Abstract: In Alberta’s oil sands, there is a missing market for an important input (water) in the production of an important good (oil). According to conventional wisdom, market creation improves social welfare under such circumstances. If the creation of a water market grants entitlements (quotas) to current users, oil firms become vertically integrated. In the presence of market power in the water market, oil firms may overinvest in capacity to deter entry in the downstream market. Entry deterrence may be optimal when incumbent oil firms sell their entire water quotas; not otherwise. Creation of a water market may be socially undesirable.

JEL: D43, D62, K20, L13

Keywords: missing market; water; oil; vertical integration; entry deterrence; water quotas.
I. Introduction

The oil sands in Northern Alberta, Canada are among the largest proven oil reserves in the world. The extraction of these unconventional oil reserves via mining is far more water intensive than traditional crude oil. In fact, the sole source of water for oil sands mining is fresh water, used at an average rate of 2.5 barrels of fresh water per barrel of oil produced. Total oil sands production and extraction used over 1.2 billion barrels of water in 2013 to produce 0.7 billion barrels of oil (Alberta Environment and Sustainable Resource Development, 2014). According to the Pembina Institute, the total oil sands water usage in 2011 would be equivalent to the annual residential water use of 1.7 million Canadians. Aside from a one-time application fee for a water licence, water is effectively a free commodity for use in the oil production process; thus, there is potential for overuse of this essentially free input. The creation of a water market in the Athabasca region would force oil producers to consider the costs associated with water usage when making their production decisions. This paper examines the potential benefits and costs associated with the implementation of such a water market, taking the region’s market structure and characteristics into account. An important aspect is that current oil producers would operate in both the upstream water market and in the downstream oil industry, with the potential for the vertically integrated oil firms to utilize market power in the upstream market to deter entry to the downstream industry.

Market creation has long been touted by economists as a way to incorporate the full costs of utilizing a resource, including way to account for the cost of, or to internalize, externalities associated with operating in the market. An important example of a market that was created with the purpose of internalizing an externality is the market for tradable permits for release of pollutants into the air. Under the US Clean Air Act, polluting industries must purchase permits that grant them the ‘right’ to emit a given quantity of pollutant into the air. Prior to the creation of this market, polluters had free access to air quality, a valuable input.

Coase (1960) advocates for the allocation of property rights where no market is currently in existence; stating that, in the absence of transaction costs, these property rights can be traded in such a way that a Pareto efficient outcome will result (Berta & Bertrand, 2014). Arrow (1969) suggests the externalities exist because there is no market for such items and, through the creation of competitive markets for these externalities, an efficient allocation will result (Berta &
However, there is no guarantee that creation of a new market to internalize these externalities will be competitive. Markets will only function efficiently if a number of assumptions are met; often the existence of the externality for which market is missing is caused by the mere fact that the market would be inefficient. As Berta and Bertrand (2014) explain:

*Arrow (1969), in fact, wants to stress the causes of missing markets and tries to explain precisely what impedes this mode of internalization. Externalities are usually not appropriable. This is why Arrow, when studying the reasons why markets for externalities are missing, underlines that “[i]t is not the mere fact that one man’s consumption enters into another man’s utility that causes the failure of the market to achieve efficiency. . . . Pricing demands the possibility of excluding non-buyers from the use of the product, and this exclusion may be technically impossible or may require the use of considerable resources”* (Arrow, 1969)

In fact, Arrow himself also stresses the fact that “markets for externalities usually involve small numbers of buyers and sellers” (Arrow, 1969), indicating his belief that it is likely we will see the existence of certain firms exercising market power in such situations. In the presence of market power, the allocation of resources is generally not Pareto efficient.

One likely result from the creation of an oligopolistic market is that incumbent firms would earn positive profits in the market, making the market desirable for new entrants. However, when market power is combined with excess profits, the threat of entry may create incentives for incumbent firms to over invest in capacity to deter entry. Early literature on strategic entry deterrence (see, e.g., Dixit (1979) and Spence (1977)) finds that investment in capacity by a monopolist can be used to deter entry as it creates a credible threat to the entrant’s future profits. Gilbert and Vives (1986) extend the analysis to an oligopoly. They show that it can be profitable for incumbents to invest in capacity to deter entry, provided the entrant faces a fixed cost, or entry cost.

Although early work on entry deterrence focuses on capacity choices made by firms that operate in a single market, there have since been significant theoretical contributions predicting that vertical integration may provide incumbents with an advantage to increase entry costs. Salop and Scheffman (1987), for example, show that dominant firms may over-purchase necessary inputs.
of production. This is a profitable strategy if the product price increase from this manipulation exceeds the firm’s average cost increase. Electricity firms operating in the European Union’s Emission Trading System market use the permit market to raise their rivals’ costs in just such a fashion (Hintermann, 2011).

Vertical integration has long been a topic of economic debate as to whether integration improves efficiency or whether it restricts competition. Kuhn and Vives (1999) examine the welfare impact of vertical integration of an upstream monopolist when the downstream industry is imperfectly competitive. They show that vertical integration actually increases output downstream, due to cost efficiencies from eliminating double marginalization; however, they neglect to consider any potential strategic effects on the market. Salinger (1988) finds that an upstream monopolist can limit access to the upstream input, if vertical integration results in market foreclosure which will cause a resultant price increase in the downstream market. De Fontenay and Gans (2005), however, show that there is a greater incentive for vertical integration under upstream competition than under upstream monopoly, implying that competition enhances the potential for strategic vertical integration. Normann (2011) finds empirical evidence supporting the hypothesis that markets containing vertically integrated firms are less competitive – prices are generally higher in vertically-integrated than in non-integrated industries. Such findings suggest that market power in the input market may allow firms to raise input prices to the point where a non-integrated firm will not enter the downstream market.

The oil sands mining operations are by far the largest water users in the Athabasca region. The high volume of water usage by relatively few firms in a concentrated geographic area has the potential to impact the water utilization of other groups within the region. As such, one can argue that there is a missing market for water usage in the region and that the creation of such a market is likely to generate inefficiency associated with market power and vertical integration. In our model, we endow three types of current water users with water licenses: (i) the local community, (ii) the First Nations community1, and (iii) the oil producers. Each group is assumed to be

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1 The First Nations communities in Canada are governed by slightly different regulations, which are federal and not provincial, and as such they do not actually hold any water licenses. However the Constitution Act of 1982 does protect Aboriginal rights to Aboriginal title, which includes traditional and new economic uses of water (Walkem, 2007). Therefore, although these rights are not explicitly licensed, our model recognizes that the First Nations in the Fort McMurray, Fort Chipewyan and surrounding areas do have a claim to water rights. Under the water market
grandfathered into the market with an initial endowment equal to the amount of water allocated to them under the current licensing system. Under the market these three groups are assumed to hold the total regional water supply, which they are able to sell to each other or to new water demanders. Oil producers operate in a vertically integrated industry, in which water usage and capacity are complementary inputs. Oil firms compete in the oligopolistic upstream water market with the local community and the First Nation. Oil firms also compete with each other (as well as with many other oil producers in the world) in the downstream market. Since the oil sands operators do not have much market power in the global oil market, we assume that this market is perfectly competitive. Hence, all market power that incumbent oil firms have in our model stems from their operation in the upstream water market.

We initially consider the case where there is no entry into the market and find that the water price is equal to marginal cost and the market functions efficiently. When entry occurs in the downstream industry, the entrant is not initially endowed with a water allocation. Unlike the incumbent oil firms, the entrant must purchase water in order to operate. Since entry increases the number of water demanders relative to the number of water suppliers, we find that entry creates an inefficient mark-up on the water price. We also find that entry deterrence may be an optimal strategy only if the increase in water demand, motivated by the potential entrant’s additional demand and the incumbents’ strategic capacity manipulations, causes the water allocation for the incumbent oil firms to bind. If entry does not cause the water allocation for the incumbent oil firms to bind, accommodating entry is always optimal. In any event, the water price in equilibrium is higher than the marginal cost.

Under the current water allocation system, entrants to the oil industry apply for a water allocation to the regulator. Subject to supporting evidence for the amount of water required, the regulator will grant a new water allocation, according to the water needs of the firm. In the absence of water scarcity, it is unlikely that the regulator will deny an application for a water allocation. Although the Canadian Association of Petroleum Producers is forecasting the annual volume of mining production to nearly double over the next 15 years, there will be no water scarcity at the model, the First Nations are assumed to be grandfathered into the market with an initial endowment equal to their current water utilization.
current water utilization per barrel\(^2\). Therefore, the water allocations of oil firms are not likely to be binding under the current system and the water allocations are likely to be efficient as firms will use water according to its marginal cost as they presently do.

