Are the Responses of the U.S. Economy Asymmetric to Positive and Negative Money Supply Shocks?*

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Abstract:

We investigate whether the United States economy responds asymmetrically to positive and negative money supply shocks of different magnitude, using a test recently introduced by Kilian and Vigfusson (2011) based on impulse response functions. We use quarterly data, over the period from 1967:1 to 2014:1, and the new CFS Divisia monetary aggregates, making a comparison among the narrower monetary aggregates, M1 M2M, MZM, M2, and ALL, and the broad monetary aggregates, M4+, M4-, and M3. We show that there is no statistically significant evidence of asymmetry in the response of the U.S. economy to positive and negative money supply shocks of different magnitude.

JEL classification: C32, C43, E52.

Keywords: Monetary policy; Money shocks; Divisia monetary aggregates.

1 Introduction

As Milton Friedman (1968, p. 1) put it in his Presidential Address at the 88th meeting of the American Economics Association, the inability of the presumed easy monetary policy to stimulate the level of economic activity in the United States during the Great Depression led to the opinion that “monetary policy was a string. You could pull on it to stop inflation but you could not push on it to halt recession. You could lead a horse to water but you could not make him drink.” That is to say, monetary policy has asymmetric effects on output and in particular that positive monetary policy shocks have smaller real effects than negatives ones. This type of asymmetry, known as the “traditional Keynesian asymmetry,” to use a phrase coined by Ravn and Sola (2004), can be derived under the assumption of downward sticky and upward flexible nominal wages and prices together with credit rationing. In its more extreme form, it states that only negative monetary policy shocks have real effects and that expansionary monetary policy could be neutral if economic agents are not willing to borrow and invest.

The “pushing on a string” view about monetary policy was widespread until Friedman and Schwartz (1963) highlighted that U.S. monetary policy during the Great Depression had, in fact, been tight and not easy as previously believed. As Florio (2004, p. 410-411) put it, in her review of the literature regarding the asymmetric effects of monetary policy, “during the period 1929–33 one third of the banks failed or merged and the quantity of money in the USA fell by one third. This reduction in the monetary aggregate was not ascribed to the reluctance of firms and households to borrow but to the behavior of the Federal Reserve that didn’t provide the banking system promptly with the needed liquidity. In that period, despite the fall in nominal interest rates, prices were falling, so real interest rates were actually very high. The downturn was so severe and lasting because the banking crisis in 1930–31 disrupted the credit system and this prevented the public from borrowing the funds they needed to invest.”

Florio (2004) surveys a number of empirical studies on the asymmetric effects of monetary policy and also reviews its possible theoretical explanations. As Florio (2004, p. 409) puts it, “it is widely recognized that monetary policy has real effects on the economy over short horizons. The idea that these real effects are asymmetric is, by contrast, less well established in the literature. In particular, the reduction in output following a negative monetary policy shock appears larger than the expansion induced by a positive shock of similar size.” In view of the serious implications for the sources of business cycles and for discriminating among competing models, our objective in the present paper is to test for asymmetric effects of monetary policy on the level of economic activity in the United States, using recent state-of-the-art advances in macroeconometrics and economic measurement.

A potential problem is that our sample period (from 1967:1 to 2014:1) is long enough to span several monetary policy regimes and it is not clear how to identify the stance of monetary policy throughout this period. For example, in the 1970s the Fed was using the federal funds rate as its operating instrument and monetary aggregates as intermediate targets. From October 1979 to October 1982, the Fed de-emphasized the federal funds rate as an operating instrument and used nonborrowed reserves as the primary operating instrument. Between October 1982 and the early 1990s, the Fed targeted on borrowed reserves and abandoned monetary aggregates as a guide for monetary policy. Finally, since the early 1990s the Fed has been using the federal funds rate as the primary operating instrument, but in the aftermath of the global financial crisis, the federal funds rate has reached the zero lower bound and the Fed is now focusing on its balance sheet instead, using quantitative measures of monetary policy.

In this paper, we follow the seminal work by Cover (1992) and use monetary aggregate policy instruments to measure the stance of monetary policy. In doing so, we use quarterly data, over the period from 1967:1 to 2014:1, and the new Divisia monetary aggregates, maintained within the Center of Financial Stability (CFS) program Advances in Monetary and Financial Measurement (AMFM). We are interested in whether positive money supply shocks have smaller output effects than negative money supply shocks (the traditional Keynesian asymmetry, also known as sign asymmetry) and in whether large money supply shocks have smaller effects than smaller money supply shocks (size asymmetry). We use a methodology, recently introduced by Kilian and Vigfusson (2011), that allows us to test for sign and size asymmetries, and we provide a comparison among eight CFS Divisia monetary aggregates — the narrower monetary aggregates, M1 M2M, MZM, M2, and ALL, and the broad monetary aggregates, M4+, M4-, and M3. See also Barnett and Chauvet (2009) and Serletis and Rahman (2009, 2015) for other studies that investigate the performance of the Divisia monetary aggregates.

The outline of the paper is as follows. Section 2 reviews the related empirical literature regarding the macroeconomic effects of positive and negative money supply shocks on the level of economic activity. In section 3 we briefly discuss the Kilian and Vigfusson (2011) test of the null hypothesis of symmetric impulse responses based on impulse response functions of a structural VAR model. Section 4 presents the data and discusses the empirical findings while section 5 discusses international data and evidence. The final section concludes the paper.

2 Related Empirical Literature

Cover (1992) is a seminal paper on the macroeconomic effects of positive and negative money supply shocks. His empirical methodology involves the simultaneous estimation of

$$m_t = \Phi(L)m_{t-1} + \boldsymbol{\theta}\boldsymbol{x}_{t-1} + \varepsilon_t \tag{1}$$

$$y_t = \boldsymbol{\phi}\boldsymbol{z}_t + \beta^+\varepsilon_t^+ + \beta^-\varepsilon_t^- + \xi_t \tag{2}$$

where m_t is the growth rate of the money supply and y_t the growth rate of real output. Equation (1) is a money supply process, with $\Phi(L)$ being a lag polynomial, $\boldsymbol{\theta}$ a vector of parameters, and $\boldsymbol{x}_{t-1}$ a vector of predetermined regressors that reflect possible endogenous policy responses (it includes variables such as unemployment, the money growth rate, the output growth rate, changes in the Treasury bill rate, and the government budget surplus). Equation (2) is the aggregate output equation, with $\boldsymbol{\phi}$ being a vector of parameters and $\boldsymbol{z}_t$ a vector of regressors (and includes lagged changes in output and lagged changes in the interest rate). In equation (2), ε_t^+ and ε_t^- are the positive and negative unanticipated money supply shocks from equation (1), defined as

$$\varepsilon_t^+ = \max(\varepsilon_t, 0) \quad \text{and} \quad \varepsilon_t^- = \min(\varepsilon_t, 0).$$

