The implications of risk and irreversibility for the measurement of marginal effective tax rates on capital

KENNETH J. McKENZIE University of Calgary

Abstract. The implications of risk and irreversibility for the measurement of marginal effective tax rates (METR) on capital are examined. It is shown that when capital is irreversible, the METR is an increasing function of systematic and unsystematic, capital and income risk. The tax system may thus distort investments in risky capital to a much greater extent than is implied by previous research that ignored irreversibilities. METR calculations based upon the Canadian corporate tax system are provided.

Les conséquences du risque et de l'irréversibilité pour la mesure des taux marginaux effectifs de taxation sur le capital. L'auteur examine les conséquences du risque et de l'irréversibilité pour la mesure des taux marginaux effectifs de taxation sur le capital (TMET). On montre que quand le capital est irréversible, les TMET sont une fonction croissante du risque systématique et non-systématique attaché au capital et au revenu. Le système fiscal peut donc distorsionner les investissements dans le capital à risque d'une façon beaucoup plus importante qu'on a pu le suggérer dans les études antérieures qui ont ignoré les irréversibilités. On fournit des calculs de TMET fondés sur le système canadien de fiscalité des sociétés.

I. INTRODUCTION

The concept of the cost of capital has played an important role in the modelling of tax policy and investment behaviour. Recently, the concept has assumed a central role in the tax reform process through the closely related idea of the marginal effective tax rate (METR) on capital. The METR provides a summary measure of the cumulative tax distortion on a marginal investment decision. Calculations of METRs are widely used as an indication of the extent to which the tax system may

I wish to thank Jack Mintz, Ron Kneebone, an anonymous referee, and seminar participants at the University of Calgary and the University of Alberta for helpful comments and suggestions.

Canadian Journal of Economics Revue canadienne d'Economique, XXVII, No. 3
August août 1994. Printed in Canada Imprimé au Canada

0008-4085 / 94 / 604–19 $1.50

inpinge upon the investment decisions of firms.[1] Indeed, the concept is now so well accepted that governments routinely use METR calculations as an important input in the formulation of tax policy (see, e.g., Department of Finance 1987).

The theory underlying the calculation of METRS is the partial equilibrium, dynamic neoclassical investment model of Jorgenson (1963). This model may be used to generate an expression for the tax-adjusted user cost of capital, which may in turn be used to determine the METR. Early studies using this approach assumed perfect certainty or incorporated risk in an ad hoc manner. More recently, some researchers have attempted to examine the implications of risk for the measurement of tax distortions. Jog and Mintz (1989), for example, show that depending upon its type the presence of risk can dramatically increase the METR on capital, implying that tax systems discourage firms from undertaking some types of risky investments.

While the incorporation of uncertainty has done much to increase our understanding of the nature of the distortions caused by the tax system, other potentially important considerations have been ignored. In particular, the public finance literature has tended to assume that investments are fully and costlessly reversible. This characterization may be inappropriate for some types of capital. For example, a great deal of capital is task or industry specific and is often so specialized as to be valuable only if used in a certain type of production. If this is the case, the conversion of capital to alternative uses is extremely costly, if not impossible – capital is irreversible. No research has been undertaken to determine the implications of irreversibility for the measurement of METRS. Given that METRS have become the 'standard' way of measuring tax distortions, this is a potentially serious shortcoming of the existing literature.

A number of papers analyse the implications of irreversibility for discrete investments made under uncertainty, including Brennan and Schwartz (1985), McDonald and Siegel (1985 and 1986), Pindyck (1988), and Dixit (1989). Pindyck (1991) provides a recent review of this research. In one of the few explicit considerations of taxes in these models, Mackie-Mason (1990) examines the role that non-linear taxes may play in influencing investment decisions of this type.

The problem with analysing tax distortions using the approach adopted in these papers is that they consider 'once-off' investments, examining a firm's decision to invest in a specific project. Although they provide some valuable insights, they are not true dynamic models of investment in the Jorgenson sense, and the results are therefore not directly comparable to the traditional literature. More recently, Pindyck (1988), Bertola (1988), and Bertola and Caballero (1991) have examined incremental irreversible investment decisions under uncertainty. In particular, Bertola (1988) and Bertola and Caballero (1991) describe an approach that generalizes the derivation of the Jorgenson user cost of capital to an environment with

1 The literature is extensive. Just a few examples are Auerbach (1983), King and Fullerton (1984), Boadway, Bruce, and Mintz (1984), Daly and Jung (1987), and McKenzie and Mintz (1992). Boadway (1987) provides a review of the theory and measurement of effective tax rates.

irreversible capital and risk. In this paper I use this approach to examine the implications of irreversibility and different types of risk for the measurement of METRS. METRS are calculated for Canada that are directly comparable to calculations made under the more common assumption of full certainty and reversibility.

The results suggest that the failure properly to account for risk and irreversibility has important implications for the measurement of tax distortions. In particular, the METR on irreversible capital is higher than that on fully reversible capital, the magnitude depending upon the level and type of risk. Previous studies of risk and taxation that ignore irreversibility, such as Jog and Mintz (1989), conclude that under full loss offsetting the METR is an increasing function of systematic capital risk, but is invariant to other types of risk. I show that if capital is irrversible, the METR depends upon both systematic and unsystematic, income and capital risk – and is increasing in all four types of risk. The implication is that to the extent that capital is inflexible in its use, the tax system may discourage investment in risky capital to a greater degree than was previously thought. As such, tax policy formulated on the basis of METRS measured in the 'standard' way may be misguided.