The closest examples of markets similar to the water market we propose here are the markets for clean air, which are created via the assignment of rights to tradable pollution permits. The Acid Rain Program in the United States, established under the Clean Air Act Amendment capped the sulfur dioxide emissions of electricity producing firms and set out to reduce the emissions annually until they met their stated emissions targets (Sovacool, 2011). Under this legislation, firms are initially grandfathered into the market with an allocation equivalent to their current emissions levels. These firms have to undertake measures to reduce their emissions, or they have to purchase additional permits from firms with excess allocations, in order to meet the annual emissions targets. Criticisms of this program suggest that there are several issues with the efficacy of the policy, including high transactions costs and price volatility which may indicate an exercise of market power of certain firms (Sovacool, 2011). “Efficiency of the electricity markets depends, to a large extent, on the presence of market power in this market and in the one for rights to pollute” (Disegni-Eshel, 2005). A similar market exists for carbon emissions permits, for electricity and other industries in the European Union, called the Emissions Trading System (ETS). Heuson (2010) examines electricity firms operating in the EU ETS and finds that many firms have the ability to exercise market power locally to pass abatement costs along to the consumers by producing less power than would be socially optimal to abate costs, as opposed to buying the additional permits required to produce the optimal amount of electricity. These permit markets are a clear example of the creation of a missing market created to internalize an externality. As Arrow conjectured: “markets for externalities usually involve small numbers of buyers and sellers” (Arrow, 1969); we see that the concentration of a few players in these permit markets can cause a failure of these markets to operate as intended.

As in the market for clean air where property rights to pollute are assigned according to the current amount each firm pollutes, our model assumes that water users are grandfathered in and

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\(^2\) The authors used the production forecast from CAPP (CAPP, 2014) and the current water utilization per barrel (Oil Sands Information Portal, 2014) to provide a forecast for the industry water needs for the next 15 years. The projected water demand will not exceed the current water allocations which are currently not causing a disruption to the seasonal flows of the river (according to the seasonal caps as a percentage of the total flow rate), even in the low flow periods of the winter season.
assigned water rights equivalent to their current licenses. Although this may not be the most efficient initial allocation property rights (Disegni-Eshel, 2005), it is often the way the rights are assigned in practice, due to the size and economic impact of these firms and due to political pressure. One main distinction between the water market we examine here and the permit markets for clean air is the incorporation of the cost of externalities. Clean air markets account for the cost of pollution, thereby internalizing the cost of this externality in the permit cost. We have chosen to focus this paper solely on the missing market for water in order to focus on the industrial organization aspect of the problem. While externalities in the form of environmental costs are present, we have ignored them to focus on the strategic water utilization. The environmental cost of water utilization is an area which requires further research. However, even when ignoring the inefficiencies caused by the presence of an externality, this research highlights the finding that the water market may still be inefficient.

Arrow and Coase both provide motivations for market creation to account for areas which are missing markets, yet each has a slightly different take on what is required for these markets to operate efficiently. Arrow suggests that these markets must be competitive, indicating the necessity of many buyers and sellers who are price takers. Coase however, suggests that an efficient equilibrium can be reached via bilateral bargaining and that the costs of reaching an agreement may indeed increase with the number of players in the market due to higher transactions costs. Thus Coase and Arrow disagree on what is fundamentally required for the market to function effectively: our suggested water market is an intermediate solution amongst the proposals of Coase and Arrow. We are suggesting a regional water market with relatively few players where some players in the market supply but also consume water. Each one of these players simultaneously maximizes the sum of consumer and producer surpluses. This is fundamentally different from what we have been able to find in the literature. Coase advocates for the assignment of property rights to one party, either the producer or the consumer, who then must bargain with the other side for the optimal allocation. The distinction here is that each party is maximizing only a producer or a consumer surplus, not both. Similarly, in the market for clean air we see that the consumers of clean air are the general public. The government, on behalf of

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3 Jasechko et al. found that increasing pollution levels in the Athabasca River are correlated to increasing water usage by the oil industry (Jasechko, Gibson, Birks, & Yi, 2012). This is of concern, especially to the First Nations who use the river as a food source and for their livelihood.
the public, assigns the rights to pollute to the producers, who maximize only their producer surplus. Thus, our model introduces this new, novel design to the newly created market: we in fact see that, in the case of no entry, an efficient outcome is reached where water price is equal to marginal cost, indicating that the market allocation is efficient. With entry, there is an additional player (the entrant) that only maximizes its water consumer surplus. This implies that with entry the optimal actions of all players deviate from maximization of the sum of consumer and producer surpluses, leading to equilibrium outcomes in which the water price exceeds its marginal cost.

The remainder of this paper will be organized as follows: Section II will define the model and describe the results for the case where there is no entry in the market, which results in an efficient market outcome where price equals marginal cost; it will then go on to discuss the case where there may be entry in the market. The findings here are that, in the absence of a binding constraint on water supply, accommodation will be optimal whereas if the water supply constraint is binding, it may be optimal for firms to deter entry. Section III will provide a brief numerical example. Section IV is a discussion of the findings and section V will conclude the paper.

II. Model

Consider an economy with consumers and suppliers of water, operating in an upstream water market, including oil producing firms. Vertically integrated oil firms use this water as an input to produce oil in the downstream market. The following model consists of four stages: Stages 1 and 2 are based on a Stackelberg leader-follower model, where the incumbent oil producers make their choice of capacity in stage 1 and a potential entrant makes his choice of capacity (and entry) in stage 2. In stage 3, water suppliers choose the quantities of water to supply the market, based on a Cournot framework. In stage 4 we derive the water demands for the consumers, to find the oil producer’s water demand function.

A. No entry

We begin by deriving the water demand of consumers, which is stage 4. There are $n_1$ final consumers from the local community and $n_2$ final consumers from the First Nation, who
consume water and a basket of consumer goods. These final consumers choose their water demand \( a_i \) to maximize the objective function:

\[
v_i = x_i + a_i \left( b - \frac{a_i}{2} \right) - pa_i
\]

where the second term denotes the benefits derived from water consumption and \( x_i \) is the net benefit from consumption of all other consumer goods. Note that \( p \) is the water price and \( b \) is a constant, which represents the consumer’s maximal willingness to pay for water.

There are also intermediate consumers of water who use the water as an input to some production process. There are \( n_2 \) intermediate First Nation consumers who use water for transportation, fishing, etc. and \( J \) intermediate consumers who use the water as an input to the oil production process. The first nations choose \( a^f \) to maximize their production function, given by

\[
a^f \left( \gamma - \frac{a^f}{2} \right) - pa^f,
\]

where \( \gamma \) is a positive constant. Each oil producer chooses \( a^o \) to maximize its profit function, given by

\[
e(a^o; k_j) - pa^o - k_j r - F,
\]

where \( e(a^o; k_j) \) denotes the firm’s energy production function, \( k_j \) is the amount of capital (i.e., capacity), \( r \) is the price per unit of capacity and \( F \) is a fixed cost. We assume that

\[
e(a^o; k_j) = a^o \left( s - \frac{a^o}{2} \right) + k_j (z - k_j) + a^o k_j,
\]

where \( s \) and \( z \) are positive constants which represent the maximal marginal products of water and capacity, respectively. Water and capacity are modeled as complimentary inputs with the inclusion of the multiplicative \( a^o k_j \) term. We also assume that the price of energy is equal to 1.

Each individual in any group will choose to consume water to maximize his or her respective objective function, leading to choices that equate marginal benefits from consumption of water to the marginal costs of doing so. Assuming interior solutions, the first order conditions can be used to derive the water demand functions for the local community, First Nation, and oil producers, respectively:

\[
a_i(p) = b - p \quad \text{(1.1)}
\]

\[
a^f(p) = \gamma - p \quad \text{(1.2)}
\]

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4 Imperial Oil’s Kearl mine is not yet considered fully operational, but is one example of the massive scale of an oil sands project. The Kearl project has a $12.9 billion dollar price tag and its expected capacity is 40 million barrels per year (Lewis, 2014); the project has a water allocation equivalent to over 600 million barrels of water per year (Alberta Environment and Sustainable Resource Development, 2014). These oil sands mining projects are large scale operations which yield a massive capital cost and are challenged with selling their oil at market prices. Given the current mining technology and the scale of these projects, we see that water is an integral component to the production of oil: the resulting production function faced by the oil producers is increasing in both capital investment and in water. This suggests that water and capital are complementary inputs to the production process.
\[ a_j^o(k_j, p) = s + k_j - p \quad (1.3) \]

We assume that \( p < \min\{b, y, s\} \). Equation (1.3) reveals that capacity \( k_j \) is a strategic complement to water. This implies that the water demand for the oil producers is stated as an increasing linear function of capacity as well as being a function of price.

Let \( Q = Q^o + Q^f + Q^l \) denote the total quantity of water supplied in the market, where \( Q^o, Q^f, \) and \( Q^l \) are the total quantities supplied by the oil industry, the first nation and the local community, respectively. Adding up the total water demands for all consumers from 1.1 to 1.3, we obtain the market clearing condition for the water market: \( Q = bn_R + n_2y + Js + K_I - p(n_R + n_2 + J) \). Solving for the water price, we have:

\[ p(Q, K_I) = \frac{n_Rb + n_2y + Js + K_I - Q}{N} \quad (2) \]

that is, the inverse water market demand function, where \( N = n_R + n_2 + J \) for \( n_R = n_1 + n_2 \) and \( K_I = \sum_1^I k_j \).

Stage 3: The water endowments (quotas) are \( Y^o \) for the oil companies, \( Y^f \) for the First Nation and \( Y^l \) for the local community. The total available water supply is given by \( Y = Y^l + Y^f + Y^o \).

