Conditional Correlation Demand Systems*

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Abstract:

We address the estimation of singular demand systems with heteroscedastic disturbances. As in Serletis and Isakin (2017) and Serletis and Xu (2019) we assume that the covariance matrix of the errors of the demand system is time-varying, and contribute to the literature by considering the constant conditional correlation (CCC) and dynamic conditional correlation (DCC) parameterizations of the variance model. We derive a number of important practical results and also provide an empirical application to support our methodology.

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1 Introduction

The consumer demand systems literature that developed after the publication of Theil's (1965) important paper, along with Diewert (1974), Chrisensen *et al.* (1975), Theil (1975), and Theil (1976), have advanced dramatically over the years. For authoritative surveys of that literature, see Brown and Deaton (1972), Blundell (1988), Lewbel (1997), and Barnett and Serletis (2008). More recently, Serletis and Isakin (2017) and Serletis and Xu (2019) merge the flexible demand systems literature with recent advances in the financial econometrics literature, by relaxing the homoscedasticity assumption in empirical demand analysis, assuming that the covariance matrix of the errors of the demand system is time varying.

Unlike static demand systems, demand systems with heteroscedastic disturbances provide a way of dealing with empirical regularity in demand system analysis and also allow applied economists to extract information about the variances and covariances of demand responses. Models of this kind are particularly promising for empirical use, because any elasticities calculated from the demand system are affected by the variances and covariances of the demand responses, and hence estimating a demand system with heteroscedastic disturbances will give different estimates than a static demand system.

Serletis and Isakin (2017) and Serletis and Xu (2019) consider the VECH and BEKK parameterizations for the covariance matrix, and analytically prove the invariance of the maximum likelihood estimator with respect to the choice of the good deleted from a singular demand system. They also prove a number of important practical results regarding how to recover the mean and variance equation parameters (and their standard errors) of the full demand system from those of any subsystem obtained by deleting an arbitrary good.

In this paper, we extend the work in Serletis and Isakin (2017) and Serletis and Xu (2019) and consider the constant conditional correlation (CCC) and dynamic conditional correlation (DCC) parameterizations of the variance model. We derive a number of important practical results and provide an empirical application in order to support our methodology. We show how the results in Serletis and Isakin (2017) and Serletis and Xu (2019) can be applied to the CCC and DCC parameterizations of the variance model.

2 Singular Demand Systems

Consider the following demand system in budget share form

$$\boldsymbol{s}_t = \boldsymbol{s}(\boldsymbol{v}_t, \boldsymbol{\theta}) + \boldsymbol{\epsilon}_t \tag{1}$$

where $\boldsymbol{s}_t = (s_{1t}, \ldots, s_{nt})'$ is a vector of budget shares with the jth element being $s_j = p_j x_j / y$, $\boldsymbol{v}_t = (v_{1t}, ..., v_{nt})'$ is a vector of expenditure normalized prices with the jth element being $v_j = p_j / y$, $\boldsymbol{\epsilon}_t = (\epsilon_{1t}, ..., \epsilon_{nt})'$ is a vector of classical disturbance terms, and $\boldsymbol{\theta}$ is the parameter vector to be estimated. Note that the demand system is singular, since the the shares sum to one (and the errors sum to zero). See Barnett and Serletis (2008) for an up-to-date survey of the state-of-the art in consumer demand analysis.