In the context of equations (1) and (2), the null hypothesis of asymmetry is $H_0 : \beta^+ = \beta^-$. Rejection of the restriction that β^+ equals β^-, together with β^+ being insignificantly different from zero and β^- significantly different from zero, supports the hypothesis of asymmetric effects. Cover (1992) used quarterly data for the United States over the period from 1951:1 to 1987:4, the simple-sum M1 monetary aggregate as the key monetary variable, and three different money supply processes in (1) to investigate the effects of positive and negative unanticipated money supply shocks. He found that positive money supply shocks do not have an effect on output while negative money supply shocks do have an effect on output.

Cover's (1992) finding was further confirmed by a number of other authors, including DeLong and Summers (1988), Thoma (1994), Kandil (1995), and Karras (1996), among others. However, this evidence of asymmetric effects of positive and negative money supply shocks also generated considerable controversy. For example, Ravn and Sola (2004) use the same data and methodology as Cover (1992) and argue that Cover's findings are not very robust and the asymmetry of the effects of positive and negative money supply shocks disappears when the regime shift in U.S. monetary policy in 1979 is controlled for. Also, Weise (1999) investigates whether monetary policy has asymmetric effects on output and prices using a more general, nonlinear VAR approach. As Weise (1999, p. 86) puts it, "the paper finds that while monetary shocks have dramatically different effects depending on the state of the economy, there is no evidence to support the hypothesis that positive and negative monetary shocks have different effects. There is some evidence that large and small shocks have different effects, and that positive and negative monetary shocks have different effects when the shocks are large."

3 A Statistical Test of Symmetric Effects

Most of the empirical studies that investigate the asymmetric effects of money supply shocks use slope-based tests of the type discussed in the previous section. Recently, however, Kilian and Vigfusson (2011) in their investigation of the effects of oil price shocks, question the use of slope-based tests to test for nonlinearities and asymmetries and propose a test of symmetric impulse responses to positive and negative shocks of different sizes, based on impulse response functions. They demonstrate that slope-based tests are not informative with respect to whether the asymmetry in the impulse responses is economically or statistically significant, and also that slope-based tests cannot allow for the fact that the degree of asymmetry of the response functions by construction depends on the magnitude of the shock.

The Kilian and Vigfusson (2011) symmetry test based on impulse response functions involves estimating the following nonlinear structural VAR model

$$m_t = \alpha_{10} + \sum_{j=1}^{p} \beta_{11}(j) m_{t-j} + \sum_{j=1}^{p} \beta_{12}(j) y_{t-j} + u_{1t} \tag{3}$$

$$y_t = \alpha_{20} + \sum_{j=0}^{p} \beta_{21}(j) m_{t-j} + \sum_{j=1}^{p} \beta_{22}(j) y_{t-j} + \sum_{j=0}^{p} \delta_{21}(j) \tilde{m}_{t-j} + u_{2t} \tag{4}$$

where $\tilde{m}_t$ is a nonlinear function of the money growth rate as follows

$$\tilde{m}_t = \max \left[0, \ln M_t - \max \left\{ \ln M_{t-1}, \ln M_{t-2}, \ln M_{t-3}, \cdots, \ln M_{t-12} \right\} \right]$$

and M_t denotes a money supply measure. Above, $\tilde{m}_t$ is interpreted as the net money supply increase over the previous three years (12 quarters) and is meant to filter out increases in the supply of money that represent corrections for recent declines — see Hamilton (2003). As noted by Kilian and Vigfusson (2011), the inclusion of additional macroeconomic variables in the VAR does not affect the econometric points of interest and is not required for consistently estimating the vetted relationship under the maintained assumption of predetermined (or contemporaneously exogenous) money supply (in our case).

The null hypothesis of symmetric impulse responses of y_t to positive and negative money supply shocks of the same size is

$$H_0 : I_y(h, \delta) = -I_y(h, -\delta) \quad \text{for } h = 0, 1, ..., H.$$

It tests whether the response of y_t to a positive shock in the money growth rate of size δ is equal to the negative of the response of y_t to a negative shock in the money growth rate of the same size, $-\delta$, for horizons $h = 0, 1, ..., H$. This is a Wald test with a χ^2_{H+1} distribution with the variance-covariance matrix obtained from the unrestricted model. In applying the methodology, we set $p = 8$ in equations (3) and (4) and report results in the next section based on $10,000$ impulse responses ($R = 10,000$) and 100 histories ($T = 100$). For a detailed discussion of the methodology, see Kilian and Vigfusson (2011).

4 The Data and Empirical Evidence

We use quarterly United States data, over the period from 1967:1 to 2014:1, on two variables: money and real GDP. To measure money, we use the best available monetary aggregates, those being the new Divisia monetary aggregates, maintained within the Center of Financial Stability (CFS) program Advances in Monetary and Financial Measurement (AMFM). They are called CFS Divisia monetary aggregates, are documented in detail in Barnett *et al.* (2013), and are available at www.centerforfinancialstability.org/amfm.php. They are rigorously founded in economic aggregation and index-number theory and represent a significant improvement over the St. Louis Fed's Divisia monetary aggregates, called MSI (monetary services indices). See Barnett *et al.* (2013) for more details.

In addition to using the new CFS Divisia data we also make comparisons among eight CFS Divisia monetary aggregates: the narrower monetary aggregates, M1 M2M, MZM, M2, and ALL, and the broad monetary aggregates, M4+, M4-, and M3. The real GDP series is from the Federal Reserve Economic Database (FRED), maintained by the Federal Reserve Bank of St. Louis. We conduct a number of unit root and stationarity tests (which includes the ADF, DF-GLS, and KPSS tests) in the logarithms of each series as well the first differences of the logarithms. We conclude that the first differences of the logarithms are stationary, and in what follows we apply the Kilian and Vigfusson (2011) test of the null hypothesis of symmetric responses to positive and negative money supply shocks in the stationary m_t and y_t series.

