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NEUTRAL EX ANTE INCOME TAXATION IN THE PRESENCE OF ADJUSTMENT COSTS AND RISK*

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The investment neutrality of a full loss offset income tax which grants ex ante, or historical cost, depreciation allowances is examined in the concurrent presence of income and capital risk and convex capital adjustment costs. In general the neutral ex ante tax depreciation rate is shown to be time varying and stochastic, and reflects both the systematic and unsystematic, capital and income risk characteristics of the investment.

I. INTRODUCTION

The design of tax systems which do not distort the investment decisions of firms is of interest to both public finance economists and policy makers. Of particular concern is the neutrality of a proportional income tax, as this is the general approach to the taxation of business income followed in most countries. The benchmark result in this case was derived by Samuelson [1964] and Johansson [1969]. Ignoring complications such as adjustment costs and uncertainty, they established that a tax levied on economic income, which consists of net cash flows less the true economic depreciation of capital, is neutral with respect to the investment decisions of firms.

More recently, a number of studies have sought to extend this well-known result in two directions. Hartman [1978] and Abel [1983b] have considered the neutrality of an income tax in the presence of convex capital adjustment costs, while Mintz [1981, 1982], Bulow and Summers [1984], Gordon [1985], and Fane [1987] have examined income tax neutrality under conditions of uncertainty. These extensions confirm that, with the appropriate modifications, the Samuelson-Johansson result is quite general and robust to the introduction of either adjustment costs or risk. Surprisingly, the bulk of this literature has considered adjustment costs and risk separately, with few models

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examining the implications of their simultaneous presence.[1] In reality, of course, both risk and adjustment costs can have an important influence on the investment decisions of firms: capital cannot be adjusted instantaneously without incurring large costs, and future output and factor prices are unknown to the firm. It would thus appear useful to re-examine the neutrality of the proportional income tax in the simultaneous presence of adjustment costs and risk.

Using a state-contingent approach Fane [1987] determined that the Samuelson-Johansson result can be extended to a risky environment with risk averse investors provided that prices (including state-contingent prices and the risk-free pre-tax interest rate) are unchanged by the introduction of the income tax. If this is the case, he showed that an income tax is neutral if allowable tax depreciation is equal to true *ex post,* replacement cost, economic depreciation. While Fane [1987] does not explicitly incorporate adjustment costs into his analysis, they fit trivially into his framework without altering the results. While this is an important result, most actual tax systems grant depreciation allowances based upon the historical or original cost of the firm's capital, rather than on its actual *ex post* replacement value.[2] In a deterministic model without adjustment costs this poses no particular difficulty, as the *ex ante* tax depreciation rate can be set equal to the economic rate of depreciation, which is known with certainty. When capital adjustment costs and uncertainty co-exist, however, the neutrality conditions, specifically those concerning the appropriate *ex ante* tax depreciation rate, become very complicated. In this paper I analyze and discuss the nature of these complexities. The paper thus provides an alternative extension of the Samuelson-Johansson result into an environment with risk and capital adjustment costs to Fane [1987]: I assume that the tax depreciation rate is determined *ex ante,* whereas he assumes that allowable depreciation is calculated *ex post.*

When examining taxation in a risky environment, it is important to distinguish between two sources of risk: *income risk* involves uncertainty about the future income stream of the firm, while *capital risk* involves uncertainty regarding the economic depreciation of the firm's capital. The modern literature on risk and taxation (which has tended to ignore adjustment costs) has established the widely accepted result that if the tax system provides full loss offsets the presence of income risk does not matter (see, for example, Mintz [1981, 1982], and Bulow and Summers [1984]). An income tax that is neutral in a deterministic setting is therefore also neutral in a risky setting as long as full loss offsets are granted; in particular, no alterations to the tax depreciation rate nor any other explicit accounting for income risk is required. As suggested by

1 Although Gordon and Wilson [1989] incorporate both adjustment costs and risk, they are not interested in neutrality so much as the welfare costs of distortionary income taxes.

2 Tax systems actually have elements of both *ex ante* and *ex post* deductibility of depreciation: at the firm level, rates are set *ex ante,* but the taxation of capital gains at the individual level takes place on an *ex post* basis. In this paper, I concentrate on the implications of the former.

Bulow and Summers [1984], and illustrated by Jog and Mintz [1989], if depreciation allowances are determined *ex ante* this is not the case for capital risk. The use of *ex ante* depreciation means that tax depreciation allowances do not fluctuate with unanticipated changes in the value of capital. In this case, even if full loss offsets are permitted, a higher tax depreciation rate must be granted to account for capital risk. The higher tax depreciation rate reflects a *systematic* capital risk premium. Of course, if true economic depreciation is deducted *ex post,* the distinction between capital and income risk is not important, as deductions then vary with actual changes in the market value of the capital.

I show that the simultaneous presence of convex adjustment costs and income and capital risk alters the neutrality conditions for an *ex ante* income tax in several ways. In general, the neutral *ex ante* tax depreciation rate is time varying, stochastic and reflects both the income and capital risk characteristics of the firm's investments. In sharp contrast to the prevailing wisdom, income risk does matter, even if there are full loss offsets. Moreover, I show that both *systematic* and *unsystematic* risk matter, not just systematic (capital) risk as shown by previous authors.