Recently, Serletis and Isakin (2017) and Serletis and Xu (2019) merge the demand systems literature with the financial econometrics literature, by relaxing the assumption that the errors in (1) are homoscedastic and instead assuming that

$$\boldsymbol{\epsilon}_t|\Psi_{t-1} \sim N\left(\mathbf{0}, \boldsymbol{H}_t\right) \tag{2}$$

where $\mathbf{0}$ is an n-dimensional null vector and the $n \times n$ covariance matrix $\boldsymbol{H}_t$ is measurable with respect to information set Ψ_{t-1}. To avoid singularity, they delete (any) one good (or, equivalently, equation from (1)) and consider the VECH and BEKK parameterizations of the corresponding $(n-1) \times (n-1)$ covariance matrix $\boldsymbol{\Phi}_t$. They show that any subsystem of (1) obtained by deleting an arbitrary good is observationally equivalent to any other subsystem, and hence maximum likelihood estimation generates parameter estimates that are invariant to the equation deleted, consistent with the invariance claims in Barten (1969) and McLaren (1990) under the homoscedasticity assumption. They also prove a number of other important practical results regarding how to recover the mean and variance equation parameters (and their standard errors) of the full demand system from those of any subsystem of (1) obtained by deleting an arbitrary good.

However, there have not been many prior studies of how to deal with singular demand systems when results are not invariant to the equation deleted, although Barnett (1981) and Barnet *et al.* (1981) address the especially difficult problem of singularity caused by implicit function systems of demand systems that are not in closed form.

3 CCC Demand Systems

Because of computational difficulties with multivariate volatility models, Bollerslev (1990) introduced the constant conditional correlation (CCC) model which significantly reduces the number of free parameters. It does so, by allowing the variances and covariances to vary over time, but restricts the conditional correlations among the $\boldsymbol{\epsilon}_t$ elements to be constant.

Assuming that the $n \times n$ conditional correlation matrix, $\boldsymbol{\rho}$ (which is symmetric with unit diagonal elements), is time invariant, then we can write

$$\boldsymbol{H}_t = \boldsymbol{V}_t \boldsymbol{\rho} \boldsymbol{V}_t \tag{3}$$

where $\boldsymbol{V}_t$ is a diagonal matrix of GARCH volatilities, $\boldsymbol{V}_t = \text{diag}\left(\sqrt{h_{1,t}}, ..., \sqrt{h_{n,t}}\right)$. Thus, the time varying covariances in $\boldsymbol{H}_t$ are of the form

$$h_{ij,t} = \rho_{ij}\sqrt{h_{i,t}}\sqrt{h_{j,t}}$$

where ρ_{ij} is the correlation of the series i and j, for $i, j \in (1, ..., n)$ and $i \neq j$.

We assume a GARCH(1,1) specification for $\boldsymbol{H}_t$ and write the full CCC demand system as

$$\boldsymbol{s}_t = \boldsymbol{s}\left(\boldsymbol{v}_t, \boldsymbol{\theta}\right) + \boldsymbol{\epsilon}_t \tag{1}$$

$$\boldsymbol{\epsilon}_t|\Psi_{t-1} \sim N\left(\mathbf{0}, \boldsymbol{H}_t\right) \tag{2}$$

$$\text{diag}\left(\boldsymbol{H}_t\right) = \boldsymbol{C} + \boldsymbol{A}\text{diag}\left(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\right) + \boldsymbol{B}\text{diag}\left(\boldsymbol{H}_{t-1}\right) \tag{4}$$

$$h_{ij,t} = \rho_{ij}\sqrt{h_{i,t}}\sqrt{h_{j,t}} \tag{5}$$

where $\boldsymbol{C}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ are diagonal matrices. Again, the error terms of the demand system consisting of equations (1), (2), (4), and (5) sum to zero and we drop an arbitrary good to avoid singularity. In this case, however, the estimated subsystems are not observationally equivalent (under the ML estimator), because the covariance matrix of each subsystem is a nonlinear function of the subsystem's variances and covariances.[1] More importantly, it is not possible to recover the parameter estimates (and their standard errors) of the mean and variance equations of the full demand system from any of the estimated subsystems, as was the case with the *vech* and BEKK parameterizations for the conditional covariance matrix, $\boldsymbol{H}_t$.

However, we can still obtain the parameter estimates of the mean and variance equations of the full demand system using a two-step approach, similar to that suggested by Engle (2002) in the context of dynamic conditional correlation (DCC) models.