Figures 1 to 8 show the empirical responses of the real GDP growth rate to one and two standard deviation positive and negative money supply shocks for each of the eight CFS Divisia monetary aggregates: the narrower monetary aggregates, M1 M2M, MZM, M2, and ALL, and the broad monetary aggregates, M4+, M4-, and M3. In particular, these figures plot the response of the real GDP growth rate to a positive shock, $I_y(h, \delta)$, and the negative of the response to a negative shock, $-I_y(h, -\delta)$. In Table 1, we also report p-values of the null hypothesis of symmetric impulse responses of the real GDP growth rate to positive and negative money shocks, $H_0 : I_y(h, \delta) = -I_y(h, -\delta)$. In fact, since the test depends on the size of the shock, we report results for both small or typical shocks (one standard deviation shocks, $\delta = \hat{\sigma}$) and large shocks (two standard deviation shocks, $\delta = 2\hat{\sigma}$).

As can be seen in Figures 1-8, both positive and negative money supply shocks are nonneutral. Moreover, the impulse responses confirm the procyclical nature of money; they indicate that positive money supply shocks (both typical and large) lead to an increase in real output while negative money supply shocks lead to a decline in real output. There is also evidence of size symmetry, as the impulse responses indicate that the effectiveness of money supply shocks increases with size. Finally, as can be seen in Table 1, the null hypothesis of symmetric responses to positive and negative money supply shocks of different magnitude cannot be rejected at all horizons. On the basis of these tests, we postulate that the responses of the U.S. economy to positive and negative money supply shocks are symmetric.

5 International Data and Evidence

We used the new CFS Divisia monetary aggregates to investigate whether the United States economy responds asymmetrically to positive and negative money supply shocks of different magnitude. A number of central banks throughout the world also produce Divisia monetary data that could be used for similar investigations. These central banks are: the European Central Bank, the Bank of Israel, the Bank of Japan, the National Bank of Poland, the Bank of England, the International Monetary Fund, and the Federal Reserve Bank of st. Louis. In fact, as an indication of the level of international acceptance of the Divisia approach to monetary aggregation, see the International Monetary Fund's official data document, *Monetary and Financial Statistics: Compilation Guide*, 2008, pp. 183-184.

Over the years, the performance of the Divisia monetary aggregates has also been investigated by a large number of studies (using methods different than the ones we used in this paper), more recently by Serletis and Gogas (2014), Hendrickson (2014), Belongia and Ireland (2014, 2015a, 2015b), Anderson *et al.* (2015), and Serletis and Koustas (2015), among others. See also the online library at http://www.centerforfinancialstability.org/amfm.php linking to Divisia monetary aggregates data and studies for over 40 countries throughout the world, including Australia, Canada, China, India, Japan, and the United Kingdom, among others.

In this regard, Jadidzadeh and Serletis (2015) argue that broad Divisia monetary aggregates potentially can resolve a number of paradoxes associated with the measurement of money, solve what Chrystal and MacDonald (1994) refer to as the "Barnett critique"— the economic measurement problems associated with the failure to find significant relations between money and the economy — and lead to the development of policy rules based on the quantity of money, consistent with the Chicago monetary tradition formulated by Simons (1936) and Friedman (1960) — see Tavlas (2015) for a discussion of that tradition.

6 Conclusion

We investigate whether the United States economy responds asymmetrically to positive and negative money supply shocks of different magnitude, using the methodology recently introduced by Kilian and Vigfusson (2011) based on impulse response functions. We use quarterly data, over the period from 1967:1 to 2014:1, and the new CFS Divisia monetary aggregates. We present clear evidence of symmetry, punching a hole in the traditional Keynesian asymmetry and those macroeconomic theories that predict asymmetries in the relationship between real aggregate activity and money supply shocks.

Our results corroborate the evidence in Serletis and Gogas (2014) and have significant implications for the conduct of monetary policy. In particular, in the aftermath of the global financial crisis and Great Recession, the federal funds rate has reached the zero lower bound and the Federal Reserve has lost its usual ability to signal policy changes via changes in interest-rate policy instruments. The evidence of a symmetric relationship between economic activity and Divisia money supply shocks elevates Divisia aggregate policy instruments to the center stage of monetary policy, as they are measurable, controllable, and in addition have predictable effects on goal variables.

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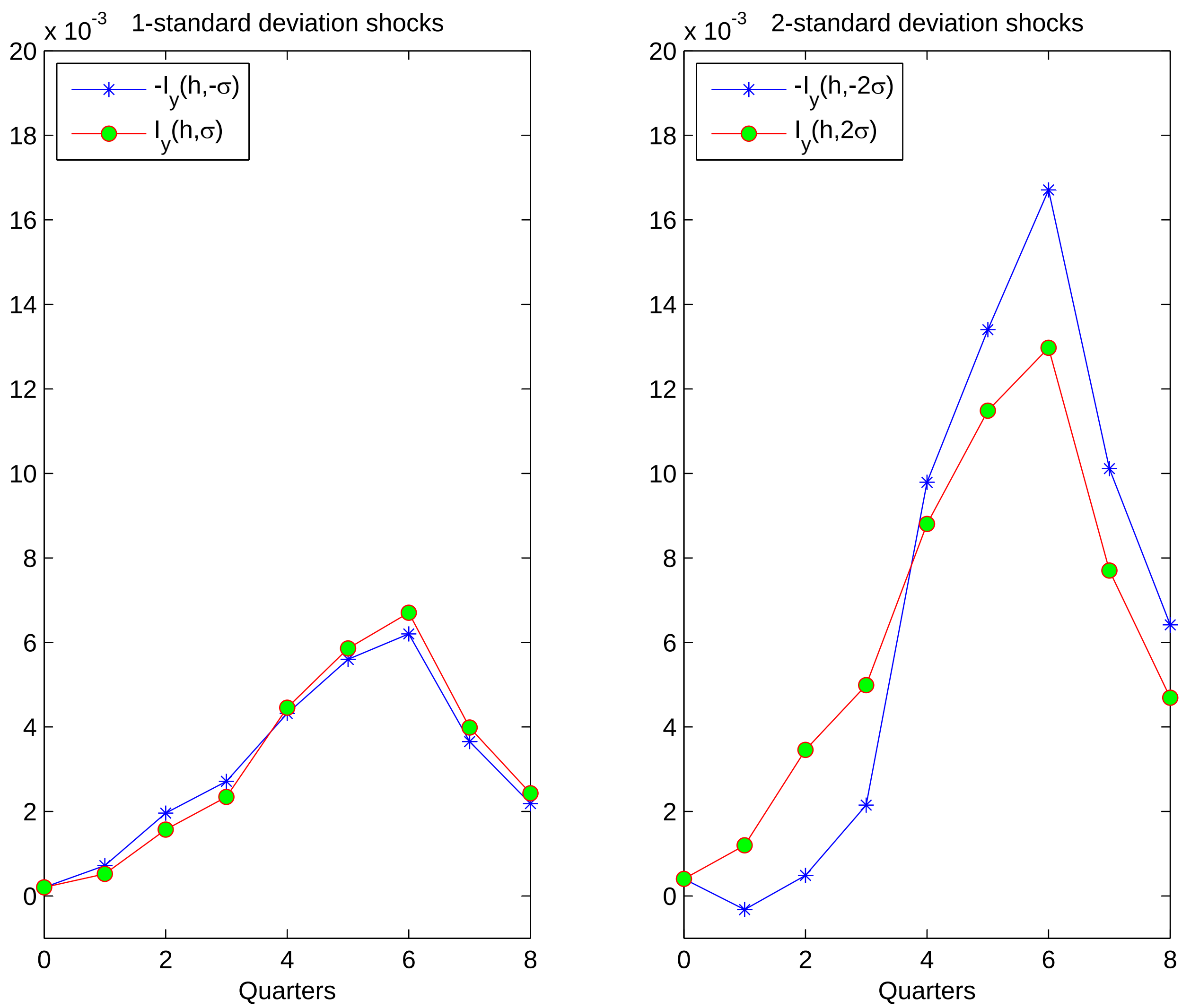

