Non-Renewable Resource Stackelberg Games*

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Abstract

The market structure for many mineral industries can be described as oligopoly with potential for Stackelberg leadership. This paper derives and analyzes dynamically consistent extraction equilibria in a two-period discrete-time "Truly" Stackelberg (TS) model of non-renewable resource extraction, where firms move sequentially within each period and where both the leader and follower have market power. We show how the leader may be able to manipulate extraction patterns by exploiting resource constraints. Whether the leader wants to speed up its own production relative to the Cournot-Nash (CN) equilibrium depends on the shape of its iso-profit curve, which is affected by the two firms' relative stock endowments and relative production costs. If the leader extracts faster, then the follower extracts slower, but in aggregate the industry extracts faster. Unlike static Stackelberg games, the follower does not necessarily have a second mover disadvantage.

Key Words: Non-Renewable Resources, Stackelberg, Cournot-Nash, Dominant-Firm/Competitive-Fringe

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1 Introduction

This paper is motivated by evidence that the supply-side of the market for many major minerals is dominated by a few large firms surrounded by one or more other relatively large firms. Table 1 shows that the market structure for ten important minerals is clearly oligopolistic, since the top five firms account for more than half of world production for four minerals (diamonds, nickel, platinum, and tin), and for more than a third of world production for seven minerals, including aluminium, copper, and phosphate.[1] Furthermore, in the platinum, diamond, nickel, and tin industries the largest firm controls nearly a fifth of the market or more, and the size of the second largest firm in these industries suggests many mineral industries may more closely resemble a Stackelberg leader-follower relationship rather than either a Dominant-Firm/Competitive-Fringe (DF) or a Cournot-Nash (CN) framework. For these highly concentrated mineral markets with a potential for Stackelberg leadership, the evolution of market production is especially important for those manufacturers that depend upon a reliable supply of these minerals. Therefore, it is important to understand how the equilibrium extraction patterns are determined under such a market structure.

This paper derives the dynamically consistent equilibrium to a two-period discrete-time non-renewable resource model in which the leader moves before the follower within each period in choosing its output and in which both the leader and the follower act as price searchers. This Truly Stackelberg (TS) model differs from CN models by the leader-follower sequencing of production choices within each period, and it differs from DF models both by the leader-follower sequencing of production choices and by the price searching behavior of the follower. While the DF and CN equilibria have been extensively characterized in the literature, the TS equilibria has no antecedent.[2] Relative to both the CN and DF equilibria, the TS game exhibits a much richer set of equilibria.

When firms have stocks sufficient to last at most two periods, the equilibria may be analyzed using simple piece-wise best-response functions in two-dimensional graphs.

[1] The 4-firm HHI index for these minerals range from 104 for silver (very competitive) to 2163 for platinum, which is sufficiently concentrated that in the U.S. government approval would be required for mergers.

[2] There exists a fourth class of games which we do not consider here: sequential-move DF equilibria, where the leader moves first within each period but the follower is a price searcher. These equilibria, however, are quite different from the TS equilibria we study. In those games, with a price taking follower, the follower's reaction to an increase in the leader's output is to decrease his output by one unit for every unit increase in the leader's production. This has the effect that the leader's strategic effect on the follower exactly offsets the effect he has upon market price, hence both the leader and follower implicitly act as price takers in equilibria in which the follower produces over both periods.

Table 1: Market Share of the World's Top 5 Companies for Major Minerals Production (% of World Production).

Mineral	1st	2nd	3rd	4th	5th	Top Five
Aluminium	Rusal	Alcan	Alcoa	CHINALCO	Norsk Hydro	HHI=382
	11%	10%	9%	8%	4%	42%
Copper	Codelco	Freeport	BHP Billiton	Xstrata	Rio Tinto	HHI=303
	11%	9%	7%	6%	4%	37%
Diamond	Anglo	Alrosa Gropu	Botswana	BHP Billiton	Rio Tinto	HHI=1230
	24%	20%	13%	7%	6%	70%
Gold	Barrick	Newmont	AngloGold	Freeport	GoldCorp	HHI=173
	9%	7%	5%	3%	3%	27%
Nickel	Norilsk	CVRD	Jinchuan	Xstrata	BHP Billiton	HHI=668
	19%	13%	8%	7%	5%	52%
Phosphate	Morocco	Mosaic	Tunisia	PotashCorp	Jordan	HHI=403
	15%	11%	5%	4%	4%	39%
Platinum	Anglo	Impala	Lonmin	Norilsk	Aquarius	HHI=2179
	36%	25%	11%	11%	4%	87%
Silver	BHP Billiton	Fresnillo	KGHM	Pan American	Goldcorp	HHI=104
	6%	5%	5%	3%	3%	22%
Tin	Yunnan Tin	PT Timah	Minsur	Thaisarco	Malaysia	HHI=940
	18%	16%	14%	10%	8%	66%
Zinc	Korea Zinc	Nyrstar	Hindustan	Xstrata	Glencore	HHI=170
	8%	7%	5%	4%	4%	28%

Source: Calculated by the authors based on U.S.G.S. (2011). HHI is the Herfindahl-Hirschman Index, HHI=$\sum_{i=1}^{5} s_i^2$, where s_i is the percent share of world production from each firm.

This allows us to clearly highlight the effect of differences in resource constraints and extraction costs on the equilibrium selection by the leader. In addition, using discrete periods allows us to differentiate between sequential-move and simultaneous-move games. In continuous-time models, it is impossible to distinguish between CN and TS games, since moves are implicitly made simultaneously. These two features allow us to delineate which results are due to strategic manipulation by the leader in a sequential game and which are due to the follower having market power. Furthermore, even though our games end after two periods, these two periods characterize the final subgame to any sequence of greater length. Thus, our results allow us to infer how games with three or more periods might unfold.

The paper makes four main contributions. First, within the CN, DF and TS games, we show the effects relative resource stock constraints and relative marginal production costs have both upon the order and the rate of extraction. In all of these games there exist equilibria in which one or both firms are at a corner solution, where a firm either produces its entire stock in period 1 or withholds all of its stock for production in period 2.[3] Thus, this feature of the non-renewable resource equilibria transcends

[3]Corner solutions also occur in static Stackelberg models (i.e., where firms are not constrained by resource stocks). In static Stackelberg games with linear demand, $p = \bar{p} - q_1 - q_2$, and constant marginal production costs, c_1 and c_2, to firms 1 and 2, respectively, it can be shown that for $c_2 < 2c_1 - \bar{p}$, only firm 2 produces,

assumptions about whether players are both price searchers and whether players move simultaneously or sequentially within each period.

Second, we find that in the sequential-move TS game situations arise in which the leader is able to *select* among several feasible equilibria in a manner that does not occur in either CN or DF games. In the TS game, the leader is able to manipulate extraction patterns by exploiting the resource endowment constraints. Specifically, there are instances in the TS game where even though equilibria in which both firms produce over both periods are feasible (and would occur if the game were a simultaneous-move CN game), the leader prefers, and has the power to select an equilibrium in which either the follower or the leader exhausts his entire stock in only one period. These issues of equilibrium selection do not arise in simultaneous-move CN or DF non-renewable resource games, since in those games, equilibria are uniquely determined by the intersection of best-response functions.

Third, we show that both firms' iso-profit curves becomes asymptotic when the firm's first period production equals a certain fraction of its stocks. For values of first period production less than this fraction of the stock, firm 1's profit is decreasing in its own first period production, which is opposite to what occurs in static Stackelberg games. In static Stackelberg games — where firms are unrestricted in total output by a stock constraint — if relative costs are such that the leader can force the follower into zero production, then those conditions continue to hold even if the game is extended to two or more periods. In contrast, with non-renewable resources, as long as both firms' marginal production costs are less than the choke price, both firms will eventually produce. Therefore, in non-renewable TS games, the issues revolve around the timing of production, rather than the existence of production.

Fourth, we analyze the effects of allowing the leader to move first has upon the aggregate extraction rate, consumers' welfare and the follower's profits. Relative to the CN, the leader wants to increase its first period production if the follower's resource stock is relatively small or if the follower's cost is relatively large, and the leader wishes to decrease its first period production if the follower's resource stock is relatively large or if the follower has sufficient cost advantage. When both firms have market power, an increase in the leader's first period production by one unit causes the follower to lower his first period production by less than one unit. Therefore, if the leader extracts faster, the follower extracts slower and total extraction increases, all else constant. However, unlike static Stackelberg games, we show that the follower does not necessarily have a

and for $c_2 > \frac{\bar{p}+2c_1}{2}$ only firm 1 produces.

second mover disadvantage.

Oligopolistic extraction of non-renewable resources have been analyzed extensively since the oil price shocks in the 1970s. The supply side of the oil market has been described by either the DF framework, with a cartel facing a large number of competitive fringe producers (Salant, 1976), or the CN framework, with oligopolists moving simultaneously (Loury, 1986; Polasky, 1992). The simultaneous DF framework has been analyzed by Pindyck (1978), Salant (1982), Ulph and Folie (1980b), Groot et al. (1992), and Benchekroun et al. (2009), inter alia. The sequential DF framework has been explored by Gilbert (1978), Ulph and Folie (1980a), Newbery (1981, 1992), Groot et al. (2000, 2003), and Benchekroun and Withagen (2012). These papers distinguish between open-loop Nash equilibria, where each supplier chooses the extraction path as a function of the initial resource stock and time, and closed-loop (or feedback) subgame perfect Nash equilibria, where extraction strategies depend on current stocks. All of these papers, however, use a continuous time framework, which implicitly assumes that firms move simultaneously within each period. We find that the CN and DF equilibria in simultaneous-move games are qualitatively similar and attribute this to the fact that both are Nash equilibria found by equating best-responses. Eswaran and Lewis (1985), using a discrete-time framework, show that the open-loop and feedback CN equilibria are identical only when each firm's decision rule is independent of the stocks of all other firms, and Hartwick and Brolley (2008) show that such a condition is satisfied as long as initial resource stocks guarantee all the firms extract in every period. However, as stated in Benchekroun et al. (2010), none of the literature has fully characterized equilibrium extraction patterns of non-renewable resources under the TS setting for a finite number of players. This is what the current paper provides: we show the effect sequential moves and market power by the follower have upon a two-period discrete time non-renewable resource model.

The remainder of the paper is organized as follows. Section 2 describes the model and derives the necessary conditions that characterize each equilibrium. Section 3 examines the properties of the Nash equilibrium for the simultaneous-move, CN and DF models. Section 4 derives the equilibria for the sequential-move, TS model in which both the leader and follower have market power, and determines the effect, relative to the CN equilibrium, the TS equilibrium has upon on the rate of extraction, consumers' surplus, and follower's profits. Section 5 concludes. Derivations of the necessary conditions for the CN, TS, and DF equilibria plus proofs of propositions are placed in the On-line Appendix.

2 Model Set Up

Let firm 1 denote the leader in the TS game or the dominant firm in the DF game and let firm 2 denote the follower in the TS game or the competitive fringe in the DF game. In the CN game, of course, the distinction between firm 1 and firm 2 is important only insofar as stocks or costs of production may differ between the two firms. The stocks held by firm 1 are S_1 and the stocks held by firm 2 are S_2. Demand in each period is linear, with $p_t = \bar{p} - Q_t$, where Q_t is total production in period $t = 1, 2$. Firm 1 faces marginal costs of production c_1 and firm 2 faces marginal costs of production c_2. Both firms will produce eventually so long as $0 < c_1, c_2 < \bar{p}$. A dollar earned in period 2 is discounted at rate $\beta \equiv \frac{1}{1+r}$, where r is the interest rate.

Let q_1 and q_2 denote the first period production by firms 1 and 2, respectively. Since we restrict our attention to two-period games, second period production is simply $S_i - q_i$, $i = 1, 2$. Firms' discounted profits are:

$$\pi_1 = (\bar{p} - c_1 - q_1 - q_2)\, q_1 + \beta \left[\bar{p} - c_1 - (S_1 - q_1) - (S_2 - q_2)\right] (S_1 - q_1)\,, \tag{1a}$$

$$\pi_2 = (\bar{p} - c_2 - q_1 - q_2)\, q_2 + \beta \left[\bar{p} - c_2 - (S_1 - q_1) - (S_2 - q_2)\right] (S_2 - q_2)\,. \tag{1b}$$

Consider the behavior of firm 2. For firm 2, first period production is governed by

$$\left\{ \begin{array}{c} q_2 = S_2 \\ 0 < q_2 < S_2 \\ q_2 = 0 \end{array} \right\} \quad \text{as} \quad \frac{\partial \pi_2}{\partial q_2} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0, \tag{2}$$

where

$$\frac{\partial \pi_2}{\partial q_2} = \begin{cases} \bar{p} - c_2 - q_1 - 2q_2 - \beta\left[\bar{p} - c_2 - (S_1 - q_1) - 2(S_2 - q_2)\right] & \text{if CN or TS} \\ \bar{p} - c_2 - q_1 - q_2 - \beta\left[\bar{p} - c_2 - (S_1 - q_1) - (S_2 - q_2)\right] & \text{if DF} \end{cases}\,. \tag{3}$$

Taking the production choice of firm 1 as given, firm 2 produces its entire stock in period 1 ($q_2 = S_2$) when the present value of its marginal profits in the first period is greater than that in the second period; firm 2 produces its entire stock in period 2 ($q_2 = 0$) when the present value of its marginal profits in the first period is less than that in the second period; and firm 2 produces in both periods ($0 < q_2 < S_2$) only if its marginal profits are equal in present value across the periods. When firm 2 is a price taker (DF), marginal profits in each period are the difference between price and marginal cost; when firm 2 is a price setter (CN or TS), marginal profits are the difference between firm 2's marginal revenue and marginal costs.