According to (4), we have

$$h_{j,t} = c_j + a_j \epsilon_{j,t-1}^2 + b_j h_{j,t-1}, \quad \text{for } j = 1, ..., n. \tag{6}$$

If we add $\epsilon_{j,t}^2$ to the both sides of the above equation, we get

$$h_{j,t} + \epsilon_{j,t}^2 = c_j + a_j \epsilon_{j,t-1}^2 + b_j h_{j,t-1} + \epsilon_{j,t}^2, \quad \text{for } j = 1, ..., n$$

which after rearranging yields

$$\epsilon_{j,t}^2 = c_j + (a_j + b_j)\epsilon_{j,t-1}^2 - b_j \zeta_{j,t-1} + \zeta_{j,t}, \quad \text{for } j = 1, ..., n. \tag{7}$$

where $\zeta_{j,t} = \epsilon_{j,t}^2 - h_{j,t}$. Note that $h_{j,t}$ is the forecast of $\epsilon_{j,t}^2$ based on lagged information

$$h_{j,t} = E\left(\epsilon_{j,t}^2 | \Psi_{t-1}\right), \quad \text{for } j = 1, ..., n$$

and thus $\zeta_{j,t}$ is the error associated with that forecast. It follows that $\zeta_{j,t}$ is a white noise process — see Hamilton (1994) — and equation (7) looks like an autoregressive-moving-average (ARMA) process in $\epsilon_{j,t}^2$. If we estimate equation (7) as an ARMA(1,1) model, we can find the ARCH effect, a_j, and the GARCH effect, b_j, for all $j = 1, ..., n$.

Now the estimation of the ARCH and GARCH effects in the full demand system requires that we use the squared residuals, $\epsilon_{j,t}^2$ for $j = 1, ..., n$, and estimate an arbitrary subsystem assuming homoscedasticity. Consider, for example, the estimation of the subsystem obtained by deleting the ith good and (thus) the error vector $\boldsymbol{u}_t = (\epsilon_{1,t}, ..., \epsilon_{i-1,t}, \epsilon_{i+1,t}, ..., \epsilon_{n,t})$. According to the adding-up property of the demand system, the missing ϵ_{it} can be found by

$$\epsilon_{it} = 0 - \sum_{k=1}^{n} \epsilon_{kt}, \quad \text{for } k \neq i.$$

Thus, we can obtain $\boldsymbol{\epsilon}_t$ by estimating an arbitrary subsystem. Note that the first stage estimation of an arbitrary subsystem is just a classical estimation of a singular demand

[1] The invariance proved in Serletis and Isakin (2017) and Serletis and Xu (2019) relies on variance models which are linear. The invariance does not hold for the conditional correlation demand system, since the likelihood functions of the subsystems of such singular demand system are dependent on the equation deleted.

system following Barten (1969). Moreover, Barten (1969) proves that the resulting estimators are identical to those that would be acquired by estimating the entire singular demand system simultaneously using the Moore-Penrose generalized inverse, which can be inverted for a singular matrix. This result provides immediate availability of the known asymptotic properties of the estimators. We then estimate the ARMA(1,1) model for each $\epsilon_{j,t}^2$, $j = 1, ..., n$, to obtain the ARCH and GARCH effects in the demand of each good.

Since the standard error of the ARCH effect a_j, for $j = 1, ..., n$, is not available in the estimation of the ARMA(1,1) model, it can be calculated as follows

$$\text{Standard error of } a_j = \sqrt{\text{Var}(a_j + b_j) + \text{Var}(b_j) - 2\text{Cov}(a_j + b_j, b_j)}$$

where $\text{Var}(a_j + b_j)$, $\text{Var}(b_j)$, and $\text{Cov}(a_j + b_j, b_j)$ are all available in the covariance matrix of the corresponding ARMA(1,1) estimation.