In McKenzie [1993] I examine the implications of an alternative specification of adjustment costs and risk to that considered here – linear adjustment costs with irreversible investment. I show that under that specification the neutral tax depreciation rate is also a function of both systematic and unsystematic income and capital risk. Taken together, these results imply that the traditional models used to examine the impact of taxes under uncertainty are too simplistic, and ignore potentially important interactions.

The remainder of the paper is organized as follows. In Section II the model is developed. In Section III the neutrality conditions are examined. Concluding comments are given in Section IV.

II. THE MODEL

A dynamic neo-classical model of the firm in continuous time is modified to incorporate an income tax and uncertainty. In order to abstract from non-neutralities caused by the differential taxation of debt and equity, it is assumed that the personal and corporate tax systems are perfectly integrated and the firm is 100% equity financed.[3] Moreover, full loss offsetting, or full refundability, is assumed.[4] Most actual

[3] If the investment were partly debt financed, the interest would be deducted by the firm and counted as income for the lender(s). So long as the tax rate on the borrower and the lender is the same, which is consistent with the assumption of full integration, total taxable income generated by the project would be unchanged, as would the neutrality conditions.

[4] Full loss offsetting may be achieved either by granting immediate refunds or by carrying forward losses at an interest rate. Under fairly general conditions, Fane [1987] established that the losses should be carried forward at the risk-free interest rate.

tax regimes are not perfectly integrated nor do they grant full loss offsets, however it is already well known that investment neutrality cannot be achieved under an income tax if these conditions do not hold; by assuming that they hold from the outset the discussion focuses on the determination of the neutral *ex ante* tax depreciation rate.

As indicated above, when depreciation allowances are determined *ex ante* the distinction between capital and income risk is important. Income risk is modeled by assuming that the price of the firm's output follows exogenously determined geometric Brownian motion:

$$dp(s)/p(s) = \mu_p ds + \sigma_p dz_p(s) \tag{1.1}$$

where μ_p is the expected instantaneous rate of growth in the output price, σ_p is the per-unit-time standard deviation of that growth rate, and dz_p is the random increment of standard Wiener processes. Similarly, capital risk is represented by geometric Brownian motion for the supply price of a unit of the firm's capital,

$$dq(s)/q(s) = \mu_q ds + \sigma_q dz_q(s) \tag{1.2}$$

where μ_q and σ_q are the instantaneous growth rate and standard deviation in the supply price of a unit of capital. Note that the current values of p and q are known, with their rates of growth becoming increasingly uncertain over time. It is assumed that the Wiener processes governing p and q are uncorrelated; this is merely for expositional simplicity, and does not significantly alter the results.[5]

The firm chooses the level of investment at each instant in time so as to maximize its value to risk averse owners. To determine the value of the firm's investment program in this environment I invoke the assumptions of Merton's [1973] intertemporal asset pricing model (ICAPM),[6] and employ an asset valuation approach originally developed by Constantinides [1978]. Viewing the investment program as an asset, the equilibrium after-tax expected rate of return on the firm must equal the after-tax risk-free interest rate plus an adjustment for systematic (market) risk. The value of the firm therefore reflects the risk aversion of its owners through its required rate of return. When this is the case, Constantinides [1978] showed that the investment program may be evaluated by discounting the expected (after-tax) income stream of the firm by the risk-free (after-tax) rate of interest, and modifying the equations of motion

5 It is easy to extend the model to incorporate other sources of income risk, say a variable factor of production with a stochastic price, or other sources of capital risk, perhaps a stochastic physical rate of depreciation.

6 Essentially we require investors with additive von Neumann-Morganstern preferences, homogeneous expectations, a stationary investment opportunity set, state variables which follow Ito diffusion processes as in equation set (1), and perfect capital markets.

for the stochastic state variables, p and q, by subtracting a systematic risk premium from the expected rates of growth (the μ's). This is equivalent to discounting each component of the cash flow stream by the appropriate risk-adjusted rate of return required by risk-averse investors in capital market equilibrium. This is a simple and straightforward way to incorporate investor risk aversion into the model.

Using this approach, the value of the firm's investment program is equivalent to that determined by

$$V(0) = \underset{\{I(t)\}}{\text{Max}}\, E_0 \int_0^\infty e^{-r(1-\tau)t}\{p(t)F[K(t)] - [q(t)I(t) + c(t)C(I(t))] - \text{Tax}(t)\}dt \tag{2.1}$$

