricing functions, $\hat{P} = \hat{P}(z,\mu,K,W^c)$ and $r = r(z,\mu,K,W^c)$; and a value function, $v = v(z,\mu,k,K,\hat{m},W^c)$ such that:

(1) Given the functions r, $\hat{P}$, H^F, and K', the functions k' and $\hat{m}'$ solve the household's dynamic maximization problem;

(2) Given the functions r and $\hat{P}$, the functions H^F and K^F solve the firm's profit maximization problem;

(3) Markets clear

$$K^F(z,\mu,K,W^c) = K$$

$$\hat{m}'(z,\mu,K,K,1,W^c) = 1;$$

(4) Individual decision rules are consistent with aggregate outcomes

$$k'(z,\mu,K,K,1,W^c) = K'(z,\mu,K,W^c).$$

3. Imperfectly Competitive Model

This economy consists of final-goods firms, an intermediary, households, and the government. The final-goods firms produce output using technology that is subject to technology shocks. Households are composed of infinitely lived-agents, who gain utility from leisure and two types of consumption goods, a credit good and a cash good. The cash good is a result of introducing money into the economy via a cash-in-advance constraint. The key characteristic of this economy is that households have market power in the labor market. Hence, households choose nominal wages and are monopolistic

[9] We use K^F to emphasize the capital chosen by the firm as part of it maximization problem.

suppliers of differentiated labor services. An intermediary, such as an employment agency, transforms these labor services into an aggregate labor input used by the final-goods producers. Moreover, we introduce nominal wage rigidity into this monopolistic structure, thus households choose nominal wages in advance. Finally, we have a government sector, whose role is to inject money into the economy through a lump-sum transfer.

3.1 Final-Goods Firms

The final-goods firm behaves very similarly to that in the Basic model, except that the final-goods firms here purchase labor hours from the intermediary. Hence, the production function representing the technology is the Cobb-Douglas formulation as in (1). The technology is subject to a technology shock z_t that evolves according to the law of motion in (2). The first-order conditions lead to the real rental rate (3) and real wage rate (4), but we will use $\bar{W}_t$ instead of W_t^c as the nominal wage that the firm takes as given when making its decision.

3.2 Intermediary

Following Christianno, Eichenbaum and Evans (2001), we assume that the household is a monopolistic supplier of a differentiated labor input h_t^i. An intermediary, such as an employment agency, uses the following Dixit-Stiglitz technology to transform individual labor services into an aggregate labor input H_t that it sells to the final-goods producers:

$$H_t = \left(\int_0^1 \left(h_t^i \right)^{\frac{1}{\lambda}} di \right)^{\lambda}, \qquad 1 \le \lambda \le \infty, \tag{21}$$

where λ governs the elasticity of substitution between different h_t^i.

The intermediary chooses h_t^i by maximizing profits $\Pi_t^{IC} = \bar{W}_t H_t - \int_0^1 W_t^i h_t^i$, subject to the technology (21), where W_t^i is the individual wage rate. From the first-order condition for the intermediary, it follows that the demand for h_t^i is given by

$$h_t^i = \left(\frac{\bar{W}_t}{W_t^i}\right)^{\frac{\lambda}{\lambda-1}} H_t, \tag{22}$$

where, $\bar{W}_t$ is related to the individual wage rate via

$$\bar{W}_t = \left(\int_0^1 \left(W_t^i\right)^{\frac{1}{1-\lambda}} di\right)^{1-\lambda}. \tag{23}$$

Equation (23) follows from the intermediary's zero-profit condition.

3.3 Households

The economy consists of a continuum of heterogeneous infinitely-lived households indexed by i, where $i \in [0,1]$. Households are assumed to be homogenous with respect to consumption and asset holdings, and heterogeneous with respect to the wage rate and hours worked. Household i maximizes the following modified utility function:

$$U_i = E_t \sum_{t=0}^{\infty} \beta^t [\alpha \log c_{1t} + (1-\alpha) \log c_{2t} - \psi h_t^i]. \tag{24}$$

The utility function (24) differs from that in (5) by reflecting the fact that households choose individual wages and obtain disutility from h_t^i. As in the Basic model, in addition to leisure, households obtain utility from two consumption goods. One is a credit good, acquired through capital and labor income, and the other is a cash good that can only be attained through cash. Hence, the households must satisfy the CIA constraint (7).

