

Introduction to *Internally Consistent Modeling,  
Aggregation, Inference, and Policy*

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## Abstract

This special issue of the *Journal of Econometrics* honors William A. Barnett's exceptional contributions to unifying economic theory with rigorous statistical inference to interpret economic data and inform public policy. It is devoted to papers that advance microeconometrics, macroeconometrics, and financial econometrics to build models to interpret evidence.

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This special issue of the *Journal of Econometrics* honors William A. Barnett's contributions in the fields of economic theory and econometrics. William Barnett is an eminent scientist and an outstanding econometrician and macroeconomist. He is the leading figure in the study of monetary and financial aggregation using index number and aggregation theory. In his seminal *Journal of Econometrics* paper in 1980, "Economic monetary aggregates: An application of index number and aggregation theory," produced while he was on the staff of the Federal Reserve Board's elite Special Studies Section, he fired the first volley arguing that official simple-sum monetary aggregates, constructed by the Federal Reserve, produce an internal inconsistency between the implicit aggregation theory and the theory relevant to the models and policy within which the resulting data are nested and used. That incoherence has been called the Barnett Critique, with emphasis on the resulting inference and policy errors and the induced appearances of function instability.

Barnett (1980) applied economic aggregation and index number theory to construct monetary aggregates consistent with Diewert's (1976) class of superlative quantity index numbers. Barnett's monetary aggregates are Törnqvist-Theil discrete time Divisia quantity indices, named after Francois Divisia, who first proposed the continuous time index in 1925 for aggregating over goods; Barnett (1980) proved how the formula could be extended to include monetary assets. Barnett has also extended the field of index number theory to include risk in Barnett (1995), Barnett *et al.* (1997), and Barnett and Wu (2005), using weaker assumptions in each successive derivation. He extended index number theory to multilateral international financial aggregation in Barnett (2007), for multicountry economic unions. Moreover, Barnett and Choi (2008) identified a generalized superlative index number class, including some indexes that cannot be found by Diewert's (1976) approach which requires algebraic representation of the aggregator function.

Thirty years later, the Federal Reserve Board and many other central banks around the world continue officially to produce and supply low quality monetary statistics, inconsistent with the relevant aggregation and index-number theory. This practice misleads central banks, as well as financial firms, mortgage lenders, and mortgage borrowers, regarding the levels of systemic risk in the economy, and also misleads economists regarding the appearance of instability of policy-relevant functions in the economy. These data deficiencies exist despite the fact that dozens of central banks throughout the world produce Divisia monetary aggregates for internal use, often available only to the central bank's highest ranking officials. Noteworthy exceptions include the Bank of England, the Federal Reserve Bank of St. Louis, the National Bank of Poland, the Bank of Israel, and the International Monetary Fund, which advocate and provide Divisia monetary aggregate data to the public. The European Central Bank's Governing Council also uses the Divisia monetary aggregates, but on a confidential basis, without making the data available to the public. The Bank of Japan produces Divisia monetary aggregates, but only for internal use. As an indication of the level of international acceptance of Barnett's approach among sophisticated economists, see the International Monetary Fund's official data document, *Monetary and Financial Statistics: Compilation Guide*, 2008, pp. 183-184.

The second volley in the Barnett critique was fired in Barnett *et al.* (1984), replicating prior studies which had found "puzzles" in monetary economics and which presented evidence of functional instability in the economy's financial sector. They found that all puzzles and appearances of instability disappeared, when the simple-sum monetary aggregates were

replaced by Divisia monetary aggregates. The same conclusion has been found repeatedly in subsequent research by economists throughout the world. More recently, Barnett documents the relevancy of the Barnett Critique in understanding the sources of the risk misperceptions that led to the global financial crisis and Great Recession. In Barnett and Chauvet (2011a) and his 2012 MIT Press book, *Getting it wrong: How faulty monetary statistics undermine the Fed, the financial system, and the economy*, he argues that economic agents, under the misperception that the business cycle had permanently ended during the Great Moderation, displayed an incorrect assessment of systemic risk and significantly increased their leverage and risk-taking activities. This led to the credit-driven, asset-price bubble in the U.S. housing market, with prices departing significantly from fundamental values. When the bubble burst, it brought down much of the financial system and not only led to an economic downturn in the United States, but also to a global recession.