subject to,

$$dp(t)/p(t) = (\mu_p - \lambda\beta_p)dt + \sigma_p dz_p(t) \tag{2.2}$$

$$dq(t)/q(t) = (\mu_q - \lambda\beta_q)dt + \sigma_q dz_q(t) \tag{2.3}$$

$$dc(t)/c(t) = \mu_c dt \tag{2.4}$$

$$dK(t) = [I(t) - \delta K(t)]dt \tag{2.5}$$

$$\text{Tax}\ (t) = \tau\{F[K(t)] - D(t)\}. \tag{2.6}$$

The value function of the firm at the current time 0 is $V(0)$, where E_0 is the expectations operator conditional on information at time 0. The risk-free interest rate, which is assumed to be constant over time, is r. The firm's (concave) production function is $F[K(t)]$, where $K(t)$ is the amount of capital in place at time t. The supply price of a unit of uninstalled capital is $q(t)$ and the level of gross investment is $I(t)$. The adjustment costs required to make $I(t)$ units of capital "usable" at time t are represented by the convex function $c(t)C[I(t)]$, where $C'[I(t)] > 0$ for $I(t) > 0$, $C''[I(t)] > 0$ and $C(0) = 0$; $c(t)$ is a term representing the autonomous change in adjustment costs over time. In general adjustment costs may be either internal or external to the firm and may take many forms – installation costs, resources diverted from other productive activities to plan and activate investments, monopsony power in the capital input market, etc. The type of adjustment costs considered here may best be thought of as internal installation costs. Thus the function $c(t)C[I(t)]$ is separable from the production function and depends only on the gross level of investment. This corresponds to Gould's [1968] formulation. This form was chosen so as to simplify the analysis, as it gives rise to simpler (but still complex) neutrality conditions than a more general formulation. If adjustment costs for the firm's capital did not exist, the firm's dynamic optimization problem would be uninteresting, because capital could be adjusted instantaneously to its optimal level. Of course in reality capital cannot be costlessly adjusted, but rather machinery, equipment and buildings must be installed and uninstalled, which takes time and money. It is assumed that capital is fully reversible, and so can be sold at its full replacement value in well functioning second-hand markets (in terms of the above notation, uninstalled capital can be both bought and sold for a unit price of $q(t)$). In this case, the adjustment cost function must be convex to be meaningful, as a linear function would imply instantaneous adjustment. An alternative, more complicated, specification would involve irreversible capital and linear adjustment costs. An emerging literature has investigated investment decisions under this specification, however there have been few examinations of tax effects in this type of model.[7]

Eqs. (2.2) and (2.3) are the modified equations of motion for the stochastic state variables, p and q. As discussed above, investor risk aversion is incorporated into the model by subtracting a systematic risk premium, $\lambda\beta_i$, from the expected growth rates; where $\lambda = (ER_m - r)$ is the expected excess return on the market over the risk-free interest rate and β_i is the ICAPM "beta" for state variable i, $i = p, q$.[8] Note that the expectations operator, E_o, is conditioned on the modified stochastic processes rather than the original equations of motion in equation set (1). Eq. (2.4) describes the movement of the autonomous adjustment cost parameter c over time, assumed for simplicity to be deterministic. Eq. (2.5) describes the accumulation of physical capital, where δ is the constant proportional physical rate of depreciation.[9]

Eq. (2.6) defines the firm's tax liability at time t. The tax rate is τ, which is assumed to be constant over time. The tax base consists of the gross revenues of the firm, $p(t)F[K(t)]$, less a depreciation allowance, $D(t)$. The depreciation deduction is

7 Exceptions are Mackie-Mason [1990] and McKenzie [1992, 1993].

8 $\beta_i = \sigma_{im}/\sigma_m^2$, where σ_{im} is the covariance between the market and state variable i, and σ_m^2 is the variance of the market portfolio.

9 It is also implicitly assumed that the government is no better at spreading risks than are individuals, and that all government revenues are returned to individuals in stochastic (lump sum) transfers. Thus, the imposition of the income tax does not alter the aggregate risk borne by the private sector. This type of assumption is common in most studies of the impact of taxation upon risk taking, either implicitly or explicitly. See, for example, Bulow and Summers [1984], Mintz [1981], Gordon [1985], and Gordon and Wilson [1989]. Moreover, by specifying prices and the pre-tax interest rate exogenously, I abstract from the general equilibrium effects of income taxation under uncertainty. This is consistent with the original Johansson- Samuelson analysis, as well as with the more recent analysis of Fane [1987].

assumed to be determined on an *ex ante* declining balance basis, using the original cost of the firm's capital and not its actual replacement cost. Allowance is made for the fact that the tax depreciation rate may change over time. Specifically, the gross investment expenditure undertaken at time s is written-off at the rate $\alpha(s)$; this rate applies in perpetuity to the investment expenditure undertaken at time s, but may change for investments made in the future.[10] Therefore, at time $s+u$, $u>0$, the depreciation deduction attributable to each dollar of investment spending undertaken at time s is $\alpha(s)e^{-(u-s)\alpha(s)}$. The total depreciation deduction at time t for all investment expenditures undertaken up to that time is:

$$D(t) = \int_0^t \alpha(s)\{q(s)I(s) + c(s)C[I(s)]\}e^{-\alpha(s)(t-s)}ds.$$

Note that the total cost of investment, the basic purchase cost plus adjustment (or installation) costs, is depreciated for tax purposes; it is easy to show that this is the correct treatment of adjustment costs under the type of income tax considered here.[11] It is important to stress that although the tax depreciation rate which applies to new investments is allowed to fluctuate over time, the rate associated with previous investment expenditures does not change; therefore, $\alpha(s,u)$, the allowed depreciation rate at $s+u$ for an investment undertaken at time s, is independent of u, but not of s : $\alpha(s,u)=\alpha(s)$ for $u>0$. Since the interest rate, r, and the tax rate, τ, are assumed to be constant, as is the tax depreciation rate for time $s, \alpha(s)$, once announced, the present value of the *ex ante* tax depreciation deductions on a dollars worth of gross investment expenditure undertaken at time s is

$$Z(s) = \alpha(s)/[r(1-\tau)+\alpha(s)]. \qquad (2.7)$$

The tax system described above thus effectively allows the firm to expense a fraction $Z(s)$ of gross investment spending at time s, therefore the problem faced by the firm may be written as

$$V(0) = \operatorname*{Max}_{\{I(t)\}} E_0 \int_0^\infty e^{-r(1-\tau)t}\{p(t)F[K(t)](1-\tau) - [q(t)I(t)+c(t)C(I(t))](1-\tau Z(t))\}dt \qquad (3)$$

10 This sort of "grandfathering" is in fact a common feature of many tax regimes, where changes in tax depreciation rates apply only to new investments, with old investments continuing to be written-off at the old rates.

11 This is one reason for using the separable adjustment cost function. If a more general non-separable formulation were used, adjustment costs would have to be expensed as they could not be separated out and depreciated. Most tax codes require identifiable installation costs to be depreciated.

subject to (2.2) - (2.5).

The parameters set by the tax authority are τ and $\alpha(s)$. In the next section, values for these tax parameters which will ensure the neutrality of the income tax with respect to the firm's investment decisions will be determined. Notationally it turns out to be easier to focus upon the neutral $Z(s)$ rather than $\alpha(s)$, with eq. (2.7) allowing the neutral tax depreciation rate to be recovered from the neutral present value deduction.

III. THE NEUTRALITY OF AN EX ANTE INCOME TAX

Investment is chosen at each instant in time so as to maximize the value of the firm to its risk averse owners. Pontryagin's Stochastic Maximum Principle gives the following necessary conditions for an optimum:[12]

$$V_K(t) = q^G(t)[1 - \tau Z(t)] \tag{4.1}$$

$$V_K(t)r(1-\tau) = p(t)(1-\tau)F_K[K(t)] - \delta V_K(t) + (1/dt)E_t dV_K(t) \tag{4.2}$$

where,

$$q^G(t) \equiv q(t) + c(t)C'[I(t)]. \tag{4.3}$$

The marginal price of a unit of installed capital, q^G, is the sum of the basic purchase price of a unit of uninstalled capital and marginal adjustment costs.

Eq. (4.1) is the usual optimality condition. Investment is undertaken at each instant in time so as to equate the shadow value of a unit of installed capital, V_K, to its marginal cost, $q^G(1-\tau Z)$, which is expressed net of the present value of the tax savings arising from the depreciation deductions.

Eq. (4.2) is an equilibrium condition expressing the equality between the after-tax risk-free return on an additional unit of installed capital (the left hand side), and its expected return (the right hand side). The expected return on a unit of installed capital is equal to the contemporaneous after-tax marginal revenue product, $p(1-\tau)F_K(K)$, less the value of physical depreciation, δV_K, plus the expected increase in the value of an additional unit of installed capital per-unit-time, $(1/dt)E_t dV_K$. Substituting (4.1) into (4.2), stochastically differentiating (4.1) and substituting the result into (4.2), and rearranging gives,

$$F_K[K(t)] = \frac{q^G(t)}{p(t)}\Big\{\{r(1-\tau) + \delta - (1/dt)E_t[dp^G(t)/q^G(t)]\} \left[\frac{1-\tau Z(t)}{1-\tau}\right] + \frac{\tau}{1-\tau}(1/dt)E_t dZ(t)\Big\}. \tag{5}$$

[12] With a concave production function and convex adjustment costs, these are also sufficient. Subscripts on small or Greek letters are an index, while subscripts on capital letters denote partial derivatives. $(1/dt)E_t d(\cdot)$ is Îto's differential operator.

Eq. (5) is the stochastic analog of the familiar condition where the firm undertakes investment at each instant in time so as to equate the contemporaneous marginal product of capital to its user cost. It implicitly defines an optimal investment function, as the marginal cost of installed capital, q^G, depends upon the level of investment. Note that in developing (5), allowance is made for the fact that $\alpha(t)$ (and thus $Z(t)$) may vary over time in a stochastic fashion.