We assume that the water endowment for each group is greater than the sum of their marginal willingness to pay, that is:

\[ Y^l > n_1b, \quad Y^f > n_2(y + b), \quad Y^o > Js, \text{ where } Y^o = \sum_1^I y^o \]

As we will demonstrate below, these conditions ensure that the water quotas do not bind in a static model where there is no entry in the oil and gas industry. This is a modeling strategy. It allows us to consider a case in which the water quota constraint for the oil producer binds when there is potential entry. We will show that a binding water quota constraint for the oil industry is a necessary condition for incumbents to have incentives to strategically deter entry.

a. Oil producers choose \( q^o_j \in [0, y^o] \) to maximize \( \Pi_j \):

\[ \Pi_j = (p(Q, K_I) - c)q^o_j + \left( \frac{1}{2} \right)(s + k_j - p(Q, K_I))^2 + k_j(z - r - k_j) - F \quad (3.1) \]

We assume that \( z > r \) and let \( Q^o = \sum_1^I q^o_j \)

b. The local government choose \( Q^l \in [0, Y^l] \) to maximize:
\[ W^l = (p(Q, K_l) - c)Q^l + J \Pi_j + \frac{n_1}{2} \left( b - p(Q, K_l) \right)^2 \]  
(3.2)

c. The First Nations government chooses \( Q^f \in [0,Y^f] \) to maximize:

\[ W^f = (p(Q, K_i) - c)Q^f + \frac{n_2}{2} \left( \gamma - p(Q, K_i) \right)^2 + \frac{n_2}{2} \left( b - p(Q, K_i) \right)^2 \]  
(3.3)

The payoffs for each group (equations (3.1) to (3.3) above) include the profits earned from the sale of water plus the consumer surplus associated with the consumption of water for each player. When price equals the marginal cost of water, there will be zero contribution to each payoff from the sale of water. However, when the price of water is greater than the marginal cost, each group will find it profitable to sell water as they gain a profit of \( p - c \) for each unit of water sold. The remaining terms in the payoff functions denote the revenues/benefits from using water, less the cost incurred, measured in terms of consumer surplus foregone.

Assuming interior solutions, the optimization problems yield the following first order conditions:

\[(p - c) + [q_f^o - a_f^o(p, k_j)] \frac{dp}{dQ} = 0 \]  
(4.1)

\[(p - c) + [Q^l - n_1 a_1(p)] \frac{dp}{dQ} = 0 \]  
(4.2)

\[(p - c) + [Q^f - n_2 a_f(p) - n_2 a_1(p)] \frac{dp}{dQ} = 0 \]  
(4.3)

Solving for the groups’ total water supplies, we obtain the water supply, given as follows:

\[ Q^o = Js + K_i - JNc + (N - 1)Jp \]  
(5.1)

\[ Q^f = n_2(\gamma + b) - Nc + p(N - 2n_2) \]  
(5.2)

\[ Q^l = n_1 b - Nc + p(N - n_1) \]  
(5.3)

Adding up (5.1) to (5.3) yields

\[ Q = n_R b + n_2 \gamma + Js + K_i - Nc(J + 2) + pN(J + 1) \]  
(6)

Using the producer demand function (equation 2) derived in stage 4; this can be solved for price to get \( p^* = c \).

**Proposition 1.** The subgame perfect equilibrium is efficient.
Intuitively, each group of players (oil producers, local government and First Nation) makes choices that, in aggregation, maximize the sum of producer and consumer surpluses. The complementarity of water and capacity here will have no impact on the downstream market; although $K_I$ appears in the supply function (6) and in the producer demand function (2), the two cancel out in equilibrium, resulting in a water price that is not dependent on the choice of capacity. Thus, in the absence of entry into the market, vertical integration will have no impact on the equilibrium outcome in the downstream market as oil producers will use water at the same cost as they do under the current system since $p^* = c$.

**B. Entry in the oil and gas industry**

The model of the previous section assumes that there is no entry in the oil and gas industry. We now consider a situation where there is potential entry into the oil and gas industry. The only difference with respect to the previous model is that the entrant’s water demand must be included in the derivation of the producer demand function as he will need water to enter the oil market. The entrant will be denoted by the subscript $J + 1$ and the water demand for the entrant will have the same form as for the incumbents: $a^p_{J+1}(p; k_{J+1}) = s + k_{J+1} - p$.

**B1. The case with slack water quotas**

Consider the case where there is an entrant and the water quotas are not binding (as we assumed with the choice of quantities above). Here we obtain the following inverse demand function:

$$p = \frac{nRb + n2y + (J + 1)s + K_I + k_{J+1} - Q}{N + 1}$$

(7)

Suppliers choose their quantities to supply as above, only now they take into account the demand of the entrant when choosing a quantity to supply. The outcome of stage 3 is that the entry drives up the water price so that $p > c$, resulting in the following function for the water price:

$$p = \left[\frac{s + k_{J+1} + (J + 2)(N + 1)c}{JN + 2N + J + 3}\right]$$

Due to the complementarity of water and capacity, the price is dependent on the entrant’s choice of capacity and will increase with scale of entry. The price is again not dependent on the
incumbent producer’s choice of capacity, indicating that choice of capacity is not strategic for the incumbents.

In stage 2, the entrant will choose capacity $k_{J+1}$ to maximize $\Pi_{J+1}$:

$$\Pi_{J+1}(p, k_{J+1}) = \frac{a^o_{J+1}(p, k_{J+1})^2}{2} + k_{J+1}(z - r - k_{J+1}) - F$$

$$s - k_{J+1} - p + z - r + (p - s - k_{J+1}) \frac{dp}{dK} = 0$$ (8.1)

In stage 1, the incumbents will choose capacity $k_j$ to maximize $\Pi_j$:

$$\Pi_j(p, q^o_j, k_j) = (p - c)q^o_j + \frac{a^o_j(p, k_j)^2}{2} + k_j(z - r - k_j) - F$$

$$s - k_j - p + z - r + (q^o_j - s - k_j + p) \frac{dp}{dK} = 0$$ (8.2)

In fact, the choice of capacity $k_j$ by the incumbents is the same as in the case where there is no entry into the industry. When the water allocations given to the industry are not binding, the incumbent firms have no incentive to deter entry to the market. This result is surprising as it indicates that, in the absence of a binding constraint on the water supply, the incumbents are actually better off by allowing entry and earning additional profits from the sale of water.

**B2. The cases when the oil industry’s water quota binds**

Now we consider the case where there is a binding constraint on the water supply. We look at the case $Q^o = Y^o$, where the oil producers wish to supply more water in the market than they have available. The incumbent oil firms are able to deter entry to the industry, provided that their water constraint is binding ($Q^o = Y^o$). As we will discuss in the section v, this creates the necessary condition that will make the water price dependent on the incumbent producers’ choice of $k_j$. For simplicity, we will just examine the least restrictive case where the only suppliers facing a supply constraint are the oil firms, however all cases have been calculated and are available from the authors, upon request.
Let $Q = Y^o + Q^l + Q^f$, so that only the oil firms face a binding constraint on the water supply. There are two possible outcomes: accommodation and deterrence. Although it is possible for the incumbents to deter entry when the water supply is limited by water availability, we will examine the payoffs between the cases where an entrant is allowed into the market versus when they are deterred.

Accommodation is analyzed first. The same first order conditions are used to derive the water demands in stage 4. As before, water demand can be stated as a function of price, only here, it becomes a function of $Y^o$, not $Q^o$:

\[
p = \frac{n_R b + n_2 \gamma + (J + 1)s + K_I + k_{J+1} - Y^o - Q^l - Q^f}{N + 1} \quad (9)
\]

Since the water quota $Y^o$ here is fixed, the price function is now dependent on the capacity choices of the oil producers. This can be seen after stage 3 when we can simplify the price to:

\[
p = \frac{(J + 1)s + K_I + k_{J+1} - Y^o + 2(N + 1)c}{2N + J + 3} \quad (10)
\]

We saw in the prior cases that the choice of water quantity was dependent on the incumbent’s capacity, but the capacity reduced out of the price function when we solved for the quantities, resulting in a price and was not dependent on the capacity choice. When the water demand exceeds the supply, the choice of price becomes strategic, as the function for price is increasing in capacity. Thus the incumbent producers can select a higher $K_I$ to drive up the water price.

As before, when there was no constraint on the water supply, the entrant will choose capacity to maximize profits $\Pi_{j+1}$, subject to this revised producer demand function (10) and the first order conditions (8.1). The incumbents choose capacity to maximize $\Pi_j$, according to the same first order conditions as before (8.2), subject to the above price function (10) to obtain:

Water price:


Entry deterrence will occur when the entrant cannot make positive profits in equilibrium: the entrant will not enter the market when the incumbents choose a level of capacity \( K_I \), such that \( \Pi_{J+1} = 0 \). Once again, we have the same producer demand function (9) as in the accommodation case above where \( Y^o = Q^o \). Subject to the producer demand function (9), the incumbents choose a higher level of capacity than they would in the case of entry, subject to:

\[ \Pi_{J+1} = \frac{a_{J+1}^o(p, k_{J+1})^2}{2} + k_{J+1}(z - r - k_{J+1}) - F = 0 \]

Since the entrant’s profits are zero, he will not find it profitable to enter the market and we find that the incumbent’s deterrence capacity is:

\[ K_I^{Det} = Y_o + 2(N + 1)(s - c) + \frac{(2N + J + 2)(z - r)}{2} - \frac{((z - r)^2 + 4\beta)^{1/2}(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)} \]

Where:
\[ \beta = \frac{2(2N + J + 2)^2 F - (z-r)^2(2N + J + 3)^2}{2(4N^2 + 4N J + 16N + J^2 + 8J + 14)} \]

The water price becomes:

\[ p_{Det} = s + \frac{z - r}{2} - \frac{(z - r)^2 + 4\beta}{2(2N + J + 2)^2} \left( \frac{1}{2}(4N^2 + 4N J + 16N + J^2 + 8J + 14) \right) \]

We find that the deterrence capacity here is higher than the accommodation capacity for the incumbent firms. The equilibrium price is lower in the case where deterrence is optimal, since the entrant will demand no water in equilibrium when we have deterrence. Due to the fact that the capacity choice of the incumbents is higher under deterrence than accommodation, we can see that the price would in fact be higher than the accommodation case should the entrant decide to enter the market when the incumbents choose this capacity level.