During 2011, as the implications of his new book became known, Barnett was appointed Director, Center for Financial Stability, in New York City. He manages a program building on his research in monetary and financial aggregation. The program he directs can be found at <http://www.centerforfinancialstability.org/amfm.php>, along with an online library linking to Divisia monetary aggregates data and studies for over 40 countries throughout the world. The CFS Divisia monetary aggregates, documented in detail by Barnett *et al.* (2013), are currently the best measures of the money supply. They indicate that policy was more expansionary than indicated by the official simple-sum monetary aggregates during the asset bubbles, and more contractionary than indicated by interest rates, following the financial crisis leading into the Great Recession. For example, as can be seen in Figure 1, Divisia M4 is currently growing at a low rate, historically inconsistent with healthy economic recovery.

Barnett has also made fundamental contributions to the associated field of flexible-functional-form modeling. The earliest modern consumer-demand-systems model, having a provable local approximation property and rigorous connection with internally consistent aggregation over goods and prices, was Theil and Barten's Rotterdam model, which used Divisia and Frisch indexes to aggregate over goods quantities and prices. The model was derived directly from the relevant first order conditions. That model was not vulnerable to the Barnett Critique. But the model's link to aggregation over consumers was questionable, since the available theory conditioned on an unreasonable assumption, implying that the model collapsed to Cobb-Douglas. Barnett (1979a) proved that the model's aggregation over consumers could be accomplished under remarkably weak assumptions, but with the addition of a remainder term, having properties that he explored. Barnett (1979b) further derived and applied the model's test for blockwise weak separability, which is the necessary condition for quantity aggregation, and he derived the remainder term after aggregation over consumers.

At that time, there also was a problem with the available econometric theory. The relevant class of models for most of this literature is the closed form nonlinear systems of equations, but proofs of the asymptotic properties of the maximum likelihood estimator, under regularity conditions relevant to nonlinear demand systems, did not exist. The available proofs assumed linear structure or independent and identically distributed endogenous variables. Those assumptions are not relevant to nonlinear regression systems. Barnett (1976) was the first to prove the asymptotic normality and efficiency of the MLE for that

class of models, paving the ground for future research in the field.

Flexible functional forms, providing second-order local approximations, have most commonly been derived from second-order Taylor series approximations. Global regularity, including monotonicity and curvature, cannot be imposed on those available models without causing them to collapse to highly restrictive special cases. Barnett (1983) proposed the use of the second-order Laurent series and identified a parsimonious special case, called Minflex Laurent, which retains the flexibility property. He applied the new modeling approach to estimate the demand for money in a manner consistent with aggregation theoretic monetary aggregation. Barnett and Lee (1985) proved that the second-order Laurent series and its Minflex parsimonious special case have better economic properties, over a very large region, than the second-order Taylor series flexible functional forms. This research on consumer demand motivated subsequent research on production by Diewert and Wales (1987), resulting in their Generalized Barnett production model.

In his insightful analysis, Gallant (1981) introduced the concept of asymptotic global flexibility, using semiparametric estimation converging globally to unknown functions. In this approach, the order of the approximation is linked to sample size, and sample size is permitted to go to infinity. Gallant used the Fourier series for that purpose. But the Fourier series basis functions span the neoclassical function space from outside that space, since the basis functions are periodic and therefore not neoclassically regular. As a result, when the series expansion is truncated by finite sample size, the model usually violates neoclassical regularity conditions. Barnett *et al.* (1991) proved that the Müntz-Szatz series expansion can be used for seminonparametric inference, while keeping the basis functions inside the neoclassical function space. The resulting model is called the Asymptotically Ideal Model (AIM) and continues to be a state of the art approach motivating sophisticated research.

Some of Barnett's most forward-looking research remains beyond the state of the art of mainstream applied research to this day. Barnett (1977a) produced the only currently known globally regular blockwise-weakly-separable demand system. Aggregation is recursive within the utility tree, thereby assuring internal consistency between the aggregate data and the model within which the data are used. Barnett (1977b) also derived and proved identification of the structural form consistent with the household production function approach and showed how it can be specified with flexible functional forms. In both of these challenging approaches, the resulting model is a nonlinear system of simultaneous equations, requiring more difficult estimation than has become customary with less sophisticated approaches.