Setting $\frac{\partial \pi_2}{\partial q_2} = 0$ and solving for q_2 yields firm 2's best-response function:

$$q_2 = \begin{cases} 0 & B_2(q_1) \leq 0 \\ B_2(q_1) \quad \text{if} & 0 < B_2(q_1) < S_2 \\ S_2 & B_2(q_1) \geq S_2 \end{cases} . \tag{4}$$

where

$$B_2(q_1) = \begin{cases} \frac{(\bar{p}-c_2)(1-\beta)+\beta(S_1+2S_2)}{2(1+\beta)} - \frac{1}{2}q_1 & \text{if CN or TS} \\ \frac{(\bar{p}-c_2)(1-\beta)+\beta(S_1+S_2)}{(1+\beta)} - q_1 & \text{if DF} \end{cases} \tag{5}$$

is the interior portion (i.e., where both firms have positive production) of firm 2's best-response function. When firm 2 is a DF price taker, rather than a CN or TS price setter, $B_2(q_1)$ has both a higher intercept and a steeper slope when plotted as a function of q_1. While static Stackelberg games have piece-wise best-response curves, with $q_2 = B_2(q_1)$ when $B_2(q_1) > 0$ and $q_2 = 0$ when $B_2(q_1) \leq 0$, the stock constraint on total production in the non-renewable resource model gives a third possible interval to the best-response function, with $q_2 = S_2$ when $B_2(q_1) \geq S_2$.

Firm 1's behavior depends upon whether the game is played as a simultaneous-move CN or DF game, or as a sequential-move TS game. Simultaneous-move games are solved by finding the Nash equilibrium. Sequential-move games are solved by finding the subgame-perfect Nash equilibrium. In each case, firm 1's first period output is chosen to satisfy

$$\left\{ \begin{array}{c} q_1 = S_1 \\ 0 < q_1 < S_1 \\ q_1 = 0 \end{array} \right\} \quad \text{as} \quad \frac{d\pi_1}{dq_1} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0, \tag{6}$$

where

$$\frac{d\pi_1}{dq_1} = \begin{cases} \frac{\partial \pi_1}{\partial q_1} & \text{if CN or DF} \\ \frac{\partial \pi_1}{\partial q_1} + \frac{\partial \pi_1}{\partial q_2}\frac{dq_2}{dq_1} & \text{if TS} \end{cases} , \tag{7}$$

with

$$\frac{\partial \pi_1}{\partial q_1} = \bar{p} - c_1 - 2q_1 - q_2 - \beta\left[\bar{p} - c_1 - 2\left(S_1 - q_1\right) - \left(S_2 - q_2\right)\right], \tag{8}$$

$$\frac{\partial \pi_1}{\partial q_2} = -[q_1 - \beta(S_1 - q_1)], \tag{9}$$

and

$$\frac{dq_2}{dq_1} = \begin{cases} -1/2 & \text{if } 0 < q_2 < S_2 \text{ and TS} \\ 0 & \text{if } q_2 = 0, \text{ or } q_2 = S_2, \text{ or DF, or CN} \end{cases} . \tag{10}$$

Thus, firm 1, like firm 2, has two corner solutions as well as an interior solution which might occur, depending on whether firm 1's marginal profits are positive, negative, or zero at equilibrium quantities. Unlike firm 2, however, firm 1 always behaves as a price setter. In addition, unlike firm 2, firm 1's choice depends upon the total derivative in (7). This is equal to the partial derivative in the simultaneous-move CN and DF games, but includes the additional "strategic" term, $\frac{\partial \pi_1}{\partial dq_2}\frac{dq_2}{dq_1}$, when the game is a sequential-move TS game. This strategic term tells how a change in firm 1's first period production affects firm 1's profits through the induced change in firm 2's first period production. When $\frac{dq_2}{dq_1} = 0$, however, as occurs whenever firm 2 is at a corner solution where $q_2 = 0$ or $q_2 = S_2$, the sequential-move TS and simultaneous-move game CN game yield identical equilibria. While CN equilibria differ from the TS equilibria only by the presence of the strategic term in firm 1's choice conditions in the TS equilibria, the DF equilibria differ from the TS equilibria both by whether firm 2 is a price taker in DF versus a price setter in TS, and by how firm 1's necessary conditions include the strategic term in the TS but not in the DF.[4]

For both the CN and DF equilibria, each firm moves simultaneously, so the Nash equilibrium is defined by the intersection of best-response functions. Setting $\frac{dq_2}{dq_1} = 0$ and solving $\partial\pi_1/\partial q_1 = 0$ for q_1 yields firm 1's best-response function:[5]

$$q_1 = \begin{cases} 0 & & B_1(q_2) \leq 0 \\ B_1(q_2) & \text{if} & 0 < B_1(q_2) < S_1 \\ S_1 & & B_1(q_2) \geq S_1 \end{cases} , \tag{11}$$

where

$$B_1(q_2) = \frac{(\bar{p} - c_1)(1-\beta) + \beta(2S_1 + S_2)}{2(1+\beta)} - \frac{1}{2}q_2 \tag{12}$$

is the interior portion of firm 1's best-response function.

Since we restrict our attention to two-period games, the stocks held by the firms must be such that in equilibrium no firm wishes to produce in period 3. In terms of

[4] A fourth possibility, not explored here, is that firm 2 acts as a price taker and firm 1 moves sequentially. In that case, we see from the price-taker condition in (3) that $dq_2/dq_1 = -1$ when firm 2 is at an interior solution.

[5] In the Figs. 4 - 8, we plot $B_1^{-1}(q_2)$, which becomes vertical at $q_1 = 0$ and $q_1 = S_1$.

the endogenous period 2 outputs, $S_1 - q_1$ and $S_2 - q_2$, this requires that each firm's marginal profit in period 2 is greater than the present value of marginal profit of taking one more unit to period 3, given that neither firm is producing in period 3:

$$\beta\left(\bar{p}-c_i\right) \leq \begin{cases} \bar{p}-c_i-\left(2-\frac{dq_2}{dq_1}\right)(S_i-q_i)-(S_j-q_j) & \text{if firm 1 in TS} \\ \bar{p}-c_i-(S_i-q_i)-(S_j-q_j) & \text{if firm 2 in DF} \\ \bar{p}-c_i-2(S_i-q_i)-(S_j-q_j) & \text{otherwise} \end{cases}, \quad i \neq j = 1,2. \tag{13}$$

The Nash equilibrium in the CN and DF models occurs at the intersections of the two firm's best-response functions. In these cases the iso-profit curves of the firms are useful for welfare analysis, but are not otherwise necessary for finding the equilibrium. In the TS model, however, the iso-profit curves for firm 1 play an important role in that firm's equilibrium selection. An iso-profit curve for firm 1, which graphs combinations of first period output by firm 1 and firm 2 that yield a constant level of profits, Π_1, for firm 1, is given by:[6]

$$q_2 = \frac{-(1+\beta)q_1^2 + [(1-\beta)(\bar{p}-c_1) + (1+\beta)S_1 + \beta S_2]q_1 + \beta\bar{p}S_1 - \Pi_1}{(1+\beta)q_1 - \beta S_1}. \tag{14}$$

Equation (14) shows that firm 1's iso-profit curve becomes asymptotic at $q_1 = \beta S_1/(1+\beta)$. Since $0 < \beta/(1+\beta) < 1$, this asymptote occurs within the feasible values of q_1. This property of the iso-profit curve highlights an important difference between two-period non-renewable resource Stackelberg models and simple static Stackelberg models. When $q_1 > \beta S_1/(1+\beta)$, the iso-profit curve for firm 1 is the familiar bell-shaped curve that occurs in static Stackelberg models where firm 1's iso-profit curves have slope zero when they cross firm 1's interior best-response curve. In this case profits to firm 1 are increasing as q_1 increases along firm 1's best-response curve. But for $q_1 < \beta S_1/(1+\beta)$, firm 1's iso-profit curve is a U-shaped curve which has profits increasing as q_1 *decreases* along firm 1's best-response curve. In addition, we see from (9) that the effect firm 2's production has upon firm 1's profits also depends upon which side of the asymptote the equilibrium occurs. When $q_1 > \beta S_1/(1+\beta)$, firm 1's profits are decreasing in firm 2's first period production (as occurs in static Stackelberg games), but when $q_1 < \beta S_1/(1+\beta)$, firm 1's profits are increasing in firm 2's first period production.

Finally, for the purpose of evaluating welfare, let $Q = q_1 + q_2$ denote the total

[6] The iso-profit curve for firm 2 is obtained by reversing firm subscripts in (14).

Table 2: Simultaneous-Move Equilibria: CN and DF

Equilibrium	Period 1 Production	First Order Necessary Conditions	CN Cost Restrictions	DF Cost Restrictions	Cost Regions
12	$q_1 = S_1,\, q_2 = S_2$	$\frac{\partial\pi_1}{\partial q_1} > 0,\, \frac{\partial\pi_2}{\partial q_2} > 0$	None	None	$(i), (ii), (iii)$
$12 \to 12$	$0 < q_1 < S_1,\, 0 < q_2 < S_2$	$\frac{\partial\pi_1}{\partial q_1} = 0,\, \frac{\partial\pi_2}{\partial q_2} = 0$	See Text	See Text	$(i), (ii), (iii)$
$12 \to 1$	$0 < q_1 < S_1,\, q_2 = S_2$	$\frac{\partial\pi_1}{\partial q_1} = 0,\, \frac{\partial\pi_2}{\partial q_2} > 0$	$c_2 < \frac{c_1+\bar{p}}{2}$	$c_2 < \frac{c_1+\bar{p}}{2}$	$(ii), (iii)$
$12 \to 2$	$q_1 = S_1,\, 0 < q_2 < S_2$	$\frac{\partial\pi_1}{\partial q_1} > 0,\, \frac{\partial\pi_2}{\partial q_2} = 0$	$c_2 > 2c_1 - \bar{p}$	$c_2 > c_1$	$(i), (ii)$
$1 \to 2$	$q_1 = S_1,\, q_2 = 0$	$\frac{\partial\pi_1}{\partial q_1} > 0,\, \frac{\partial\pi_2}{\partial q_2} < 0$	$c_2 > \frac{c_1+\bar{p}}{2}$	$c_2 > \frac{c_1+\bar{p}}{2}$	(i)
$1 \to 12$	$0 < q_1 < S_1,\, q_2 = 0$	$\frac{\partial\pi_1}{\partial q_1} = 0,\, \frac{\partial\pi_2}{\partial q_2} < 0$	$c_2 > \frac{c_1+\bar{p}}{2}$	$c_2 > \frac{c_1+\bar{p}}{2}$	(i)
$2 \to 1$	$q_1 = 0,\, q_2 = S_2$	$\frac{\partial\pi_1}{\partial q_1} < 0,\, \frac{\partial\pi_2}{\partial q_2} > 0$	$c_2 < 2c_1 - \bar{p}$	$c_2 < c_1$	(iii)
$2 \to 12$	$q_1 = 0,\, 0 < q_2 < S_2$	$\frac{\partial\pi_1}{\partial q_1} < 0,\, \frac{\partial\pi_2}{\partial q_2} = 0$	$c_2 < 2c_1 - \bar{p}$	$c_2 < c_1$	(iii)

production of both the leader and follower in period 1, and let $S = S_1 + S_2$ denote total stocks. Then consumers' surplus (CS) is the discounted present value of the stream of consumer surplus:

$$\begin{aligned} CS(Q) &= \int_0^Q (\bar{p} - z)\mathrm{d}z - (\bar{p} - Q)Q + \beta\left\{\int_0^{S-Q} (\bar{p} - z)\mathrm{d}z - [\bar{p} - (S - Q)](S - Q)\right\} \\ &= Q^2/2 + \beta(S - Q)^2/2. \qquad (15) \end{aligned}$$

From this expression we see that consumers' surplus is increasing in first period production if, and only if, $Q > \beta S/(1 + \beta)$.

3 Simultaneous-Move Equilibria: CN and DF

Since the literature has focused on either the CN or DF simultaneous-move equilibria, we begin by briefly considering these equilibria. In both cases, each firm moves simultaneously within periods. Both firms act as price setters in the CN equilibrium, but only the dominant firm acts as a price setter in the DF equilibrium. Since the moves are made simultaneously, equilibria in each game are found by equating the two best-response functions, (4) and (11). Given that both firm's best-response curves are each piecewise linear functions, each equilibrium—for a given set of costs and stocks—is unique.

Depending upon cost and stock conditions, Table 2 shows that there exist up to eight possible types of two-period equilibria for both the CN and DF games. The notation describing each equilibrium indicates which firms are active in each period.