We see that deterrence will only occur when there is less slack in the constraint on the water supply: the scarcer water is for the incumbents, the more likely they are to deter entry. In addition to this, entry deterrence requires a sufficiently large fixed cost, in the absence of a large enough fixed cost it becomes too expensive for the incumbents to invest in a capacity level high enough to deter entry.

III. Example

To illustrate the outcomes from the model when there is a constraint on the water supply, we look at the following example. Suppose \( J = 6, n_1 = 200 \text{ and } n_2 = 60 \), and also that \( c = 2, b = 8, \gamma = 12, s = 38, z = 840 \text{ and } r = 590 \). We examine the effect on capacity choice and the profits of the incumbent producers under two variations of fixed costs and two variations of water constraint \( Y^o \). The table below lists the water price, fixed cost, water constraint, and the profit of an incumbent firm under each of 3 cases.

| Table 1: |
|-----------|-----------------|-----------------|
| Variable  | Value Under Accommodation | Value Under Deterrence |

16
### Table

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$3.81</td>
<td>$3.81</td>
<td>$3.10</td>
<td>$3.25</td>
<td>$6.84</td>
<td>$3.25</td>
</tr>
<tr>
<td>F</td>
<td>41,125</td>
<td>40,000</td>
<td>41,125</td>
<td>41,125</td>
<td>40,000</td>
<td>41,125</td>
</tr>
<tr>
<td>Y*</td>
<td>1,260</td>
<td>1,260</td>
<td>1,650</td>
<td>1,260</td>
<td>1,260</td>
<td>1,650</td>
</tr>
<tr>
<td>K_I</td>
<td>1,704</td>
<td>1,707</td>
<td>1,712</td>
<td>1,758</td>
<td>3,657</td>
<td>2,110</td>
</tr>
<tr>
<td>Profit</td>
<td>$221</td>
<td>$1,346</td>
<td>$370</td>
<td>$248</td>
<td>-$52,893</td>
<td>-$1,876</td>
</tr>
</tbody>
</table>

Case A is an example of the conditions required to make entry deterrence optimal. The profits are greater when entry is deterred for this level of water quota and fixed cost. We can see that a change to either the water constraint or the fixed cost will affect the optimality of entry deterrence: entry deterrence will not be optimal in case B or in case C. Case B looks at a lower fixed cost to the industry; when the fixed cost is too low, we can see that it becomes too costly for the incumbents to deter entry, resulting in negative profits, therefore accommodation will be optimal. In case C, we consider a higher water quota than case A, while leaving the fixed cost the same. If the water quota is not sufficiently small, it will also be too costly for the incumbents to invest in the entry deterring capacity and they will prefer to accommodate entry into the market.

### IV. Discussion

The objective of this paper has been to examine whether the creation of a market for water in Northern Alberta would create efficiencies, or whether it would be a source of inefficiency for the oil mining industry. We look at four main outcomes which would be possible if oil producers were vertically integrated and operating in the water market. When there is no entry into the industry, we find that the market is operating efficiently, as demonstrated by price equal to marginal cost. Such a price is analogous to the situation we have under the current legislation: in the current system, users do not pay for water and will use it according to that marginal cost required to fetch the water. Although an efficient market, this no entry case provides no incentive to reduce consumption of water or to conserve, thus it is just as efficient as the current water licensing system.

Accommodated entry into the oil market is also considered, when the water quotas of the incumbent producers are not binding. Entry into the industry brings the price of water up beyond
the marginal cost, allowing the water suppliers to earn a positive profit from selling the water. They are thus better off in the situation where there is entry than in the case where there is no entry into the market. However, as the incumbent oil producers’ profits are increasing with water sales, we see an increase in their market power which comes from the mark-up of charging a water price greater than the marginal cost. This mark-up suggests that the market is not competitive and puts a potential entrant at a disadvantage compared to the incumbent producers. The water price with entry is increasing with the capacity of the entrant, which is intuitive: due to the complementarity of water and capacity, we would expect a larger scale of entry to require a greater amount of water, thus driving up the price even higher. The incumbents’ equilibrium investment in capacity in this case is the same as in the case of no entry: their choice of capacity is not strategic here since the water price is not dependent on the capacity level of the incumbents.

We next consider entry accommodation when the water quotas of the incumbent oil producers are binding. The limitation placed on the water suppliers regarding the quantity available to supply the market drives up the price of water, beyond the price we saw in the case where the quota did not bind. The imposition of the constraint on the supply of water by the incumbents results in a producer demand function, and hence a water price, that is dependent on the capacity choice of the incumbent producers. Since these firms are constrained in their water availability, the capacity choice they make becomes strategic. Given the current technology, a greater capacity will require more water for each producer, driving up the water price in equilibrium. Although entry is not deterred in this case, the higher water price and the strategic link between the price and capacity do provide motivation for incumbent producers to act strategically. In fact, we see that a reduction in the water quota to a quantity lower than the amount the incumbent producers had been using when unconstrained will effectively drive up the water price while reducing the profits and capacity levels of the incumbents when a new entrant enters the market.

Finally, we analyzed the possibility of entry deterrence when the water quotas of the incumbent oil producers are binding. Incumbent producers can exercise their market power by choosing a level of capacity that is so high that the entrant will not find it profitable to enter the industry. The capacity choice in this case is higher than all of the other cases, indicating that the incumbents have chosen their capacity level, not due to efficiencies in the returns on capacity,
but they have chosen a high capacity in order to drive up the resultant water price and prevent entry into the market. It will be optimal to deter entry when the profits in the case of entry are lower than the profits will be in the case of entry deterrence. Deterrence is typically optimal only when the entrant faces a high enough fixed cost. If the fixed cost is too low, it will be optimal to accommodate entry.

With this model, we show that it is both necessary and sufficient for the incumbents to have a binding constraint on their water endowment, in order for the choice of capacity to become strategic.

**Proposition 2:** A binding water quota for the oil producers $Q^o = Y^o$ is both necessary and sufficient for capacity to be a strategic variable in the water market.

Proof:

In the case of entry, we have producer demand function:

$$p = \frac{n_R b + n_2 y + (J + 1)s + K_I + k_{J+1} - Q^l - Q^f - Q^o}{N + 1}$$

We can see that the price is increasing in the total capacity of the incumbents $K_I$. However, when the water constraint is non-binding, the incumbents will choose their water supply to maximize profits, and will select the quantity according to:

$$Q^o = Js + K_I - J(N + 1)c + Njp$$

Since this contains the term $K_I$, as does the price function, the expression will reduce to obtain:

$$p = \frac{n_R b + n_2 y + s + J(N + 1)c + k_{J+1} - Q^l - Q^f}{(N + NJ + 1)}$$

This is no longer a function of $K_I$, indicating that the choice of capacity is not strategic. Note that neither $Q^l$ or $Q^f$ is a function of $Q^o$ or $K_I$, as such they will have no effect on whether or not the choice of capacity is strategic. When we have $Q^o = Y^o$ as a fixed value, the oil firms do not choose $Q^o$ according to the formula above, since they sell their entire allocation. Therefore the capacity term will not cancel out of the price function and the choice of capacity will depend on $K_I$ and will thus be strategic. Q.E.D.
This proposition demonstrates two key facts about the water usage under a market model. First, it highlights the fact that the choice of capacity is a strategic compliment to water supply when the water quota of incumbent oil producers binds. Binding water quotas allow oil producers to exercise market power in the water market, necessitating an increase in capacity downstream, to deter the entry of potential competitors. Second, it underscores the fact that this market power can be exercised in the absence of a shortage in total water supply. The water endowments may be binding or non-binding for the local community and the First Nation as these do not impact the interaction between the choice of capacity and the water price. However, the existence of a binding quota on the water available to incumbent oil producers is sufficient to enable them to exercise this market power, although there is no actual water scarcity in the region. It does not matter that the local community and First Nation may have extra water available; the oil producers may strategically manipulate the market through their capacity choice downstream to create artificial water scarcity where none exists.

Deterrence causes additional inefficiencies due to the fact that the incumbent producers actually incur an additional cost to deter entry. In the other equilibrium outcomes, where entry is accommodated, the optimal choices of both capacity and water demand are chosen at the level where the marginal cost to use an additional unit is equal to the marginal benefit from using another unit of the input. When the incumbents choose their capacity level to deter entry, they choose a level that is not on their reaction function. That is, they choose a level of capacity where the marginal cost is equal to the marginal benefit plus some additional premium $\sigma$.

