Resource allocation, affluence and deadweight loss when relative consumption matters *

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Abstract

We explore the link between affluence and well-being using a simple general equilibrium model with a pure Veblen good. Individuals derive utility from the pure Veblen good based solely on how much they consume relative to others. In equilibrium, consumption of the pure Veblen good is the same for everyone, so the Veblen good contributes nothing to utility. Hence, resources devoted to the Veblen good provide us with a measure of deadweight loss. We ask: Under what preference conditions does the proportion of productive capacity devoted to the pure Veblen good increase as an economy becomes more affluent? In a relatively general preference framework we derive a sufficient condition for which the Veblen good crowds out standard forms of consumption and leisure, resulting in an inverse relationship between affluence and utility. With additional structure on the model we are able to fully characterize the behavior of deadweight loss and utility as an economy becomes more affluent.

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1 Introduction

In recent years, a great deal of attention has been paid to what is sometimes called the Easterlin paradox: in wealthy societies, there is no discernible positive correlation between average perceived well-being and per capita income. The evidence is consistent with the suggestion that over the twentieth century we have developed an unhealthy preoccupation with per-capita income and productivity as measures of aggregate well-being[1].

We analyze the link between affluence and well-being using a simple general equilibrium model in which there are two resources, labour and a non-human resource, and three goods, leisure, a standard consumption good and a *pure Veblen good*. Individuals derive utility from the pure Veblen good based solely on how much they consume relative to others. In equilibrium, everyone consumes the same amount of the Veblen good, so the Veblen good contributes nothing to utility. For this reason, resources allocated to production of the Veblen good provide us with a measure of deadweight loss. Our research question is: Under what preference conditions does the proportion of productive capacity devoted to the pure Veblen good increase as an economy becomes more affluent?

Whether the Veblen good is more problematic in affluent economies depends on the relative magnitudes of two effects. The marginal private effect (MPE) is driven by the fact that an individual's marginal utility of the Veblen good diminishes as her consumption of it increases. MPE is, of course, negative. The marginal social effect (MSE) is driven by the relationship between an individual's marginal utility of the Veblen good and the average amount of the Veblen good consumed by others. MSE is positive or negative as this relationship is direct or inverse. We show that a positive MSE that is larger than the absolute value of MPE is a sufficient condition for proportionate deadweight loss to be increasing in affluence. In fact, this condition provides a stark illustration of the Easterlin paradox: when

[1] Notable discussions include Galbraith (1969) and more recently Helliwell (2003).

per-capita income increases the Veblen good crowds out standard forms of consumption and leisure, resulting in an inverse relationship between affluence and utility.

When MSE is positive but smaller than the absolute value of MPE, the answer to our question depends on how rapidly marginal utility of the Veblen good diminishes relative to that of leisure and standard consumption. This can be fully characterized in terms of a measure of curvature equivalent to the Arrow-Pratt measure of risk aversion, which we refer to as Arrow-Pratt curvature. This provides a measure of the rate at which marginal utility is decreasing that is comparable across the three goods. When the Arrow-Pratt curvature for the Veblen good is sufficiently small relative to the Arrow-Pratt curvature for leisure and the standard good, proportionate deadweight loss will increase with affluence. Conveniently, Arrow-Pratt curvature is a cardinal concept, and in principle it is measurable. So our analysis may offer a new framework for empirical work on the Easterlin paradox.

The economy in our model can become more affluent in two distinct ways: through technological change that enhances labour productivity, or through an increase in either the quantity or the productivity of the non-human resource. We find that the relationship between deadweight loss and affluence may depend on the source of the affluence, so this distinction is an important one.

This paper builds primarily on the work of Brekke et al. (2003) and Eaton and Eswaran (2009). Brekke et al. (2003) explore the hypothesis of Hirsch (1976) that as per-capita income rises an ever greater proportion of resources will allocated to positional consumption. In a model with a standard consumption good and a positional good they find that the extent to which the Hirsch hypothesis is true depends on assumptions made regarding individual preferences—specifically, on the substitutability between the positional good and the standard good and on the relationship between social position and consumption. Using a specific functional form, they contrast two cases: when utility is based on the difference between individual consumption of the positional good and the mean for society and when utility is based on the ratio between individual consumption of the positional good and the mean for society. They find that when the difference matters, society ends up in a relative consumption trap, in which all incremental resources are allocated to positional consumption. In this model, affluence is driven by an exogenous increases in income. Eaton and Eswaran (2009) introduce positional consumption using a pure Veblen good. In their model affluence is driven by an increases in labour productivity. They find there is a threshold level of wealth beyond which productivity gains are dissipated in ever increasing production of the Veblen good.

In this paper we make three contributions. First, the more general preference framework in our model allows us to cleanly delineate the conditions under which Veblen goods are, and are not, more problematic in more affluent economies. The second contribution is to distinguish the source of affluence. An economy can become affluent through technical change that enhances labour productivity and/or the productivity of non-human resources. As we show, the response of deadweight loss to increasing affluence depends in important ways on the source of the affluence. To our knowledge this is the first study to make this distinction. Finally, since our approach allows us to state all results in terms of parameters that are potentially estimable, it provides a different framework for empirical work.

This study also builds on a growing body of literature which shows that consumption referencing has important economic implications. A number of theoretical studies have found that preferences over relative consumption may provide an explanation for the Easterlin paradox (see, for example, Hirsch (1976), Sen (1983), Frank (1985, 2000), Hopkins and Kornienko (2004)). In addition, there is a great deal of empirical support for the proposition that relative consumption is an important determinant of well-being and economic decision making. Fliessbach et al. (2007) use magnetic resonance imaging to show that regions of the brain which process reward (the ventral striatum) are less active when an individual's payoff is low relative to that of others, providing physical evidence that social context directly impacts how we perceive and value reward. Strong empirical evidence suggests that relative income or wealth is important for individual utility (see Clark et al. (2008) for a review of this literature). Controlling for own income, one's subjective well-being is inversely related to the income of one's neighbors (Luttmer (2005), Barrington-Leigh and Helliwell (2008)), but increasing in the income of one's socio-economic group relative to the mean in their geographic area (Dynan and Ravina, 2007). Relative consumption offers a causal link between the wealth or income of ones' neighbors, which is private information, and ones' utility.

The remainder of the paper is laid out as follows. In Section 2 we present the model. In Section 3 we focus on interior equilibria of the model and in Section 4 on the corner equilibria. In Section 5 we look at a special case in which preferences for all three goods exhibit constant Arrow-Pratt curvature. Imposing this structure on utility allows us to talk about the behavior of deadweight loss more generally, rather than deriving conditions for specific points. A brief discussion and concluding remarks are presented in Section 6.