In addition to the CIA constraint, the household must satisfy the budget constraint. The total real income that a household has available to carry out transactions

is income earned from labor and capital, and real currency holdings. This income is used to invest, to acquire purchases of the two consumption goods, and to obtain real money balances that are carried over to next period for future transactions. The budget constraint for the households is a slightly modified version of (8) given by

$$(25) \qquad c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{P_t} \leq \frac{W_t^i}{P_t} h_t^i + r_t k_t + \frac{m_t}{P_t} + \frac{T_t}{P_t},$$

where i_t is investment, and capital evolves according to the law of motion (9).

In this economy, households have market power in the labor market. In particular households choose wages – nominal wages – in advance. We postulate that at the beginning of *each* period, households choose the nominal wage for a time that is j periods ahead in the future. This is similar to the arrangement in Cho and Cooley (1995). In the case of j=2, at time t, households choose a nominal wage for the period t+2, at t+1 they choose a wage for period t+3, and this process repeats over time. See Figure 1 for an illustration.

The first-order condition with respect to W_t^i is:

$$(26) \qquad W_t^i = E\left(\left(\lambda P_t \frac{\psi c_{2t}}{1-\alpha}\right) \middle| \omega_{t-j}\right),$$

where $\omega_{t-j} = \left\{z_{t-j}, \mu_{t-j}, k_{t-j}, K_{t-j}, W_{t-j}^i, .., W_{t-1}^i, \bar{W}_{t-j}, .., \bar{W}_{t-1}\right\}$ is the information set available to households at time t-j.[10] According to equation (26), households set their nominal wage equal to a constant markup over the expected value of marginal rate of substitution between consumption and leisure, which is equal to the wage prevailing in the spot labor market under perfect competition.

[10] The wage in (26) is chosen at time t-j. At time t households choose $\bar{W}_{t+j}^i$, which we obtain by updating (26) forward j periods.

In addition to choosing W_t^i, households choose k_{t+1} and m_{t+1}. The resulting first-order conditions are given by (10) and (11).

3.4 Government

The government plays exactly the same role as in the Basic model. It prints money, injects it into the economy and satisfies the budget constraint (13), where G_t is assumed to be zero. The government issues money according to the rule (14), where μ_t evolves according to the autoregressive process (15).

3.5 Dynamic Programming

The problem solved by the household is to maximize the expected life-time utility (21), subject to (2)-(4), (7), (9) – (11), (13) – (15), (21) – (23), (25) – (26), where (7) and (25) are assumed to hold with equality, and the laws of motion for aggregate capital stock, $K_{t+1} = (1-\delta)K_t + I_t$. To ensure that all variables converge to a steady state, m_t and P_t are eliminated from the problem by using $\hat{m}_t$ and $\hat{P}_t$, where $\hat{m}_t \equiv m_t / M_t$ and $\hat{P}_t \equiv P_t / M_{t+1}$.

The dynamic programming problem solved by the household is (primes denote next period values)

(27)

$$
\begin{aligned}
&v\left(z,\mu,k,K,\hat{m},W_1^i,..,W_{j-1}^i,\bar{W}_1,..,\bar{W}_{j-1}\right) \\
&\quad = \max_{\hat{m}'k'w_j^i}\Big[\alpha\log c_1 + (1-\alpha)\log c_2 - \psi h^i \\
&\qquad + \beta E\left(v\left(z',\mu',k',K',\hat{m}',W_2^i,..,W_j^i,\bar{W}_2,..,\bar{W}_j\right)\middle| z,\mu,k,K,\hat{m},W_1^i,..,W_{j-1}^i,\bar{W}_1,..,\bar{W}_{j-1}\right)\Big]
\end{aligned}
$$

subject to:

(i) $$c_2 + k' + \frac{\hat{m}'}{\hat{P}} = \frac{W_1^i}{\hat{P}}h^i + rk + (1-\delta)k,$$

(ii) $$c_1 = \frac{\hat{m} + e^{\mu} - 1}{e^{\mu}\hat{P}},$$

(iii) $z' = \gamma z + \varepsilon'$,

(iv) $\mu' = \eta\mu + \xi'$,

(v) $r = r\left(z, \mu, K, \bar{W}_1, .., \bar{W}_{j-1}\right)$,

(vi) $h^i = \left(\frac{\bar{W}_1}{W_1^i}\right)^{\frac{\lambda}{\lambda-1}} H^F$,

(vii) $H^F = H^F\left(z, \mu, K, \bar{W}_1\right)$,

(viii) $K' = K'\left(z, \mu, K, \bar{W}_1, .., \bar{W}_{j-1}\right)$ and $\hat{P} = \hat{P}\left(z, \mu, K, \bar{W}_1, .., \bar{W}_{j-1}\right)$,

where $\bar{W}_1$ is the period-1 aggregate nominal wage determined through (23), and W_1^i is the period-1 individual wage determined via (26). Equation (i) is the resource constraint, and (ii) is the cash-in-advance constraint. Equations (iii) and (iv) are the laws of motion for the technology shock and the money growth shock, respectively. Equation (v) states the price of capital, and (vi) gives the labor demand function for the intermediary. Finally, (vii) is the aggregate labor hours chosen by the firm, and (ix) gives the perceived functional relationship between the aggregate state, and aggregate capital and the price level.

A <u>recursive competitive equilibrium</u> consists of a set of household's decision rules, $k' = k'(z, \mu, k, K, \hat{m}, W_1^i, .., W_{j-1}^i, \bar{W}_1, .., \bar{W}_{j-1})$, $\hat{m}' = \hat{m}'(z, \mu, k, K, \hat{m}, W_1^i, .., W_{j-1}^i, \bar{W}_1, .., \bar{W}_{j-1})$ and $w_j^i = w_j^i(z, \mu, k, K, \hat{m}, W_1^i, .., W_{j-1}^i, \bar{W}_1, .., \bar{W}_{j-1})$; decision rule determining the aggregate value, $K' = K'(z, \mu, K, \bar{W}_1, .., \bar{W}_{j-1})$; decision rules for the final goods firm, $H^F = H^F(z, \mu, K, \bar{W}_1)$ and $K^F = K^F(z, \mu, K, \bar{W}_1)$; decision rules for the intermediary, $h^i = h^i(z, \mu, k, K, \hat{m}, W_1^i, .., W_{j-1}^i, \bar{W}_1, .., \bar{W}_{j-1})$; pricing functions $\hat{P} = \hat{P}(z, \mu, K, \bar{W}_1, .., \bar{W}_{j-1})$ and

$r = r(z, \mu, K, \bar{W}_1, .., \bar{W}_{j-1})$; and a value function, $v = v(z, \mu, k, K, \hat{m}, W_1^i, .., W_{j-1}^i, , \bar{W}_1, .., \bar{W}_{j-1})$, such that[11]:

(1) Given the functions r, $\hat{P}$, K', and h^i, the functions k', $\hat{m}'$, and W_j^i solve the household's dynamic maximization problem;

(2) Given the functions r, $\hat{P}$, and $\bar{W}_1$, the functions H^F and K^F solve the final good-firm's profit maximization problem;

(3) Given the functions H^F, $\bar{W}_1$, and W_1^i, the function h^i solves the intermediary's profit maximization problem;

Zero profits for the intermediary implies $\bar{W} = \left(\int_0^1 \left(W^i \right)^{\frac{1}{1-\lambda}} di \right)^{1-\lambda}$;

(4) Markets clear

$$K^F(z, \mu, K, \bar{W}_1) = K,$$

$$\hat{m}'(z, \mu, K, K, 1, \bar{W}_1, .., \bar{W}_{j-1}, \bar{W}_1, .., \bar{W}_{j-1}) = 1;$$