In section II the basic framework of the model is set up and the solution to the firm's investment problem is presented and discussed. In section III an expression for the METR on capital is developed and illustrative estimates are presented for Canada under a number of assumptions regarding the reversibility of capital and level and type of risk. Section IV is devoted to comparative statics, where the impact of changes in various risk parameters on the METR is explored. Section V contains a summary and conclusions.

II. BASIC FRAMEWORK

With a few modifications the basic model is identical to that described in Bertola (1988) and Bertola and Caballero (1991), augmented to include taxation. Since my interest is in applying this approach to an examination of the implications of risk and irreversibility for the measurement of METRS, I shall only briefly outline the basic framework of the model and identify the changes that must be made to account for taxes.

The technology of an all equity financed firm is given by a homogeneous Cobb-Douglas production function:

$$Q(t) = [L(t)^{\alpha}K(t)^{1-\alpha}]^{\phi}, \qquad 0 < \alpha < 1,\ \phi > 0, \tag{1}$$

where $Q(t)$ is the firm's output, $L(t)$ is the amount of labour employed, $K(t)$ is the amount of capital installed at time t, α is labour's share of total costs, and ϕ is a returns-to-scale parameter, with $\phi >=< 1$ representing increasing, constant or decreasing returns to scale.

The firm faces a constant elasticity demand function for its output:

$$B(t) = D(t)Q(t)^{\mu-1}, \qquad 0 < \mu\phi < 1, \tag{2}$$

where $B(t)$ is the price of the firm's output at time s, and $D(t)$ is a shift parameter for the demand curve. The price elasticity of demand is constant at $1/(\mu - 1)$.

The firm's after-tax instantaneous operating profits are given by

$$\Pi^T(t) = (1 - \tau)\big(B(t)Q(t) - w(t)L(t)\big), \tag{3}$$

where τ is the corporate tax rate and $w(t)$ the cost of a unit of labour. Equation (3) reflects the fact that labour costs are deducted immediately for tax purposes, and it assumes that there are full loss offsets.[2]

The firm is assumed to be able to adjust labour instantaneously: for a given amount of capital it will do so in order to maximize (3) subject to (1) and (2). This gives a conditional labour demand function which can be substituted back into (3) and rearranged to get the following expression for after-tax operating profits:

$$\Pi^T(t) = (1 - \tau)\Pi(t) \tag{4.1}$$

$$\Pi(t) \equiv hK(t)^{\gamma} D(t)^{x_D} w(t)^{x_w} \tag{4.2}$$

$$\gamma \equiv \frac{(1 - \alpha)\phi\mu}{1 - \alpha\phi\mu}, \qquad 0 < \gamma < 1 \tag{4.3}$$

$$x_D \equiv \frac{1}{1 - \alpha\phi\mu} > 1 \tag{4.4}$$

$$x_w \equiv -\frac{\alpha\phi\mu}{1 - \alpha\phi\mu} < 0 \tag{4.5}$$

$\Pi(t)$ is the before-tax operating profit function. The parameter h is a constant that depends on α, μ, and ϕ.

Capital is assumed to have value only if used in production, and it is therefore completely irreversible. As such, the firm can instantaneously increase its capital stock at any point by paying a unit price $P(t)$, but it can disinvest only by allowing its capital to depreciate over time. As indicated above, the firm pays taxes on the flow of operating income, but these taxes are reduced by various credits and deductions associated with capital acquisitions. Let ψ denote the investment tax credit (ITC) rate and φ the declining balance tax depreciation rate. Tax depreciation allowances are determined ex ante, based on the original cost of the firm's assets. If the firm purchases a unit of capital at price $P(t)$, the ITC reduces its tax liability dollar by an amount $\psi P(t)$ at time t. The present value of the flow of tax depreciation deductions on the remaining $(1 - \psi)P(t)$ is $P(t)(1 - \psi)\tau\varphi/(r + \varphi)$, where r is the risk-free rate of interest. The assumption that there are full loss offsets means that the firm will be able to claim the depreciation deductions with certainty; as such, the flow of tax

2 The assumption of full loss offsetting is common in the METR literature. See Mintz (1988) for an example of how METRs may be calculated under imperfect loss offset. Also, although I ignore inflation in the development of the model, it is included in the empirical analysis of section III.

depreciation deductions is discounted by the risk-free interest rate.[3] The tax credits and deductions associated with the purchase of a unit of capital effectively lower its unit price, in present value terms, to $P(t)[1-\psi-(1-\psi)\tau\varphi/(r+\varphi)]$. Since the firm is assumed to be all equity financed, there are no deductions associated with debt finance.[4]

Bulow and Summers (1984) stress the importance of distinguishing between different types of uncertainty when the impact of taxes on risk taking is examined. In particular, they distinguish between two types of risk: uncertainty regarding the future level of the firm's operating income, referred to as *income risk*, and uncertainty regarding the replacement value of the firm's capital, referred to as *capital risk*. I assume that the source of income risk is uncertainty regarding the future values of the demand parameter $D(t)$, while capital risk involves uncertainty regarding the future unit price of capital $P(t)$.[5] This notion is formalized by assuming that these variables follow geometric Brownian motion in continuous time, which means that their current values are known and expected to grow at a constant rate, but that the growth rates fluctuate, becoming increasingly uncertain over time.

$$dP(t) = P(t)\vartheta_p ds + P(t)\sigma_p dW_p(t) \tag{5.1}$$

$$dD(t) = D(t)\vartheta_D ds + P(t)\sigma_D dW_D(t), \tag{5.2}$$

where ϑ_i is the drift in state variable $i = D, P, \sigma_i$ is its standard deviation, and dW_i is the increment in a standard Weiner process.[6] Wages, $w(t)$, are assumed to grow at the deterministic rate ϑ_w.