2 The General Model

There are three goods in our model: leisure, a standard consumption good, and a pure Veblen good. Veblen wrote about both the conspicuous consumption of leisure and of goods. In our model, individuals care about relative consumption of the pure Veblen good, but not relative consumption of leisure. This is consistent with Veblen's prediction that conspicuous consumption would entirely displace conspicuous leisure (Veblen, 1899, pp.53–57). It is also consistent with recent empirical evidence that individuals care only about absolute consumption of leisure, but relative consumption of visible goods (Carlsson et al., 2007). Further evidence suggests that leisure is crowded out as individuals attempt to increase their relative consumption of goods, both in recent years (Bowles and Park, 2005; Neumark and Postlewaite, 1998), and historically (Huberman and Minns, 2007).

2.1 Assumptions

The economy consists of a large number of identical individuals, with preferences represented by the additively separable utility function

$$U(x_i, y_i, v_i) = F(x_i) + G(y_i) + V(v_i, v), \tag{1}$$

where x_i is individual i's consumption of leisure, y_i her consumption of a standard good (i.e. a composite commodity), and v_i her consumption of a pure Veblen good. The quantity v is the mean consumption of the pure Veblen good in the economy. $F(x_i)$ is the contribution of leisure to individual i's utility, $G(y_i)$ the contribution of the standard good, and $V(v_i, v)$ the contribution of the pure Veblen good.

We adopt the standard increasingness and concavity assumptions regarding the utility generated by private consumption of these three goods.

Assumption 1. *increasingness and concavity*

$$\begin{aligned} F'(x_i) > 0, &\quad F''(x_i) < 0. \\ G'(y_i) > 0, &\quad G''(y_i) < 0. \\ V_1(v_i, v) > 0, &\quad V_{11}(v_i, v) < 0. \end{aligned}$$

In addition, we assume that the standard good and leisure are essential.

Assumption 2. *standard good and leisure essential*

$$\lim_{x_i \to 0} F'(x_i) = \infty, \qquad \lim_{y_i \to 0} G'(y_i) = \infty.$$

In the spirit of Veblen and the literature on consumption referencing, we assume that the individual's utility from the Veblen good is a decreasing function of average consumption of the Veblen good in the economy.

Assumption 3. *consumption referencing*

$$V_2(v_i, v) < 0.$$

Imagine an initial situation in which everyone is consuming the same amount of the Veblen good. It will then be the case that $v_i = v$ for all individuals. Now suppose that everyone's consumption of the Veblen good changes by the same amount, Δv. No one's relative consumption of the Veblen good has changed, and accordingly, if the Veblen good is *pure*, no one's utility from the Veblen good will have changed. This will be the case if $V_1(v, v) = |V_2(v, v)|$, or if $V_1(v, v) + V_2(v, v) = 0$. Accordingly, we adopt the following assumption:

Assumption 4. *a pure Veblen good*

$$V_1(v, v) + V_2(v, v) = 0.$$

Each individual is endowed with one unit of time, which is allocated to leisure and work, and one unit of a non-human resource. The technology for producing the standard and Veblen goods is linear in both the quantity of labor and the quantity of the non-human resource, and thus is completely described by two exogenous productivity parameters, $a > 0$ and $w > 0$. Specifically, r units of the non-human resource produce ar units of either the Veblen good or the standard good, and l units of labor produce wl units of either the Veblen good or the standard good. Notice that a is the productivity of the non-human resource[2],

[2]For simplicity, we will interpret a as the productivity of the non-human resource. However, given the

and w is the productivity of the labor or human resource.

All markets are competitive, so the general equilibrium prices of labor and the non-human resource are w and a, while the equilibrium prices of the standard and Veblen goods are unity.

2.2 The Representative Individual's Choice Problem

Since she is endowed with one unit of the human resource (time) and one unit of the non-human resource, in general equilibrium the representative individual's *productive potential* or full income—to use Becker's terminology—is $w + a$. Therefore, the individual's budget constraint is

$$wx_i + y_i + v_i \leq w + a$$

The representative individual's choice problem is then

$$\begin{aligned} \max_{x_i,y_i,v_i} \quad & U(x_i, y_i, v_i) = F(x_i) + G(y_i) + V(v_i, v) \\ \text{subject to} \quad & wx_i + y_i + v_i \leq w + a, \\ & x_i \leq 1, \quad v_i \geq 0 \end{aligned}$$

We have not included non-negativity constraints for leisure or the standard good in the formulation of the choice problem since optimal quantities of leisure and the standard good will be strictly positive, because they are essential.

There are four sorts of equilibrium corresponding to this choice problem, depending on whether the solution values of x_i and v_i are positive or 0. For the moment we focus on just

linearity of production in the model we could just as easily talk about a as a composite parameter that captures both the productivity and quantity of the resource. To see this let q denote quantity of the resource and b its productivity. Then aggregate productive potential is just $a \equiv qb$. So we can interpret comparative static results wrt a in either way.

one of them, the case in which the solution values for $x_i < 1$ and $v_i > 0$ (addressing the remaining three in Section 4.) In doing so we are implicitly assuming that labor productivity, w, is large enough so that the individual does not allocate all of her time to leisure, and that productive potential, $w + a$, is large enough so that she spends a positive amount of the Veblen good.

2.3 The Interior Equilibrium of the Model

As all agents are identical, in equilibrium everyone will choose the same quantities, $x_i = x^\star$, $y_i = y^\star$ and $v_i = v^\star$. Hence, the interior equilibrium of the model is characterized by the following conditions:

$$\frac{F'(x^\star)}{w} = G'(y^\star) = H'(v^\star) \quad (2)$$

$$wx^\star + y^\star + v^\star = w + a \quad (3)$$

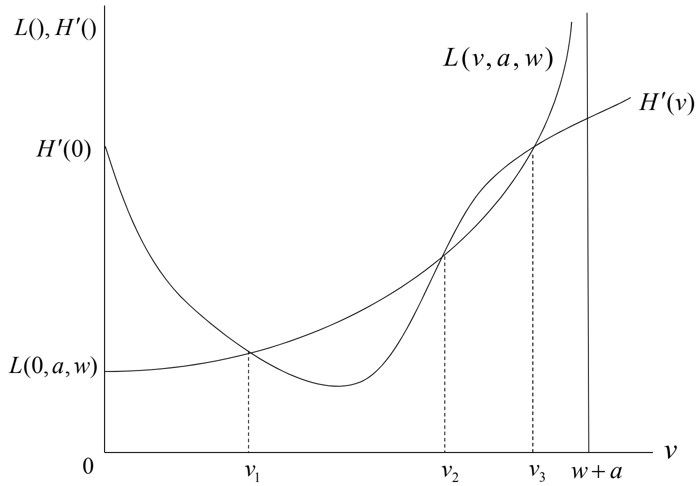
where $H'(v)$ is defined as the marginal utility of v when all individuals consume the same quantity of it. That is,

$$H'(v) \equiv V_1(v, v)$$

A graphic representation of the equilibrium will prove to be helpful. For this purpose, consider the following optimization problem. Fix the representative person's expenditure on the Veblen good, and optimally allocate the remainder of her resources to leisure and the standard good. Let $\hat{x}$ be the solution to this problem, where v is a parameter. Then $\hat{x}$ is characterized by:

$$\frac{F'(\hat{x})}{w} = G'(a + w(1 - \hat{x}) - v)$$

Figure 1: Interior Equilibrium



so $\hat{x}$ is a function of v, a and w: $\hat{x}(v, a, w)$. Then

$$L(v, a, w) \equiv \frac{F'(\hat{x}(v, a, w))}{w} = G'(a + w(1 - \hat{x}(v, a, w)) - v)$$

where $L(v, a, w)$ is defined as the marginal value of an additional unit of y or x, given that choices are optimal holding v fixed.