Figure 1. Responses to 1- and 2-standard deviation positive and negative Divisia M1 shocks.

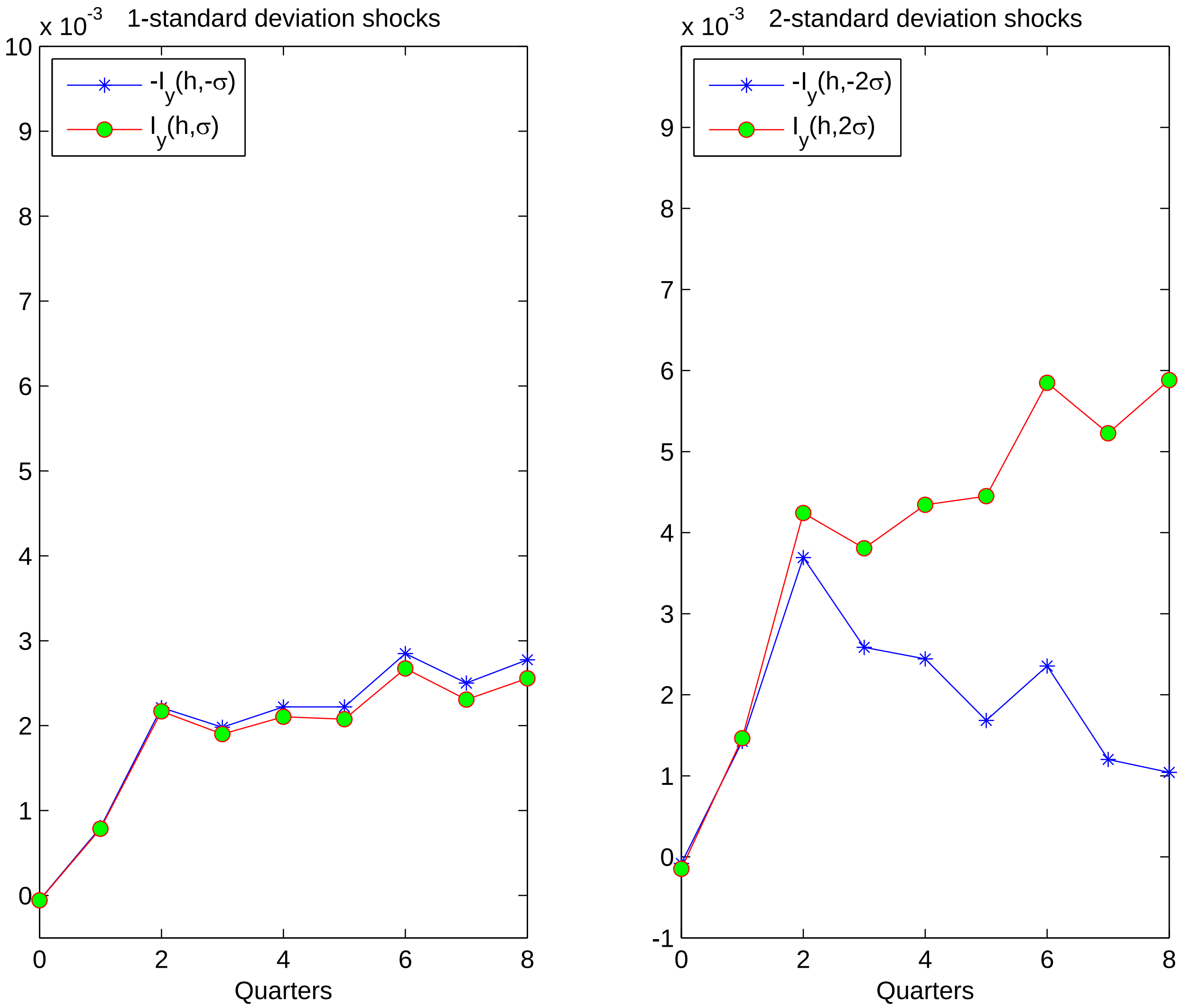

Figure 2. Responses to 1- and 2-standard deviation positive and negative Divisia M2M shocks