(5) Individual decision rules are consistent with aggregate outcomes.

$$k'(z, \mu, K, K, 1, \bar{W}_1, .., \bar{W}_j, \bar{W}_1, .., \bar{W}_j) = K'(z, \mu, K, \bar{W}_1, .., \bar{W}_j),$$

$$h^i(z, \mu, K, K, 1, \bar{W}_1, .., \bar{W}_j, \bar{W}_1, .., \bar{W}_j) = H^F(z, \mu, K, \bar{W}_1).$$

4. Solution Method

The method used to solve both dynamic programming problems is identical, and is discussed in detail by Hansen and Prescott (1995). Accordingly, we will present a very

[11] We use K^F to emphasize the capital chosen by the firm as part of it maximization problem.

brief discussion of the procedure. We begin with approximating the return function by a quadratic return function through the use of Taylor series expansions.[12] After the approximation, we are left with a linear quadratic dynamic programming problem. The method of successive approximations is used to compute the optimal value function. This implies iterating on the value function until it converges to an optimal value function, by first selecting a v_0. Once convergence is reached, the first order conditions from the last iteration can be used to form the decision rules.

5. Calibration

We use post-war U.S. data, from 1954:1 to 1999:4, to calibrate this model. The capital's share of total income, θ is calibrated to be 0.4, and δ is set equal to 0.025 corresponding to a 10% annual depreciation rate. The value of β is set to match the capital-output ratio, thus the quarterly value of β is 0.989. The value of ψ (=2.3) is chosen so that at the steady state, households spends 1/3 of their time working. To calibrate the value of α, the parameter that determines the importance of the cash good to the credit good, we use the household's first-order condition together with $c_{1t} + c_{2t} = c_t$ to obtain

$$\frac{c_t}{c_{1t}} = \frac{1}{\alpha} + \frac{1-\alpha}{\alpha} R_t, \tag{28}$$

where R_t is the interest rate, and c_{1t} represents the aggregate real money balances held at period t, given that the CIA constraint (7) holds with equality. Thus, c_t/c_{1t} is the velocity of money with respect to consumption, VEL. Empirically, c_t is consumption of nondurable goods and services, while m_t is the portion of M1 that is held by the

[12] The advantage of approximating the return function by a quadratic equation is that the decision rules obtained from the dynamic programming problem are linear.

households. The interest rate we use is the rate on a three-month treasury bill (RTB). We estimate the following regression and obtain

(29) $$VEL_t = 1.1481 + 0.1273\ RTB_t.$$

$$(0.0304) \qquad (0.0116)$$

The coefficient in front of R_t is estimated to be 0.1273, which corresponds to $\alpha = 0.89$.

The values of γ and σ_z are chosen following Prescott (1986). Given that the Solow residual is highly persistent, $\gamma = 0.95$, and $\sigma_z = 0.007$. The money growth rate, η, standard deviation σ_μ, and the average growth rate of money $\bar{\mu}$ are selected following Cooley and Hansen (1995). We take logs of equation (14), combine it with equation (15), leading to an estimated autoregressive process as follows

(30) $$\Delta \log M_t = 0.0052 + 0.552 \Delta \log M_{t-1} + \xi_t,\ \hat{\sigma}_\mu = 0.0089,$$

$$(0.001) \qquad (0.062)$$

which implies that $\eta = 0.55$, $\sigma_\mu = 0.0089$ and $\bar{\mu} = 0.012$. The two shocks in this economy z_t and μ_t are assumed to have zero correlation.

Finally, we pin down the parameter λ. The Dixit-Stiglitz technology in (18) implies that $\frac{\lambda}{\lambda - 1}$ is the elasticity of substitution between differentiated labor inputs. Using firm-level data, Griffin (1996) estimates the value of $\frac{\lambda}{\lambda - 1}$ to be between 2 and 6, corresponding to values of λ of 2 and 1.2. We set $\lambda = 1.6$, picking a value within this range. On the other hand, Christianno, Eichenbaum and Evans (2001) set λ to be 1.05, which is below the minimum value of Griffin's study. We pick this second value of λ as a lower bound.