Using eq. (5), I may now establish the following proposition:

PROPOSISION 1: For given prices and the risk-free interest rate, the income tax is neutral with respect to the investment decision of the firm if the present value of the *ex ante* tax depreciation deductions on a dollars worth of gross investment expenditure at each instant in time satisfies the following differential equation:

$$\text{(P.1)} \qquad Z(t) = \frac{\delta - (1/dt)E_t[dq^G(t)/q^G(t)]}{r(1-\tau) + \delta - (1/dt)E_t[dq^G(t)/q^G(t)]} + \frac{(1/dt)E_t dZ(t)}{r(1-\tau) + \delta - (1/dt)E_t[dq^G(t)/q^G(t)]}$$

where,

$$\text{(P.2)} \qquad \delta - (1/dt)E_t[dq^G(t)/q^G(t)] = \delta - \mu - [F(t) + G(t) + H(t)]$$

is the *ex ante* economic rate of depreciation, and

$$\text{(P.3)} \qquad F(t) \equiv \frac{c(t)C''[I(t)]}{q^G(t)}(1/dt)E_t dI(t)$$

$$\text{(P.4)} \qquad G(t) \equiv -[q(t)/q^G(t)]\lambda\beta_q$$

$$\text{(P.5)} \qquad H(t) \equiv (1/2)[c(t)C'''(I(t))/q^G(t)][I_p^2(t)p(t)^2\sigma_p^2 + I_q^2(t)q(t)^2\sigma_q^2].$$

Proof: To see that the neutral $Z(t)$ satisfies (P.1), substitute (P.1) into (5) and note that the resulting optimality condition for investment is independent of the tax parameters. For a derivation of (P.3) - (P.5), see the Appendix.[13]

[13] To obtain (P.2) it is assumed for simplicity that $\mu_q = \mu_c = \mu$ (see the Appendix for the general case). Also, general price inflation has been ignored in the development of the model. It may easily be incorporated into the analysis without changing the nature of the conclusions. In the presence of inflation the expected income stream should be discounted by the real after-tax risk-free rate of interest and the depreciation allowances should be indexed for inflation. It should also be noted that the conditions stated in Proposition 1 are sufficient, not necessary, for neutrality.

Without parameterizing the production and adjustment cost functions, the differential eq. (P.1) cannot generally be solved. Nevertheless, some intuition may be gained for the result by noting that the neutral depreciation deduction is composed of two terms. Consider the first term on the right hand side of (P.1). In a deterministic model without adjustment cost, the neutral present value depreciation deduction is a constant which satisfies $Z = \alpha/[r(1-\tau)+\alpha]$, with α set equal to $\delta - \dot{q}/q = \delta - \mu$. This is the well known Johansson-Samuelson neutrality requirement that the *ex ante* tax depreciation rate equal the economic rate of depreciation, which in this case is simply the physical rate of depreciation less rate of capital gain on the firm's assets. In the presence of adjustment costs and risk, the stochastic analog to the economic rate of depreciation, I call it the *ex ante* economic rate of depreciation, is $\delta - (1/dt)E_t[dq^G(t)/q^G(t)]$, which is given in (P.2) and differs from the traditional expression due to the terms $F(t)$, $G(t)$, and $H(t)$. These additional terms will be discussed in detail below, however note at this point that the first term in (P.1) incorporates the idea that the neutral *ex ante* tax depreciation rate must reflect the *ex ante* economic depreciation rate: indeed, if the second term in (P.1) is zero, in the spirit of Johansson and Samuelson neutrality would require that the *ex ante* tax depreciation rate equal the *ex ante* economic depreciation rate exactly, or $\alpha(t) = \delta - (1/dt)E_t[dq^G(t)/q^G(t)]$.

In general the second term in (P.1) is not equal to zero. The term $(1/dt)E_t dZ(t)$ is the mean change per-unit-time in the present value of the *ex ante* depreciation deductions. From eqs. (P.3) - (P.5), it is evident that the *ex ante* economic rate of depreciation depends upon the contemporaneous values of $q(t)$ and $p(t)$, and on the level of investment, $I(t)$, all of which evolve stochastically over time. This, of course, means that the neutral $Z(t)$ also evolves stochastically over time. In general $Z(t)$ will have a systematic trend and a random element. Using the modified equations of motion for the state variables expressed in equation set (2), and expanding using the Ito differential operator, the mean change per-unit-time in $Z(t)$ is:

$$(1/dt)E_t dZ(t) = \sum_i Z_i(t)i(t)(\mu - \lambda\beta_i) + Z_K(t)[I(t) - \delta K(t)]$$
$$+ (1/2)[Z_{pp}(t)p(t)^2\sigma_p^2 + Z_{qq}(t)q(t)^2\sigma_q^2], \quad i = p, q, c.$$

If, for example, $Z(t)$ were expected to increase from time t to time $t+u$, then the firm would have an incentive to postpone its investment expenditure from t to $t+u$ in order to get the higher deduction. In order for the tax system to be neutral with respect to the investment decision, an upward adjustment must be made to the current present value depreciation deduction to take account of this; this is the reason for the second term in (P.1).