Finally, regarding the conditional correlations, we can collect the $\zeta_{j,t}$, for $j = 1, ..., n$, residuals from the ARMA(1,1) estimations and use them to find $h_{j,t}$, which is equal to $\epsilon_{j,t}^2 - \zeta_{j,t}$. We can also calculate the standardized residuals, $\tilde{\epsilon}_{j,t}$, as

$$\tilde{\epsilon}_{j,t} = \frac{\epsilon_{j,t}}{\sqrt{h_{j,t}}}, \quad \text{for } j = 1, ..., n.$$

It is also to be noted that the calculation of the standardized residuals requires the non-negativity of $h_{j,t}$, since the square root of $h_{j,t}$ won't exist otherwise. As Hamilton (1994) suggests the non-negativity of $h_{j,t}$ will be satisfied if $c_j > 0$, $a_j > 0$, and $b_j > 0$, for $j = 1, ..., n$. After obtaining the standardized residuals, we can calculate the correlation matrix of the standardized residuals, which is actually $\boldsymbol{\rho}$.

4 DCC Demand Systems

Although the CCC model has appealing features, such as simplified estimation and a reduced number of free parameters, the assumption of constant conditional correlation is restrictive and is a concern in empirical applications. In this regard, Engle (2002) proposed the dynamic conditional correlation (DCC) model, which allows the conditional correlation matrix to vary over time.

In the DCC model the conditional correlation matrix in equation (3) is time-varying

$$\boldsymbol{\rho}_t = \left(\text{diag}\boldsymbol{\Theta}_t\right)^{-1/2} \boldsymbol{\Theta}_t \left(\text{diag}\boldsymbol{\Theta}_t\right)^{-1/2}$$

where

$$\boldsymbol{\Theta}_t = \boldsymbol{S}\left(1 - \rho_1 - \rho_2\right) + \rho_1 \widetilde{\boldsymbol{\epsilon}}_{t-1} \widetilde{\boldsymbol{\epsilon}}_{t-1}' + \rho_2 \boldsymbol{\Theta}_{t-1}$$

and $\boldsymbol{S}$ is the unconditional covariance matrix of the standardized residuals, $\widetilde{\boldsymbol{\epsilon}}_t$. It is also required that $\rho_1 \geq 0$, $\rho_2 \geq 0$, and $0 \leq \rho_1 + \rho_2 < 1$ to ensure the stability of $\boldsymbol{\Theta}_t$.

Thus the full DCC demand system can be written as

$$\boldsymbol{s}_t = \boldsymbol{s}(\boldsymbol{v}_t, \boldsymbol{\theta}) + \boldsymbol{\epsilon}_t \quad (1)$$

$$\boldsymbol{\epsilon}_t | \Psi_{t-1} \sim N(\mathbf{0}, \boldsymbol{H}_t) \quad (2)$$

$$\boldsymbol{H}_t = \boldsymbol{V}_t \boldsymbol{\rho}_t \boldsymbol{V}_t \quad (8)$$

$$\boldsymbol{\rho}_t = (\text{diag}\boldsymbol{\Theta}_t)^{-1/2} \boldsymbol{\Theta}_t (\text{diag}\boldsymbol{\Theta}_t)^{-1/2} \quad (9)$$

$$\boldsymbol{\Theta}_t = \boldsymbol{S}(1 - \rho_1 - \rho_2) + \rho_1 \widetilde{\boldsymbol{\epsilon}}_{t-1} \widetilde{\boldsymbol{\epsilon}}'_{t-1} + \rho_2 \boldsymbol{\Theta}_{t-1}. \quad (10)$$

The estimation of the DCC demand system is similar to that for the CCC demand system. In fact, since the DCC model uses the same variance equations as the CCC model, which are given by (6), we can estimate the variance equations of the DCC model by the same modified two-step approach that we discussed in the previous section for the CCC model. We can also calculate the standardized residuals, and estimate equation (10) to obtain the time-varying conditional correlations and the covariance terms.