**Proposition 3**: Under entry deterrence, the choice of capacity is greater than the efficient capacity such that $MB + \sigma = MC$

**Proof:**

\[
MB + \sigma = MC \iff \\
(s + k_j - p + z - r + \sigma = 2k_j \iff \\
s + z - r - k_j - p < 0
\]
\[
s + z - r - Y^o - 2(N + 1)(s - c) - \frac{(2N + J + 2)(z - r)}{2}
\]
\[
+ \frac{(2N + J + 2)((z - r)^2 + 4\beta)^{\frac{1}{2}}(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)^2}
\]
\[
- s - \frac{z - r}{2} + \frac{((z - r)^2 + 4\beta)^{\frac{1}{2}}(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)^2}
\]
\[
< 0
\]

\[
\left[ \frac{4F - (z - r)^2}{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)} \right]^{1/2}
\]
\[
< \frac{2(2N + J + 2)[Y^o + 2(N + 1)(s - c)]}{(2N + J + 3)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}
\]
\[
+ \frac{(2N + J + 2)(2N + J + 1)(z - r)}{(2N + J + 3)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}
\]

\[
\frac{4F}{(z - r)^2} - 1
\]
\[
< \left( \frac{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{(z - r)^2} \right) \left( \frac{(2N + J + 2)}{(2N + J + 3)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)} \right)
\]
\[
* \left( 2[Y^o + 2(N + 1)(s - c)] + (2N + J + 1)(z - r) \right)^2
\]

\[
4F - (z - r)^2
\]
\[
< \left( \frac{(2N + J + 2)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{(2N + J + 3)} \right)
\]
\[
* \left( 2[Y^o + 2(N + 1)(s - c)] + (2N + J + 1)(z - r) \right)^2
\]

Based on \( \Pi_j^* \) from case 1, the following condition on the fixed cost must be satisfied for the incumbents to generate a non-negative profit.

\[
F < \left( \frac{1}{2} \right) (s - c)^2 + \left( \frac{1}{2} \right) (s - c + (z - r))^2
\]

\[
4F - (z - r)^2 < \left( 2(s - c) + (z - r) \right)^2
\]
Observe that
\[
(2(s - c) + (z - r))^2 < \left( \frac{(2N + J + 2)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{(2N + J + 3)} \right) * \{2[Y^o + 2(N + 1)(s - c)] + (2N + J + 1)(z - r)\}^2 \tag{13}
\]

The above expression (13) is true since \(\frac{(2N + J + 2)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{(2N + J + 3)} > 1\) and
\[
(2(s - c) + (z - r))^2 < (2[Y^o + 2(N + 1)(s - c)] + (2N + J + 1)(z - r))^2
\]

Thus, we can combine (12) and (13) to see that expression (11) above will be true.

Q.E.D.

Taken together, these four cases demonstrate that the water market will create inefficiencies, as opposed to incentives to use water more efficiently. The magnitude of the inefficiencies will be predicated on whether or not the water quotas allocated to the incumbent oil firms are sufficiently large. When there is a lot of slack in the water constraint so that the firms have an excess of water to supply, a slight mark-up in the price is evident. However, the creation of the market and the allocation of quotas produce an incentive for firms to generate artificial water scarcity, through the strategic choice of capacity, in order to deter entry and lessen competition for other scarce inputs.

V. Conclusion

This paper analyzes the impacts of the creation of a market for water which causes oil producing firms to become vertically integrated. These firms operate in an otherwise competitive international downstream oil industry. The establishment of an upstream water market will assign rights for water usage, creating buyers and sellers of water. We have demonstrated that a group of sellers (oil producers) can artificially create scarcity for potential entrants to the industry through the strategic complementarity of water and capacity. Thus, upstream market power can be exerted to deter entry downstream. Even with entry accommodation, we see that the water market will result in a water price greater than the marginal cost, an inefficient...
outcome. The only time where we see the market operating efficiently (with a water price equal to marginal cost) is when there is no entry, which is unlikely to be the case in the competitive oil industry over time.

Under the current water licensing system, it is natural to think that a regulator would allocate additional water to accommodate downstream entry, in the absence of scarcity\(^5\). This suggests that the water quotas will never be binding under the current system, as the quota will be increased to accommodate water needs. Therefore it is likely that industry water allocations will be more efficient under the current system than in a market system as the current method is better able to adapt to water needs of the industry. The market is efficient in the absence of entry, but so is the current system and the current system will be more efficient in the case of entry, thus the current system dominates in efficiency.

As discussed previously, this water market is similar to the clean air permit markets operating in the United Stated and in Europe. While these markets focus on internalizing the cost of the pollution externality, our model seeks only to examine a water market prior to the inclusion of any environmental costs. Our findings are poignant since they suggest that the potential exists in such markets for the exercise of market power and inefficiencies, simply due to vertical integration and market power generated by the market creation itself; implying that using such markets to incorporate the environmental costs in the future may have unintended consequences of entry deterrence. These findings may have application in the clean air market, particularly for electricity production, an industry where market power is often evident. Both the incorporation of externality costs and the application of these findings to other industries are areas which require further research.

\(^5\) As discussed in the introduction, empirical evidence suggests that according to current and forecasted utilization, the region will not see water scarcity within the forecast period of approx. years.
References


VI. Appendix

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Section 1: Entry is undesirable

Water Demand:

i. Local Community: \(n_1\) consumers

ii. First Nations: \(n_2\) consumers

iii. Oil Producers: \(J\) producers

Final Water Consumers: \(n_1 + n_2 = n_R\)

Water Suppliers:

- Local Community is endowed with \(Y_l\) units of water
- First Nations are endowed with \(Y_f\) units of water
- Oil Producers are endowed with \(Y_o\) units of water

Let \(Y = Y_l + Y_f + Y_o\) be fixed.

In this scenario, the quantities sold will be less than the endowments \(Y_i\).

Stage 3: Find water demand

Utility of a representative final consumer \(i\):

\[ x_i + a_i \left( b - \frac{a_i}{2} \right) \]

Budget Constraints:

\[ u_i = x_i + p^o a_i \quad \forall \ i = 1, ..., n_R \]

Where:

- \(x_i\) = Utility derived from the basket of consumer goods
- \(a_i\) = Water demand (final)
- \(a^o\) = Water usage (intermediate, oil producers)
- \(a^f\) = Water usage (intermediate, First Nation)
- \(p\) = Price of water
- \(u_i\) = Income or wealth

Utility Maximization for final water consumer \(i\):

\[
\max_{[a_i]} (x_i + a_i \left( b - \frac{a_i}{2} \right) - p a_i)
\]

Then \(0 = -p + b - a_i \Rightarrow a_i(p; b) = b - p\)

Income Maximization for intermediate First Nations production

\[
\max_{[a^f]} (a^f \left( \gamma - \frac{a^f}{2} \right) - p a^f)
\]

Then \(0 = -p + \gamma - a^f \Rightarrow a^f(p; \gamma) = \gamma - p\)

Profit maximization for oil producers (in the oil market):

\[
\Pi^o_s = e(a^o; k_j) - p a^o - k_j r = a^o \left( s - \frac{a^o}{2} \right) + k_j (z - k_j) + a^o k_j - p a^o - k_j r - F
\]

Where

- \(k_j\) = Capacity or capital cost for producer \(j\), \(z\) and \(s\) are constants
r= price of capital and F= fixed cost

$$\max_{\{a^o\}}(\Pi^o_s) = \max_{\{a^o\}} \left[ a^o \left( s - \frac{a^o}{2} \right) + k_j(z - k_j) + a^o k_j - p a^o - k_j r - F \right]$$

Then \( 0 = s + k_j - a^o - p \Rightarrow a^o(p; k_j) = s + k_j - p \)

Let: \( K_i = \sum_1^I k_j \)

Supply of Water

For final consumers: \( n_R \cdot a_i(p; b) \)

For intermediate First Nation: \( n_2 \cdot a^f(p; \gamma) \)

For incumbent oil producers: \( J \cdot a^o(p; k_j) \)

Market clearing condition:

\[
Q = n_R \cdot a_i(p; b) + n_2 \cdot a^f(p; \gamma) + J \cdot a^o(p; k_j)
\]

\[
Q = n_R(b - p) + n_2(\gamma - p) + K_i + J(s - p)
\]

\[
Q = bn_R + n_2\gamma + Js + K_i - p(n_R + n_2 + J)
\]

Let \( N = n_R + n_2 + J \)

Producer demand function

\[
p(Q, K_i) = \frac{n_R b + n_2\gamma + Js + K_i - Q}{N}
\]

Where \( \frac{dp}{dQ} = -\frac{1}{N} \)

Stage 2: All groups choose water supply simultaneously

a. Oil producers will choose \( q_j^o \in (0, y^o) \) to maximize \( \Pi_j \), where

\[
Q^o = \sum_{s=1}^I q_j^o \text{ and } Y^o = \sum_{i=1}^I y^o
\]

\[
\max_{\{q_j^o\}} \left( (p - c)q_j^o + a^o \left( s - \frac{a^o}{2} \right) + k_j(z - k_j) + a^o k_j - p a^o - k_j r - F \right)
\]

\[
q_j^o = s + k_j - Nc + (N - 1)p
\]