Given this framework, the firm's problem is to choose an investment program that maximizes its value, determined by the expected present value of its cash flows. The specification of the firm's discount rate is important in this regard. In general this is the rate of return required by the owners of the firm and reflects their degree of risk aversion, rate of time preference, etc. One possibility is simply to

3 Some earlier METR studies use a risk-adjusted rate to discount depreciation deductions under a full loss offset tax system (see, e.g., Boadway, Bruce, and Mintz 1984). As these deductions will be received with certainty, this system is clearly inappropriate.

4 The assumption of all equity finance is made because there is no generally accepted theory of firm financial behaviour conducive to the model used here. The deductibility of debt interest in most corporate tax regimes favours debt over equity finance, while the typically higher personal tax rates on interest favour equity over debt. Without some form of 'market imperfection,' corner solutions result where firms choose either all debt or all equity finance. If imperfections are introduced, it is not obvious what sort of asset pricing model one should use to determine firm value in an uncertain environment; for example, in the presence of bankruptcy costs and debt interest deductibility at the corporate level, the use of the CAPM would not be appropriate. This problem is 'assumed away' by consideration of the case of the all equity financed firm only.

5 It is straightforward to include other sources of income risk, such as uncertainty regarding future wage levels and labour productivity, and other sources of capital risk, such as uncertainty regarding the physical rate of depreciation in capital.

6 For simplicity it is assumed that the Weiner processes governing the stochastic movements in P and D are uncorrelated; it is straightforward to extend the analysis to allow them to co-vary. Doing so would not substantially alter the results. For a treatment of the basic mathematics of Weiner processes see Malliaris and Brock (1982).

assume that the owners of the firm are risk neutral, in which case the appropriate discount rate is the risk-free interest rate, r. Unfortunately, this assumption hides some effects which prove to be important in the analysis of tax distortions under uncertainty. The approach taken here is to follow Constantinides (1978), who shows that if we invoke the assumptions of Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAPM), capital market equilibrium requires that the firm value its investment program *as if* preferences were risk neutral, and *as if* the equations of motion for the stochastic state variables were modified by replacing the drift terms ϑ_i with $\vartheta_i - h_i$ for $i \equiv D, P$, where $h_i = \lambda\beta_i$ is a systematic (or market) risk adjustment with λ the expected return on the market portfolio in excess of the risk-free interest rate and β_i the ICAPM 'beta' for an asset with risk characteristics identical to state variable i. This amounts to discounting each component of the cash flow stream by its own risk-adjusted discount rate.[7]

With the above modifications the problem is similar to that posed in Bertola and Caballero (1991). Modifying their solution to account for these changes is straightforward and gives an investment rule that generalizes the more familiar Jorgenson neoclassical result: if at any point the gross of corporate tax rate of return on a marginal unit of capital is less than the *appropriately measured* tax-adjusted user cost of capital, let the capital stock depreciate, otherwise increase the capital stock instantaneously so as to maintain equality between the gross rate of return and the tax-adjusted user cost:

$$\frac{\partial\Pi(t)/\partial K(t)}{P(t)} \begin{cases} \leqq c\Gamma\ \forall t \\ = c\Gamma\ \forall t \text{ when gross investment is positive,} \end{cases} \tag{6.1}$$

where

$$c \equiv r + \delta - \vartheta_p + h_p + (1/2)\sigma^2 H \tag{6.2}$$

$$\Gamma \equiv [1 - \psi - (1 - \psi)\tau\psi/(r + \varphi)]/(1 - \tau), \tag{6.3}$$

and

$$\sigma^2 \equiv \sigma_p^2 + x_D^2\sigma_D^2 \tag{6.4}$$

$$H \equiv \frac{-a + [a^2 + b]^{(1/2)}}{\sigma^2} \tag{6.5}$$

$$a \equiv x_w\vartheta_w + x_D\vartheta_D - x_D h_D + (1/2)x_D(x_D - 1)\sigma_D^2 + \delta(1 - \gamma) \\ - \vartheta_p + h_p - (1/2)\sigma^2 \tag{6.6}$$

$$b \equiv 2[r + \delta - \vartheta_p + h_p]\sigma^2. \tag{6.7}$$

The term $c\Gamma$ is the tax-adjusted user cost of irreversible capital: c is the user cost of capital in the absence of taxes, defined in (6.2), and Γ is the tax adjustment, defined

7 See Mackie-Mason (1990) for an application of this approach in a model with corporate taxes.

in equation (6.3). As long as the present value of the tax credits and deductions is less than the purchase price of capital (which would typically be the case), $\Gamma > 1$ and the imposition of the corporate tax increases the cost of capital. Equations (6.4)–(6.7) define various terms contained in the user cost of capital expression.