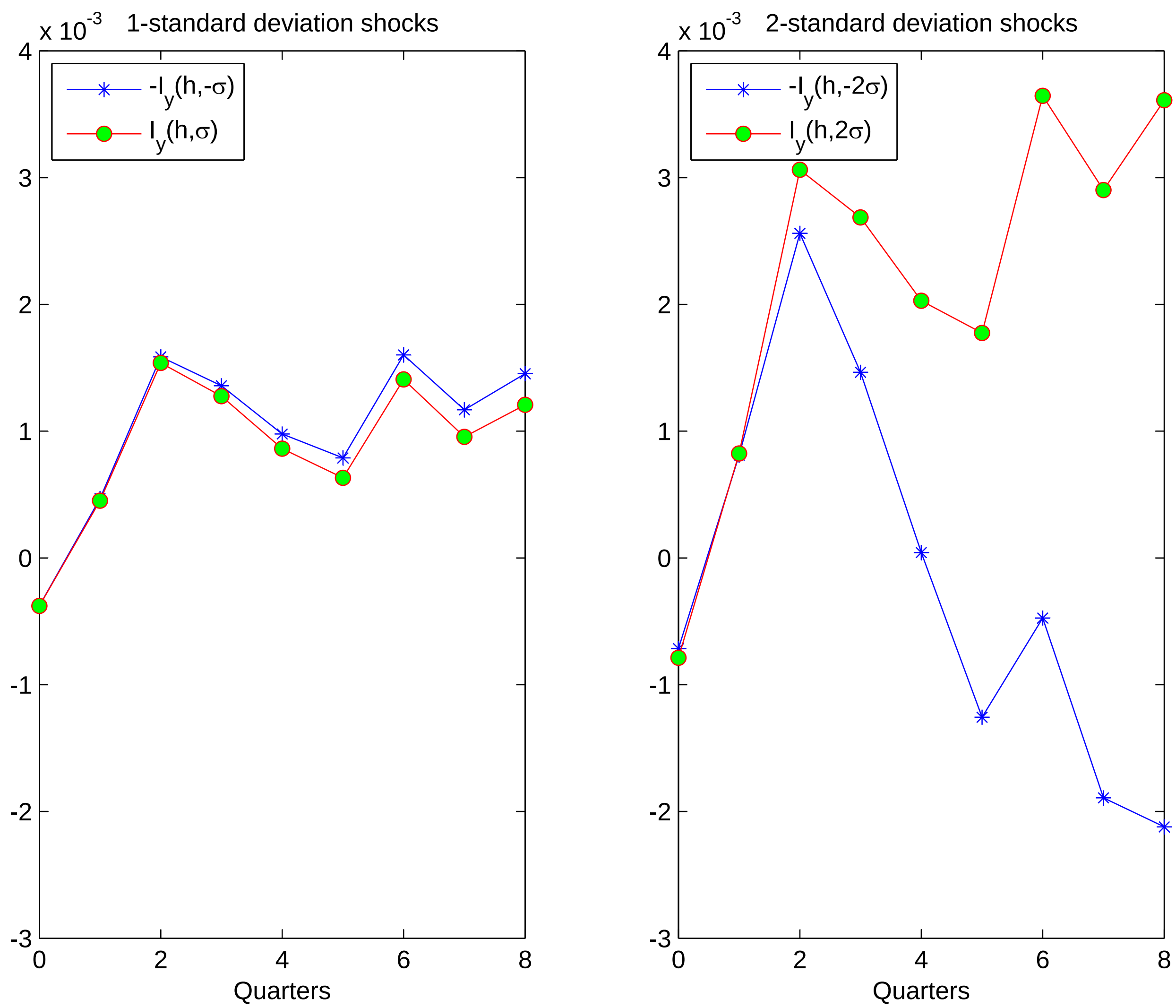

Figure 3. Responses to 1- and 2-standard deviation positive and negative Divisia MZM shocks