Thus, "$1 \to 2$" implies that firm 1 produces only in period 1 while firm 2 produces only in period 2, while "12" implies both firms exhaust in period 1, "$12 \to 12$" implies both firms produce in each of period 1 and 2, and so on.[7] The necessary conditions for the CN equilibria are derived in Appendix A and the necessary conditions for the DF equilibria are derived in Appendix D, which are available in the on-line supplement.

There exist stocks S_1 and S_2 such that the "12" equilibrium occurs for all feasible marginal costs, i.e., for all $0 \leq c_1, c_2 < \bar{p}$ for both the CN and DF games. But the remaining equilibria occur only for particular relative costs. Similar to previous literature, we show that in the both the CN and DF equilibria, there are three relevant cost regions, in order of decreasing relative costs to firm 2. For the CN equilibria, these are (i) $\frac{c_1+\bar{p}}{2} < c_2 \leq \bar{p}$, (ii) $2c_1 - \bar{p} \leq c_2 \leq \frac{c_1+\bar{p}}{2}$, and (iii) $0 \leq c_2 < 2c_1 - \bar{p}$. For the DF equilibria, the cost regions are (i) $\frac{c_1+\bar{p}}{2} < c_2 \leq \bar{p}$, (ii) $2c_1 - \bar{p} \leq c_2 \leq c_1$, and (iii) $0 \leq c_2 < c_1$.

For each of the three cost regions, the necessary conditions delineate a unique set of feasible stocks in the S_1-S_2 space which yield each equilibria. These are shown in Fig. 1a-1c for the CN equilibria by cost region, with each equilibrium ordering of production shaded differently. The unshaded areas in the positive orthant marked "exhaustion in 3 or more periods" represent stock values which in equilibrium require three or more periods to extract. The S_i values marked by $\underline{S}_i^{CN}$ represent the lower boundaries of the equilibrium $12 \to 12$ in the stock i direction.

In Fig. 1a, drawn for cost region (i) where firm 2 has much higher costs than firm 1, depending upon the relative stocks up to five two-period equilibria may occur: 12, $1 \to 2$, $1 \to 12$, $12 \to 12$, and $12 \to 2$. Equilibrium $12 \to 12$, however, exists in cost region (i) only if c_2 is sufficiently small.[8] In all of these equilibria, firm 1, which has the cost advantage, produces in period 1. Firm 2 only produces in period 1 when both firms have very small stocks (equilibrium 12) or when firm 2 has large stocks relative to firm 1 (equilibrium $12 \to 2$). When costs are relatively equal, as in cost region (ii) as depicted in Fig. 1b, only four two-period equilibria exist for all cost values within this range: 12, $12 \to 1$, $12 \to 2$, and $12 \to 12$. In each equilibrium both firms produce in period 1. In cost region (iii), where firm 2 has the cost advantage, there are again up to five equilibria in which production occurs over two periods: 12, $12 \to 1$, $12 \to 12$,

[7]Equilibrium "$. \to 12$" in which $q_1 = q_2 = 0$ does not exist because if both firms are willing to produce their entire stock in the same period as the other firm, then both do better by producing in the first period, since period 2 profits are discounted at rate $\beta < 1$.

[8]In cost region (i), the upper bound for equilibrium $12 \to 12$ is the exhaustion constraint for firm 2. When evaluated at $\underline{S}_1^{CN}$, this exhaustion condition is greater than $\underline{S}_2^{CN}$ only if $\frac{c_1+\bar{p}}{2} < c_2 < [(2+3\beta)\bar{p}+2c_1]/(4+3\beta)$.

$2 \to 1$, and $2 \to 12$.[9] These equilibria are depicted in Fig. 1c. In each equilibrium in cost region (*iii*), firm 2 produces in period 1, and firm 1 only produces in period 1 when both firms have very little stock (equilibrium 12) or when firm 1 has relatively large stocks (equilibrium $12 \to 1$).

Fig. 2 compares the CN and DF equilibria for the case where costs are in cost region (*ii*) for both the CN and DF equilibria, i.e., where $c_1 < c_2 < (c_1 + \bar{p})/2$. In Fig. 2, areas 12, $12 \to 2$, $12 \to 1$, and $12 \to 12$ have the same extraction pattern in both games. However, in both areas $12 \to 2$ and $12 \to 12$, where firm 2 produces in both periods, first period production is greater under the DF equilibrium than in the CN equilibrium.[10] In areas 12 and $12 \to 1$, production is unchanged, since in both firm 2 exhausts in period 1. Thus, for those stocks such that the DF equilibrium sequence remains the same as the CN equilibrium, production either remains constant or increases in the DF equilibrium. We also show in Appendix D that for these equilibria where production sequencing changes, the DF equilibrium exceeds total production in period one. Similar comparisons can be made in the other cost regions.

While Fig. 2 emphasizes the differences between the CN and DF equilibria, there are two important similarities between the equilibria. First, for each cost region, each equilibria occupies a mutually exclusive S_1-S_2 space. For example, in Fig. 1a the lower boundary of the $12 \to 12$ region in the S_1 direction is $\underline{S}_1^{CN}$, and this boundary also forms the upper bound in the S_1 direction for equilibrium $12 \to 2$. Similarly, $\underline{S}_2^{CN}$ forms both the lower bound in the S_2 direction for equilibrium $12 \to 12$ and the upper bound in the S_2 direction for equilibrium $1 \to 12$. Similar statements may be made for the boundaries to each equilibrium within each cost region in both the CN and DF games. This is summarized in the following:

Proposition 1. *For both the CN and DF games, for given relative costs and stocks, there exists one, and only one, equilibrium sequencing of production.*

Proof. Both the CN and DF equilibria are the intersection of the piece-wise linear best-response curves. In q_2-q_1 space, the slope of firm 1's best-response curve (12) is either infinite (when $B_1(q_2) = 0$ or when $B_1(q_2) = S_1$) or $-1/2$ (when $0 < B_1(q_2) < S_1$),

[9] In this case, equilibrium $12 \to 12$ exists only for sufficiently high c_2. Evaluating the exhaustion condition for firm 1 in equilibrium $12 \to 12$ at $\underline{S}_1^{CN}$ yields a value greater than $\underline{S}_2^{CN}$ only if $c_2 > \frac{(4+3\beta)c_1-(2+3\beta)\bar{p}}{2} > \frac{c_1+\bar{p}}{2}$.

[10] In area $12 \to 2$, $Q^{CN} - Q^{DF} = \frac{S_1-(1-\beta)(\bar{p}-c_2)}{2(1+\beta)} < 0$, which is less than zero since $S_1 < (1-\beta)(\bar{p}-c_2)$. In area $12 \to 12$, $Q^{CN} - Q^{DF} = \frac{(1-\beta)(2c_2-\bar{p}-c_1)}{3(1+\beta)} < 0$, which is less than zero since $c_2 < (\bar{p}+c_1)/2$ in cost region (*ii*).

while the slope of firm 2's best-response curve (12) is either zero (when $B_2(q_1) = 0$ or $B_2(q_1) = S_2$) or -2 (when $0 < B_2(q_1) < S_1$) for the CN and is either zero or -1 (when $0 < B_2(q_1) < S_1$) for the DF. □

This proposition is the result of both the CN and DF equilibria being defined as the intersection of piece-wise linear best-response curves.

Second, the sequencing of production is related to the relative costs of extraction, but is independent of the relative stocks held by the firms. To show this, we first define *diversified production* as follows:

Definition 1. *An extraction path is said to be diversified if both firms extract in the first period and continue to extract until their initial resource endowment is exhausted.*

Proposition 2. *If the marginal production costs of the two firms are relatively equal (i.e.,* $2c_1 - \bar{p} \leq c_2 \leq (\bar{p} + c_1)/2$ *in CN or* $c_1 \leq c_2 \leq (\bar{p} + c_1)/2$ *in DF), then there exists a diversified extraction path in either the CN or DF games, respectively, independent of each firm's resource endowment. If firm* i *has much lower costs than firm* j*, for* $i \neq j, i, j = 1, 2$*, then firm* i *always produces in period 1, and firm* j *only produces in period 1 when both firms have very little stock or when firm* j *has large stocks relative to firm* i*.*

Proof. Given in the text. □

These propositions show that while the CN and DF equilibria differ in many ways, they share a similar qualitative nature. In both cases, for each cost and stock combination, there exists a unique extraction path. Furthermore, diversified extraction always occurs when marginal extraction costs are similar. When costs are very different, the low cost producer always begins extraction in period 1 while the high cost producer only begins production in period 1 if his stocks are sufficiently high to warrant production in both periods, given the actions of the low cost producer.

Finally, note that in both the CN and DF games, each of the eight possible equilibrium paths holds within a convex set of $\{S_1, S_2\}$ values. Nevertheless, the set of two-period equilibria taken as a whole is a convex set only in cost region (ii). In both cost regions (iii) and (i), the two-period equilibrium set of stocks is not convex. This occurs because the exhaustion condition for equilibrium $1 \to 12$ is flatter than the exhaustion condition for equilibrium $12 \to 12$, as shown in Fig. 1a for the CN case, while the exhaustion condition for equilibrium $2 \to 12$ is steeper than the exhaustion condition for equilibrium $12 \to 12$ in Fig. 1c.

4 Truly Stackelberg Equilibria

Now, we turn to the "truly Stackelberg" game, where firm 1 chooses its output first within each period in a sequential game and where both firms have market power. The TS game differs from the CN game where quantity choices are made simultaneously within each period. The TS game also differs from the DF game, where both firms move simultaneously and where the follower is a price taker. While in the DF game, it is firm 2 whose behavior changes relative to the CN game, in the TS game, it is firm 1 whose behavior changes relative to the CN game.

Unlike the DF game, where firm 2's behavior changes in every possible equilibrium, firm 1's behavior changes in the TS game only when it is able to influence firm 2's behavior. Thus, only the three equilibria where the follower produces in both periods, equilibria $12 \rightarrow 12$, $12 \rightarrow 2$, and $2 \rightarrow 12$, do the TS necessary conditions change, relative to the CN equilibrium, by allowing the leader to move first in a sequential game. In these three equilibria, movements along firm 2's best-response curve imply that $dq_2^*/dq_1 = -1/2$. In all other TS equilibria, $dq_2^*/dq_1 = 0$, implying that in these cases the TS necessary conditions are identical to the CN necessary conditions.[11] At these three TS equilibria, firm 1 is no longer constrained to being on its best-response curve. In addition, there are two new TS equilibria, discussed below, which we denote as $12 \rightarrow 1^{*TS}$ and $1 \rightarrow 12^{*TS}$ which, unlike their CN counterparts, also occur at a point off firm 1's best-response curve. The necessary conditions for the three TS equilibria which differ from the CN equilibria are derived in Appendix B.

Fig. 3 shows how the TS and CN equilibria differ in the stocks space S_1-S_2. As in the CN graphs, regions in which different equilibria occur are shaded differently. CN regions are marked as in Fig. 1. Equilibria in which the TS equilibrium differs from the CN equilibrium are indicated with the superscript "TS". Period 2 exhaustion condition boundaries for the CN equilibrium are shown as solid lines.[12] As in Fig. 1, stocks for which exhaustion occurs in three or more periods are unshaded.

In every cost region the minimum S_1 value for which the $12 \rightarrow 12^{TS}$ equilibrium

[11] In a sequential-move DF framework, since the follower has no market power, the reaction of firm 2 to an increase in production from firm 1 is that $dq_2^*/dq_1 = -1$. Thus, when firm 2 produces in both periods, firm 1 effectively behaves as though it too is a price taker. As a result, no equilibrium exists in which both firms extract over successive periods, except when their marginal extraction costs are equal.

[12] Period 2 TS exhaustion condition boundaries are not shown in Fig. 3. As in the DF equilibria, the strategic behavior by firm 1 causes there to exist areas which are feasible in the two-period TS game, which required three periods in the CN game.

occurs in the TS equilibria relative to the CN equilibria satisfies $\underline{S}_1^{TS} > \underline{S}_1^{CN}$.[13] In contrast, $\underline{S}_2^{TS}$ may be greater than or less than $\underline{S}_2^{CN}$. And cost region (ii) is split into two parts in the TS game, labelled $(ii)(a)$ and $(ii)(b)$, respectively, with $\underline{S}_2^{TS} > \underline{S}_2^{CN}$ in cost region $(ii)(a)$ and $\underline{S}_2^{TS} < \underline{S}_2^{CN}$ in cost region $(ii)(b)$.[14]

We say that a "*gap*" occurs when $\underline{S}_2^{TS} > \underline{S}_2^{CN}$, so that for $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$, some stock values S_2 in equilibrium $12 \to 12^{CN}$ are not in equilibrium $12 \to 12^{TS}$. This occurs in cost regions (i), $(ii)(a)$, and (iii) as depicted in Figs. 3a, 3b, and 3d, respectively. We say that an "*overlap*" occurs when two TS equilibria are feasible. This occurs when $\underline{S}_2^{TS} < \underline{S}_2^{CN}$ in cost region $(ii)(b)$ as shown in Fig. 3c so that for a given $S_1 \geq \underline{S}_1^{TS}$, all S_2 values that were in equilibrium $12 \to 12^{CN}$ are in equilibrium $12 \to 12^{TS}$. This also occurs in cost regions $(ii)(a)$ and $(ii)(b)$, where for $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$, both equilibria $12 \to 2^{TS}$ and equilibria $12 \to 1^{CN}$ are feasible.