The resulting parameter values are summarized in Table 4.1.

θ	β	δ	ψ	α	η	$\bar{\mu}$	σ_μ	γ	σ_z	λ
0.4	0.989	0.025	2.3	0.89	0.55	0.012	0.0089	0.95	0.007	1.3/2.52

TABLE 1
Parameter Values

6. Simulation Results

The quantitative results reported in Table 2 are sample means computed for 100 simulations. Each simulation consists of 184 periods, the same number as our U.S. sample. All the resulting artificial time series and actual data are passed through the H-P filter. We present standard deviations and cross-correlations for the U.S. economy as well as for the Basic and Imperfectly Competitive (IC) models. The first row of Table 4.2 shows business cycle statistics for the U.S. economy. This is followed by results for the Basic model and the IC model with a spot labor market under perfect competition.

The rest of the section presents simulation results for the Basic and the IC models with nominal wage rigidity. Specifically, we report results where nominal wages are contractually updated or adjusted by the household every quarter (j=1) as well as those when wages are changed every year (j=4). We focus on comparing the results of the Basic model with nominal wage contracts (henceforth BNW), with the results of the Imperfectly Competitive model under nominal wage rigidity (henceforth ICNW model).

Without nominal wage contracts, the Basic model (no wage rigidity) does quite well in matching the business cycle properties of real variables, but does a poor job on the nominal side of the economy. The correlations between output and nominal interest rate as well as inflation are very weak and negative. As indicated by Cooley and Hansen

(1989), this is due to the lack of a propagation mechanism for monetary shocks. We obtain similar results in the IC model irrespective of the value of λ.

We start by looking at the volatility of real variables. Once we introduce nominal wage rigidity, the standard deviations of real variables – particularly hours and output – increase dramatically. Incorporating nominal wage rigidity introduces a transmission mechanism for monetary shocks that leads to unrealistically high fluctuations in real variables as compared to the U.S. data. This is true for both the Basic model and the IC model. In the Basic model, the nominal wage is set in advance, at time t-j. When the wage is adopted at time t, labor demand determines actual labor hours. Hence, firms are able to adjust to shocks – monetary and technology – that occur in between the time t-j and t when choosing labor hours. This leads to a high volatility in hours and transfers to a high volatility in output. Similar reasoning leads to high volatility in hours in the IC model. Households choose a wage in advance (t-j), and at time t the intermediary chooses h_i – taking nominal wage as given – due to the monopolistic structure of the labor market. The final-goods firm then chooses H_t, also taking the nominal wage as given. In this case, both the intermediary and the final-goods firm are able to adjust to the technology and money supply shocks that occur in between t-j and t. This adjustment leads to the higher volatility. When $\lambda = 1.05$, we find the results to be almost identical to the Basic model, and a further increase of λ to 1.6 produces only slight differences.

Similar to the results of Cho and Cooley (1995), we find output and productivity to be negatively correlated in our models. Without nominal wage rigidity, the correlation closely matches the data. However, we obtain a negative correlation (-.47) in the Basic model with nominal wage contracts, which rises as the length of the contract increases.

Similarly, we find a negative correlation in the IC model under nominal wage rigidity. The results are almost identical when $\lambda = 1.05$, and change only marginally when λ is set at 1.6. The negative correlation between output and productivity in the Basic model as well as the IC model is a result of labor fluctuating more than output. Nominal wage rigidity creates a channel through which hours are affected more than output. When nominal wages are fixed, a rise in the price level (primarily due to a positive monetary shock) causes a fall in the MP_L, which in turn increases labor hours. This additional channel, through which hours increase, causes hours to fluctuate more than output, hence leading to a negative correlation between output and productivity.