The user cost of irreversible capital without taxes (c) differs in an important way from the more familiar neoclassical expression for fully reversible capital, $c^n = r + \delta - \vartheta_p + h_p$, owing to the term $(1/2)\sigma^2 H$, which reflects an adjustment for the cost of irreversibility. To understand the intuition behind this adjustment, recall that when capital is fully reversible, the standard neoclassical result is that the firm employs capital up to the point where the marginal unit earns just enough to cover its cost, c^n, which consists of the opportunity cost of financing the investment (r) plus the physical rate of depreciation on a unit of capital (δ) less the capital gain on a unit of capital (ϑ_p) plus a risk premium (h_p) to compensate risk-averse investors for the imputed cost of bearing systematic capital risk. In the conventional analysis, because capital is fully reversible and adjustment costs are linear, the capital stock can be instantaneously adjusted in response to movements in the stochastic state variables so that the equality between the rate of return on a marginal unit of capital and the user cost is maintained at each instant.

If capital is irreversible, when the firm is faced with an adverse movement in the state variables, it does not immediately lower its capital stock but allows it to depreciate at rate δ. When this is the case, the ability to delay an incremental investment has value in a risky environment. The firm implicity holds a continuum of 'real investment options,' each of which may be exercised by incrementally increasing its capital stock. Waiting to install a unit of capital is costly in that it delays the realization of operating profits, but it is beneficial in that it postpones the payment of the unit price and allows the firm to learn more about the evolution of the stochastic state variables. These effects are summarized in the term $(1/2)\sigma^2 H$, which represents the 'option value' associated with waiting to invest in a unit of irreversible capital (see Pindyck 1991). Part of the cost of investing currently in a unit of capital is the opportunity cost associated with exercising this option. To be profitable, an incremental unit of capital must cover this additional cost, which is reflected by an upward adjustment in the user cost of capital.[8] The addition to the user cost of capital due to irreversibility depends upon parameters representing four different 'types' of risk – unsystematic capital risk (σ_p), unsystematic income risk (σ_D), systematic capital risk (h_p), and systematic income risk (h_p).[9]

III. THE MARGINAL EFFECTIVE TAX RATE ON IRREVERSIBLE CAPITAL

Corporate taxes distort the firm's investment rule in two ways: the return on a marginal unit of capital is lowered because the operating income it generates is

8 Inspection of equation (6.5) confirms the $H > 0$; Bertola (1988) shows that convergence actually requires that $H > 1$. Since $\sigma^2 > 0$, it follows that (1/2) $\sigma^2 H$ is positive.

9 By 'systematic' risk I mean 'market' risk, since the h_i parameters reflect the covariance of the state variables with the market as a whole. 'Unsystematic' risk refers to 'non-market risk,' since σ_p and σ_D are the individual standard deviations of the state variables.

taxed, and the effective price of a unit of capital is decreased, owing to the ITC and the flow of tax depreciation deductions. Both of these effects are reflected in the tax adjustment to the user cost of capital expression, Γ. A summary measure of the cumulative tax distortion may be determined by calculating the marginal effective tax rate (METR) on capital.

To derive an expression for the METR, recall that when investment is positive, condition (6.1) holds with equality, and the gross of tax rate of return on a marginal unit of capital is equal to its tax-adjusted user cost. To determine the METR I follow the established approach (see Boadway 1987) by defining the *gross* of corporate tax, *net* of depreciation and risk rate of return on a marginal unit of capital as follows:

$$r^g \equiv \frac{\partial \Pi(s)/\partial K(s)}{P(s)} - G, \tag{7.1}$$

where,

$$G \equiv \delta - \vartheta_p + h_p + (1/2)\sigma^2 H. \tag{7.2}$$

The term G is the 'risk-adjusted' ex ante economic rate of depreciation.

Similarly, let r^n denote the rate of return required by savers *net* of corporate taxes, depreciation and risk. The METR is then defined as

$$T = \frac{r^g - r^n}{r^g}. \tag{8}$$

To calculate T, estimates of both r^g and r^n are required.

To determine r^n, I invoke the 'open economy arbitrage' assumption of Boadway, Bruce, and Mintz (1984) (see also McKenzie and Mintz 1992). The assumption recognizes that as a small open economy Canada is a price taker on international capital markets and therefore treats the required after-corporate-tax rate of return on equity as given. This means that the risk-free interest rate, r, and the systematic risk adjustments to the growth rates in the stochastic state variables, h_p and h_D, can be treated as exogenous. In this environment, all investments must yield the internationally determined risk-free rate of interest net of corporate taxes, depreciation, and risk, and r^n therefore must equal r.

The results of the investment model described above may be used to determine r^g. The investment rule expressed in equation set (6) says that when investment occurs, the gross rate of return is equal to the tax-adjusted user cost of capital. Using this rule and equation set (7), we see that when investment takes place, the gross of corporate tax, net of depreciation and risk rate of return,

$$r^g = r\Gamma + G(\Gamma - 1). \tag{9}$$

It will prove useful in the interpretation of subsequent results to consider the design of a 'neutral' corporate tax of the imputed profits form. Under an imputed profits tax all of the economic costs of holding the capital are deducted as they accrue. This system includes a deduction for the full opportunity cost of finance, including the imputed cost of equity, so that r is replaced with the after-tax interest rate $r(1-\tau)$. A deduction for the risk-adjusted economic rate of depreciation G is also required. Setting the tax depreciation rate φ equal to G, in conjunction with full cost of finance deductibility and no ITC, will yield a METR of zero – investment decisions will not be distorted by the tax system.