5 Volatility Spillovers

With the CCC and DCC demand systems, we can also investigate volatility spillover effects by changing the variance equation (6) to

$$h_{j,t} = c_j + \sum_{k=1}^{n} a_k \epsilon^2_{k,t-1} + \sum_{k=1}^{n} b_k h_{k,t-1}, \quad \text{for } j = 1, ..., n.$$

This specification allows past shocks and volatilities to show up in estimating the current volatility of each good. Adding $\epsilon^2_{j,t}$ to both sides of this equation, and rearranging, yields

$$\epsilon^2_{j,t} = c_j + \sum_{k=1}^{n} (a_k + b_k) \epsilon^2_{k,t-1} - \sum_{k=1}^{n} b_k \zeta_{k,t-1} + \zeta_{j,t}, \quad \text{for } j = 1, ..., n \quad (11)$$

which is a VARMA(1,1) model.

After estimation of (11), the standardized residuals could be computed and the conditional correlation matrix could be constructed, as we discussed in the CCC or DCC sections. It is to be noted, however, that in this case the non-negativity of $h_{j,t}$ for $j = 1, ..., n$ needs to be assured.

6 Empirical Application

We follow Serletis and Xu (2019) and use the basic translog demand system with three goods ($n = 3$) for demonstration purposes. The basic translog reciprocal indirect utility function of Christensen *et al.* (1975) is given by

$$\log h(\boldsymbol{v}) = \alpha_0 + \sum_{k=1}^{n} \alpha_k \log \nu_k + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{jk} \log \nu_k \log \nu_j \quad (12)$$

where $\boldsymbol{v}$ is the income-normalized price vector and $\beta_{ij} = \beta_{ji}$ for $i, j \in [1, n]$. The BTL share equations, derived using the logarithmic form of Roy's identity, are

$$s_i = \frac{\alpha_i + \sum_{k=1}^{n} \beta_{ik} \log \nu_k}{\sum_{k=1}^{n} \alpha_k + \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{jk} \log \nu_k}, \quad \text{for } i = 1, ..., n. \tag{13}$$

As the share equations are homogeneous of degree zero in the (α and β) parameters, estimation requires some parameter normalization. We use the normalization

$$\sum_{i=1}^{n} \alpha_i = 1.$$

We use the same data used by Serletis and Xu (2019). It consists of monthly time series data on monetary asset quantities and their user costs, recently produced by Barnett *et al.* (2013) and maintained within the Center of Financial Stability (CFS) program Advances in Monetary and Financial Measurement (AMFM). The sample period is from 1967:2 to 2015:3 (a total of 579 observations). For a detailed discussion of the data and the methodology for the calculation of user costs, see Barnett *et al.* (2013) and http://www.centerforfinancialstability.org. In particular, we model the demand for three monetary assets: large time deposits, x_1, small time deposits at commercial banks, x_2, and demand deposits, x_3. As we require real per capita asset quantities for the empirical work, we divide each quantity series by the CPI (all items) and total population.

As already noted, the invariance of the maximum likelihood estimator with respect to the choice of the good to be deleted cannot be established in the case with the conditional correlation models, because these models are nonlinear. In this case, we follow the two-step approach discussed earlier. We use all available observations, from 1967:2 to 2015:3, and estimate the CCC demand system, consisting of equations (1), (2), (4), and (5), and the DCC demand system, consisting of equations (1), (2), (8), (9), and (10), and report the estimation results in Tables 1 and 2, respectively.

The estimates of the variance equations of the CCC model in Table 1 show that there are significant ARCH and GARCH effects in the demand for each of large time deposits, x_1, small time deposits at commercial banks, x_2, and demand deposits, x_3. They suggest that, for each asset, shocks have big effects and that volatility is persistent. The conditional correlation coefficients, given by ρ_{12}, ρ_{13}, and ρ_{23}, indicate a positive relationship between x_1 and x_2 ($\rho_{12} = 0.133$), and that each of x_1 and x_2 is negatively related to x_3 ($\rho_{13} = -0.866$ and $\rho_{23} = -0.450$).