In Figure 1 we illustrate the graph of $L(v, a, w)$, across feasible values of v. The graph must satisfy certain obvious properties. It must intersect the vertical axis at a point above the horizontal axis ($L(0, a, w) > 0$), it must be upward sloping ($L_1(v, a, w) > 0$), and it must be asymptotic to the vertical line $v = w + a$ (because the standard good and leisure are essential.)

In the figure we also illustrate the graph of $H'(v)$. Because it is determined by the relative strength of two possibly opposing effects, the slope of this graph is not constrained—it can be positive, zero or negative. Recall that $H'(v)$ is just $V_1(v,v)$, so

$$H''(v) = \underbrace{V_{11}(v,v)}_{MPE} + \underbrace{V_{12}(v,v)}_{MSE}$$

For convenience, we call the first effect the *marginal private effect* (MPE) and the second the *marginal social effect* (MSE) of an infinitesimal increase in v. By assumption the MPE is negative, but MSE could be positive, and the two effects would then be opposed. If a positive MSE exceeds the absolute value of MPE, the graph of $H'(v)$ is upward sloping. In this case, as v increases the marginal value to each individual of the Veblen good increases as well.

Any point of intersection of the two graphs determines an equilibrium value of v. In Figure 1 there are three such values. However, the v_2 equilibrium is unstable. Roughly speaking, if we begin at v_2, a small change of v in either direction creates forces that lead to further adjustments in the same direction. For example, if quantity of the Veblen good is increased by a small amount, a situation is created in which the marginal value of resources devoted to the Veblen good exceeds their marginal value in either of the other uses, leading to a further increase in quantity of the Veblen good. In contrast, the v_1 and v_3 equilibria are stable. For our purposes, unstable equilibria are uninteresting, and are therefore ignored[3].

Before moving into a detailed analysis of the equilibrium we define two measures which will prove useful, Arrow-Pratt curvature and deadweight loss.

2.4 Arrow-Pratt Curvature (APC)

The Arrow-Pratt measure of the relative curvature of a function plays a pivotal role in our analysis. Arrow-Pratt is most often interpreted as a measure of relative risk aversion. More

[3] See A for the stability conditions. In the appendix we also show that, given assumptions 1–3, there always exists at least one stable equilibria

generally, it is a measure of the relative curvature of a function, and that is the interpretation that we use. Arrow-Pratt has the desirable property of being a cardinal measure, and unlike marginal utility, it is in principle an estimable quantity (see Arrow (1974)).

We use the relative curvature of three functions. The first two are $F(x)$ and $G(y)$, the contributions of leisure and the standard good to the individual's utility, and the third is $H(v)$, which is simply the indefinite integral of $H'(v)$. The relative curvature of $G(y)$, which we denote by $r_y(y)$, is defined as follows:

$$r_y(y) \equiv \frac{-yG''(y)}{G'(y)}$$

$r_x(x)$ and $r_v(v)$, the relative curvatures of $F(x)$ and $H(v)$, are similarly defined.

2.5 Deadweight Loss

Because, in equilibrium, all individuals choose the same quantity of the Veblen good the following derivative is of interest:

$$\frac{\partial V(v,v)}{\partial v} = V_1(v,v) + V_2(v,v) = 0$$

The second equality follows from Assumption 4—the pure Veblen good assumption. When everyone consumes the same amount of the pure Veblen good, the private marginal utility of the good ($V_1(v,v)$) is exactly offset by its social marginal disutility ($V_2(v,v)$). Hence, the equilibrium utility of the representative person is independent of the quantity consumed of the Veblen good. Regardless of how much or how little of the Veblen good is consumed in equilibrium, it contributes nothing to the well-being of the people who consume it.

It follows then that resources devoted to the pure Veblen good are simply squandered.

The share of resources devoted to the Veblen good is just

$$s_v = \frac{v^*}{a + w}$$

Notice that s_v is the proportionate deadweight loss in this economy.

With this background, it may be helpful to pose our research question yet again. Under what conditions on preferences do either or both of the following inequalities hold?

$$\frac{\partial s_v}{\partial a} > 0, \qquad \frac{\partial s_v}{\partial w} > 0$$

3 Interior Solution

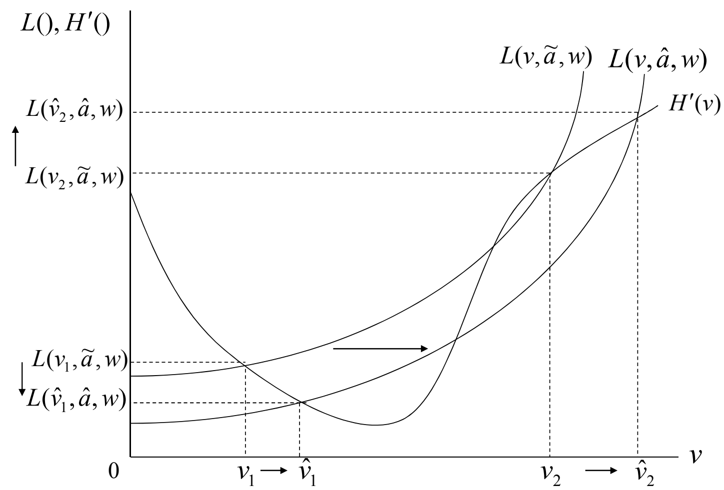
In Figure 2 we illustrate the comparative statics of this system[4]. Consider an increase in the productivity of the non-human resource, from $\widetilde{a}$ unit to $\hat{a}$. Clearly this leads to a downward shift in $L(v, a, w)$. The initial equilibrium may be at either v_1 or v_2. At either equilibrium, consumption of the Veblen good unambiguously increases as a increases.

If the initial equilibrium is v_1, where MSE does not dominate MPE, marginal utility of the standard good and leisure decrease in response to the increase in a. Therefore, it is clear that both $y^\star$ and $x^\star$ increase as a result of the increase in a, and equilibrium utility increases. In this case, without further assumptions, nothing can be said about the sign of $\partial s_v/\partial a$—proportionate deadweight loss may either increase or decrease as a increases.