Consider the characteristics of the neutral tax depreciation rate. In the case where there is no uncertainty, the neutral tax depreciation rate is simply $\delta - \vartheta_p$, which is the physical rate of depreciation less the rate of capital gain on a unit of capital; this is the standard 'textbook' result (see, e.g., Boadway and Wildasin). Under conditions of uncertainty, but with full reversibility, an additional term, h_p, must be added to account for the imputed cost of bearing systematic capital risk. The reason for this addition is that when tax depreciation allowances are determined ex ante, based upon the original purchase price of the capital, deductions do not fluctuate with unanticipated changes in the replacement price of capital. As such, the government does not automatically share in the firm's capital risk and an explicit deduction must be made to account for the imputed cost of bearing it. Note that no adjustment need be made to account for the cost of bearing income risk when capital is fully reversible, because under the assumption of full refundability the firm's tax liability fluctuates perfectly with changes in its operating income. Income risk is thus implicitly deducted under a full loss offset tax system, since the government fully shares in this type of risk through fluctuations in its tax revenues. This relationship is the basis for the observation by Bulow and Summers (1984) that full loss offset tax systems that use historical depreciation may discriminate against capital, but not against income, risky investments (see also Auerbach 1987).

When, in addition to being risky, capital is also irreversible, a third term, $(1/2)\sigma^2 H$, must be added to the neutral tax depreciation rate to account for the added cost associated with holding irreversible capital. This additional cost should be deducted under an imputed profits tax. Note that the additional deduction reflects all four 'types' of risk.[10] The implication is that to the extent that existing tax regimes do not account for this added cost, they will discourage investment in irreversible capital by increasing the gross of tax rate of return required for investment to take place.

The Canadian corporate tax system is not a pure imputed profits regime. This is so because the full opportunity cost of holding capital is not deducted; only debt finance costs are deductible, with no deduction for the imputed cost of equity, and tax depreciation rates do not typically reflect the risk-adjusted economic rate of depreciation. The Canadian corporate tax system is thus decidedly non-neutral;

10 In McKenzie (1992) I consider the neutrality of an imputed profits tax under slightly different conditions – full reversibility but convex, rather than linear, capital adjustment costs. In that environment it is shown that the neutral tax depreciation rate is also a function of all four types of risk. It is thus evident that the traditional view that only systematic capital risk 'matters' in full loss offset tax systems is too simplistic.

TABLE 1
Marginal effective tax rates (per cent)

	Riskless reversible	Risky reversible	Risky irreversible $\sigma^2 = 0.02$	$\sigma^2 = 0.05$	$\sigma^2 = 0.10$
AFF	32.4	44.6	46.0	47.7	50.2
MAN	28.6	37.2	38.9	41.0	43.8
CON	35.4	43.2	44.7	46.6	49.3
TRS	35.0	44.7	47.1	50.0	53.7
COM	38.1	46.4	48.4	50.8	54.0
WST	34.1	42.7	45.1	47.9	51.5
RTT	31.6	36.6	41.9	44.6	48.1
SER	42.4	48.7	50.4	52.6	55.4
TOT	31.7	41.5	43.2	45.4	48.4

NOTES
AFF = Agriculture, Forestry, Fishing; MAN = Manufacturing; CON = Construction; TRS = Transportation and Storage; COM = Communications; WST = Wholesale Trade; RTT = Retail Trade; SER = Services; TOT = Weighted Average Total.
See the appendix for parameter and data assumptions.

calculating METRS provides an indication of the extent to which the tax system departs from neutrality.

The importance of properly accounting for irreversibility and risk is illustrated in table 1, which presents METR calculations for three scenarios: (1) the 'traditional' risk-free (or pure income risky) case with full reversibility; (2) the capital risky case with full reversibility; and (3) the capital and income risky case with irreversibility. To calculate METRS some degree of aggregation is necessary. Table 1 presents METR estimates for eight broad industry groups. Although the framework developed above is based upon the dynamic optimizing decisions of an individual firm, the parameter values used in the METR calculations presented in the table are based on industry weighted averages for investments in buildings, machinery, and equipment in Canada.[11]

An examination of the risk-free, fully reversible case in the first column of table 1 reveals that the METRS range from a low of 28.6 per cent in Manufacturing to a high of 42.4 per cent in Services, with an average METR of 31.7 per cent overall. The differences among sectors are due to variations in the tax rates, tax depreciation rates, and economic rates of depreciation.