4.1 Leader's choice over equilibria

Key to the analysis of the TS game is determining how the leader *selects* amongst the feasible equilibria when choices arise. Once we have established how the leader selects amongst equilibria when that opportunity arises, we analyze the welfare effects of his choice.

Equilibrium Selection when Firm 2 Produces Over Both Periods.

Our first result is that when feasible, in each TS equilibria in which firm 2 produces over two periods, firm 1 can improve its profits. When the CN equilibrium $12 \to 12^{CN}$ has $q_1^{CN} > \beta S_1/(1+\beta)$ in cost regions (i) and (ii), firm 1's profits are increasing in its own output. Therefore, if firm 1 has sufficient stocks, it can increase its profits by moving to equilibrium $12 \to 12^{TS}$, and if it is stock constrained, it can increase its profits by moving to equilibrium $12 \to 2^{TS}$. When the CN equilibrium $12 \to 12^{CN}$ has $q_1^{CN} < \beta S_1/(1+\beta)$ in cost region (iii), firm 1's profits are decreasing in its own output. Therefore, if firm 2 has sufficient stocks, firm 1 can increase its profits by moving to equilibrium $12 \to 12^{TS}$, and if firm 2 is stock constrained, firm 1 can increase its profits

[13] This occurs because equilibrium $12 \to 12^{TS}$'s boundary at its left side is either with equilibrium $12 \to 2^{TS}$ [in cost regions (i) and (ii)] or with equilibrium $2 \to 12^{TS}$ [in cost region (iii)]. Since the effect of an increase in q_1 is a reduction in q_2 at rate $-1/2$ in each of these equilibria, the lower bound for which S_1 yields equilibrium $12 \to 12^{TS}$ shifts to the right relative to the CN.

[14] In cost regions (i) and (iii), $\underline{S}_2^{TS} > \underline{S}_2^{CN}$, but in CN cost region (ii), $\underline{S}_2^{TS} < \underline{S}_2^{CN}$ if, and only if, $c_2 < \bar{c}_2 \equiv \frac{(4\beta+6)c_1+(4\beta+3)\bar{p})}{8\beta+9}$ [found by equating (A.4b) and (B.2b)].

by moving to equilibrium $12 \to 1^{*TS}$. To sum up, we have the following Proposition.

Proposition 3. *In the TS game, if firm 2 produces over two periods, firm 1 can always improve its profits through its strategic effect on firm 2's production.*

Proof. See Appendix C. □

This result is shown in Fig. 4. In both panels, $q_1^{12\to 12^{CN}} > \beta S_1/(1+\beta)$, so that firm 1's profits increase as q_1 increases along firm 1's best-response curve. In cost region (ii), where $q_2^{12\to 12^{CN}} > \beta S_2/(1+\beta)$, so that firm 2's profit are increasing as q_2 decreases, if the interior equilibrium $12 \to 12^{TS}$ is feasible, as occurs when $S_1 > \underline{S}_1^{TS}$ and $S_2 > \underline{S}_2^{TS}$, firm 1 can increase its profits by increasing its first period output, as shown in Fig. 4a. When $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$, however, firm 1 does not have sufficient stocks to obtain equilibrium $12 \to 12^{TS}$, yet it can still obtain higher profits than at equilibrium $12 \to 12^{CN}$ by exhausting its own stocks in period 1, in equilibrium $12 \to 2^{TS}$. This is shown in Fig. 4b, which is drawn for cost region (i), where firm 2's output satisfies $q_2^{12\to 12^{CN}} < \beta S_2/(1+\beta)$, so that firm 2's profit are increasing as q_2 decreases along its best-response curve.[15]

Equilibrium Selection in the "Overlap" Case, Cost Region $(ii)(b)$.

Next, consider firm 1's equilibrium selection problem when there is an 'overlap,' so that $\underline{S}_2^{TS} < \underline{S}_2^{CN}$. This occurs in cost region $(ii)(b)$ depicted in Fig. 3c. When $\underline{S}_2^{TS} < S_2 < \underline{S}_2^{CN}$, both equilibrium $12 \to 12^{TS}$ and $12 \to 1^{CN}$ are feasible. Therefore, firm 1 chooses whichever equilibrium yields the highest profits for itself.

In equilibrium $12 \to 12^{TS}$, firm 1's profits are given by (1a) evaluated at (B.1) from Appendix B, yielding

$$\pi_1^{12\to 12^{TS}} = \frac{(1-\beta)^2(\bar{p}-2c_1+c_2)^2 - 8\beta S_1[S_1+S_2-2(\bar{p}-c_1)]}{8(1+\beta)}. \tag{16}$$

Under equilibrium $12 \to 1^{CN}$, firm 1's profits are given by (1a) evaluated at $q_2 = S_2$ and q_1 satisfying (11), yielding

$$\pi_1^{12\to 1^{CN}} = \frac{[S_2-(1-\beta)(\bar{p}-c_1)]^2 - 4\beta S_1[S_1+S_2-2(\bar{p}-c_1)]}{4(1+\beta)}. \tag{17}$$

[15] Similar graphs may be drawn for cost region (iii), where $q_1^{12\to 12^{CN}} < \beta S_1/(1+\beta)$, so that firm 1's profits increase as q_1 decreases along firm 1's best-response curve. Firm 1's choice in the each of these TS equilibria, however, depends only on how its profits are affected by a increase in its own first period output and how much stock it holds.

Equating these and solving for S_2 yields

$$S_2 = \bar{S}_2 \equiv (1-\beta)(\bar{p}-c_1) \pm \frac{(1-\beta)(\bar{p}-2c_1+c_2)}{\sqrt{2}}. \tag{18}$$

At $\bar{S}_2$, firm 1's profits in equilibrium $12 \to 12^{TS}$ and equilibrium $12 \to 1^{CN}$ are equal. The derivative of the difference in profits is $d\Delta\pi_1/dS_2 = \frac{(1-\beta)(\bar{p}-c_1)-S_2}{2(1+\beta)}$, which is positive at the smaller root and negative at the larger root. Therefore, above the smaller root, $\bar{S}_2 = (1-\beta)(\bar{p}-c_1) - \frac{(1-\beta)(\bar{p}-2c_1+c_2)}{\sqrt{2}}$, equilibrium $12 \to 12^{TS}$ is chosen by firm 1 and below $\bar{S}_2$, equilibrium $12 \to 1^{CN}$ is chosen by firm 1.

The equilibrium selection problem can be seen most clearly using best-response graphs, as shown in Fig. 5. In Fig. 5a, $S_2 > \bar{S}_2$, so firm 1's profits are greater at equilibrium $12 \to 12$ than at equilibrium $12 \to 1$, even though both are feasible. In Fig. 5b, $S_2 < \bar{S}_2$, so firm 1's profits are greater at equilibrium $12 \to 1$, where firm 2 is forced to exhaust in the first period, than at equilibrium $12 \to 12$, even though both are feasible. When $S_2 = \bar{S}_2$, firm 1 is indifferent between the two equilibria.

Firm 1 also faces an equilibrium selection problem in cost regions $(ii)(a)$ and $(ii)(b)$ depicted in Figs. 3b and 3c for values $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$. In these areas, equilibria $12 \to 2^{TS}$ and $12 \to 1^{CN}$ are each feasible.

Evaluating π_1 at equilibrium $12 \to 2^{TS}$ using (B.3) from Appendix B yields

$$\pi_1^{12\to 2^{TS}} = \frac{S_1[2(1+\beta)(\bar{p}-c_1)-(1-\beta)(\bar{p}-c_2)-(1+2\beta)S_1-2\beta S_2]}{2(1+\beta)}. \tag{19}$$

Equating $\pi_1^{12\to 2^{TS}}$ and $\pi_1^{12\to 1^{CN}}$ and solving for S_2 yields

$$S_2 = \bar{\bar{S}}_2(S_1) \equiv (1-\beta)(\bar{p}-c_1) \pm \sqrt{2S_1(1-\beta)(\bar{p}-2c_1+c_2)-2S_1^2}. \tag{20}$$

The boundary $\bar{\bar{S}}_2(S_1)$, unlike $\bar{S}_2$, depends upon S_1. Again, the smaller root, $\bar{\bar{S}}_2(S_1) \equiv (1-\beta)(\bar{p}-c_1) - \sqrt{2S_1(1-\beta)(\bar{p}-2c_1+c_2)-2S_1^2}$, forms the boundary between equilibria $12 \to 2^{TS}$ and $12 \to 1^{CN}$, with $12 \to 2^{TS}$ chosen by firm 1 above this boundary and $12 \to 1^{CN}$ chosen by firm 1 below this boundary.[16]

Fig. 6 shows the equilibrium selection problem faced by firm 1 in cost regions $(ii)(a)$ and $(ii)(b)$ when $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$, where the choice is between equilibria $12 \to 2^{TS}$ and $12 \to 1^{CN}$. In Fig. 6a, $S_2 > \bar{\bar{S}}_2(S_1)$, so firm 1's profits are greater at

[16] $d\Delta(\pi_1^{12\to 2} - \pi_1^{12\to 1})/dS_2 = \frac{(1-\beta)(\bar{p}-c_1)-S_2}{2(1+\beta)}$. This is positive at the smaller root and negative at the larger root. Furthermore, as shown in Fig. 3c, $\bar{\bar{S}}_2(\underline{S}_1^{TS}) = \bar{S}_2$.

equilibrium $12 \rightarrow 1$ than at equilibrium $12 \rightarrow 1^{CN}$, even though both are feasible. In Fig. 6b, $S_2 < \bar{\bar{S}}_2(S_1)$, so firm 1's profits are greater at equilibrium $12 \rightarrow 1^{CN}$, than at equilibrium $12 \rightarrow 2^{TS}$, even though, again, both are feasible.

These equilibria are summarized as follows:

Proposition 4. *In the overlap areas in cost region (ii), where two TS equilibria are feasible, firm 1 selects the equilibrium which maximizes its profits. This results in equilibrium* $12 \rightarrow 1^{CN}$ *when firm 2's stocks are small, and either* $12 \rightarrow 2^{TS}$ *when* S_1 *is small and* $12 \rightarrow 12^{TS}$ *when* S_1 *is large.*

Proof. Derived in the Text. □

In the overlap areas in cost region $(ii)(b)$, where $S_1 > \underline{S}_1^{TS}$ and $\underline{S}_2^{CN} < S_2 < \underline{S}_2^{TS}$, firm 1's equilibrium selection is determined by the stocks held by firm 2. If firm 2's stocks are large ($S_2 > \bar{S}_2$), then firm 1 can induce firm 2 to produce in period 2 with a small increase in its own period 1 output. But when firm 2's stocks are smaller than $\bar{S}_2$, the required increase in firm 1's period 1 production to induce firm 2 to produce in period 2 is too large to be profitable. In the overlap area between $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$ in cost regions $(ii)(a)$ and $(ii)(b)$, both stocks constrain firm 1's choice. Only when both stocks are large [i.e., $S_2 > \bar{\bar{S}}_2(S_1)$], can firm 1 profitably induce firm 2 to produce over both periods by producing its own entire stock in period 1.

Equilibrium Selection in the "Gap" Cases, Cost Regions (i), $(ii)(a)$, and (iii).

In the overlap cases, firm 1 selected between two feasible TS equilibria for each stock value within the overlap area. In the 'gap' cases, in contrast, stock values within the gap satisfy the necessary conditions for none of the TS equilibria so far considered. But there does exist a feasible CN equilibrium for each given stock values. Thus, firm 1's equilibrium selection problem in these cases involves finding whether or not there exists a TS equilibrium which is both feasible and profit dominates the CN equilibrium.

In cost region (iii), where firm 2 has a significant cost advantage over firm 1, a gap occurs in Fig. 3d for values of $S_1 > \underline{S}_1^{CN}$ and for $\underline{S}_2^{CN} < S_2 < \underline{S}_2^{TS}$.[17] In this gap area, equilibrium $12 \rightarrow 12^{CN}$, which is feasible, is dominated by equilibrium $12 \rightarrow 1^{*TS}$. At equilibrium $12 \rightarrow 1^{*TS}$, as in equilibrium $12 \rightarrow 1^{CN}$, firm 1 chooses q_1 to induce firm

[17] For $S_2 > \underline{S}_2^{TS}$ and $S_1 < \underline{S}_1^{TS}$, equilibrium $2 \rightarrow 12^{TS}$ dominates equilibrium $12 \rightarrow 12^{CN}$ by Proposition 3. For $S_2 < \underline{S}_2^{CN}$, equilibrium $12 \rightarrow 1^{CN}$ is the best firm 1 can do.

2 to exhaust in period 1. What differs, however, is that in equilibrium $12 \to 1^{CN}$, firm 1 chose q_1 such that firm 1's iso-profit curve is tangent to firm 2's best-response curve along the $q_2 = S_2$ portion of that curve, which because $q_2 = S_2$ occurs where firm 1's best-response curve intercepts firm 2's best-response curve (see Fig. 6b). In contrast, in equilibrium $12 \to 1^{*TS}$, firm 1 chooses $q_1^{12\to 1^{*TS}}$ such that $B_2(q_1) = S_2$ for all $q_1 \leq q_1^{12\to 1^{*TS}}$. Thus, as shown in Fig. 7, firm 1's iso-profit curve is not tangent to firm 2's best-response curve, since $q_1^{12\to 1^{*TS}}$ occurs at the kink in firm 2's best-response curve where $B_2(q_1^{12\to 1^{*TS}}) = S_2$.