If instead the initial equilibrium is v_2, where MSE dominates MPE, the marginal utility of leisure and standard consumption increase in response to the increase in a, implying that both $y^\star$ and $x^\star$ decrease in response to the increase in a. It follows that equilibrium utility will also decrease, since in equilibrium only the standard good and leisure create utility. In addition, since consumption of both leisure and the standard good decrease in response to

[4] A formal derivation of comparative statics is available in B.

Figure 2: Comparative Statics



an increase in the productivity of the non-human resource, it is the case that $\partial v^\star/\partial a > 1$. In other words, when MSE dominates MPE *more than all* of the added productive potential $(\widetilde{a} - \hat{a})$ is squandered on increased production of the pure Veblen good. Therefore, in this case, proportionate deadweight loss increases as a increases.

We can also comment fairly generally on the relationship between the representative individual's utility and a. Differentiating equilibrium utility, $U(x^\star, y^\star, v^\star)$, with respect to a, we get:

$$\frac{dU(x^\star, y^\star, v^\star)}{da} = F'(x^\star)\frac{dx^\star}{da} + G'(y^\star)\frac{dy^\star}{da} + V_1(v^\star, v^\star)\frac{dv^\star}{da} + V_2(v^\star, v^\star)\frac{dv^\star}{da}$$

From equilibrium condition (1.2) we know $F'(x^\star)/w = G'(y^\star) = V_1(v^\star, v^\star)$, and the budget constraint implies that $wdx^\star/da + dy^\star/da + dv^\star/da = 1$. Using these equalities the marginal change in equilibrium utility with respect to a can be written as:

$$\frac{dU(x^\star, y^\star, v^\star)}{da} = V_1(v^\star, v^\star) + V_2(v^\star, v^\star)\frac{dv^\star}{da}$$

The change in equilibrium utility that results from an increase in a is the sum of the change in utility due to the change in private consumption of the Veblen good, $V_1(v^\star, v^\star)$, and the change in utility due to a change in social consumption of the Veblen good, $V_2(v^\star, v^\star)dv^\star/da$. Since $V_1(v^\star, v^\star) = |V_2(v^\star, v^\star)|$, we see that

$$\frac{dU(x^\star, y^\star, v^\star)}{da} = \left(1 - \frac{dv^*}{da}\right) V_1(v^\star, v^\star)$$

So the sign of $dU(x^\star, y^\star, v^\star)/da$ is determined by the magnitude of $dv^\star/da$ about unity, which depends on the sign of $H''(v)$. When the MSE does not dominates the MPE, consumption of v increases by less than the full increase in a. Therefore, utility increases as a result of greater consumption of y and x. Conversely, if the MSE dominates the MPE, consumption of v increases by more than the full increase in a, crowding out consumption of y and x. As a result, utility decreases with a.

Results are summarized in Proposition 1.

Proposition 1 (Comparative Statics for a)**.**
i) If at the equilibrium $H''(v) < 0$ (MSE is less than MPE), then

$$\frac{dx^\star}{da} > 0, \qquad \frac{dy^\star}{da} > 0, \qquad \frac{dv^\star}{da} > 0 \qquad \frac{dU}{da} > 0 \qquad \frac{ds_v}{da} \gtreqless 0.$$

ii) if at the equilibrium $H''(v) > 0$ (MSE is greater than MPE), then

$$\frac{dx^\star}{da} < 0, \qquad \frac{dy^\star}{da} < 0, \qquad \frac{dv^\star}{da} > 1 \qquad \frac{dU}{da} < 0 \qquad \frac{ds_v}{da} > 0.$$

iii) if at the equilibrium $H''(v) = 0$ (MSE is equal to MPE), then

$$\frac{dx^\star}{da} = 0, \qquad \frac{dy^\star}{da} = 0, \qquad \frac{dv^\star}{da} = 1 \qquad \frac{dU}{da} = 0 \qquad \frac{ds_v}{da} > 0.$$

Proof. *See C.1 for proof*

We can clarify the behavior of dead-weight-loss when $H''(v) < 0$. This is captured in Proposition 2.

Proposition 2 (Comparative Statics for a Continued)**.** *Consider an equilibrium where $H''(v) < 0$ (MSE is less than MPE), and define: $s_y = y^\star/(w + a)$, $s_{wx} = wx^\star/(w + a)$ and*

$$\bar{r}_v \equiv \frac{(s_y + s_{wx}) r_x(x^\star) r_y(y^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

i) if $r_v(v^\star) = \bar{r}_v$; then $ds_v/da = 0$.

ii) if $r_v(v^\star) < \bar{r}_v$; then $ds_v/da > 0$.

iii) if $r_v(v^\star) > \bar{r}_v$; then $ds_v/da < 0$.

Proof. *See C.2 for proof*

Notice the sign of ds_v/da depends on the value of $r_v(v^\star)$ relative to $r_x(x^\star)$ and $r_y(y^\star)$. Proposition 2 allows us to draw some interesting conclusions. First, as a increases without bound, implying that s_{wx} is approaching zero, s_v will increase with a if and only if $r_v(v^\star) < r_y(y^\star)$. Second, if $r_x(x^\star) = r_y(y^\star)$ then, again, s_v will increase with a if and only if $r_v(v^\star) < r_y(y^\star)$. Third, because $r_x(x^\star)$ and $r_y(y^\star)$ are strictly positive, there always exists a feasible $\bar{r}_v > 0$. Finally, a sufficient condition for increasing deadweight loss as resource productivity increases is that $r_v(v^\star) < \min\{r_x(x^\star), r_y(y^\star)\}$. If, for any equilibrium defined by the triplet $(x^\star, y^\star, v^\star)$, $r_v(v^\star) < \min\{r_x(x^\star), r_y(y^\star)\}$ then deadweight loss is strictly increasing in resource productivity.

An increase in labor productivity, say from $\tilde{w}$ to $\hat{w}$, also generates a downward shift in the locus $L(v, w, a)$, as in Figure 2. At the initial equilibrium v_1 increasing w results in a new equilibrium similar to $\hat{v}_1$. At the new equilibrium we can conclude that the consumption of y has increased, as $L(v_1, a, \tilde{w}) > L(\hat{v}_1, a, \hat{w})$, but the change in equilibrium x is ambiguous. Starting at the initial equilibrium v_2 increasing w will result in a new equilibrium similar to $\hat{v}_2$. At the new equilibrium we can conclude that the consumption of y and x unambiguously decrease.[5]

We can also draw some conclusions about equilibrium utility. Differentiating equilibrium utility, $U(x^\star, y^\star, v^\star)$, with respect to w, and noting that $G'(y^\star) = F'(x^\star)/w = V_1(v^\star, v^\star)$, yields:

$$\frac{dU(x^\star, y^\star, v^\star)}{dw} = \left(1 - x^\star - \frac{dv^\star}{dw}\right) V_1(v^\star, v^\star) \gtreqless 0$$