This special issue, prepared in William A. Barnett's honor, contains contributions by many of the world's most eminent economists. In exploring aspects of the interface between econometrics and theory, it follows and complements an earlier *Journal of Econometrics* special issue, *The Interface between Econometrics and Economic Theory*, edited by Charalambos D. Aliprantis, William A. Barnett, Bernard Cornet, and Steven Durlauf. That special issue, published in 2007, focused primarily on the relevant economic theory and econometric theory needed for internally consistent inference. A related special issue, prepared by William A. Barnett, W. Erwin Diewert, and Arnold Zellner on *Measurement with Theory*, was published in 2011 and emphasized the use of economic theory in specifying and applying aggregator functions to measure aggregates. The current special issue focuses on subsequent advances in applied research, which, in the tradition of Barnett's work, seek to advance the profession's ability to make full use of the relevant economic theory and

econometrics, in the interests of internal consistency between data, modeling, and inference; avoidance of misleading appearances of function instability; and avoidance of induced policy errors.

The papers in this issue reflect the span of William A. Barnett's exceptional scholarly contributions, reprinted in Barnett and Serletis (2000), Barnett and Binner (2004), and Barnett and Chauvet (2011b). In what follows, we very briefly describe them.

The first paper by Michael T. Belongia and Peter N. Ireland, "*The Barnett critique after three decades: A new Keynesian analysis*," presents a strong argument to the effect that monetary policy analysis has been badly remiss in not giving more emphasis to William Barnett's Divisia monetary aggregates, a situation that is enhanced by the Federal Reserve's failure to produce and disseminate reliable monetary statistics. The argument proceeds by extending a New Keynesian model to include currency and deposits as distinct monetary assets held by the private sector, and with monetary policy conducted according to a Taylor-type interest rate rule. The paper shows that, both qualitatively and quantitatively, the Barnett critique is alive and well: a simple-sum monetary aggregate fails to track the behavior of the true monetary aggregate while a Divisia aggregate of monetary services tracks the aggregation-theoretic monetary aggregator function almost perfectly.

The second paper by John Geweke and Lea Petrella, "*Likelihood-based inference for regular functions with fractional polynomial approximations*," develops fractional polynomial approximations of aggregator functions (cost, production, and utility functions). It builds on the pioneering contributions by Barnett and Jonas (1983) and Barnett *et al.* (1991) by extending the multivariate Weierstrass approximation of Evard and Jafari (1994) to fractional polynomials. It shows that regular fractional polynomials can approximate regular aggregator functions and their first two derivatives arbitrarily well. The paper also uses canonical cost function data and shows that a fully Bayesian approach to inference for the fractional polynomial approximation of a cost function is practical using standard Markov chain Monte Carlo methods.

In the spirit of William Barnett's pioneering research, the paper by Gabriella Conti, Sylvia Frühwirth-Schnatter, James J. Heckman, and Rémi Piatek, "*Bayesian exploratory factor analysis*," develops and applies a Bayesian approach to the problem of exploratory factor analysis. Classical approaches use different criteria to pick the dimension of the number of factors and to assign measurements to factors. The criteria in both steps are ad hoc rules of thumb. The paper published in this issue uses a coherent Bayesian framework to select the number of factors required to summarize a set of measurements and the assignment of measurements to factors. The method maximizes the posterior probability of each component of the model. Factors are allowed to be freely correlated. The model attains Thurstone's "perfect simple structure," which assigns each measurement to at most one factor. The authors develop new Bayesian computational algorithms and apply them to analyze a high dimensional set of measurements allowing for measurement error. They apply the algorithm to produce interpretable aggregates from a very large set of psychological measures. The authors use Monte Carlo analyses to confirm the validity of their methodology.

The paper by Erwin Diewert, "*Decompositions of profitability change using cost functions*," provides a decomposition of a production unit's profitability (defined as the value of outputs produced divided by the corresponding cost) growth over two periods into various explanatory factors. The explanatory factors are the growth in the production unit's cost

efficiency, an index of output price growth, a measure of returns to scale, technical progress, and an index of input price growth. It also shows how the decomposition of profitability growth could be rearranged to give a decomposition of total factor productivity growth into explanatory growth factors, thus relating the two important measures, productivity and profitability, to each other.