The boundary between the $12 \to 1^{*TS}$ equilibrium and the $2 \to 12^{TS}$ equilibrium, which lies above it in cost region (iii), occurs when firm 1 just has sufficient stocks to force firm 2 to produce its entire stock in period 1. Thus, equilibrium $2 \to 1$ occurs along this boundary, which is given by $S_2 = \frac{1}{2}\left[(1-\beta)(\bar{p} - c_2) + \beta S_1\right]$. This can be seen to be an extension of the boundary between $2 \to 1$ and $2 \to 12$ in Fig. 3d.

A second gap area occurs in cost region (i), where firm 1 has a significant cost advantage, as is shown in Fig. 3a. In the gap region where $\underline{S}_2^{CN} < S_2 < \underline{S}_2^{TS}$ and $S_1 > \underline{S}_1^{CN}$, equilibrium $12 \to 12^{CN}$ is feasible, but neither an interior equilibrium $12 \to 12^{TS}$ nor equilibrium $1 \to 12^{CN}$ is feasible. Equilibrium $12 \to 2^{TS}$ is feasible when $S_1 < \underline{S}_1^{TS}$. But there is one other possibility: equilibrium $1 \to 12^{*TS}$. This equilibrium occurs in the portion of firm 1's best-response curves where firm 1's profits increase as q_1 increases along firm 1's best-response curve. In $1 \to 12^{*TS}$ firm 1 induces firm 2 to produce only in period 2. It does so by choosing $q_1^{1\to 12^{TS}}$ where firm 2's best-response curve is kinked at $B_2(q_1) = 0$. This is shown in Fig. 8.

The other possibility within the gap area in cost region (i) is the equilibrium $12 \to 2^{TS}$. The boundary between equilibria $12 \to 2^{TS}$ and $1 \to 12^{*TS}$ in the gap area in cost region (i) in Fig. 3a occurs when firm 1 just has sufficient stock to force firm 2 to produce zero in period 1 while firm 1 produces its entire stock in period 1. This occurs as an extension of the upper bound to equilibrium $1 \to 2$, along the line $S_2 = -(1-\beta)(\bar{p} - c_2)/2 + \beta S_1/2$.

A third gap area occurs in cost region $(ii)(a)$ as shown in Fig. 3b for stock values such that $S_1 > \underline{S}_1^{TS}$ and for which $\underline{S}_2^{CN} < S_2 < \underline{S}_2^{TS}$. In this case, equilibrium $12 \to 12^{CN}$ is feasible. The only other candidate for the TS equilibrium is the equilibrium $12 \to 1^{*TS}$. However, unlike cost region (iii), the higher relative costs to firm 2 in cost region $(ii)(a)$ make equilibrium $12 \to 1^{*TS}$ infeasible.

We summarize these outcomes in the following:

Proposition 5. *In the gap area in cost region (i) where equilibrium $12 \to 12^{CN}$ is* *feasible, since firm 1's profits are increasing in* q_1*, firm 1 increases its first period output to the point where firm 2 ceases production in period 1, if possible, and if not, firm 1 produces its entire stock in period 1. In the gap area in cost region* (iii)*, where equilibrium* $12 \to 12^{CN}$ *is feasible, since firm 1's profits are decreasing in* q_1*, firm 1 decreases its first period output to the point where firm 2 extracts its entire stock in period 1, if possible, and if not, firm 1 produces its entire stock in period 2. In the gap area in cost region* $(ii)(a)$*, where equilibrium* $12 \to 12^{CN}$ *is feasible, that equilibrium is the best firm 1 can do.*

Proof. See Appendix C □

Thus, in the TS game, the Stackelberg leader, firm 1, has the opportunity to influence what equilibrium occurs. Sometimes firm 1 prefers the equilibrium it could attain under the CN, as in Figs. 5b and 6b, but we have identified instances where firm 1 can improve its profits relative to the CN in the TS equilibrium by selecting an equilibrium in which at least one of the firms is forced to exhaust its stock in period 1.

Like the CN and DF games, the TS game has a unique equilibrium path for every cost and stock combination. The TS equilibrium also exhibits diversified production whenever costs are in cost region (ii), independent of the relative stocks, and when in cost region (i), where firm 1 has the cost advantage, firm 1 always produces in period 1, and when in cost region (iii), where firm 2 has the cost advantage, firm 2 always produces in period 1. Thus Propositions 1 and 2 can be extended to the TS game as well.

Finally, note that unlike the CN case, not all of the equilibrium areas in which a particular sequencing of production occurs are convex sets. In particular, equilibrium $12 \to 1$ is non-convex in both cost regions $(ii)(a)$ and $(ii)(b)$.

4.2 TS vs CN

Table 3 summarizes the effects of changes in the extraction pattern in the TS equilibrium relative to the CN equilibrium. Since firm 1 is unambiguously made better off by the TS equilibrium, our focus is upon the effect of the leader's choices on the rate of extraction, consumer surplus, and firm 2's profits.

We analyze two cases in detail to illustrate the intuition. Consider the last case described in Table 3. In TS equilibrium $12 \to 1^{*TS}$, firm 1 prefers to produce less in the first period relative to the CN equilibrium $12 \to 12^{CN}$, implying that firm 2 will produce more in period 1 and total production of the two firms declines since

Table 3: Equilibrium Effects of TS Equilibrium Selection Relative to the CN Equilibrium

	Equilibrium		Change under TS Relative to CN		
Cost Region	CN	TS	Extraction Rate	Consumer's Surplus	Firm 2 Profits
Region (i)	$12 \to 12$	$1 \to 12^*$	Faster	Increased	Increased
	$12 \to 12$	$12 \to 2$	Faster	Increased	Increased
Region $(ii)(a)$	$12 \to 12$	$12 \to 2$	Faster	Increased	Uncertain
	$12 \to 1$	$12 \to 2$	Faster	Increased	Decreased
Region $(ii)(b)$	$12 \to 12$	$12 \to 2$	Faster	Increased	Decreased
	$12 \to 1$	$12 \to 2$	Faster	Increased	Decreased
	$12 \to 1$	$12 \to 12$	Faster	Increased	Decreased
Region (iii)	$12 \to 12$	$2 \to 12$	Slower	Decreased	Increased
	$12 \to 12$	$12 \to 1^*$	Slower	Decreased	Increased

$dq_2^*/dq_1 = -1/2$. Firm 2, however, obtains a larger profit as illustrated by its iso-profit curve in Fig. 7. Consumers are worse off as resources become more expensive in the first period.

In equilibrium $1 \to 12^{*TS}$, relative to the CN equilibrium $12 \to 12^{CN}$, firm 1 produces more in the first period, firm 2 produces less in period 1, and total production rises in period 1. Again, firm 2 obtains a larger profit even though its first period production declines. This is due to the fact that profits to firm 2 are increasing by moving down along firm 2's best response curve, as illustrated in Fig. 8. In this case, consumers are better off as resources become cheaper in the first period. Therefore, social welfare is improved by allowing firm 1 to move first.

To summarize, we have the following proposition.

Proposition 6. (a) *If firm 1's profit decreases with its first period production, then in the TS equilibrium where firm moves first, it will extract at a slower rate compared to the case where firms move simultaneously, and vice versa.* (b) *If the leader extracts slower, then the follower extracts faster and total society extracts slower; and vice versa.* (c) *If the leader's profit decreases with its first period production or if the follower's profit decreases with its first period production, then the follower obtains a higher profit as a second mover compared to the case where firms move simultaneously.*

Proof. See Appendix C. □

With firms move simultaneously in each period, firm 1's first period production decreases with firm 2's resource stock but increases with firm 2's production cost. Thus, based on Proposition 5, the leader's profit decreases with its first period production if firm 2's resource stock is relatively large or if firm 2's production costs are relatively small. Therefore, the leader is more likely to postpone its production if the follower's resource stock is relatively large or if the follower has the cost advantage.

4.3 A Special Case

A special case arises if the two firms have identical costs as well as identical stocks.[18] In this case, if the two firms move simultaneously, then both firms produce either only in period one or over both periods, depending on the size of their stock endowments, since the 12 region kink occurs at $\underline{S}_1^{CN} = \underline{S}_2^{CN}$ when $c_1 = c_2$. If firm 1 moves first, however, then equilibrium $12 \rightarrow 2^{TS}$ exists for $(1-\beta)(\bar{p}-c)/3 < S < (1-\beta)(\bar{p}-c)/2$. Even in this special case, there exists an advantage to moving first.

5 Discussion and Conclusions

This paper examines a two-period, discrete-time "Truly Stackelberg" model where firms move sequentially within each period and where both the leader and the follower behave strategically. This contrasts to the literature on Stackelberg games in non-renewable resources, that has primarily used continuous-time models which implicitly assume that firms move simultaneously within period. The TS game equilibria are contrasted with the simultaneous move Cournot Nash equilibria.

In the TS game, the leader is able to manipulate extraction patterns by exploiting resource endowment constraints in a fashion novel to the literature. Specifically, there are instances in the TS game where even though an equilibrium in which both firms produce over both periods is feasible (and would occur if the game were simultaneous-move), the leader prefers, and has the power to select an equilibrium in which either the follower or the leader exhausts his entire stock in only one period. We also show that with a resource stock constraint, the issues revolve around the timing of production, rather than the existence of production as in static Stackelberg games.

Whether the leader wants to speed up its production depends on the shape of its iso-profit curve, which is affected by the two firms' relative stock endowments and relative production costs. When leader's profits are increasing in its own first period

[18] The case where the two firms have identical marginal extraction costs, $c_1 = c_2$, is analyzed in cost region $(ii)(b)$, as depicted in Fig. 3c. The case where the two firms have identical stocks is given by equilibria along the 45° line in S_1-S_2 space for each cost region. However, as this 45° line could pass through almost every equilibria, depending on the relative costs, little is gained by this restriction.

output as it moves down its own best-response curve, the leader wishes to increase output relative to the CN equilibrium. The other case, which only appears in cost regions where either firm 1 or firm 2 has a significant cost advantage, the profit for the high-cost firm is decreasing in its own first period output. The leader is more likely to postpone its production if the follower's resource stock is large relative to its own or if the follower has the cost advantage. When the leader extracts faster, then the follower extracts slower and in aggregate society extracts faster.

Finally, unlike static Stackelberg games, the follower does not necessarily have a second mover disadvantage. This is because in non-renewable resource Stackelberg games, both firms must eventually produce all of their stock.