Since \( Q^o = J q_j^o \)

\[
Q^o = J q_j^o = Js + K_i - fNc + (N - 1)f p
\]

b. Local Community

The local community has indirect utility function:
\[ v_i(\cdot) = x_i + \frac{(b-p)^2}{2} \]

Where
\[ x_i = \frac{(p-c)Q^i + J\Pi_j}{n_1} \]

The local government will choose \( Q^i \in (0,Y^i) \) to maximize:
\[ W^i = (p-c)Q^i + J\Pi_j + \frac{n_1}{2}(b-p)^2 \]

First Order conditions:
\[ \frac{dp}{dQ}Q^i + (p-c) - n_1(b-p) \frac{dp}{dQ} = 0 \]
\[ Q^i = n_1b - Nc + p(N - n_1) \]

c. First Nation
First Nations will maximize its payoff:
\[ W^f = \Pi_s^f + \Pi_s^g \]
\[ W^f = (p-c)Q^f + \frac{1}{2}(y-p)^2 \]

Need to add the consumer surplus to the First Nations producer surplus.
A resident \( i \) in the First Nations will maximize their utility as follows:
\[ \max_{a_i}(u_i - pa_i + a_i(b - \frac{a_i}{2})) \] which yields: \( a_i(p; b) = b - p \)

Substitute the demand functions into the objective function to derive the indirect utility function for a representative resident \( i \).
\[ v_i(w_i, p, b) = u_i - p(b-p) + (b-p)\left( b - \frac{(b-p)^2}{2} \right) \]
\[ v_i(\cdot) = u_i + \frac{(b-p)^2}{2} \]

Where \( u_i = W^f / n_2 \)

The First Nations government chooses \( Q^f \in (0,Y^f) \) to maximize:
\[ W^f = (p-c)Q^f + \frac{n_2}{2}(y-p)^2 + \frac{n_2}{2}(b-p)^2 \]

Assuming an interior solution, the first order condition is:
\[ \frac{dp}{dQ}Q^f + (p-c) - n_2(y-p) \frac{dp}{dQ} - n_2(b-p) \frac{dp}{dQ} = 0 \]
\[ Q^f = n_2(y+b) - Nc + p(N - 2n_2) \]

So, we have:
\[ Q = Q^o + Q^f + Q^i \]
where
\[ Q^o = Js + Ki - JNc + (N-1)p \]
\[ Q^f = n_2(y+b) - Nc + p(N - 2n_2) \]
\[ Q^i = n_1b - Nc + p(N - n_1) \]
Find the quantities of water:

\[ Q = Js + K_I - JNc + (N - 1)Jp + n_2(y + b) - Nc + p(N - 2n_2) + n_1b - Nc + p(N - n_1) \]

\[ Q = n_Rb + n_2y + Js + K_I - Nc(J + 2) + N(J + 1) \left[ \frac{n_Rb + n_2y + Js + K_I - Q}{N} \right] \]

\[ Q = n_Rb + n_2y + Js + K_I - Nc \]

\[ p = \frac{n_Rb + n_2y + Js + K_I - [n_Rb + n_2y + Js + K_I - Nc]}{N} \]

\[ p^* = c \]

**Stage 1**: Oil producers choose capacity \( k_j \)

\[
\max_{k_j} \left( q^o_j(p - c) + \left( \frac{1}{2} \right) (s + k_j - p)^2 + k_j(z - k_j - r) - F \right)
\]

\[ q^o_j \frac{dp}{dK} + s + k_j - s \frac{dp}{dK} - k_j \frac{dp}{dK} - p + p \frac{dp}{dK} + z - 2k_j - r = 0 \]

\[ k_j = \frac{N(z - r) + (N - 1)s - (N - 1)p + q^o_j}{N + 1} \]

\[ k_j^* = (z - r) + s - c \]

Results:

\[ K_I^* = J(z - r) + Js - Jc \]

\[ Q^* = n_Rb + n_2y + 2Js + J(z - r) - (J + N)c \]

\[ p^* = c \]

\[ Q^{o*} = 2Js + J(z - r) - 2Jc \]

\[ Q^{f*} = n_2(y + b) - 2n_2c \]

\[ Q^{l*} = n_1b - n_1c \]

\[ \Pi_j^* = (s - c)^2 + (s - c)(z - r) + \left( \frac{1}{2} \right) (z - r)^2 - F \]

**Section 2: Entry Accommodation**

**Water Demand:**

i. Local Community: \( n_1 \) consumers

ii. First Nations: \( n_2 \) consumers

iii. Oil Producers: \( J \) producers

iv. New Entrant (oil market): \( J + 1 \)th producer

**Water Suppliers:**

- Local Community is endowed with \( Y^1 \) units of water
First Nations are endowed with $Y^f$ units of water
Oil Producers are endowed with $Y^o$ units of water

Let $Y = Y^f + Y^o$ be fixed.

Stage 4: Find water demands

Supply of Water (derived in section 1)

For final consumers: $n_R \cdot a_i(p; b)$

For intermediate First Nation: $n_2 \cdot a^f(p; \gamma)$

For incumbent oil producers: $J \cdot a^o(p; k_j)$

For entrant: $a^o(p; k_{j+1})$

Market clearing condition:

$$Q = n_R \cdot a_i(p; b) + n_2 \cdot a^f(p; \gamma) + J \cdot a^o(p; k_j) + a^o(p; k_{j+1})$$

$$Q = bn_R + n_2\gamma + (J + 1)s + K_i + k_{j+1} - p(n_R + n_2 + J + 1)$$

Let $N = n_R + n_2 + J$

Producer demand function

$$P(Q, K_i, k_{j+1}) = \frac{n_Rb + n_2\gamma + (J + 1)s + K_i + k_{j+1} - Q}{N + 1}$$

Where $\frac{dp}{dQ} = -\frac{1}{N+1}$

Stage 3: All groups choose water supply simultaneously

a. Oil producers:

$$\max_{\{q^o_j\}} \left( (p - c)q^o_j + \frac{1}{2}(s + k_j - p)^2 + k_j(z - k_j) - k_jr - F \right)$$

$$\Rightarrow \left( \frac{dp}{dQ} q^o_j + (p - c) - (s + k_j - p) \frac{dp}{dQ} \right) = 0$$

$$q^o_j = s + k_j - (N + 1)c + Np$$

Since $Q^o = Jq^o_j$

$$Q^o = Jq^o_j = Js + K_i - J(N + 1)c + Np$$

b. Local Community

The local government will choose $Q^l \in (0, Y_l)$ to maximize:

$$W^l = (p - c)Q^l + J\Pi^f + \frac{n_1}{2} (b - p)^2$$

First Order conditions:
\[
\frac{dp}{dQ} Q^l + (p - c) - n_1(b - p) \frac{dp}{dQ} = 0
\]
\[
Q^l = n_1 b - (N + 1)c + p(N + 1 - n_1)
\]

c. First Nation

The First Nations government chooses \(Q^f \in (0, Y_2)\) to maximize:

\[
W^f = (p - c) Q^f + \frac{n_2}{2} (y - p)^2 + \frac{n_2}{2} (b - p)^2
\]

Assuming an interior solution, the first order condition is:

\[
\frac{dp}{dQ} Q^f + (p - c) - n_2(y - p) \frac{dp}{dQ} - n_2(b - p) \frac{dp}{dQ} = 0
\]
\[
Q^f = n_2(y + b) - (N + 1)c + p(N + 1 - 2n_2)
\]

So, we have:

\[
Q = Q^o + Q^f + Q^l
\]

where

\[
Q^o = Js + K_l - J(N + 1)c + Njp
\]
\[
Q^f = n_2(y + b) - (N + 1)c + p(N + 1 - 2n_2)
\]
\[
Q^l = n_1 b - (N + 1)c + p(N + 1 - n_1)
\]

Find the quantities of water:

\[
Q = Js + n_2y + n_2b + K_l - (J + 2)(N + 1)c
\]
\[
+ (JN + N + J + 2) \left[ \frac{n_R b + n_2y + (J + 1)s + K_l + k_{j+1} - Q}{N + 1} \right]
\]
\[
Q = Js + n_2y + n_2b + K_l + \frac{(JN + N + J + 2)(s + k_{j+1}) - (J + 2)(N + 1)^c}{JN + 2N + J + 3}
\]

Then the price becomes:

\[
p = \left[ \frac{s + k_{j+1} + (J + 2)(N + 1)c}{JN + 2N + J + 3} \right]
\]

Stage 2: Entrant J+1 chooses capacity \(k_{j+1}\)

\[
\max_{k_{j+1}} \left( \frac{1}{2} (s + k_j - p)^2 + k_{j+1}(z - k_{j+1} - r) - F \right)
\]
\[ k_{j+1} = \frac{(N + 1)(z - r) + Ns - Np}{N + 2} \]

\[ k_{j+1}^* = \frac{(JN + 2N + J + 3)(N + 1)(z - r) + (JN + 2N + J + 2)N(s - c)}{(N + 2)(JN + 2N + J + 3) + N} \]

Interpretation: since \( P(Q, K_i, k_{j+1}) \) becomes a function of \( k_{j+1} \) only once we solve for \( Q \), this means that the incumbent’s choice of capacity does not depend on the entrant and they have no incentive to deter entry under this scenario.