The paper by Jaroslav Borovička and Lars Hansen, “*Examining macroeconomic models through the lens of asset pricing,*” develops and implements a new methodology for analyzing dynamic stochastic general equilibrium models that have implications for asset pricing. It introduces two new tools, called ‘shock-exposure’ elasticities and ‘shock-price’ elasticities, for measuring the sensitivity of cash flows and their prices to shocks, including economic shocks featured in the empirical macroeconomics literature. The shock-exposure elasticities are similar to nonlinear impulse response functions and reflect the impact of current shocks on the future distributions of the underlying macroeconomic processes. The shock-price elasticities are the pricing counterparts to impulse response functions and reflect the current period compensation for the exposure to future shocks. The ideas presented in the paper apply to nonlinear economic models (including those with stochastic volatility), and bridge the gap between macroeconomics and finance theory.

The paper by Mauro Alem and Robert M. Townsend, “*An evaluation of financial institutions: Impact on consumption and investment using panel data and the theory of risk-bearing,*” assesses the impact that various types of financial institutions have on the consumption and investment behavior of individuals in an emerging market economy. Its contribution comes through empirical results, derived from a specification that is grounded firmly in the theory of optimal risk sharing. It combines the theory with the use of the Townsend Thai data, a comprehensive panel of approximately 960 households from Thai villages. The strategy is to compare the gains from participating in the financial sector with financial autarky. The main conclusion is that some banks are particularly helpful in smoothing consumption and investment, while others are less so. The informal sector is more successful in smoothing investment and not particularly in smoothing consumption.

The paper by Helmut Herwartz and Helmut Lütkepohl, “*Structural vector autoregressions with Markov switching: Combining conventional with statistical identification of shocks,*” deals with the identification of structural vector autoregressive (SVAR) models through the use of heteroskedasticity, which takes the form of a Markov regime switching reduced form error covariance matrix. In the absence of further identification constraints, the existence of two distinct regimes on the covariance matrix delivers exact identification, provided that rank conditions are satisfied. The paper combines the identification constraints arising from heteroskedasticity with other identification constraints arising from economic considerations in order to obtain over identification and hence empirical tests of the model. The discussion is based on a VAR model for the United States containing oil prices, output, consumer prices and a short-term interest rate.

The paper by Yu-chin Chen, Stephen J. Turnovsky, and Eric Zivot, “*Forecasting inflation using commodity price aggregates,*” investigates whether asset prices contain information that may be useful in predicting inflation. It establishes that world commodity price indexes have predictive power for inflation for a number of small commodity-exporting countries that have adopted inflation targeting monetary policies. This conclusion is robust to using either aggregate indexes or sub-indexes, although the latter perform better.

The last paper by Guohua Feng and Apostolos Serletis, “*Undesirable outputs and a primal Divisia productivity index based on the directional output distance function*,” is motivated by the finding that the conventional Divisia productivity indexes all ignore undesirable (bad) outputs and proposes a new primal Divisia-type productivity index that is valid in the presence of undesirable outputs. This index is derived by total differentiation of the directional output distance function with respect to a time trend and is referred to as the Divisia-Luenberger productivity index. The paper also provides a comparison between the new Divisia-Luenberger productivity index and a representative of the conventional Divisia productivity indexes, the Feng and Serletis (2010) productivity index which is based on the radial output distance function. It concludes that failure to take into account undesirable outputs leads to misleading conclusions regarding productivity, technological change, and efficiency improvements.