From Proposition 1 it is evident that the neutral *ex ante* tax depreciation deduction in the presence of adjustment costs and income and capital risk is significantly more complicated than that obtained in the traditional deterministic paradigm which ignores adjustment costs. In particular, the neutral present value depreciation deduction (and therefore the neutral *ex ante* tax depreciation rate) evolves stochastically over time, and reflects a number of parameters which characterize the risk of the firm's investments. An important element of the depreciation deduction is the *ex ante* economic rate of depreciation, (P.2). As mentioned above, the *ex ante* economic depreciation rate differs from its more familiar deterministic counterpart in the inclusion of the terms $F(t)$, $G(t)$, and $H(t)$. Each of these terms will be discussed in turn.

Component F arises due to the presence of the convex adjustment costs. It requires that along with subtracting the expected growth rate in the supply price of capital from the physical depreciation rate, the change in marginal adjustment costs arising from changes in the level of investment must also be netted out. The economic rate of depreciation thus reflects the change in the marginal price of a unit of installed capital, gross of adjustment costs, rather than just the growth in the basic price of uninstalled capital. Abel [1983b] has established this in a deterministic setting. Since the adjustment cost function is convex, if investment is growing component F is negative and the *ex ante* economic rate of depreciation is higher than it would be in the absence of adjustment costs.

Component G reflects the systematic capital risk premium. The *expected* rate of capital gain on holding a unit of capital is μ. If μ is positive, the expectation that the supply price of capital will increase lowers the *ex ante* economic rate of depreciation as the capital the firm has on hand becomes more valuable. But, the purchase price of uninstalled capital is stochastic, which gives rise to an added cost of holding the capital if the owners are risk averse. Under the ICAPM assumption, the imputed cost of bearing capital risk is measured by $\lambda\beta_q$, which reflects the systematic risk associated with the purchase price of the capital. An income tax must allow for the deduction of this additional cost of holding capital. The use of historical depreciation for tax purposes means that the depreciation allowances do not fluctuate with changes in the replacement price of capital. Therefore, the cost of bearing capital risk must be deducted explicitly, in the form of an adjustment to the tax depreciation rate. If β_q is positive, the *ex ante* economic rate of depreciation must be adjusted upwards. This means that assets which are risky in a systematic capital risk sense require a higher *ex ante* tax write-off than do comparable riskless assets. This is the Bulow and Summers [1984] result.[14]

It is important to note that component G reflects capital risk only, with no allowance for income risk. In the conventional paradigm, which ignores adjustment costs, the standard explanation for this is that with full loss offsets taxes fluctuate perfectly with income. This means that there is an implicit deduction for the imputed cost of bearing income risk and no explicit deduction is required to maintain the neutrality

[14] See also Jog and Mintz [1989] and Auerbach [1985].

of the tax system. When adjustment costs exist, however, an explicit allowance must be made for income risk for different reasons. This is evident from component H, which reflects both capital and income risk of an unsystematic nature. Component H arises due to the *concurrent* presence of adjustment costs and risk. Its sign is the same as that of $C'''(I)$ – if marginal adjustment costs are strictly convex (concave) then H is positive (negative). To gain some insight into the meaning of this term, assume that the marginal adjustment cost schedule is convex and that the only source of uncertainty is income risk.[15] Increases in investment in response to increases in p then raise marginal adjustment costs by more than decreases in investment of the same magnitude lower them. Therefore, even when price disturbances are expected to average out to zero over time, the increases in marginal adjustment costs when firms adjust investment upwards will, on average, more than outweigh the decreases when investment falls. Responding to unanticipated price movements which balance out over time thus results in higher adjustment costs over time. The firm can reduce these costs by holding more capital, therefore making fewer upward adjustments in its capital stock. This added benefit to holding capital, a sort of precautionary benefit, can be viewed as a reduction in the *ex ante* economic depreciation rate, as the firm's existing capital is more valuable. This leads to a reduction in the neutral *ex ante* tax depreciation rate. Of course the opposite is true if marginal adjustment costs are concave (i.e., $C'''(I) < 0$).

In general, the neutral *ex ante* tax depreciation rate must embody all three of the alterations to the economic rate of depreciation arising from adjustment costs and risk. Component F is consistent with the literature on taxation and adjustment costs, G with the literature on taxation in a risky environment, while H is due to the interaction between adjustment costs and risk. Component H is particularly important, as it means that the *concurrent* presence of adjustment costs and income and capital risk results in alterations to the *ex ante* economic rate of depreciation over and above those which occur if adjustment costs or risk are considered on their own. A key implication is that, in general, costs associated with both capital and income risk must be reflected in the neutral *ex ante* tax depreciation deduction. This provides an interesting contrast to Mintz [1981, 1982] and Bulow and Summers [1984] who ignored adjustment costs and derived the result than a neutral *ex ante* income tax system need make no allowance for income risk, so long as there are full loss offsets.

If investors are risk neutral ($\beta_q = 0$), the terms involving systematic risk (G) disappear, but the terms involving unsystematic risk (H) remain. This is because there still remains an additional benefit to holding more (less) capital in a risky setting if marginal adjustment costs are convex (concave). Again contrary to the prevailing wisdom, even if the owners of the firm are risk neutral the neutrality of an *ex ante*

15 See also Pindyck [1982].

income tax requires that the (unsystematic) risk characteristics of the investment be reflected in the tax depreciation rate.