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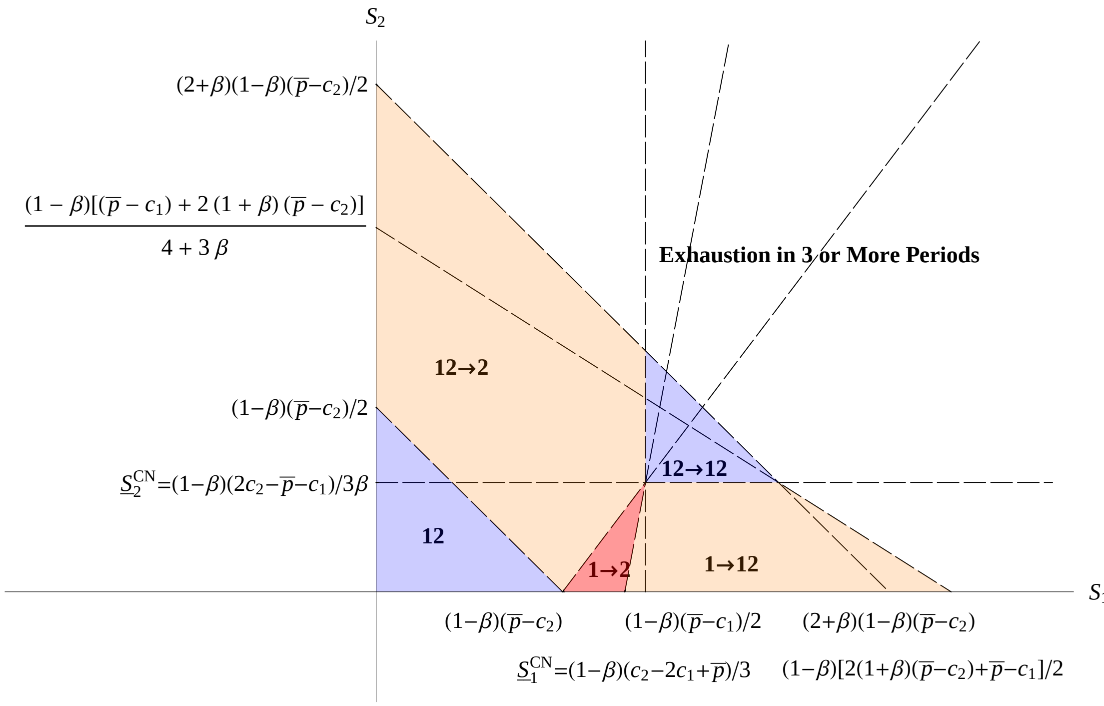
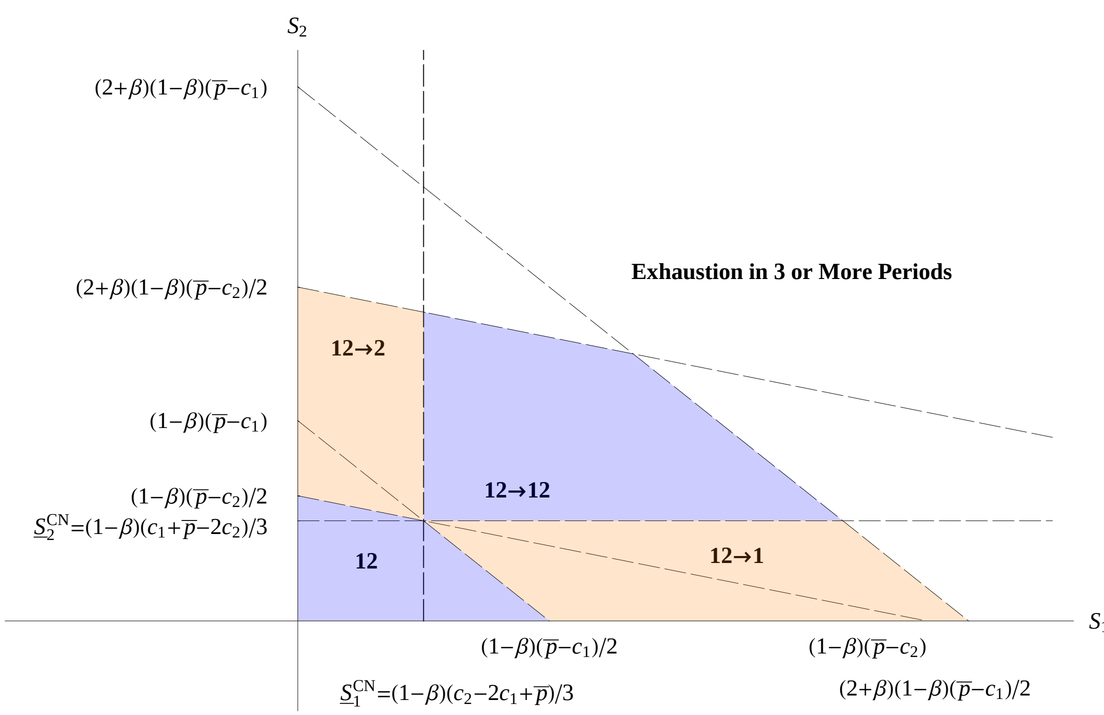
Figure 1: Simultaneous-Move, Cournot-Nash Equilibrium Stock Areas by Cost Region

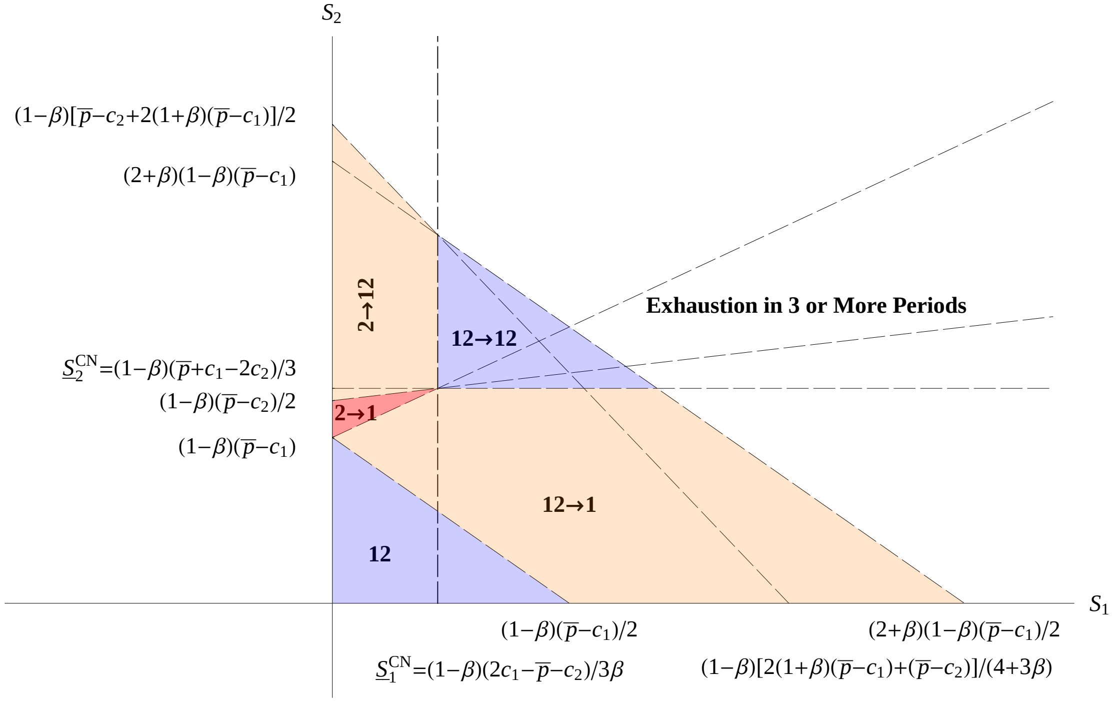


(a) CN Equilibria in Cost Region (i): $c_2 > (c_1 + \bar{p})/2$



(b) CN Equilibria in Cost Region (ii): $2c_1 - \bar{p} < c_2 < (c_1 + \bar{p})/2$

Figure 1: Simultaneous-Move, Cournot-Nash Equilibrium Stock Areas by Cost Region



(c) CN Equilibria in Cost Region (iii): $c_2 < 2c_1 - \bar{p}$

Figure 2: Comparison of CN and DF Equilibria in Cost Region (ii): $c_1 < c_2 < (c_1 + \bar{p})/2$.

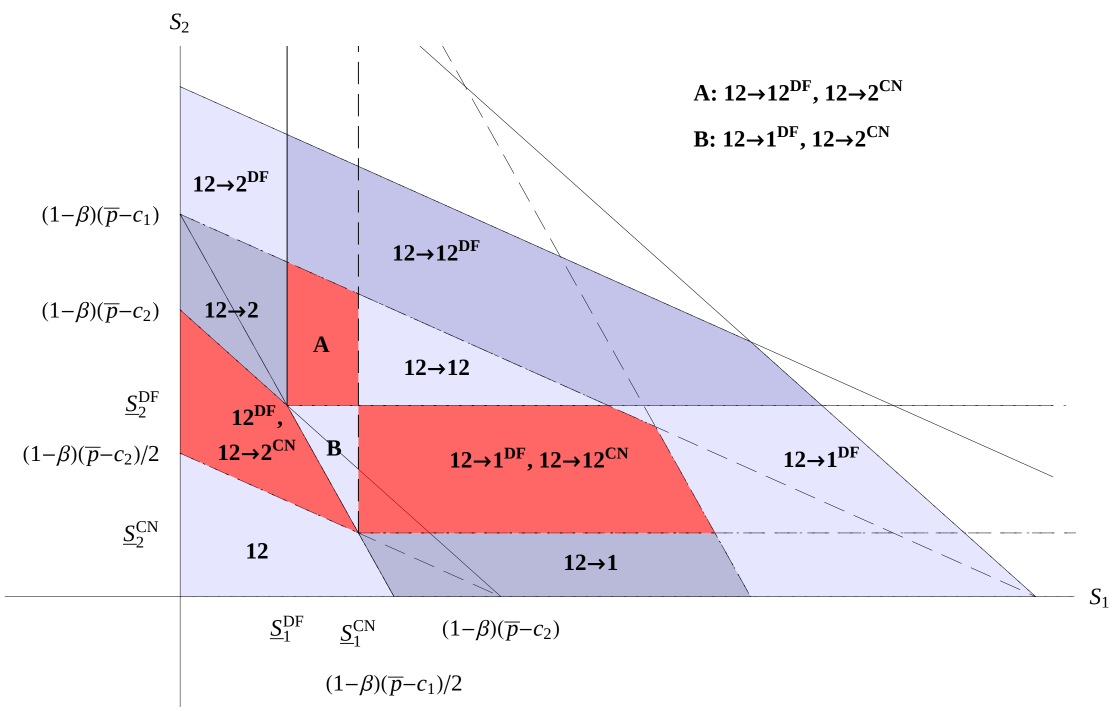
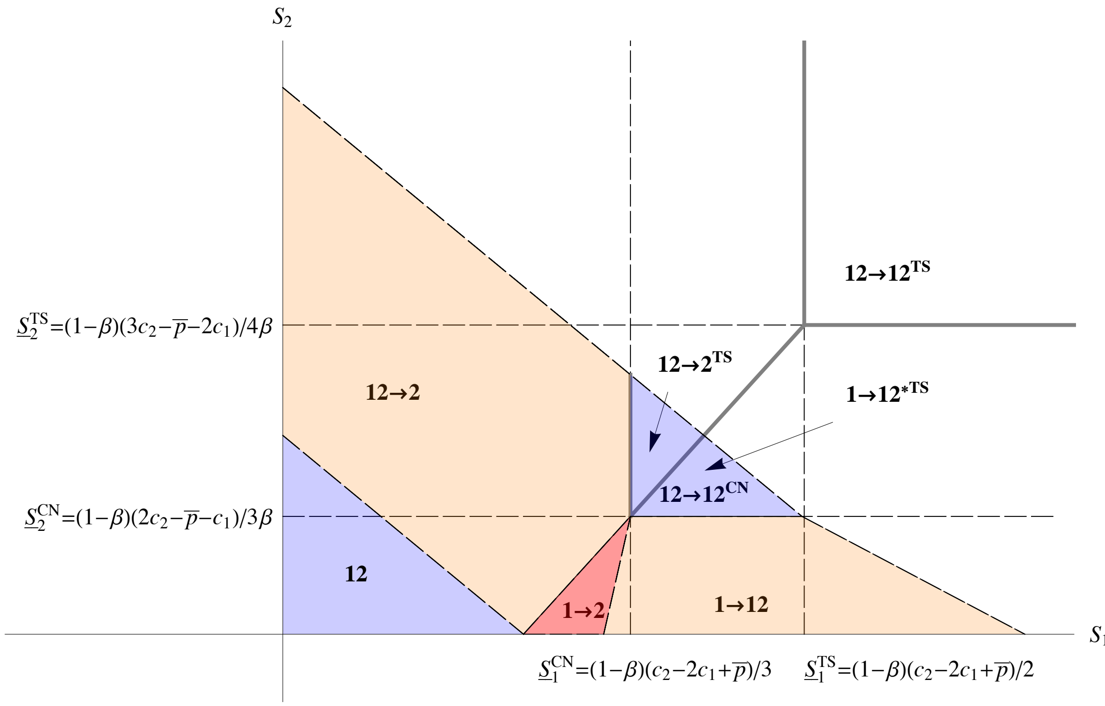
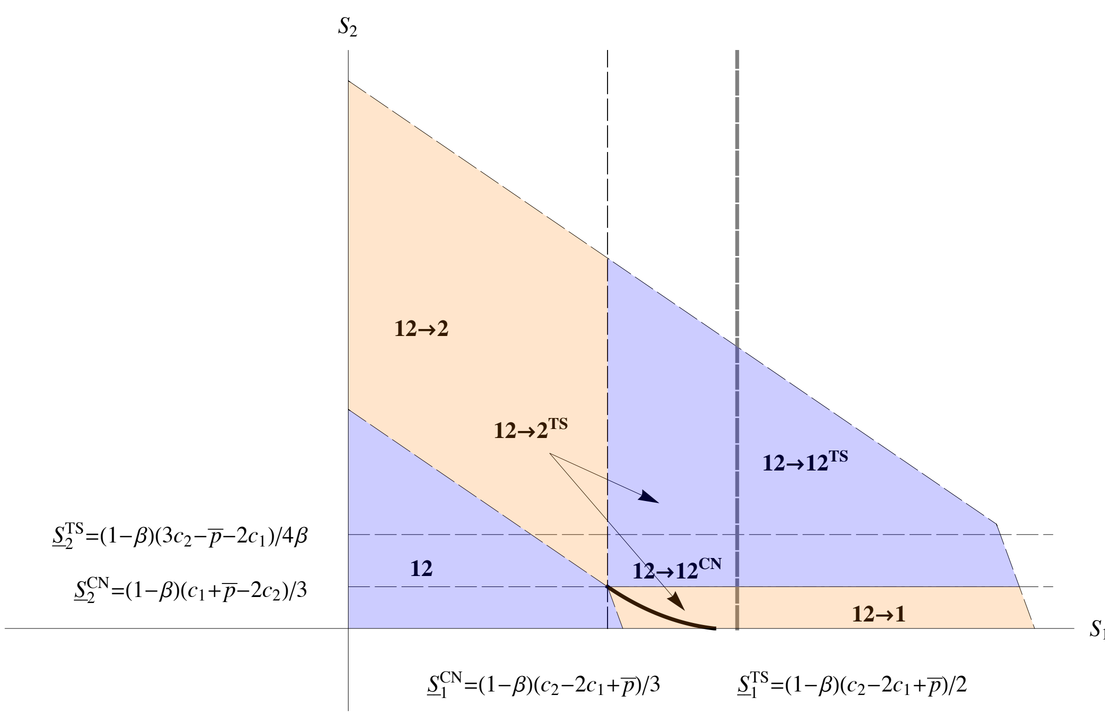