## References

- [1] Barnett, W.A., 1976. Maximum likelihood and iterated Aitken estimation of nonlinear systems of equations. *Journal of the American Statistical Association* 71, 354-360.
- [2] Barnett, W.A., 1977a. Recursive subaggregation and a generalized hypocycloidal demand model. *Econometrica* 45, 1117-1136.
- [3] Barnett, W.A., 1977b. Pollak and Wachter on the household production function approach. *Journal of Political Economy* 85, 1073-1082.
- [4] Barnett, W.A., 1979a. Theoretical foundations for the Rotterdam model. *Review of Economic Studies* 46, 109-130.
- [5] Barnett, W.A., 1979b. The joint allocation of leisure and goods expenditure. *Econometrica* 45, 1117-1136.
- [6] Barnett, W.A., 1980. Economic monetary aggregates: An application of index number and aggregation theory. *Journal of Econometrics* 14, 11-48.
- [7] Barnett, W.A., 1983. New indices of money supply and the flexible Laurent demand system. *Journal of Business and Economic Statistics* 1, 7-23.
- [8] Barnett, W.A., 1995. Exact aggregation under risk. In: Barnett, W.A., Salles, M., Moulin, H., Schofield, N. (Eds.), *Social Choice, Welfare, and Ethics*. Cambridge: Cambridge University Press.
- [9] Barnett, W.A., 2007. Multilateral aggregation-theoretic monetary aggregation over heterogeneous countries. *Journal of Econometrics* 136, 457-482.
- [10] Barnett, W.A., 2012. *Getting it wrong: How faulty monetary statistics undermine the Fed, the financial system, and the economy*. Cambridge, MA: The MIT Press.
- [11] Barnett, W.A., Binner, J., 2004. *Functional Structure and Approximation in Econometrics*, Volume 261 (Contributions to Economic Analysis). Elsevier, Amsterdam.
- [12] Barnett, W.A., Chauvet, M., 2011a. How better monetary statistics could have signaled the financial crisis. *Journal of Econometrics* 161, 6-23.
- [13] Barnett, W.A., Chauvet, M., 2011b. *Financial Aggregation and Index Number Theory*. World Scientific, Singapore.
- [14] Barnett, W.A., Choi, K.-H., 2008. Operational identification of the complete class of superlative index numbers: An application of Galois theory. *Journal of Mathematical Economics* 44, 603-612.
- [15] Barnett, W.A., Jonas, A., 1983. The Müntz-Szatz demand system: An application of a globally well behaved series expansion. *Economics Letters* 11, 337-342.

- [16] Barnett, W.A., Lee, Y.W., 1985. The global properties of the minflex Laurent, generalized Leontief, and translog flexible functional forms. *Econometrica* 53, 1421-1437.
- [17] Barnett, W.A., Wu, S., 2005. On user costs of risky monetary assets. *Annals of Finance* 1, 35-50.
- [18] Barnett, W.A., Serletis, A., 2000. *The Theory of Monetary Aggregation, Volume 245 (Contributions to Economic Analysis)*. Elsevier, Amsterdam.
- [19] Barnett, W.A., Geweke, J., Wolfe, M.D., 1991. Semionparametric Bayesian estimation of the Asymptotically Ideal Production Model. *Journal of Econometrics* 49, 5-50.
- [20] Barnett, W.A., Liu, Y., Jensen, M., 1997. The CAPM risk adjustment for exact aggregation over financial assets. *Macroeconomic Dynamics* 1, 485-512.
- [21] Barnett, W.A., Offenbacher, E.K., and Spindt, P.A., 1984. The new Divisia monetary aggregates. *Journal of Political Economy* 92, 1049-1085.
- [22] Barnett, W.A., Liu, J., Mattson, R.S., van den Noort, J., 2013. The new CFS Divisia monetary aggregates: Design, construction, and data sources. *Open Economies Review* 24, 101-124.
- [23] Diewert, W.E., 1976. Exact and superlative index numbers. *Journal of Econometrics* 4, 115-145.
- [24] Diewert, W.E., Wales, T.J., 1987. Flexible functional forms and global curvature conditions. *Econometrica* 55, 43-68.
- [25] Evard, J.C., Jafari, F., 1994. Direct computation of the simultaneous Stone-Weierstrass approximation of a function and its partial derivatives in Banach spaces, and combination with Hermite interpolation. *Journal of Approximation Theory* 78, 351-363.
- [26] Feng, G., Serletis, A., 2010. A primal Divisia technical change index based on the output distance function. *Journal of Econometrics* 159, 320-330.
- [27] Gallant, A.R., 1981, On the bias of flexible functional forms and an essentially unbiased form: The Fourier functional form. *Journal of Econometrics* 15, 211-245.

**Figure 1. Divisia M4+, Year-Over-Year Percentage Growth Rate  
(January, 2000 to April, 2014)**