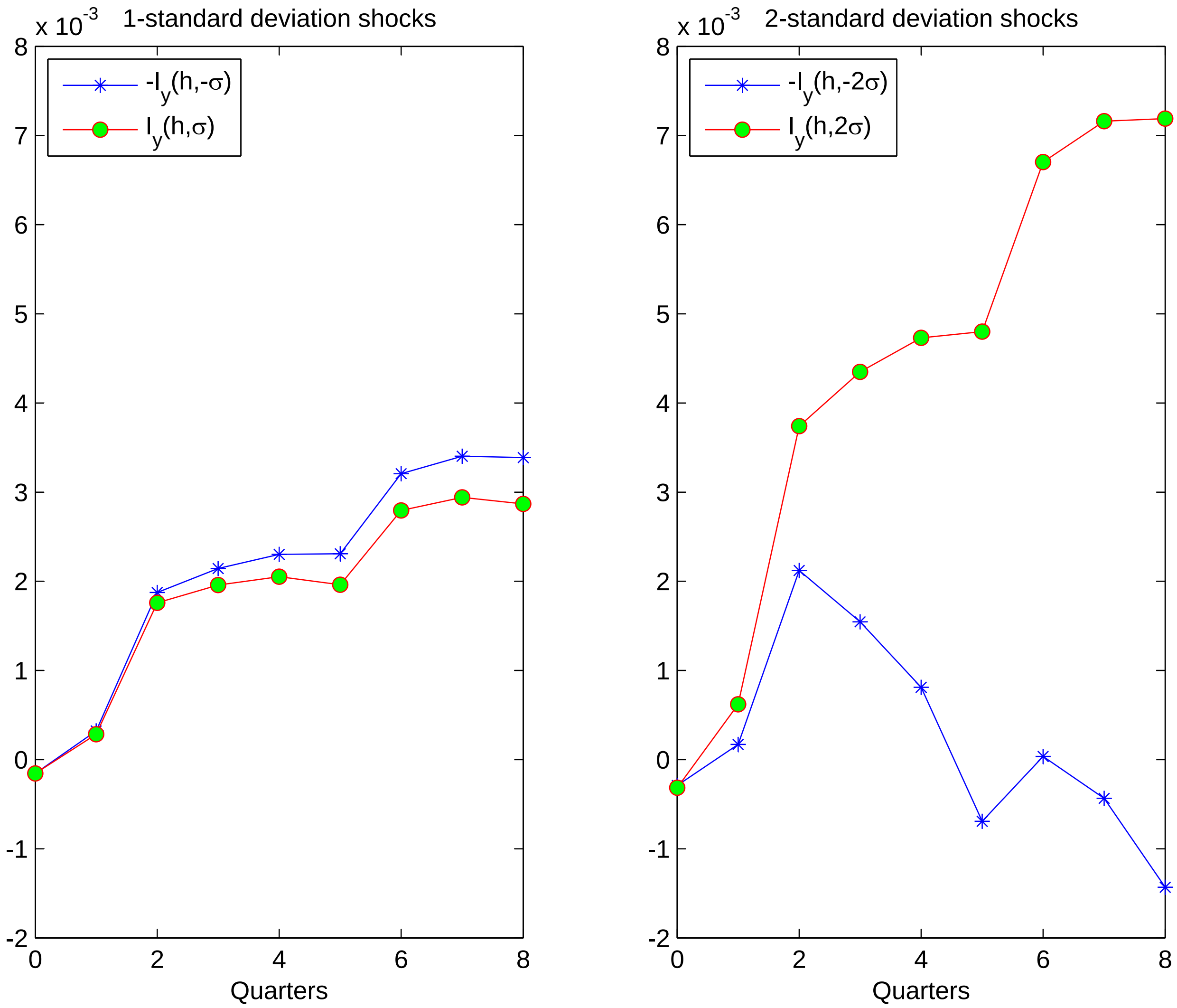

Figure 4. Responses to 1- and 2-standard deviation positive and negative Divisia M2 shocks

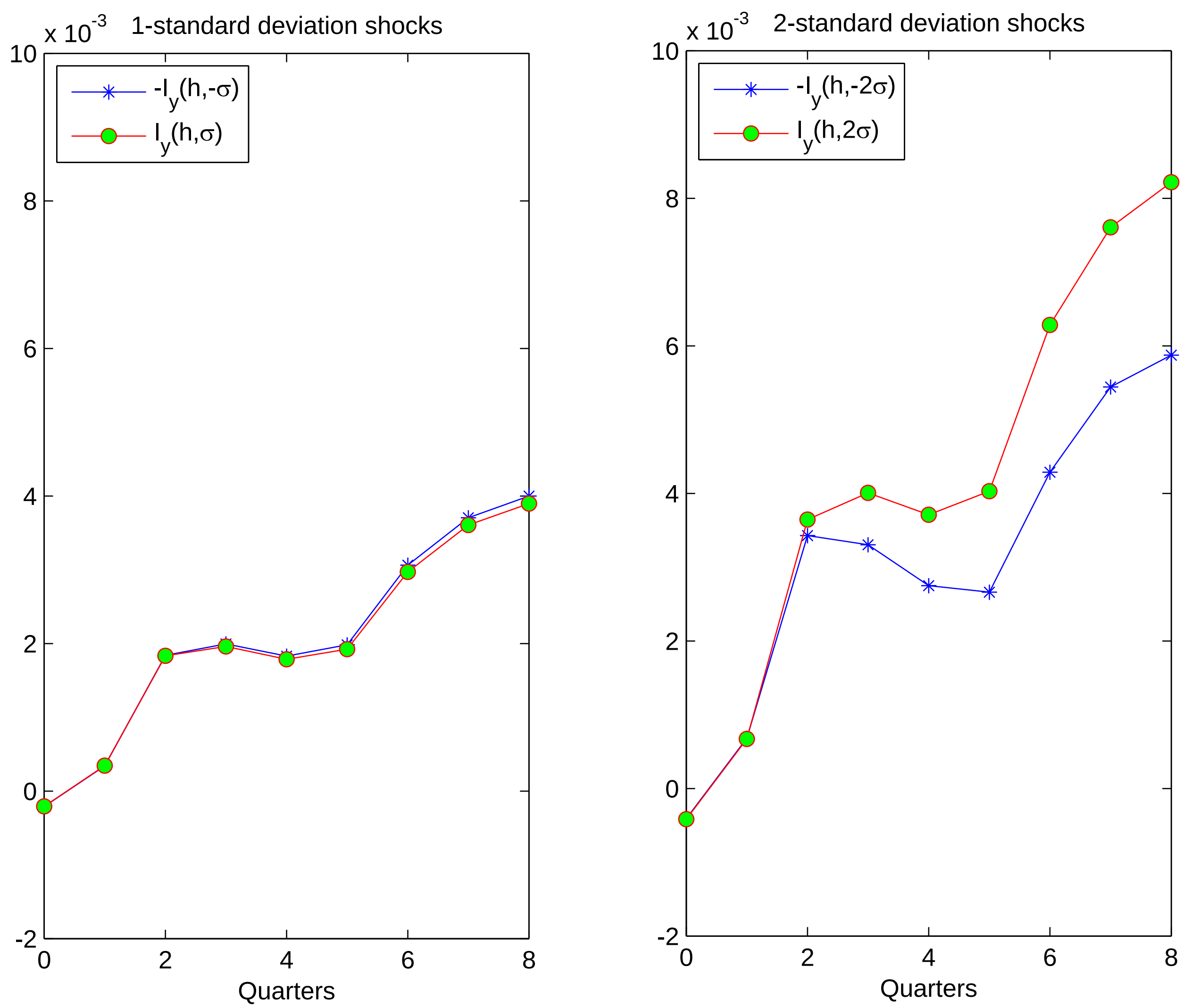

Figure 5. Responses to 1- and 2-standard deviation positive and negative Divisia ALL shocks