In sum, we find that the results on the real side of the economy are very similar between the BNW model and the ICNW model. We find only slight differences when the value of λ is set to 1.6, which we consider to be on the high end of an empirically plausible range. When $\lambda = 1.05$, as in Christianno, Eichenbaum and Evans (2001), the results are almost identical. Based on the results on the real side of the economy, we conclude that the Basic model, where nominal wages are set via a rule, is in line with the IC model, which exhibits a microeconomic foundation for nominal wage setting.

On the nominal side of the economy, similar to Cooley and Hansen (1989), our Basic model without nominal wage rigidity generates weak correlations between output and nominal variables, such as inflation and nominal interest rate. Same holds for the IC model. However, by introducing nominal wage rigidity, we integrate a monetary transmission mechanism into both models. This results in correlations between output and inflation and nominal interest rate to more closely mimic the U.S. data. Notice that the correlation between output and nominal interest rate is much more sensitive to the

value of λ. However, the differences between the BNW model and the ICNW model are still quite small.

Next, we take a look at the standard deviations of inflation and nominal interest rate. The Basic model as well as the IC model without nominal wage rigidity both yield standard deviation of nominal interest rate that is below what we observe in the data. However, when we introduce nominal wage rigidity, the fluctuations of nominal interest rate increase, and in the case of a one year contract are unrealistically high. This is true for the BNW model as well as the ICNW model. Finally, none of the models considered here are able to mimic the low volatility of inflation found in the data. With the introduction of nominal wage rigidity, the volatility decreases, but it is still very high. We obtain identical results for the BNW model and the ICNW when nominal wages are updated every quarter (j=1) and every year (j=4), irrespective of the value of λ. Hence, we find volatility of inflation to be insensitive to λ.

7. Conclusion

Our analysis shows that introducing nominal wage rigidity into a one-sector real business cycle model, where households have market power in the labor market, generates almost identical results to a model where households have no market power, and nominal wages are contracted. For example, Cho (1993) and Cho and Cooley (1995) introduce nominal wage contracts into a standard RBC model. These models have been criticized for lacking microeconomic foundation for the introduction of a nominal wage contract structure. Specifically, nominal wages agreed to in advance are based on a rule, not directly a result of optimal decision making. In contrast, in the imperfectly competitive

model individual households choose nominal wages in advance as part of their maximization problem. Hence, the imperfectly competitive formulation is a more appropriate structure to analyze nominal wage rigidity in real business cycle models.

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TABLE 2
Standard Deviations and Correlations of Output with key Macroeconomic Variables

	Standard Deviations					Correlations		
	Y	H	Y/H	inf	i	corr(Y,Y/H)	corr(Y, inf)	corr(Y,i)
U.S. Data	1.65	1.51	1.06	0.43	1.18	0.49	0.34	0.37
1954.1-1999.4								
Economies without Nominal Wage Rigidity								
Basic Model	1.698	1.359	0.430	1.291	0.673	0.838	-0.157	-0.001
IC model: lambda = 1.05	1.697	1.358	0.430	1.291	0.671	0.838	-0.157	-0.001
IC model: lambda = 1.6	1.683	1.333	0.439	1.292	0.656	0.845	-0.160	-0.001
Economies with Nominal Wage Contract Rigidity								
A: one-period nominal wage rigidity								
BNW model	2.503	3.382	1.352	1.198	1.435	-0.470	0.582	0.504
ICNW model: lambda = 1.05	2.502	3.381	1.352	1.198	1.433	-0.470	0.582	0.505
ICNW model: lambda = 1.6	2.493	3.373	1.355	1.198	1.417	-0.467	0.582	0.511
B: four-period nominal wage rigidity								
BNW model	4.214	6.616	2.732	1.174	1.827	-0.811	0.218	0.667
ICNW model: lambda = 1.05	4.213	6.615	2.733	1.174	1.818	-0.811	0.218	0.668
ICNW model: lambda = 1.6	4.208	6.606	2.736	1.174	1.735	-0.810	0.215	0.676

Note:
Y = Output = real GNP; H = Hours = total hours of work; Y/H = Productivity = ouput devided by total hours; P = Price = CPI;
inf = Inflation = changeLN(CPI); i =Nominal interest rate = 3-month treasury bill rate.