$1 - x^\star$ is the incremental change in income resulting from the change in productivity. If $v^\star$ changes by an amount greater than $1 - x^\star$ utility and labor productivity will be inversely related. To illustrate, consider a simplified example. Suppose $H''(v^\star) = 0$, so the change in $v^\star$ is given by[6]

$$\frac{dv^\star}{dw} = 1 - \left(1 + \frac{1}{r_x(x^\star)}\right) x^\star \geq 1 - x^\star \tag{4}$$

The extent to which $v^\star$ increases, and utility decreases, when w increases depends on the relative curvature of $F(x)$[7]. Because $H''(v^\star) = 0$ any increase in consumption income,

[5] At $\hat{v}_2$ $L(v_1, a, \tilde{w}) < L(\hat{v}_1, a, \hat{w})$ therefore it must be the case that y has decreased. We can unambiguously sign dx/dw at v_2 as follows. When w increases, holding x constant, $F'(x^\star)/w$ decreases. If x increases or remains constant then $F'(x)/w < G'(a + \hat{w}(1 - x) - \hat{v}_2)$. Therefore it must be the case that x decreases.

[6] $H''(v^\star) = 0$ captures the case in which indivdiuals care about the difference between their own consumption of the Veblen good and the mean consumption of the Veblen good, that is $V(v_i, v) \equiv V(v_i - v)$.

[7] In B we provide a full derivation of $dv^\star/dw$, which, when $H''(v) = 0$, reduces to:

$$\frac{dv^\star}{dw} = 1 - x^\star - \frac{wG'(y^\star)}{F''(x^\star)}$$

$(1-x)w+a$, is fully allocated to v, and the marginal utility of consumption, $G'(y^\star) = H'(v^\star)$, remains constant. Increasing w makes leisure expensive relative to consumption, and $x^\star$ declines, increasing $(1-x)w+a$. The benefits of less leisure—greater income—are dissipated in consumption of the Veblen good. $r_x(x)$ provides us with a measure of how much x is sacrificed. If $r_x(x)$ is less than one, $F'(x)$ increases slowly when x is decreased, and large amounts of leisure are sacrificed. When $r_x(x)$ is greater than one, $F'(x)$ increases rapidly when x is decreased, and relatively small amounts of leisure are sacrificed. Therefore $H''(v) = 0$ is a sufficient, but not necessary condition for utility to be non-increasing in productivity.

Proposition 3 (Comparative Statics for w)**.**

i) If at the equilibrium $H''(v) < 0$ (MSE is less than MPE), then

$$\frac{dx^\star}{dw} \gtreqqless 0, \qquad \frac{dy^\star}{dw} > 0, \qquad \frac{dv^\star}{dw} > 0, \qquad \frac{dU}{dw} \gtreqqless 0 \qquad \frac{ds_v}{dw} \gtreqqless 0.$$

ii) if at the equilibrium $H''(v) > 0$ (MSE is greater than MPE), then

$$\frac{dx^\star}{dw} < 0, \qquad \frac{dy^\star}{dw} < 0, \qquad \frac{dv^\star}{dw} > 0, \qquad \frac{dU}{dw} < 0 \qquad \frac{ds_{sv}}{dw} > 0.$$

iii) if at the equilibrium $H''(v) = 0$ (MSE is equal to MPE), then

$$\frac{dx^\star}{dw} < 0, \qquad \frac{dy^\star}{dw} = 0, \qquad \frac{dv^\star}{dw} > 0, \qquad \frac{dU}{dw} < 0 \qquad \frac{ds_v}{dw} > 0.$$

Proof. *See C.3 for proof*

Again, clarification is needed for the case where $H''(v) < 0$, which will again depend on the relative curvature of the respective components to utility. This is clarified in Proposition

substituting $F'(x^\star) = wG'(y^\star)$ and multiplying the numerator and denominator by $x^\star$, resulting in Equation 4.

4.

Proposition 4 (Comparative Statics for w Continued)**.** *Consider an equilibrium where $H''(v) < 0$ (MSE is less than MPE), and define: $s_y = y^\star/(w+a)$, $s_{wx} = wx^\star/(w+a)$ and*

$$\hat{r}_v \equiv \frac{x^\star(1 - s_y - s_{wx})(1 - r_x(x^\star))r_y(y^\star) + (s_y + s_{wx})r_y(y^\star)r_x(x^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

i) if $r_v(v^\star) = \hat{r}_v$; then $ds_v/dw = 0$

ii) if $r_v(v^\star) < \hat{r}_v$; then $ds_v/dw > 0$

iii) if $r_v(v^\star) > \hat{r}_v$; then $ds_v/dw < 0$

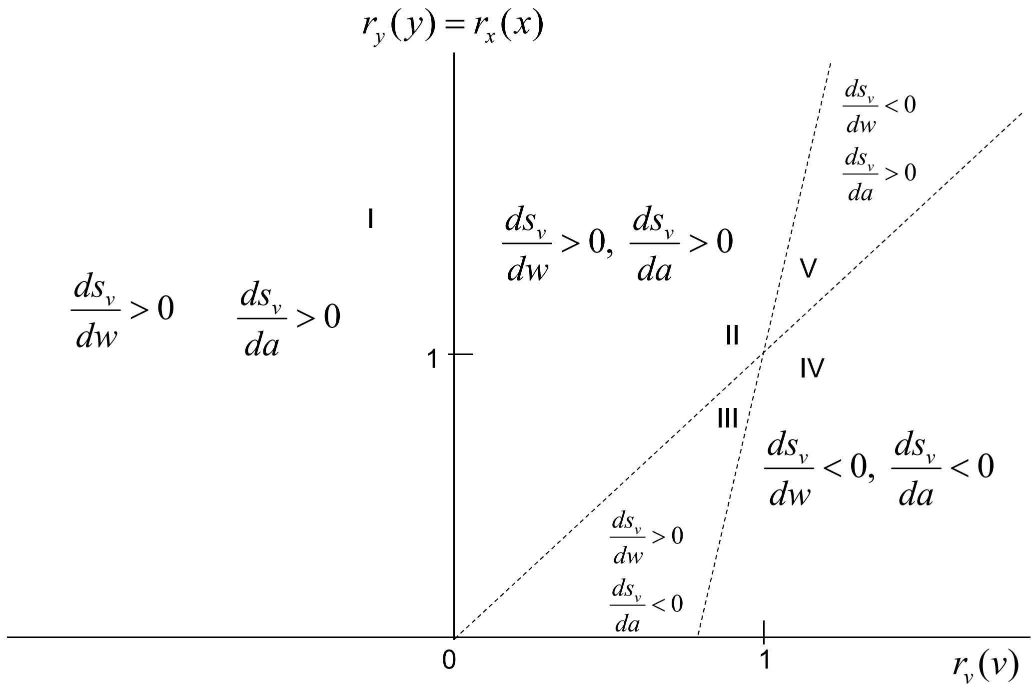
Proof. *See C.4 for proof*

There are several things we take away from Proposition 4. When $r_x(x^\star) < 1$ then $\bar{r}_v < \hat{r}_v$. Therefore if $r_x(x^\star) < 1$ then $r_v(v^\star) < \bar{r}_v$ is sufficient to conclude that deadweight loss is increasing in a and in w. If $r_x(x^\star) > 1$ then $r_v(v^\star) < \hat{r}_v$ is sufficient to conclude that deadweight loss is increasing in a and in w. For any $r_x(x^\star) \neq 1$ there exists a pair of APC values for y and v such that deadweight loss is increasing in one form of affluence but decreasing in the other.