The estimation results of the DCC model are given in Table 2. The coefficient estimates of the mean and variance equations are the same as those for the CCC model in Table 1. This is so, because the estimation procedure is the same for the CCC and DCC models; the two models differ only in terms of how the $\boldsymbol{\Theta}_t$ matrix is calculated. Panel C of Table 2 shows the dynamics of the conditional correlation coefficient matrix, $\boldsymbol{\rho}_t$, which are governed by $\rho_1 = 0.495$ (with a standard error 0.022) and $\rho_2 = 0.478$ (with standard error of 0.024), suggesting that the conditional correlations are time-varying.

7 Conclusion

We address the estimation of singular demand systems with heteroscedastic disturbances, extending the work in Serletis and Isakin (2017) and Serletis and Xu (2019). In doing so, we relax the homoscedasticity assumption and instead assume that the covariance matrix of the errors of the demand system is time-varying. We consider the constant conditional correlation (CCC) and dynamic conditional correlation (DCC) parameterizations of the variance model, and derive a number of important practical results and also provide an empirical application in order to computationally support and validate our methodology.

It is to be noted that the approach for modeling demand systems with the CCC and DCC parameterizations of the variance model does not establish a link between the elasticities of substitution and the assumed heteroscedastic error specification. The main reason is that they are available from the two different stages of estimation. The heteroscedastic error specifications in Serletis and Isakin (2017) and Serletis and Xu (2019) do affect the estimates of the elasticities of substitution. However, the CCC and DCC specifications used in this paper allow the researcher to easily extract demand variances and covariances from a demand system.

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Table 1. Estimation of the CCC model

Coefficient	Estimate	Standard error
	A. Conditional mean equation	
α_1	0.652	0.004
α_2	0.189	0.002
α_3	0.159	0.000
β_{11}	0.109	0.042
β_{12}	0.060	0.011
β_{13}	−0.178	0.015
β_{21}	0.060	0.000
β_{22}	0.186	0.004
β_{23}	−0.009	0.005
β_{31}	−0.178	0.000
β_{32}	−0.009	0.000
β_{33}	0.225	0.009
	B. Conditional variance equation	
c_1	0.005	0.002
a_1	0.478	0.051
b_1	0.473	0.042
c_2	0.002	0.001
a_2	0.753	0.047
b_2	0.219	0.043
c_3	0.009	0.003
a_3	0.559	0.048
b_3	0.403	0.042
	C. Conditional correlation	
ρ_{12}	0.133	
ρ_{13}	−0.866	
ρ_{23}	−0.450	

Note: Sample period, monthly data, 1967:2-2015:3.

Table 2. Estimation of the DCC model

Coefficient	Estimate	Standard error
	A. Conditional mean equation	
α_1	0.652	0.004
α_2	0.189	0.002
α_3	0.159	0.000
β_{11}	0.109	0.042
β_{12}	0.060	0.011
β_{13}	−0.178	0.015
β_{21}	0.060	0.000
β_{22}	0.186	0.004
β_{23}	−0.009	0.005
β_{31}	−0.178	0.000
β_{32}	−0.009	0.000
β_{33}	0.225	0.009
	B. Conditional variance equation	
c_1	0.005	0.002
a_1	0.478	0.051
b_1	0.473	0.042
c_2	0.002	0.001
a_2	0.753	0.047
b_2	0.219	0.043
c_3	0.009	0.003
a_3	0.559	0.048
b_3	0.403	0.042
	C. Conditional correlation	
ρ_1	0.495	0.022
ρ_2	0.478	0.024

Note: Sample period, monthly data, 1967:2-2015:3.