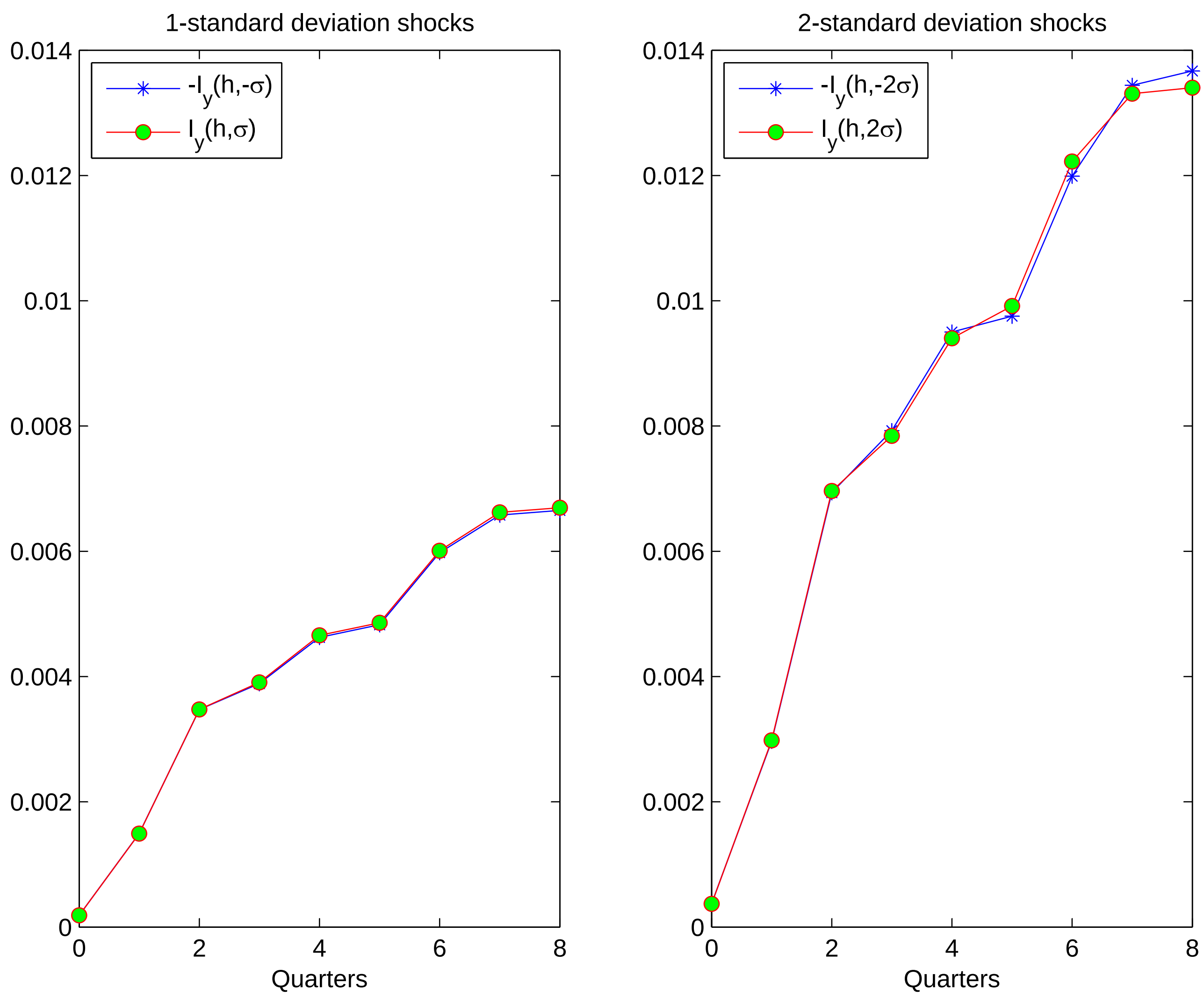

Figure 6. Responses to 1- and 2-standard deviation positive and negative Divisia M4+ shocks

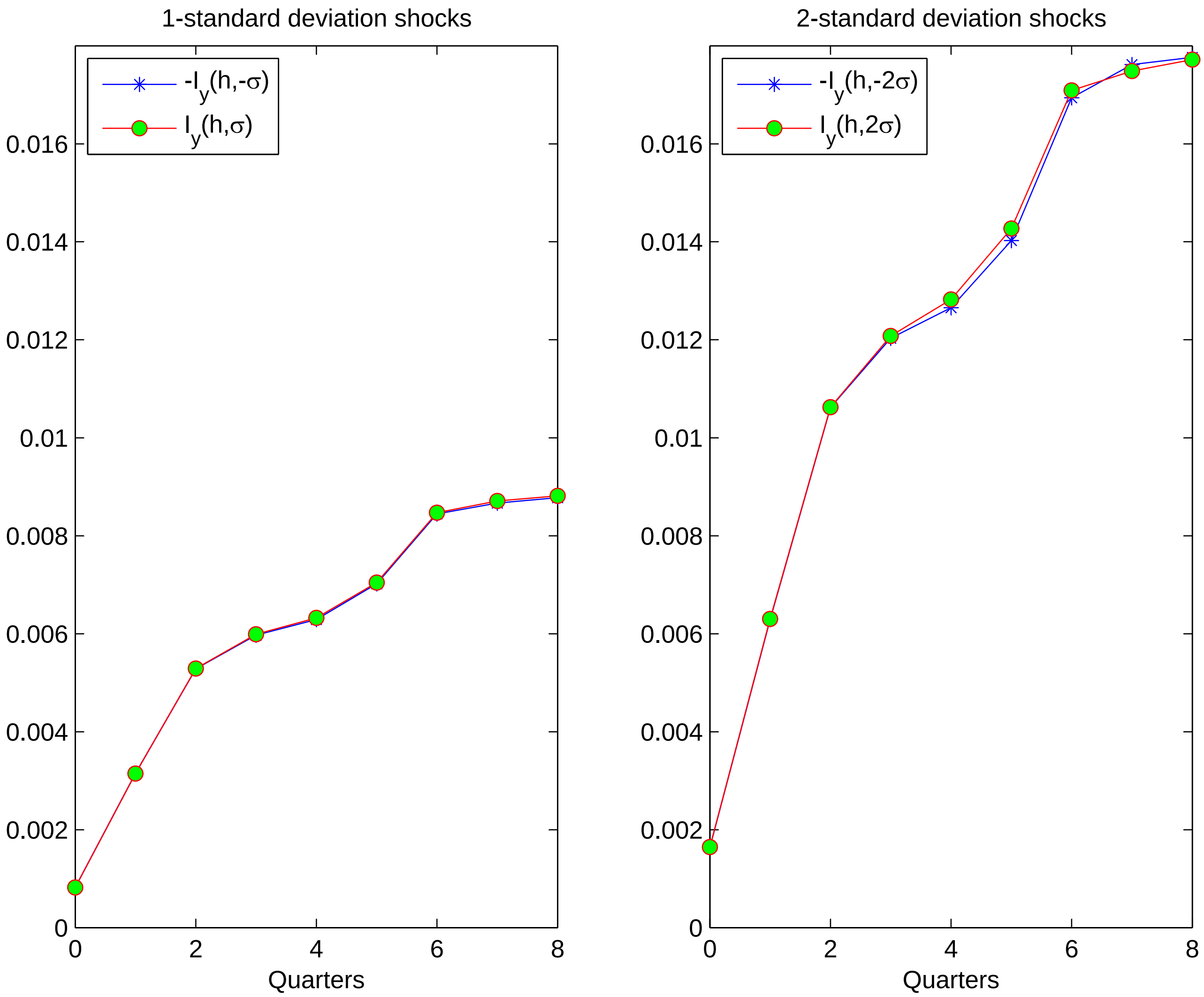

Figure 7. Responses to 1- and 2-standard deviation positive and negative Divisia M4- shocks