The comparative statics outlined in propositions 1 to 4 are very revealing. When MSE dominates MPE, as the economy grows more affluent through increases in the productivity of either labor or the non-human resource, the share of total productive capacity devoted to the pure Veblen good increases at rate that is larger than the rate at which total productive capacity is increasing. Consequently, equilibrium utility declines as productive capacity increases. This illustrates just how destructive Veblen competition can potentially be.

The results are less clear in the case where MPE dominates MSE. We are able to clear up a great deal of the ambiguity by looking at the APC for each functional component

Figure 3: Arrow-Pratt Curvature and Deadweight Loss



Horizontal axis represents possible values of $r_v(v)$ along the real number line. Vertical axis represents possible values of $r_y(y)$ along the real number line. Dashed lines represents $\bar{r}_v$ and $\hat{r}_v$ evaluated at $r_y(y) = r_x(x)$.

of utility. In Figure 3 we depict a summary of our results. To simplify the diagram we assume that $r_y(y) = r_x(x)$.[8] All possible values of $r_v(v)$ are shown along the horizontal access. Notice that when MSE dominates MPE $r_v(v) < 0$. The vertical access represents possible values of $r_y(y)$. Given the curvature assumptions for $G()$ (see Assumption 1), $r_y(y)$ is restricted to strictly positive values. The dashed line with a unitary slope corresponds to $\bar{r}_v$, from Proposition 2, and the dashed line with a slope greater than 1 corresponds to $\hat{r}_v$, from Proposition 4.

As depicted, the behavior of deadweight loss given a change in resource productivity is fairly straightforward. Increasing resource productivity will lead to greater proportionate

[8] A similar diagram can be drawn without this assumption in three dimensional space.

deadweight loss whenever $r_v(v) < r_y(y)$, as in areas I, II and V. Increasing labour productivity will increase deadweight loss in the economy if utility is represented by APCs that fall in the ares I, II or III.

In the next section we turn our attention to the corner solutions of this problem. These lead to some interesting results regarding the relationship between deadweight loss and affluence in very poor versus wealthy economies.

4 Corner Solutions

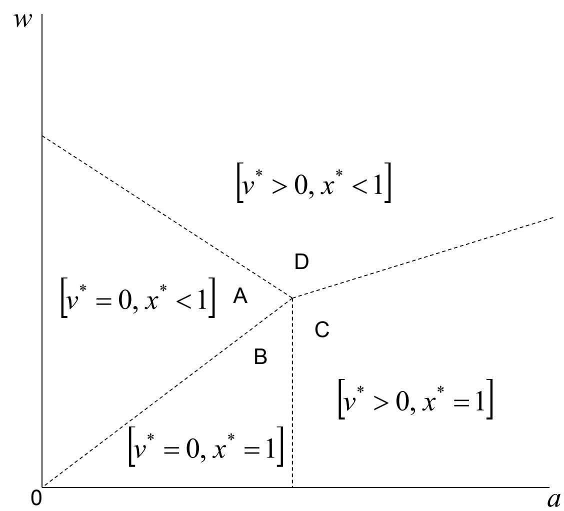
There are three non-interior equilibria that may arise. In the first of these scenarios, individuals allocate all of their income to consuming the standard good, and all of their time to leisure. Second, individuals may allocate a positive fraction of their time to paid labor, and consume only the standard good. Finally, individuals may allocate all of their time to leisure and consume a strictly positive quantity of the Veblen good. Which of these three cases an economy is in depends on the value of the productivity parameters, w and a. The three cases, and the interior equilibrium, are illustrated in (a, w) space in Figure 4.

In Case 1, illustrated by area B in Figure 4, individuals allocate all of their time to leisure ($x^\star = 1$), and none of their income to the Veben good ($v^\star = 0$). This requires that:

$$\frac{F'(1)}{w} \geq G'(a) \geq H'(0) \tag{5}$$

From these conditions it is clear that this corner equilibrium arises when the productivity of human and non-human resources are sufficiently low. We can solve for the maximum values of w and a that satisfy (5), formally defined by the set $B = \{(a, w) | 0 \leq a \leq a_B$ and $0 \leq$

Figure 4: Equilibrium Outcomes



$w \leq w_B(a)\}$ where:

$$w_B(a) = \frac{F'(1)}{G'(a)} \qquad a_B = G'^{-1}(H'(0))$$

Notice that the upper bound $w_B(a)$ is strictly increasing in a and and the upper bound a_B is a constant.

In this case, marginal increases in w do not contribute to utility or consumption since no one works, and marginal increases in a are entirely allocated to the standard good. Since non-human resources are devoted entirely to the standard good and human resources to leisure, there is no deadweight loss in this economy. In fact, the equilibrium is Pareto-efficient. The comparative statics for this case are detailed in Proposition 5.

Proposition 5 (Comparative Statics for Corner Solution: $v^\star = 0$ and $x^\star = 1$)**.**

$$\frac{dy^\star}{dw} = 0, \quad \frac{dx^\star}{dw} = 0, \quad \frac{dv^\star}{dw} = 0, \quad \frac{dU}{dw} = 0 \quad \frac{ds_v}{dw} = 0$$

$$\frac{dy^\star}{da} = 1, \quad \frac{dx^\star}{da} = 0, \quad \frac{dv^\star}{da} = 0, \quad \frac{dU}{da} > 0 \quad \frac{ds_v}{da} = 0$$

Proof. *See C.5 for proof*

In Case 2, illustrated by area A in Figure 4, individuals allocate some of their time to work and none of their income to the Veblen good. This requires that:

$$\frac{F'(x^\star)}{w} = G'(a + w(1 - x^\star)) \geq H'(0) \tag{6}$$

This scenario will only arise if w is sufficiently large such that individuals choose to work, and a is sufficiently small such that all consumption income is put towards the standard good. The productivity pair that sustains this equilibrium is formally defined by the set $A = \{(a, w) | 0 \leq a \leq a_B \text{ and } w_B(a) \leq w \leq w_A(a)\}$ where the upper bound $w_A(a)$ is related to a through the following equation:

$$w_A(a)(1 - x^\star) = G'(H'(0)) - a \tag{7}$$

Differentiating this relationship with respect to a yields:

$$\frac{dw_A(a)}{da} = \frac{1}{(1 - x^\star)} \left(w\frac{dx^\star}{da} - 1 \right) < 0 \tag{8}$$

so we know that $w_A(a)$ is decreasing in a.