If the marginal adjustment cost schedule is linear, which would be the case if adjustment costs were quadratic, component H disappears. In this case increases in investment in response to price movements raise *marginal* adjustment costs by the same amount that decreases in investment of the same magnitude lower them, and there would be no added benefit to holding capital in a stochastic environment where price shocks balance out over time. The following corollary to Proposition 1 establishes that even if marginal adjustment costs are linear, the co-existence of adjustment costs and uncertainty results in alterations to the *ex ante* rate of economic depreciation which are not present in the deterministic case.

COROLLARY 1: The *ex ante* economic rate of depreciation, and thus the neutral *ex ante* tax depreciation deduction, depends upon systematic and unsystematic capital and income risk even when marginal adjustment costs are linear.

Proof: The optimal level of investment at any instant in time is a function of the contemporaneous values of the state variables, some of which are stochastic. Applying Ito's Lemma gives:

$$dI(t) = I_K(t)dK(t) + \sum_i I_i(t)di(t) + (1/2)\sum_i \sum_j I_{ij}(t)di(t)dj(t)$$
$$+ (1/2)I_{KK}(t)[dK(t)]^2 + \sum_i I_{Ki}(t)[dK(t)][di(t)], \quad i,j = p,q,c$$

which implies that,

$$\text{(C)} \qquad (1/dt)E_t dI(t) = I_K(t)[I(t) - \delta K(t)] + \sum_i I_i(t)i(t)(\mu - \lambda\beta_i)$$
$$+ (1/2)\sum_i I_{ii}(t)i(t)^2\sigma_i^2, \quad i = p,q,c.$$

Substituting (C) into (P.3), we see that the *ex ante* economic rate of depreciation, and thus the neutral $Z(t)$, depends upon systematic and unsystematic income and capital risk even if $C'''(I) = 0$.

The marginal cost of installation depends upon the optimal response of investment to changes in the state variables. Eq. (C) shows that investment responds differently to changes in the state variables in the presence of uncertainty than in the deterministic case. The differences reflect the impact of uncertainty on the firm's expected investment profile. Eq. (C) captures the impact of both systematic and unsystematic income and capital risk on the investment profile, and hence on marginal adjustment costs and the *ex ante* economic rate of depreciation.

III. A. An Example

A particularly important implication of the above analysis, and one that contrasts sharply with previous literature, is that in the presence of adjustment costs the neutral *ex ante* tax depreciation rate must reflect the systematic and unsystematic income risk characteristics of the investment, even with full loss offsets. A simple example will clarify this point. Consider a constant returns to scale Cobb-Douglas production function:

$$F[K(t), L(t)] = L(t)^{\omega} K(t)^{(1-\omega)}, \quad \omega < 1 \tag{6.1}$$

where $L(t)$ denotes the amount of a current input (labor).

Assume the investment cost function is of the constant elasticity form,

$$C[I(t)] = c(t)I(t)^{\eta}, \quad \eta > 1. \tag{6.2}$$

If the output price, p, and the investment cost function parameter, c, follow diffusion processes like those in equation set (1),[16] and the tax system is as depicted in (2.6) (with labor costs expensed), using stochastic dynamic programming it is possible to determine the optimal investment function at time t as:[17]

$$I(t) = \left[\frac{p(t)F_K(t)}{[r(1-\tau)+\delta-\mu+\gamma]}\frac{(1-\tau)}{(1-\tau Z)}\right]^{1/(\eta-1)} \tag{7.1}$$

where F_K is the marginal product of capital and,

$$\gamma \equiv \frac{\lambda\beta_p\eta}{(\eta-1)(1-\omega)} + \frac{\omega\sigma_p^2}{2(1-\omega)^2}. \tag{7.2}$$

Here, it is assumed that $Z = \alpha/[r(1-\tau)+\alpha]$ is non-stochastic and constant. It turns out that for these functional forms the neutral *ex ante* tax depreciation rate is in fact time invariant, which can be confirmed by noting that $I(t)$ is unaffected by the parameters of the tax system if the present value of the depreciation deductions is set equal to:

$$Z = [\delta-\mu+\gamma]/[r(1-\tau)+\delta-\mu+\gamma] \tag{8}$$

[16] As in the general analysis income risk arises from the stochastic movement of the output price. In this example capital risk arises from the stochastic motion of the cost parameter c. No distinction is made between the basic cost of uninstalled capital and adjustment costs, with the total cost of investing in I units of capital given by the convex function in (6.2).

[17] Abel [1983a] solves the problem for this parameterization with no taxes assuming risk neutrality. It is straightforward to extend his approach to incorporate risk aversion modeled via the ICAPM and the type of income tax regime considered here.

which corresponds to setting $\alpha = \delta - \mu + \gamma$.