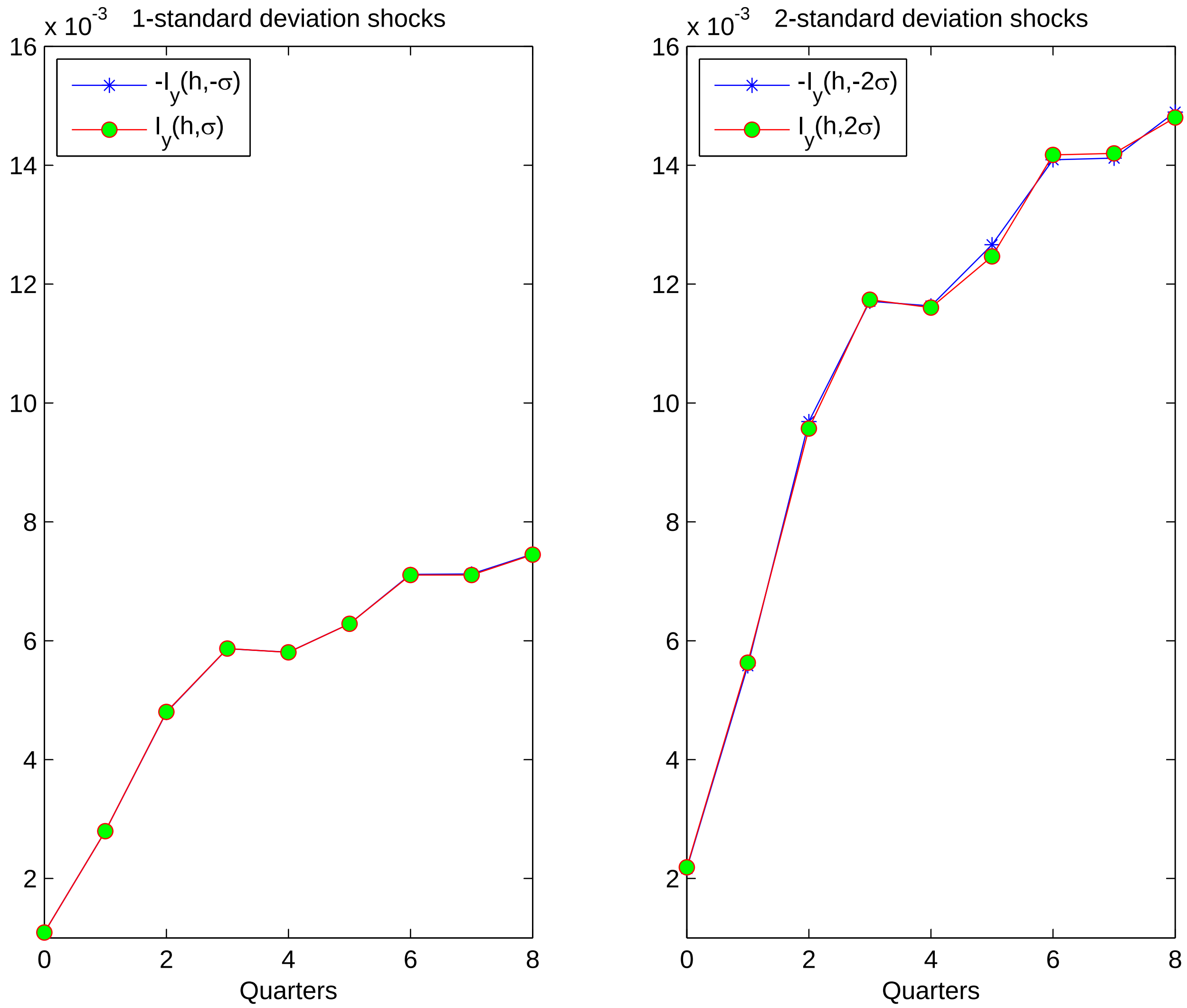

Figure 8. Responses to 1- and 2-standard deviation positive and negative Divisia M3 shocks

Table 1. p-values for $H_0 : I_y(h, \delta) = -I_y(h, -\delta), h = 0, 1, ..., 7$

A. Narrower CFS Divisia monetary aggregates

	Divisia M1		Divisia M2M		Divisia MZM		Divisia M2		Divisia ALL	
h	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$
0	0.97	0.99	0.78	0.44	0.91	0.39	0.98	0.60	0.91	0.82
1	0.38	0.39	0.94	0.72	0.94	0.68	0.83	0.70	0.99	0.97
2	0.51	0.58	0.94	0.81	0.93	0.78	0.88	0.76	0.99	0.99
3	0.33	0.52	0.95	0.85	0.96	0.80	0.95	0.84	0.99	0.98
4	0.25	0.36	0.98	0.92	0.98	0.87	0.98	0.92	0.99	0.99
5	0.32	0.48	0.99	0.95	0.99	0.92	0.99	0.95	0.99	0.99
6	0.39	0.60	0.99	0.98	0.99	0.94	0.99	0.97	0.99	0.99
7	0.39	0.66	0.99	0.99	0.99	0.96	0.99	0.99	0.99	0.99

B. Broad CFS Divisia monetary aggregates

	Divisia M4+		Divisia M4-		Divisia M3	
h	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$	$\delta = \widehat{\sigma}$	$\delta = 2\widehat{\sigma}$
0	0.99	0.93	0.95	0.98	0.97	0.77
1	0.99	0.99	0.99	0.99	0.99	0.95
2	0.99	0.99	0.99	0.99	0.99	0.98
3	0.99	0.99	0.99	0.99	0.99	0.99
4	0.99	0.99	0.99	0.99	0.99	0.99
5	0.99	0.99	0.99	0.99	0.99	0.99
6	0.99	0.99	0.99	0.99	0.99	0.99
7	0.99	0.99	0.99	0.99	0.99	0.99

Note: Results are based on 10,000 impulse responses ($R = 10,000$) and 100 histories ($T = 100$).