The second column in table 1 illustrates METRS under the assumption of capital risk but full reversibility. The key difference from column one is that the economic rate of depreciation is higher, owing to the inclusion of the systematic capital risk premium h_p. To calculate the figures for this case I follow Bulow and Summers (1984) and Jog and Mintz (1989) in assuming that fluctuations in firm values primarily reflect changes in asset values; therefore Capital Asset Pricing Model (CAPM)

11 See the appendix for a description of the data used to calculate the METRS.

estimates of a firm's 'beta' are used as an indicator of capital risk.[12] The systematic capital risk premiums used to compute the METRS are thus based upon sectoral CAPM estimates for Canada provided by Jog and Mintz (1989).[13] The inclusion of the capital risk adjustment in the economic rate of depreciation increases the METR because tax depreciation rates do not increase accordingly. On average the METR is 41.5 per cent, about 10 percentage points higher than the risk-free (or pure income risky) case. The increase is greater in industries with a large risk premium (Agriculture, Forestry and Fishing) and lower in industries with a small risk premium (e.g., Retail Trade). The conclusion, which is consistent with Jog and Mintz (1989), is that the Canadian tax system discriminates significantly against investments in capital risky assets vis-à-vis comparable riskless or income risky assets.

The last three columns in table 1 present METR calculations under the assumption of income and capital risk and irreversibility. Calculations are illustrated for three assumptions regarding the variability parameter σ^2, which is an index of *total* unsystematic risk, since it reflects both capital and income unsystematic risk (see equation (6.4)).[14] So that the figures may be compared with previous calculations, it is assumed that there is no systematic income risk ($h_D = 0$), and the systematic capital risk premiums are based upon sectoral CAPM estimates. To put the assumptions regarding the total variance parameter in perspective, note that the variance in the market portfolio is usually measured at around 4.5 per cent^2. As is evident from the table, for low levels of total unsystematic risk ($\sigma^2 = 2$ per cent^2) the METRS on irreversible investments are only about 2 percentage points higher than the METRS for fully reversible, capital risky investments. As the degree of unsystematic risk increases, however, so too do the METRS for irreversible investments, to an average of about 4 percentage points higher than the risky reversible case when $\sigma^2 = 5$ per cent^2 and about 7 percentage points higher when $\sigma^2 = 10$ per cent^2. As discussed above, the reason for this difference is that the tax system fails to make an upward adjustment in tax depreciation rates to account for the additional cost of holding irreversible capital, which increases as total unsystematic risk rises.

The conclusion is that the additional tax distortion imposed upon risky investments due to irreversibility can be significant, and that by ignoring the consequences of irreversibility, traditional estimates of METRS may understate the distortions caused by the corporate income tax.

12 The plausibility of this assumption has been questioned by Gordon (1985) and Gordon and Wilson (1989). None the less I retain it here, primarily because it suggests an easy way to obtain an empirical measure of systematic capital risk. While I recognize that this is a weak justification, it does not change the *qualitative* results of the analysis.

13 Jog and Mintz provide estimates of industry 'betas,' which will generally be lower than the beta for an individual firm. As such, the calculations shown in table 1 likely understate the METRS of individual firms within the broad industry groups.

14 The parameter σ^2 actually varies slightly across the sectors. The handings for the three cases depicted in table 1 are weighted averages.

IV. COMPARATIVE STATICS

Comparative static analysis can be conducted to determine analytically how the METR responds to changes in the parameters representing the different 'types' of risk – unsystematic capital risk (σ_p), unsystematic income risk (σ_D), systematic income risk (h_D), and systematic capital risk (h_p). This analysis serves two purposes. First, it emphasizes that the direction of the changes in METRS illustrated in table 1 are not due to the judicious choice of parameter values; second, it helps to sharpen the intuition behind the results. It should be noted that when capital is fully reversible, the METR is invariant to changes in all of the risk parameters except systematic capital risk.

To begin, note from equation (8) that the METR is increasing in r^g, which from equation set (7) is increasing in the risk-adjusted economic rate of depreciation G. As such, any of the parameters that increase the risk-adjusted economic rate of depreciation also increase the size of the tax distortion. The comparative static analysis thus focuses on the risk-adjusted economic rate of depreciation.

Partial differentiation of equation (7.2), using the definitions in equation set (6), yields the following results:[15]

$$\frac{\partial G}{\partial \sigma_p} = (1/2)\sigma_p \left[1 + (a^2 + b)^{-1/2} \left[\frac{b - a\sigma^2}{\sigma^2}\right]\right] > 0 \tag{10.1}$$

$$\frac{\partial G}{\partial \sigma_D} = (1/2)\sigma_D x_D^2 \left[1 + (a^2 + b)^{-1/2} \left[\frac{b - a\sigma^2}{\sigma^2}\right]\right] - (1/2)x_D(x_D - 1)\sigma_D\left[1 - a(a^2 + b)^{-1/2}\right] > 0 \tag{10.2}$$

$$\frac{\partial G}{\partial h_D} = (1/2)x_D\left[1 - a(a^2 + b)^{-1/2}\right) > 0 \tag{10.3}$$

$$\frac{\partial G}{\partial h_p} = 1 + (1/2)\left[\frac{a + \sigma^2}{a + H\sigma^2} - 1\right] > 0. \tag{10.4}$$

The risk-adjusted economic rate of depreciation, and therefore the size of the tax distortion as measured by the METR, is increasing in all four types of risk.