Figure 3: Sequential-Move, Truly Stackelberg Equilibrium Stock Areas by Cost Region

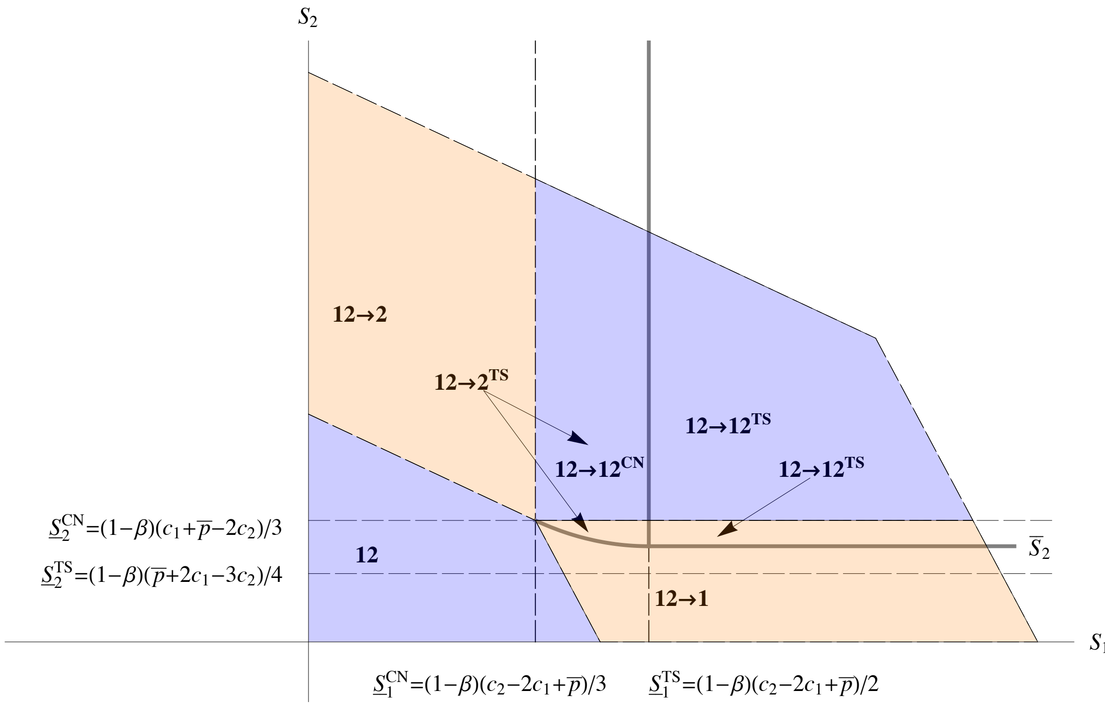


(a) TS Equilibria in Cost Region (i) : $\frac{\bar{p}+c_1}{2} < c_2$

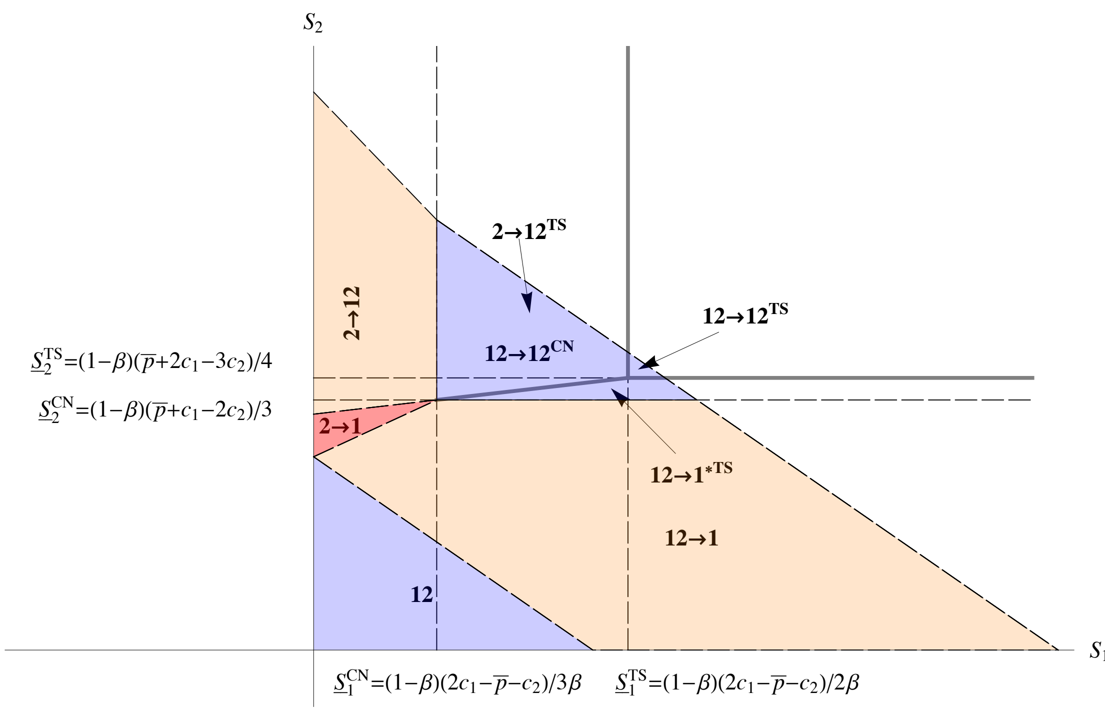


(b) TS Equilibria in Cost Region $(ii)(a)$: $\frac{(4\beta+6)c_1+(4\beta+3)\bar{p})}{8\beta+9} < c_2 < \frac{\bar{p}+c_1}{2}$

Figure 3: Sequential-Move, Truly Stackelberg Equilibrium Stock Areas by Cost Region

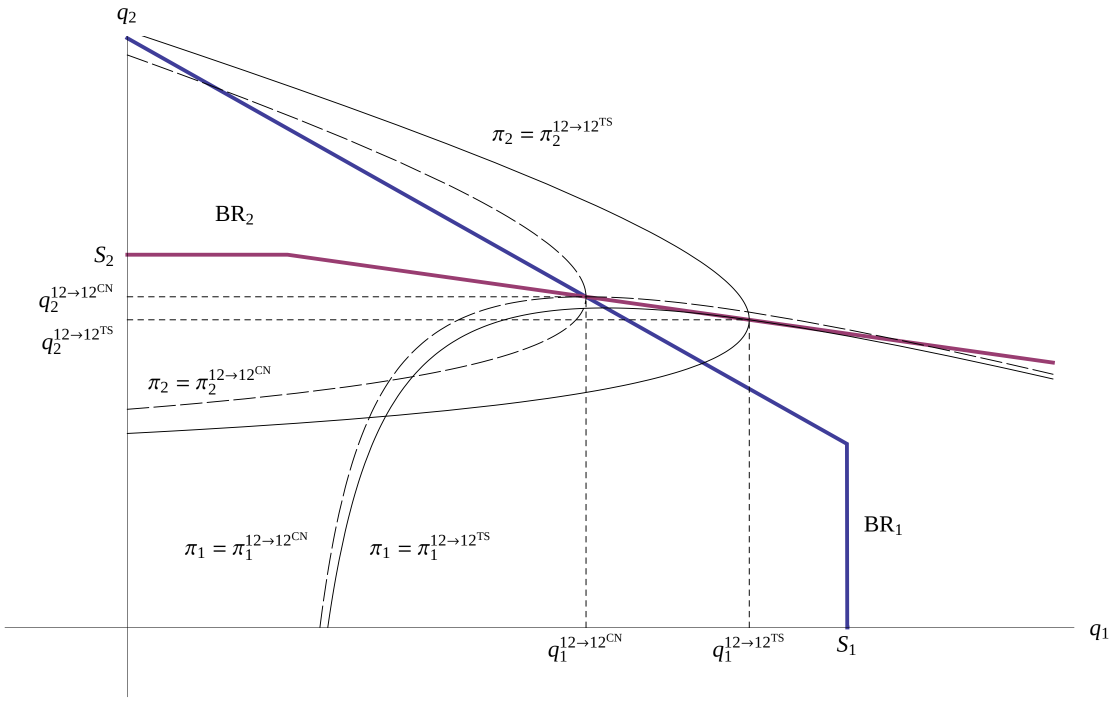


(c) TS Equilibria in Cost Region $(ii)(b)$: $2c_1 - \bar{p} < c_2 < \frac{(4\beta+6)c_1+(4\beta+3)\bar{p})}{8\beta+9}$

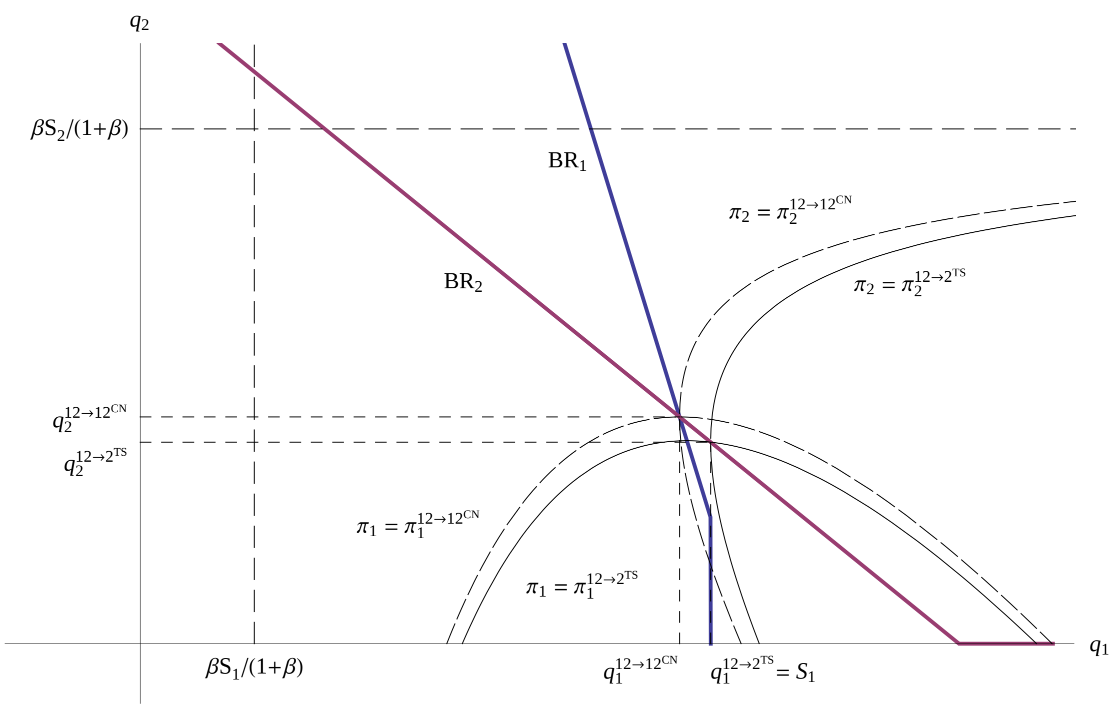


(d) TS Equilibria in Cost Region (iii): $c_2 < 2c_1 - \bar{p}$

Figure 4: Truly Stackelberg Equilibrium Selection with Firm 2 Extracts over Two Periods

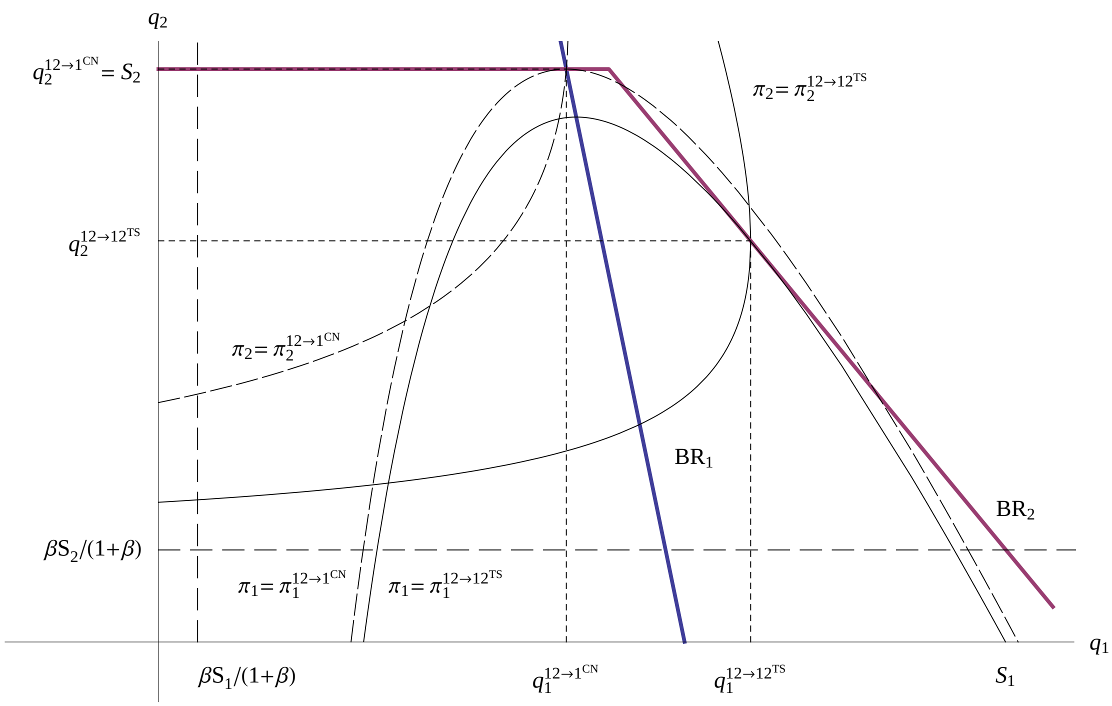


(a) Firm 1 Prefers $12 \to 12^{TS}$ to $12 \to 12^{CN}$ if $S_1 > \underline{S}_1^{TS}$ and $S_2 > \underline{S}_2^{TS}$ in Cost Region (ii)