**Stage 1**: Incumbents choose \( k_j \) to maximize profits

\[
\max_{k_j} \left( q_j^o(p - c) + \left( \frac{1}{2} \right) (s + k_j - p)^2 + k_j(z - k_j - r) - F \right)
\]

\[
q_j^o \frac{dp}{dK} + s + k_j - s \frac{dp}{dK} - k_j \frac{dp}{dK} - p + p \frac{dp}{dK} + z - 2k_j - r = 0
\]

\[
k_j = \frac{(N + 1)(z - r) + Ns - Np + q_j^o}{N + 2}
\]

\[
K_i = \frac{J(N + 1)(z - r) + Njs - Njp + Q^o}{N + 2}
\]

\[
k_i^* = J(z - r) + Js - Jc
\]

\[
k_j^* = (z - r) + s - c
\]

**Results:**

\[
k_{j+1}^* = \frac{(JN + 2N + J + 3)(N + 1)(z - r) + (JN + 2N + J + 2)[s - c]}{(N + 2)(JN + 2N + J + 3) + N} \frac{N(N + 2N + J + 3) + N}{(N + 2)(JN + 2N + J + 3) + N}
\]

\[
p^* = \frac{s + (J + 2)(N + 1)c}{JN + 2N + J + 3} + \frac{(N + 1)(z - r)}{(N + 2)(JN + 2N + J + 3) + N} + \frac{N(N + 2N + J + 3) + N}{(JN + 2N + J + 3)[(N + 2)(JN + 2N + J + 3) + N]}
\]

\[
K_i^* = J(z - r) + Js - Jc
\]

\[
k_j^* = (z - r) + s - c
\]

\[
a_i^* = (b - p^*)
\]

\[
a_f^* = (\gamma - p^*)
\]
\[ a^{**} = (s + k_j^{**} - p^{**}) \]
\[ Q^{**} = 2Js + n_2 y + n_R r + J(z - r) - Jc + \frac{(JN + N + J + 2)(s + k_{j+1}) - (J + 2)(N + 1)c}{JN + 2N + J + 3} \]
\[ Q^{o**} = J[2s - (N + 2)c + (z - r) + Np^{**}] \]
\[ Q^{o**} = J2s - J(N+2)c + J(z - r) + \frac{NJ s + NJ(J + 2)(N + 1)c}{JN + 2N + J + 3} + \frac{NJ(N + 1)(z - r)}{(N + 2)(JN + 2N + J + 3) + N} \]
\[ + \frac{N^2J(N+2N+J+2)[s - c]}{(JN + 2N + J + 3)[(N + 2)(JN + 2N + J + 3) + N]} \]
\[ Q^{f**} = n_2(y + b) - (N + 1)c + p^{**}(N + 1 - 2n_2) \]
\[ Q^{i**} = n_1 b - (N + 1)c + p^{**}(N + 1 - n_1) \]
\[ \Pi_j^{**} = p^{**} \left[ (N + \frac{1}{2})p^{**} - c(2N + 1) \right] + \left( \frac{1}{2} \right)(s - c + z - r)^2 + \frac{1}{2}s^2 + Nc^2 - F \]
\[ \Pi_{j+1}^{**} = \left( \frac{1}{2} \right)(s + k_{j+1}^{**} - p^{**})^2 + k_{j+1}^{**}(z - k_{j+1}^{**} - r) - F \]
\[ k_{j+1}^{**} = \frac{(N + 1)(z - r) + Ns - Np^{**}}{N + 2} \]
\[ \Pi_{j+1}^{**} = \left( \frac{1}{2} \right) \left( \frac{2(N + 1)(s - p^{**}) + (N + 1)(z - r)}{N + 2} \right)^2 - \left( \frac{(N + 1)(z - r) + N(s - p^{**})}{N + 2} \right)^2 \]
\[ + \frac{(z - r)[(N + 1)(z - r) + N(s - p^{**})]}{N + 2} - F \]

**Section 3: Entry Deterrence is Optimal (constrained water supply)**

Consider the case where \( Q^o = Y^o, Q^l < Y^l, Q^f < Y^f \) such that the water sales for the oil producers are constrained by their available water supply, but the First Nation and the local community have more water than they choose to supply. [Only the oil producers are constrained]. As before, water demand can be stated as a function of price:

\[ p = \frac{n_R r + n_2 y + (J + 1)s + K_1 + k_{j+1}}{N + 1} - (Y^o + Q^l + Q^f) \]

Here, as in case 2.1, the first nations and local community will chose their water to supply to maximize their profits to obtain:

\[ Q^f = n_2(y + b) - (N + 1)c + p(N + 1 - 2n_2) \]
\[ Q^l = n_1 b - (N + 1)c + p(N + 1 - n_1) \]

Then the price becomes:

\[ p = \frac{(J + 1)s + K_1 + k_{j+1} - Y^o + 2(N + 1)c}{2N + J + 3} \]
The entrant will choose a non-negative $k_{j+1}$ to maximize:

$$\Pi_{j+1} = \left(\frac{1}{2}\right)(s + k_{j+1} - p)^2 + k_{j+1}(\alpha - k_{j+1}) - F,$$

where: $\alpha = (z - r) > 0$

In the case where deterrence is optimal, we know that the entrant will not enter the market and can therefore not have positive profits, so we assume $\Pi_{j+1} = 0$.

Differentiating with respect to $k_{j+1}$ and assuming an interior solution, we have:

$$\frac{d\Pi_{j+1}}{dk_{j+1}} = (s + k_{j+1} - p)\left(1 - \frac{1}{2N + J + 3}\right) + (\alpha - 2k_{j+1}) = 0$$

These first order conditions imply that

$$s + k_{j+1} - p = \frac{(2N + J + 3)(2k_{j+1} - \alpha)}{2N + J + 2}$$

Hence

$$\Pi_{j+1} = \left(\frac{1}{2}\right)\frac{(2N + J + 3)^2(2k_{j+1} - \alpha)^2}{(2N + J + 2)^2} + k_{j+1}(\alpha - k_{j+1}) - F$$

$$\Pi_{j+1} = \frac{(2N + J + 3)^2(2k_{j+1} - \alpha)^2 + 2(2N + J + 2)^2[k_{j+1}(\alpha - k_{j+1}) - F]}{2(2N + J + 2)^2}$$

Since $\Pi_{j+1} = 0$ we have

$$(2N + J + 3)^2(2k_{j+1} - \alpha)^2 + 2(2N + J + 2)^2[k_{j+1}(\alpha - k_{j+1}) - F] = 0$$

$$(2N + J + 3)^2(4k_{j+1}^2 - 4k_{j+1}\alpha + \alpha^2) + 2(2N + J + 2)^2[k_{j+1}\alpha - k_{j+1}^2 - F] = 0$$

$$2(4N^2 + 4NJ + 16N + J^2 + 8J + 14)[k_{j+1}^2 - \alpha k_{j+1}] + \alpha^2(2N + J + 3)^2 - 2(2N + J + 2)^2F = 0$$

Let:

$$\beta = \frac{2(2N + J + 2)^2F - \alpha^2(2N + J + 3)^2}{2(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}$$

$$\left(\frac{F}{\alpha^2}\right) > \left[\frac{(2N + J + 3)^2}{2(2N + J + 2)^2}\right]$$

Then the above equation reduces to

$$k_{j+1}^2 - \alpha k_{j+1} - \beta = 0$$

So \( k_{j+1}^{opt} = (\alpha \pm (\alpha^{-2} + 4\beta)^{1/2})/2 \)
Where
\[(\alpha^2 + 4\beta)^\frac{1}{2} = \left[\frac{4(2N + J + 2)^2F - (z - r)^2(4N^2 + 4N + 8N + J^2 + 4J + 4)}{(4N^2 + 4N + 16N + J^2 + 8J + 14)}\right]^{1/2}\]

\[(\alpha^2 + 4\beta)^\frac{1}{2} = (2N + J + 2)\left[\frac{4F - (z - r)^2}{(4N^2 + 4N + 16N + J^2 + 8J + 14)}\right]^{1/2}\]

From the first order conditions we know that
\[k_{j+1} = p - s + \frac{(2N + J + 3)(2k_{j+1} - \alpha)}{2N + J + 2}\]

\[k_{j+1}^* = \frac{(2N + J + 3)^2\alpha - (2N + J + 2)[K_l - Y^o - 2(N + 1)(s - c)]}{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}\]

Combining to solve for \(K_{j}^*\):

\[\alpha + (\alpha^2 + 4\beta)^\frac{1}{2} = \frac{2(2N + J + 3)^2\alpha - 2(2N + J + 2)[K_l - Y^o - 2(N + 1)(s - c)]}{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}\]

\[\left(\alpha + (\alpha^2 + 4\beta)^\frac{1}{2}\right)(4N^2 + 4N + 16N + J^2 + 8J + 14) = 2(2N + J + 3)^2\alpha - 2(2N + J + 2)[K_l - Y^o - 2(N + 1)(s - c)]\]

\[2(2N + J + 2)K_l = 2(2N + J + 3)^2\alpha + 2(2N + J + 2)[Y^o + 2(N + 1)(s - c)] - \left(\alpha + (\alpha^2 + 4\beta)^\frac{1}{2}\right)(4N^2 + 4N + 16N + J^2 + 8J + 14)\]

\[K_{j}^* = Y^o + 2(N + 1)(s - c) + \frac{(2N + J + 2)\alpha}{2} - \frac{(\alpha^2 + 4\beta)^\frac{1}{2}(4N^2 + 4N + 16N + J^2 + 8J + 14)}{2(2N + J + 2)}\]