In this case individuals allocate a portion of their time to paid labor, and all of their income the standard good, so again there is no deadweight loss, and the equilibrium is

Pareto-efficient. The comparative statics for this case are described in Proposition 6.

Proposition 6 (Comparative Statics for Corner Solution: $v^\star = 0$ and $x^\star < 1$)**.**

$$\frac{dy^\star}{dw} > 0, \quad \frac{dx^\star}{dw} \gtreqless 0, \quad \frac{dv^\star}{dw} = 0, \quad \frac{dU}{dw} > 0 \quad \frac{ds_v}{dw} = 0$$

$$\frac{dy^\star}{da} > 0, \quad \frac{dx^\star}{da} > 0, \quad \frac{dv^\star}{da} = 0, \quad \frac{dU}{da} > 0 \quad \frac{ds_v}{da} = 0$$

Proof. *See C.6 for proof*

This is analogous to the traditional consumption choice between leisure and a standard good. Increases in a have no effect on the relative price of x and y, and consumption of both goods is increased. When income increases due to an increase in w, and $r_y(y^\star) > 1$ then $dx^\star/dw \leq 0$ if and only if $w(1 - x^\star) \leq a/(r_y(y^\star) - 1)$. If $r_y(y^\star) \leq 1$, then it is always the case that $dx^\star/dw \leq 0$.

In Case 3, illustrated by area C in Figure 4, individuals allocate all of their time to leisure ($x^\star = 1$), and some of their income to the Veblen good ($v^\star > 0$). This requires that

$$\begin{aligned} \frac{F'(1)}{w} &\geq G'(y^\star) = H'(v^\star) \\ y^\star + v^\star &= a \end{aligned}$$

The productivity pair that sustains this equilibrium is formally defined by the set $C = \{(a, w)| a_B < a < \infty \text{ and } 0 \leq w \leq w_C(a)\}$ where the upper bound $w_C(a)$ is related to a through the following equation:

$$w_C(a) = \frac{F'(1)}{G'(a - v)} \tag{9}$$

Differentiating this upper bound with respect to a we find that:

$$\frac{dw_C(a)}{da} = -\frac{F'(1)}{(G'(a-v))^2} G''(a-v^\star)\left(1-\frac{dv^\star}{da}\right) \tag{10}$$

Therefore, $w_C(a)$ is increasing in a, as drawn in Figure 4 if $dv^\star/da < 1$. As we will show below, this is only the case if $H''(v^\star) > 0$.

The comparative statics with respect to w are identical to those in Case 1. Those for a can be examined in two parts, and are detailed in Proposition (7).

Proposition 7 (Comparative Statics for Corner Solution: $v^\star > 0$ and $x^\star = 1$)**.**
i) When $H''(v^\star) < 0$ (MSE is less than MPE), then

$$\frac{dy^\star}{da} > 0, \quad \frac{dx^\star}{da} = 0, \quad \frac{dv^\star}{da} > 0, \quad \frac{dU}{da} > 0 \quad \frac{ds_v}{da} \gtreqless 0$$

ii) When $H''(v^\star) \geq 0$ (MSE is greater than MPE), then

$$\frac{dy^\star}{da} \leq 0, \quad \frac{dx^\star}{da} = 0, \quad \frac{dv^\star}{da} \geq 1, \quad \frac{dU}{da} \leq 0 \quad \frac{ds_v}{da} \geq 0$$

Proof. *See C.7 for proof*

This corner solution is analogous to the simpler model which does not incorporate a labor choice. The behavior of deadweight loss when $H''(v^\star) < 0$ will depend on the relative curvature of v compared to the relative curvature of y. Specifically, $ds_v/da > 0$ if and only if $r_v < r_y$. As in the interior case, $H''(v^\star) \geq 0$ is more detrimental, as an ever-increasing proportion of resources is allocated to the consumption v.

The four cases depicted in Figure 4 clearly highlight one way in which affluence and the consumption of the Veblen good are related. In the poor economy, depicted in Case B, there is no deadweight loss and the equilibrium is Pareto efficient. In Case A we see a moderately advanced economy in labor productivity but poor in non-labor resources. Again, there is no deadweight loss and the equilibrium is Pareto-efficient. Once the productivity of resources exceeds a threshold, which we call a_B, a positive quantity of the Veblen good is consumed in equilibrium. In Case C we depict the leisure economy, in which no-one works, and a positive quantity of the Veblen good is consumed. Finally, Case D depicts the affluent economy, with both high resource productivity and high labour productivity. In this case also a positive quantity of the Veblen good is consumed.

5 Constant Arrow-Pratt Curvature

In this section we consider a family of utility functions characterized by constant Arrow-Pratt curvature (CAPC). Imposing this structure on utility allows us to talk about the behavior of ds_v/dw and ds_v/da, when MPE dominates MSE, more generally. Rather than deriving the conditions for specific points, as in propositions 2 and 4, we can discuss the behavior of deadweight loss for all levels of w and a conditional only on CAPC parameters.

With CAPC utility, the representative individual's choice problem is specified as follows:

$$\begin{aligned}
\max_{x_i,y_i,v_i} \quad & U(x_i,y_i,v_i) = \frac{x_i^{1-\alpha}}{1-\alpha} + \frac{y_i^{1-\beta}}{1-\beta} + \frac{v_i^{1-\gamma} - \bar{v}^{1-\gamma}}{1-\gamma} \\
\text{subject to} \quad & wx_i + y_i + v_i \leq w + a, \\
& v_i \geq 0, \quad x_i \leq 1
\end{aligned}$$

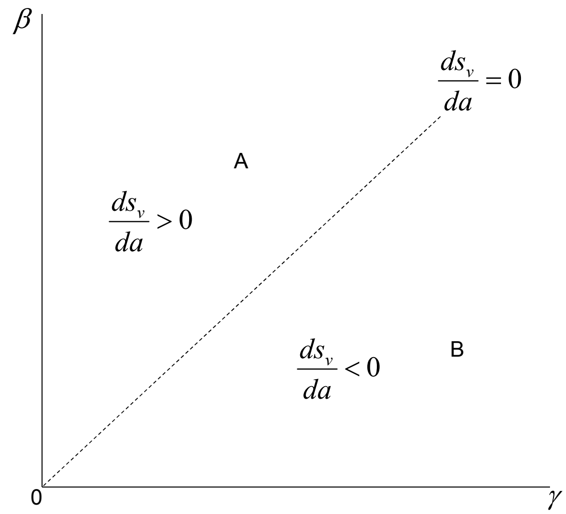
Where α, β and γ are constants:

$$r_x(x) = \alpha \qquad r_y(y) = \beta \qquad r_v(v) = \gamma \qquad \forall x, y, z \geq 0$$

Under this specification,

$$V_1(v_i, v) + V_2(v_i, v) = v^{-\gamma} - \bar{v}^{-\gamma}$$

Figure 5: Deadweight Loss and Resource Productivity



Derivatives evaluated at $w = 0$.