15 When taking the derivatives, I have assumed that each of the risk paremeters is independent. Strictly speaking, this is not likely to be the case. As pointed out by Craine (1989), for a given firm the index of total unsystematic risk and systematic risk will not usually be independent, but rather an increase in overall variability would typically be associated with an increase in systematic risk. I ignore these interactions, with two justifications. First, since an increase in one 'type' of risk would likely be reflected in an increase in another 'type,' the movements would tend to reinforce each other, and therefore the sign of the derivatives would be the same if the interactions were taken into account. Second, one way to view the comparative statics is as a comparison of the METRs on two firms that have similar risk characteristics with the exception that one of the firms has a (marginally) higher level of one 'type' of risk. The manipulations required to sign the derivatives are available from the author on request.

The intuition is straightforward. In the case of unsystematic capital risk (equation (10.1)), as the variability in the price of capital increases, the option value associated with delaying the installation of capital goes up because it becomes more likely that the price of capital might decrease in the future; such a situation is analogous to the rise in the value of a financial option as the variance in the price of the underlying security increases. This rise in the option value increases the opportunity cost of investing currently, which is reflected by an increase in the risk-adjusted economic rate of depreciation.

An increase in the variability of demand has a similar impact, with an interesting modification. The first term on the right-hand side of (10.2) is the 'financial option' effect discussed above – as the variability in demand increases the option value associated with delaying the installation of capital goes up. An increase in unsystematic income risk has an offsetting effect, however, which accounts for the second term in (10.2). The operating profit function is convex in the demand variable D (see equation set (4)). Jensen's Inequality thus implies that as the variability in D increases, expected operating profits rise, primarily because the firm is able to react to demand fluctuations by increasing or decreasing its output by adjusting its use of labour. The implication is that a rise in the variability of demand increases the expected present value of operating profits forgone by waiting to invest, which lowers the option value. It can be shown that the increase in the option value due to the first effect offsets the reduction in the option value due to the second effect, and the net result is a rise in the risk-adjusted economic rate of depreciation.

An examination of equation (10.3) shows that one of the costs of delaying investment in a unit of capital is the forgone income it would generate. A rise in systematic income risk increases the rate at which the firm discounts its operating income, which lowers the expected present value of the profits forgone by delaying an incremental investment. There is then a reduction in the cost of delaying the installation of the capital, which is reflected by an increase in the option value. This additional cost is manifested in an increase in the economic rate of depreciation.

An increase in systematic capital risk has two effects. The first has nothing to do with irreversibility and has been documented by Jog and Mintz (1989). As discussed above, the failure to deduct the imputed cost of bearing systematic capital risk in a tax system using historical depreciation allowances increases the risk adjusted economic rate of depreciation and therefore the METR; this effect accounts for the first term (the 1) on the right-hand side of (10.4), which is obviously positive. The second effect is due to irreversibility. One of the benefits of waiting to invest in a unit of capital is the delay in having to pay for it. A rise in systematic capital risk increases the rate at which the firm discounts the purchase price of capital, which lowers the benefit from delaying, or equivalently lowers the cost of investing currently. This effect is captured in the second term, which is negative (because $H > 1$). The second effect thus acts to decrease the economic rate depreciation and lower the tax distortion. The two effects work against each other. Overall, the first effect dominates and an increase in systematic capital risk increases the economic rate of depreciation.

V. SUMMARY AND CONCLUSIONS

In this paper I have used a model developed by Bertola (1988) to examine the implications of different types of risk and the irreversibility of capital for the measurement of tax distortions using the concept of the METR. Using data based upon the Canadian tax system, I show that the METR on irreversible capital is higher than that on fully reversible capital, the magnitude depending upon the level and type of risk. For investments in irreversible capital, METRS are increasing in all four types of risk. The results suggest that standard applications of the METR approach, which ignore the role of uncertainty and irreversibilities, understate the disincentive effects of corporate taxation.

In recent tax reforms governments have altered tax, ITC, and depreciation rates with the objective of making their tax systems 'more neutral' (see, e.g., Department of Finance 1987). The typical approach is to compare METRS calculated before and after the tax changes. The analysis presented here suggests that tax policy decisions guided by METRS determined in the 'standard' way may have been based upon a flawed measure. In particular, METRS on very risky investments in capital that has no alternative uses were likely underestimated relative to METRS on less risky investments in more flexible capital. The failure to account for these effects may partly explain why the Canadian tax reform of 1987, which lowered METRS measured in the standard way, did not appear to generate strong positive incentive effects; if the level of uncertainty increased as tax rates were lowered and the tax system 'streamlined,' then METRS, correctly measured, may actually have increased.

REFERENCES

Auerbach, A. (1987) 'Comment on R. Boadway's "The theory and measurement of marginal effective tax rates."' In *The Impact of Taxation on Business Activity*, ed. J.M. Mintz and D.D. Purvis (Kingston, Ontario: John Deutsh Institute)

Bertola, G. (1988) 'Adjustment costs and dynamic factor demands: investment and employment under uncertainty.' Unpublished PhD thesis, MIT

Bertola, G., and R. Caballero (1991) 'Irreversibility and aggregate investment.' Mimeo, Princeton University

Boadway, R.W. (1987) 'The theory and measurement of effective tax rates.' In *The Impact of Taxation on Business Activity*, ed. J.M. Mintz and D.D. Purvis (Kingston, Ontario: John Deutsh Institute)

Boadway, R.W., N. Bruce, and J.M. Mintz (1984) 'Taxation, inflation, and the effective marginal tax rate on capital.' This JOURNAL 17, 62–79