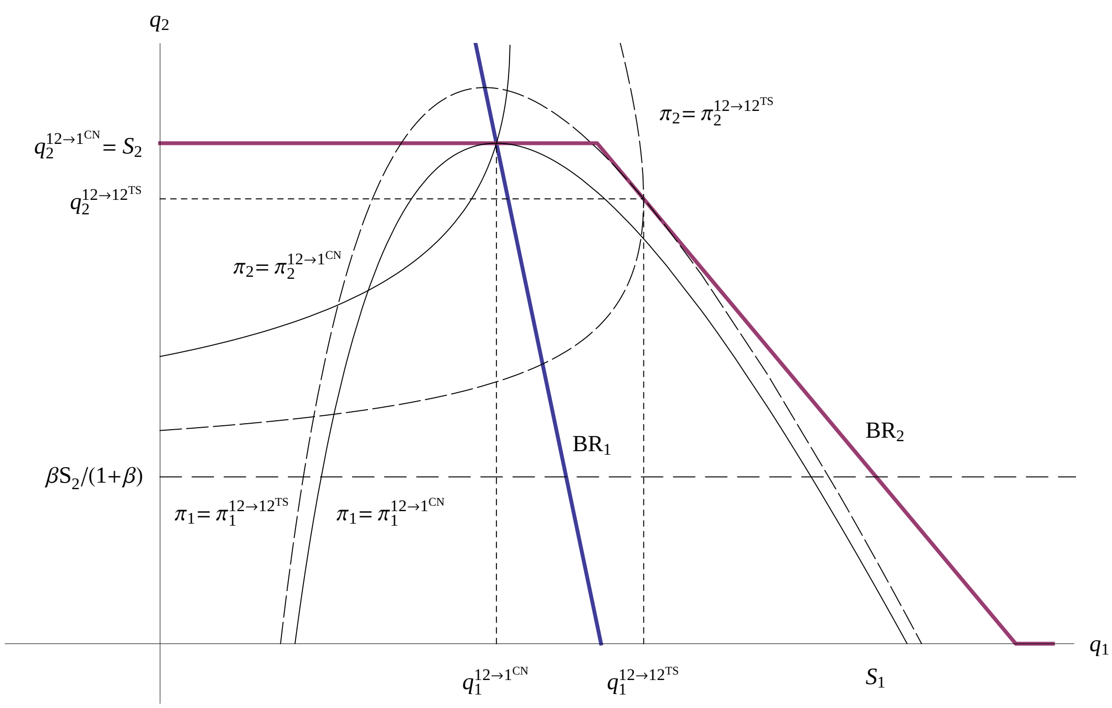


(b) Firm 1 Prefers $12 \to 2^{TS}$ to $12 \to 12^{CN}$ if $S_1 > \underline{S}_1^{TS}$ and $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$ in Cost Region (i)

Figure 5: Truly Stackelberg Equilibrium Selection: $12 \to 12^{TS}$ vs. $12 \to 1^{CN}$ in the 'Overlap' Area of Cost Region $(ii)(b)$ for $S_1 > \underline{S}_1^{TS}$

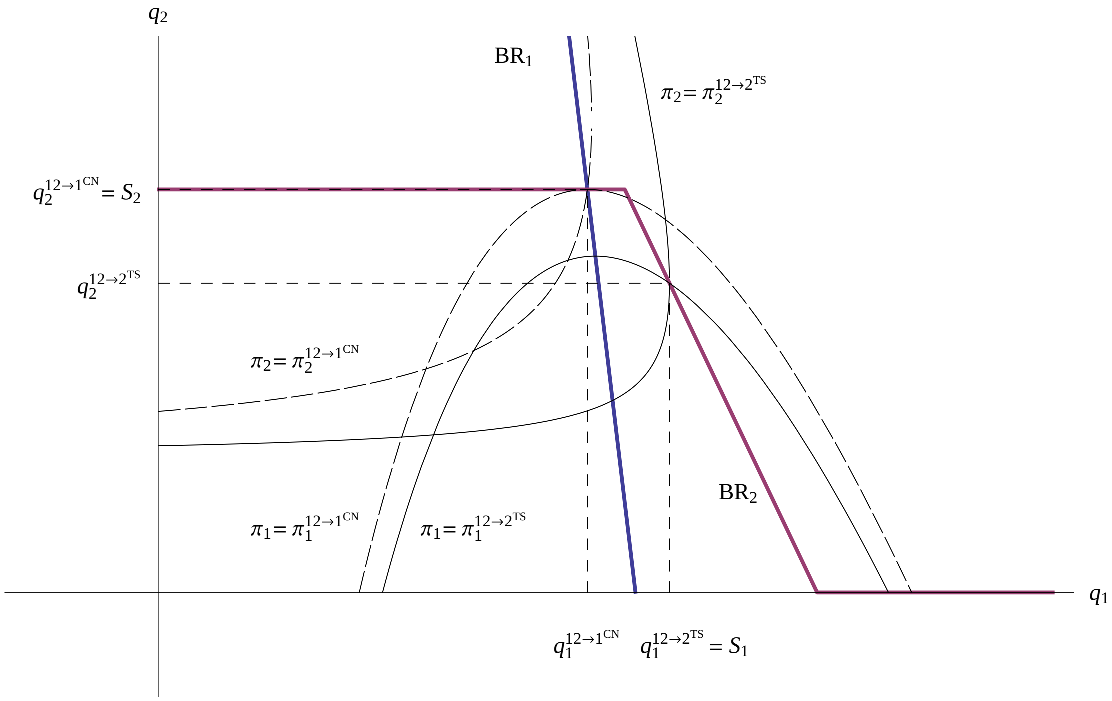


(a) Firm 1 Prefers $12 \to 12^{TS}$ to $12 \to 1^{CN}$ if $S_2 > \bar{S}_2$ and $S_1 > \underline{S}_1^{TS}$ in Cost Region $(ii)(b)$

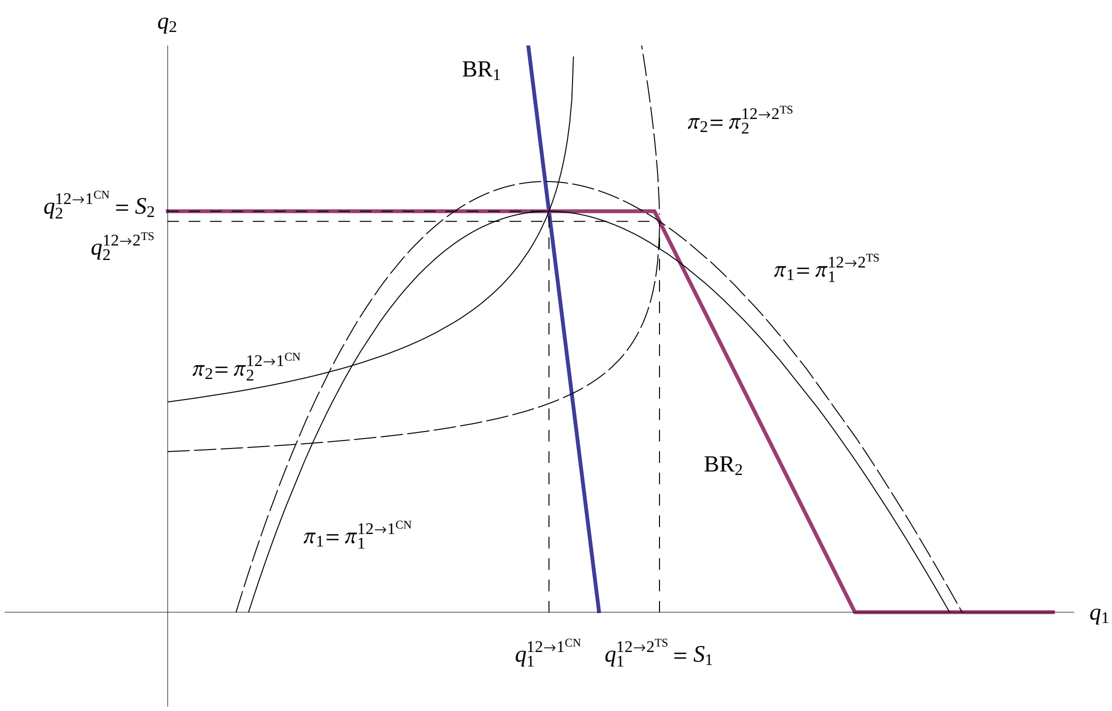


(b) Firm 1 Prefers $12 \to 1^{CN}$ to $12 \to 12^{TS}$ if $S_2 < \bar{S}_2$ and $S_1 > \underline{S}_1^{TS}$ in Cost Region $(ii)(b)$

Figure 6: Truly Stackelberg Equilibrium Selection: $12 \to 1^{CN}$ vs. $12 \to 2^{TS}$ in the 'Overlap' Areas of Cost Regions $(ii)(a)$ and $(ii)(b)$ for $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$



(a) Firm 1 Prefers $12 \to 2^{TS}$ to $12 \to 1^{CN}$ if $S_2 > \bar{\bar{S}}_2(S_1)$ and $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$ in Cost Region (ii)



(b) Firm 1 Prefers $12 \to 1^{CN}$ to $12 \to 2^{TS}$ if $S_2 < \bar{\bar{S}}_2(S_1)$ and $\underline{S}_1^{CN} < S_1 < \underline{S}_1^{TS}$ in Cost Region (ii)

Figure 7: Truly Stackelberg Equilibrium Selection: $12 \to 12^{CN}$ vs. $12 \to 1^{*TS}$ in the 'Gap' Area of Cost Region (iii)

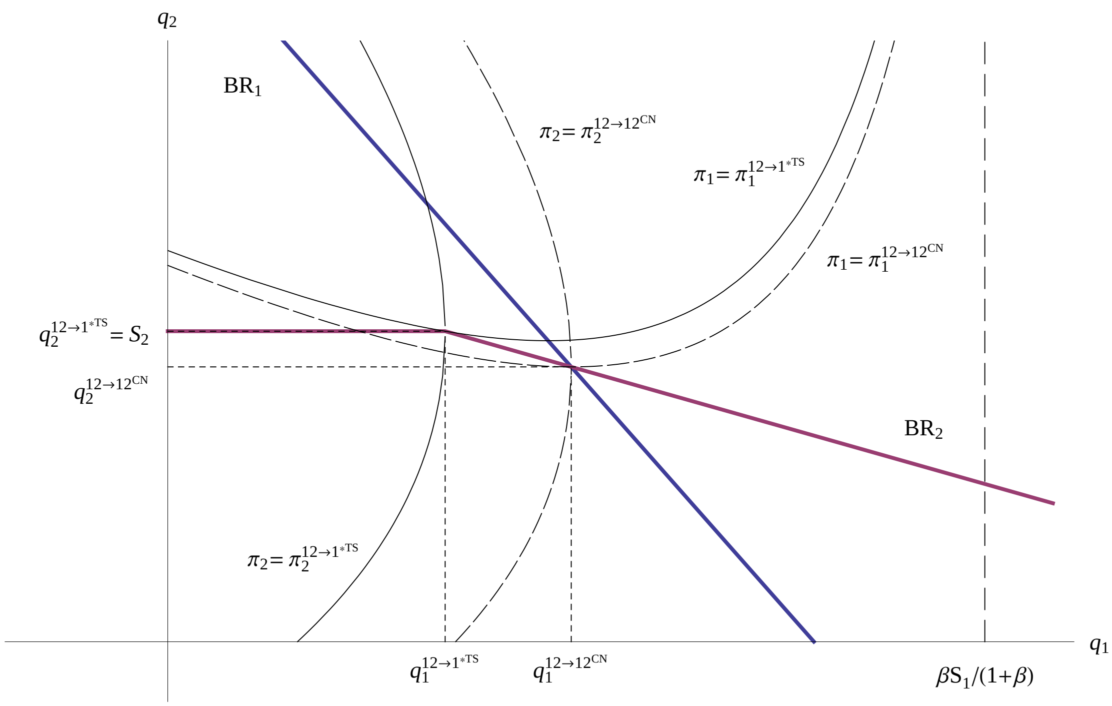


Figure 8: Truly Stackelberg Equilibrium Selection: $12 \to 12^{CN}$ vs. $1 \to 12^{*TS}$ in the 'Gap' Area of Cost Region (i)