**Results (Section 3):**

When entry is deterred, the price will become:

\[p = \frac{Js + K_l - Y^o + 2(N + 1)c}{(2N + 2 + J)}\]

\[\beta = \frac{2(2N + J + 2)^2F - \alpha^2(2N + J + 3)^2}{2(4N^2 + 4N + 16N + J^2 + 8J + 14)}\]

\[p_{j+1}^* = s + \frac{\alpha}{2} - \frac{(\alpha^2 + 4\beta)^\frac{1}{2}(4N^2 + 4N + 16N + J^2 + 8J + 14)}{2(2N + J + 2)^2}\]
\[ k_{j+1} = 0 \]

\[ K_i^{**} = Y^o + 2(N + 1)(s - c) + \frac{(2N + J + 2)\alpha}{2} - \frac{(\alpha^2 + 4\beta)(4N^2 + 4NJ + 16N + J^2 + BJ + 14)}{2(2N + J + 2)} \]

\[ Qf^{**} = n_2(y + b) - (N + 1)c + p^{***i}(N + 1 - 2n_2) \]

\[ Ql^{**} = n_1b - (N + 1)c + p^{***i}(N + 1 - n_1) \]

\[ Q^o^{**} = Y^o \]

\[ \Pi_{j+1}^{**} = 0 \]

\[ \Pi_j^{**} = Y^o(p^{**} - c) + \left(\frac{1}{2}\right)(s + k_j^{**} - p^{**})^2 + k_j^{**}(z - k_j^{**} - r) - F \]

**Section 4: Accommodation is Optimal (constrained water supply)**

Consider the case where the water constraint is binding for the oil producers, \( Q^o = Y^o \), but not for the First Nation of for the local community \( Q^o = Y^o, Q^l < Y^l, Q^f < Y^f \). The same first order conditions as before apply to the water demands. As before, water demand can be stated as a function of price:

\[ p = \frac{(J + 1)s + K_i + k_{j+1} - Y^o + 2(N + 1)c}{2N + J + 3} \]

In this case, the supply is determined by \( Q = Q^f + Q^l + Y^o \)

\[ Q^f = n_2(y + b) - (N + 1)c + p(N + 1 - 2n_2) \]

\[ Q^l = n_1b - (N + 1)c + p(N + 1 - n_1) \]

\[ Q^o = Y^o \]

**Stage 2: Choice of capacity by the entrant:**

Entrant chooses capacity \( k_{j+1} \) to maximize \( \Pi_{j+1} \):

\[ \Pi_{j+1}(p, k_{j+1}) = \frac{\alpha^o(p, k_{j+1})^2}{2} + k_{j+1}(z - r - k_{j+1}) - F \]

First Order condition: \( s - k_{j+1} - p + z - r + (p - s - k_{j+1}) \frac{1}{2N + J + 3} = 0 \)

\[ (2N + J + 4)k_{j+1} = (2N + J + 2)s + (2N + J + 3)(z - r) - (2N + J + 2)p \]


The price then becomes:

\[ p = \frac{(2N + J + 4)(J + 1)s + K_i - Y^o + 2(N + 1)c}{(2N + J + 3)(2N + J + 4) + (2N + J + 2)} + \frac{(2N + J + 2)s + (2N + J + 3)(z - r)}{(2N + J + 3)(2N + J + 4) + (2N + J + 2)} \]
Stage 1: Choice of capacity by the incumbents:

Incumbent oil producers choose capacity $k_j$ to maximize $\Pi_j$:

First Order condition: $s - k_j - p + z - r + (q^o - s - k_j + p) \frac{1}{2N + J + 3} = 0$

$(2N + J + 4)k_j = (2N + J + 2)s + (2N + J + 3)(z - r) + y^o - (2N + J + 2)p$


Results:


$$p^{*,A} = \frac{(2N + J + 4)(J + 1)s - y^o + 2(N + 1)c}{(2N + J + 3)(2N + J + 4) + (J + 1)(2N + J + 2)}$$

Section 5: Additional Analysis

Now we compare the outcomes for 4 cases: No entry, Entry when the water quotas are not binding, entry when the water quota is binding, and entry deterrence when the water quota is binding. Note that here we consider the case, where the quotas are binding for the oil producers only, not for the first nation or for the local community ($Q^o = Y^o, Q^l < Y^l, Q^f < Y^f$)

a. No entry vs Accommodated Entry:

$$p^* = c$$
\[ K_i' = f(z - r) + Js - Jc \]
\[ K_i'' = f(z - r) + Js - Jc \]
\[ p^* = \frac{s + (J + 2)(N + 1)c}{JN + 2N + J + 3} + \frac{(N + 1)(z - r)}{(N + 2)(JN + 2N + J + 3) + N} \]
\[ + \frac{N(JN + 2N + J + 2)[s - c]}{(JN + 2N + J + 3)[(N + 2)(JN + 2N + J + 3) + N]} \]

Here we see that \( p^* < p^{**} \) and \( K_i^{**} = K_i' \), since \( p^* < p^{**} \Leftrightarrow \]
\[ c < \frac{s + (J + 2)(N + 1)c}{JN + 2N + J + 3} + \frac{(N + 1)(z - r)}{(N + 2)(JN + 2N + J + 3) + N} \]
\[ + \frac{N(JN + 2N + J + 2)[s - c]}{(JN + 2N + J + 3)[(N + 2)(JN + 2N + J + 3) + N]} \]

This is true since the second two terms in the RHS are positive since \( z > r \) and \( s > c \) by assumption and
\[ c < \frac{s + (J + 2)(N + 1)c}{JN + 2N + J + 3} = \frac{s}{JN + 2N + J + 3} + \frac{(N + 2) + J + 2)c}{JN + 2N + J + 3} \]

We can see that \( \frac{(N + 2) + J + 2)c}{JN + 2N + J + 3} \approx c \) and for \( s \) sufficiently large, the above will be true.

b. Accommodation in the absence of a binding constraint vs accommodation where there is a binding water constraint: \( Q_0 = Y_0 \). We can see that both the capacity and the price are greater when the water constraint is binding:
\[ K_i^{**} > K_i'^* \Leftrightarrow \]
\[ \frac{(2N + J + 2)Js}{(2N + J + 4)} + \frac{(2N + J + 3)(z - r)}{(2N + J + 4)} + \frac{(2N + J + 3)(2N + J + 4) + (2N + J + 2)Y_0}{(2N + J + 3)(2N + J + 4)^2} \]
\[ - \frac{(2N + J + 2)[(J + 1)s - Y_0 + 2(N + 1)c]}{(2N + J + 3)(2N + J + 4)} > J(z - r) + Js - Jc \]

We can see that this will be true for \( Y_0 \) sufficiently large. If \( Y_0 \) is not sufficiently large, we will see in the next case that entry deterrence will be optimal when there is a constraint on the water price. We also predict that when the water constraint is binding, the water price will be higher:
\[ p^{**} > p^{**} \]
\[(2N + J + 4)[(J + 1)s - J\gamma^o + 2J(N + 1)c] + (J + 1)[(2N + J + 2)s + (2N + J + 3)(z - r)]
+ \frac{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)\gamma^o}{(2N + J + 3)(2N + J + 4)}
> \frac{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)[s + (J + 2)(N + 1)c]}{JN + 2N + J + 3}
+ \frac{(N + 1)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)(z - r)}{[N^2J + 2N^2 + 3NJ + 8N + 2J + 6]}
+ \frac{N(N + 2N + J + 2)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)[s - c]}{(JN + 2N + J + 3)[N^2J + 2N^2 + 3NJ + 8N + 2J + 6]}
\]

c. Accommodation vs deterrence where there is a binding water constraint: \(\gamma^o = Y^o\). We can see that both the capacity and the price are greater when entry is deterred:

\[p^{**.3} > p^{**.4} \leftrightarrow\]

\[s + \frac{\alpha^2 + 4\beta}{2} - \frac{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)^2}
> \frac{(2N + J + 4)[(J + 1)s - Y^o + 2(N + 1)c]}{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}
+ \frac{(2N + J + 2)[(J + 1)s + (2N + J + 3)(J + 1)(z - r)]}{(N^2J + 2N^2 + 3NJ + 8N + 2J + 6)} + \frac{Y^o}{(2N + J + 3)(2N + J + 4)}\]

We see that this is indeterminate and the greater price will be determined by both the value of the water quota and by the fixed cost

\[K_i^{**.3} > K_i^{**.4} \leftrightarrow\]

\[Y^o + 2(N + 1)(s - c) + \frac{(2N + J + 2)(z - r) - (4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)}
+ \frac{(2N + J + 2)[(J + 1)s - Y^o + 2(N + 1)c]}{(2N + J + 3)(2N + J + 4)}\]

We can see here that, given the size of the denominators on the right hand side above, the accommodation capacity will be lower, again dependent on the prior assumption about the fixed cost being sufficiently low.