Brennan, M.J., and E.S. Schwartz (1985) 'Evaluating natural resource investments.' *Journal of Business* 58, 135–57

Bulow, J., and L. Summers (1984) 'The taxation of risky assets.' *Journal of Political Economy* 92, 20–39

Craine, R. (1989) 'Risky business: the allocation of capital.' *Journal of Monetary Economics* 23, 201–18

Constantinides, G. (1978) 'Market risk adjustment in project valuation.' *Journal of Finance* 33, 603–16

Daly, M., and J. Jung (1987) 'The taxation of corporate income in Canada: an analysis of marginal effective tax rates.' This JOURNAL 20, 555–87
Department of Finance (1987) 'The White Paper: tax reform 1987.' and 'Tax reform 1987: income tax reform'
— (1989) 'The calculation of marginal effective corporate tax rates in the 1987 White Paper on tax reform.' Working Paper No. 89-6, Tax Policy and Legislation Branch
Dixit, A. (1989) 'Entry and exit under uncertainty.' *Journal of Political Economy* 97, 620–38
Gordon, R. (1985) 'Taxation of corporate capital income: tax revenues versus tax distortions.' *Quarterly Journal of Economics* 100, 1–27
Gordon, R., and J. Wilson (1989) 'Measuring the efficiency cost of taxing risky capital income.' *American Economic Review* 79, 427–39
Jog, V., and J.M. Mintz (1989) 'Corporate tax reform and its economic impact: an evaluation of the Phase I proposals.' In *The Economic Impacts of Tax Reform*, Canadian Tax Paper No. 84, ed. J.M. Mintz and J. Whalley (Toronto: Canadian Tax Foundation)
Jorgenson, D. (1963) 'Capital theory and investment behavior.' *American Economic Review, Papers and Proceedings*, 247–59
King, M., and D. Fullerton (1981) *The Taxation of Income from Capital: A Comparative Study of the United States, the United Kingdom, Sweden and West Germany* (Chicago: University of Chicago Press and NBER)
Mackie-Mason, J. (1990) 'Some nonlinear tax effects on asset values and investment decisions under uncertainty.' *Journal of Public Economics* 42, 301–27
Malliaris, A., and W. Brock (1982) *Stochastic Methods in Economics and Finance*, Advanced Textbooks in Economics, vol. 17 (New York: North-Holland)
McDonald, R., and D. Siegel (1985) 'Investment and the valuation of firms when there is an option to shut down.' *International Economic Review* 26, 331–49
— (1986) 'The value of waiting to invest.' *Quarterly Journal of Economics* 107–27
McKenzie, K.J. (1992) 'Neutral *ex ante* income taxation in the presence of adjustment costs and risk.' Mimeo, University of Calgary
McKenzie, K.J., and J.M. Mintz (1992) 'Tax effects on the cost of capital.' In *Pressures for Harmony: Canada-U.S. Tax Comparisons*, ed. J. Whalley and J. Shoven (Chicago: University of Chicago Press and NBER)
Merton, R. (1973) 'An intertemporal CAPM.' *Econometrica* 41, 867–87
Mintz, J.M. (1988) 'An empirical estimate of corporate tax refundability and effective tax rates.' *Quarterly Journal of Economics* 103, 225–31
Pindyck, R.S. (1988) 'Irreversible investment, capacity choice, and the value of the firm.' *American Economic Review* 78, 969–85
— (1991) 'Irreversibility, uncertainty, and investment.' *Journal of Economic Literature* 29, 1110–48

APPENDIX: DATA

To calculate the METRS in table 1 the nominal risk-free interest rate (r) is 8 per cent, the rate of inflation is 2 per cent, $\mu\phi$ is set to 0.5, the expected growth rate in the state variables (ϑ_1) is 2 per cent, the excess return on the market portfolio (λ) is 7 per cent, and the ITC rate (ψ) is set to zero for all industries (the 1987 tax reform eliminated the general ITC, leaving only regional ITCS). Systematic income risk (h_D) is assumed to be zero for all sectors. Industry-specific parameter values used in the calculations are as follows:

TABLE A1

	h_p[a]	δ[b]	α[c]	τ[d]	φ[e]
			Per cent		
AFF[f]	10.1	10.4	66.0	40.8	19.6
MAN	7.0	10.6	80.0	36.5	19.9
CON	7.0	14.0	83.0	41.4	22.9
TRS	5.0	6.1	71.0	41.5	9.7
COM	5.0	8.2	57.0	41.3	10.6
WST	5.0	7.4	81.0	41.1	12.5
RTT	5.0	7.9	82.0	41.3	16.2
SER	5.0	13.3	81.0	40.5	13.3
TOT	7.0	9.3	74.0	38.9	16.3

NOTES

a Based on sectoral CAPM estimates provided by Jog and Mintz (1989)

b Weighted average physical depreciation rates using useful service lives for buildings and machinery and equipment; based on data from Department of Finance (1989)

c From Statistics Canada, *Corporate Financial Statistics* (Cat. No. 61-207), 1989

d Differences in tax rates reflect different eligibilities for the Manufacturing and Processing Deduction, which effectively lowers the tax rate. Based upon data from Department of Finance (1989)

e Weighted average CCA (capital cost allowance) rates for buildings and machinery and equipment, based upon data from Department of Finance (1988)

f See table 1 for definitions.