This example highlights a number of features from the preceding analysis. As discussed, the neutral *ex ante* tax depreciation rate incorporates both the systematic and unsystematic income risk characteristics of the investment. To see the potential importance of this, consider the following parameter values: $\delta = .15$, $\mu = 0$, $\lambda = .07$, $\eta = 2$, $\beta_p = 1$, $\omega = .3$, $\sigma_p^2 = .05$. For these values the neutral *ex ante* tax depreciation rate is approximately 36.53%. If the income risk parameters were ignored in setting the "neutral" tax depreciation rate, α would be set to 15%. Interestingly, for this particular parameterization the *ex ante* economic depreciation rate does not reflect the capital risk characteristics of the investment and is not stochastic. Also note that if the investment cost function is a quadratic, with $\eta = 2$, the adjustments for income risk remain. Moreover, even if investors are risk neutral ($\beta_p = 0$), the neutral tax depreciation rate must still incorporate an adjustment for unsystematic income risk.

IV. CONCLUSION

This paper has analyzed the neutrality of a full loss offset income tax which grants *ex ante* depreciation allowances in the presence of convex adjustment costs and income and capital risk. It established that, in general, the neutral *ex ante* tax depreciation rate varies over time in a stochastic fashion, and reflects both the income and capital risk characteristics of the firm's investments. This is in sharp contrast to much of the prevailing literature which, ignoring adjustment costs, suggests that the neutral *ex ante* depreciation deduction need only make an allowance for capital risk, with no need to take account of income risk so long as there are full loss offsets.

By highlighting the ways in which existing income tax regimes which grant *ex ante* depreciation allowances may deviate from neutrality in the presence of adjustment costs and risk, the analysis has important implications for tax policy. Although the normative merits of neutral income taxation were not addressed, it seems important to at least establish a benchmark from which one might choose to deviate. The results are particularly important in the light of recent tax reform efforts in many countries. The reforms were motivated in part by a desire to design more "neutral" tax systems, with the setting of *ex ante* tax depreciation rates more in line with economic rates an important element of this.

The results can also be viewed as yet another "nail in the coffin" of the imputed profits approach to taxation. The informational problems associated with determining the neutral *ex ante* depreciation allowance in a stochastic environment with adjustment costs are formidable to say the least, depending upon the risk characteristics of the investment and the specific form of the adjustment cost and production functions. Such a system would clearly be impossible to implement perfectly in practice. Taxing economic income *ex post* (as in Fane [1987]) also poses serious measurement problems, as market valuations would be required for all assets in the economy. In contrast, the information required to implement a neutral cash-flow tax are minimal, even in an environment with adjustment costs and risk.[18]

APPENDIX

Proof of Proposition 1

The expected change per-unit-time in the marginal cost of installed capital is (dropping time indices):

$$(1/dt)E_t dq^G = (1/dt)E_t dq + C'(I)(1/dt)E_t dc + c(1/dt)E_t dC'(I). \tag{A1}$$

Using the modified equation of motion for q, (2.3), the expected change per-unit-time in the purchase price of a new unit of uninstalled capital is:

$$(1/dt)E_t dq = q(\mu_q - \lambda\beta_q). \tag{A2}$$

The second and third terms in (A1) are the expected change per-unit-time in marginal adjustment costs. Since c is assumed to follow a deterministic path, the second term may be written simply as:

$$C'(I)(1/dt)E_t dc = cC'(I)\mu_c. \tag{A3}$$

Expanding $dC'(I)$, ignoring higher order terms which vanish as dt becomes small, the third term in (A1) is:

$$c(1/dt)E_t dC'(I) = cC''(I)(1/dt)E_t dI + (1/2)cC'''(I)(1/dt)E_t(dI)^2. \tag{A4}$$

Eq. (A4) may be re-written by recalling that the optimal I at any instant in time is a function of time and the contemporaneous values of the state variables. Applying Ito's Lemma gives:

$$\begin{aligned} dI(p,q,c,K) = {} & I_K dK + \sum_i I_i di + (1/2)\sum_i \sum_j I_{ij} di dj \\ & + (1/2) I_{KK}(dK)^2 + \sum_i I_{Ki}(dK)(di), \quad i,j = p,q,c \end{aligned} \tag{A5}$$

which implies that,

$$(1/dt)E_t(dI)^2 = \sum_i I_i^2 i^2 \sigma_i^2, \quad i = p,q. \tag{A6}$$

Substituting (A6) into (A4) yields:

$$c(1/dt)E_t dC'(I) = cC''(I)(1/dt)E_t dI + (1/2)cC'''(I)\sum_i I_i^2 i^2 \sigma_i^2, \quad i = p,q \tag{A7}$$

[18] See, for example, Boadway and Bruce [1984] and Fane [1987].

(A1), (A2), (A3), and (A7) may be combined to give:

(A8) $$(1/dt)E_t(dq^G/q^G) = (q/q^G)\mu_q + (c/q^G)C'(I)\mu_c + (c/q^G)C''(I)(1/dt)E_t dI - (q/q^G)\lambda\beta_q + (1/2)(c/q^G)C'''(I)\sum_i I_i^2 i^2 \sigma_i^2, \quad i = p, q.$$

For simplicity it is assumed that $\mu_q = \mu_c = \mu$, which gives eqs. (P.2) - (P.5) in the text.

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