When $v_i = v$ the right hand side of this expression is equal to 0, so the pure Veblen good assumption (Assumption 4) is satisfied. Further, the change in the marginal utility of v, when the consumption of all individuals changes equally, is given by:

$$V_{11}(v_i, v) + V_{12}(v_i, v) = -\gamma v^{-(\gamma+1)} \leq 0$$

So MPE always dominates MSE.

In equilibrium, where $\bar{v} = v^{\star}$, consumers behave as if they are maximizing the utility function where $V(v, v)$ is specified by:

$$H(v) = \frac{v^{1-\gamma}}{1-\gamma}$$

To simplify, the analysis is restricted to two cases, $(w = 0, a > 0)$ and $(w > 0, a = 0)$.

Notice that when $w = 0$ no time will be allocated to labour. The marginal utility of the Veblen good, $v^{-\gamma}$, approaches infinity as v tends to zero, therefore $(w = 0, a > 0)$ is equivalent to the corner solution described by Proposition 7: $x = 1$, $v > 0$ and $x > 0$. It follows that $ds_v/da > 0$ if and only if $\beta > \gamma$. In Figure 5 we depict the behavior of deadweight loss in (γ, β) space. Intuitively, when $\beta > \gamma$ the marginal utility from consuming y falls rapidly relative to the marginal utility from consuming v and income is therefore allocated in increasingly greater proportions to consuming v.

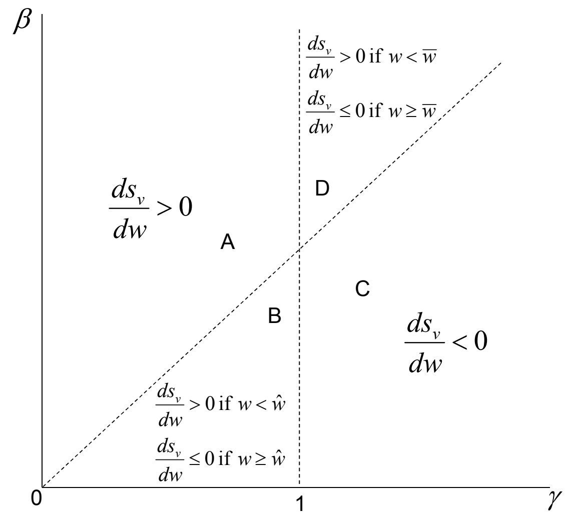
The second case, $(w > 0, a = 0)$, is more complicated. Now there is a strictly positive allocation of time to labour. Equilibrium is described by the following equations:

$$y^\star = w^{\frac{1}{\beta}} x^{\star\frac{\alpha}{\beta}} \qquad v^\star = w^{\frac{1}{\gamma}} x^{\star\frac{\alpha}{\gamma}} \qquad w = wx^\star + w^{\frac{1}{\beta}} x^{\star\frac{\alpha}{\beta}} + w^{\frac{1}{\gamma}} x^{\star\frac{\alpha}{\gamma}}$$

The behavior of deadweight loss hinges on two conditions: whether γ is greater than or less than unity, and the magnitude of γ relative to β. Interestingly, the qualitative results are independent of α. The magnitude of γ about unity determines the tradeoff between consumption and leisure, and the relative values of γ and β determine the tradeoff between consumption goods. First consider $\gamma < 1$, implying that, for some sufficiently large w, increasing labour productivity decreases leisure. If $\beta > \gamma$ then as w increases resources are allocated to consumption of the Veblen good in increasing proportions. Therefore, deadweight loss is strictly increasing. In the case where $\beta < \gamma$, at low levels for w resources are allocated to the Veblen good at an increasing rate, however for high levels of w this trend is reversed, and income is allocated at an increasing rate to the standard good as w increases. These results are formally stated in Proposition 8.

Proposition 8 (Comparative Statics when $\gamma < 1$)**.** *If $\gamma < 1$ the following comparative statics hold:*

Figure 6: Deadweight Loss and Labour Productivity



Derivatives evaluated at $a = 0$.

i) If $\beta < \gamma$ then $\frac{dx}{dw} < 0$ for all $w > 0$ and there exists a $\hat{w} \in [0, \infty)$ such that $\frac{ds_v}{dw} > 0$ when $w < \hat{w}$ and $\frac{ds_v}{dw} \leq 0$ when $w \geq \hat{w}$.

ii) If $\gamma \leq \beta < 1$ then $\frac{dx}{dw} < 0$ and $\frac{ds_v}{dw} > 0$ for all $w > 0$.

iii) If $\gamma < 1 \leq \beta$ then there exists a $\tilde{w} \in (0, \infty)$ such that $\frac{dx}{dw} > 0$ when $w < \tilde{w}$, and $\frac{dx}{dw} \leq 0$ when $w \geq \tilde{w}$ and $\frac{ds_v}{dw} > 0$ for all $w > 0$.

Proof. *See C.8 for proof*

Consider the case of $\gamma > 1$. If $\beta < \gamma$, deadweight loss is always decreasing in w; incremental resources are allocated in larger proportions to consuming y and x. In the case where $\beta > \gamma$, at low levels of w resources are allocated to the Veblen good at an increasing rate, however for high levels of w this trend is reversed, as resources are allocated at an increasing rate to leisure. These results are formally stated in Proposition 9.

Proposition 9 (Comparative Statics when $\gamma > 1$). *If $\gamma > 1$ the following comparative statics hold:*

i) If $1 < \beta \leq \gamma$ then $\frac{dx}{dw} < 0$ and $\frac{ds_v}{dw} < 0$ for all $w > 0$.

ii) If $\beta \leq 1 < \gamma$ then there exists a $\breve{w} \in (0, \infty)$ such that $\frac{dx}{dw} > 0$ when $w < \breve{w}$, and $\frac{dx}{dw} \leq 0$ when $w \geq \breve{w}$ and $\frac{ds_v}{dw} < 0$ for all $w > 0$.

iii) If $1 < \gamma \leq \beta$ then $\frac{dx}{dw} > 0$ and there exists a $\bar{w} \in [0, \infty)$ such that $\frac{ds_v}{dw} > 0$ when $w < \bar{w}$, and $\frac{ds_v}{dw} \leq 0$ when $w \geq \bar{w}$.

Proof. *See C.9 for proof*

Figure 6 provides a depiction of the deadweight loss behavior stated in Proposition 8 and Proposition 9. In Region A, where γ is less than 1 and less than β, deadweight is unambiguously increasing as labour productivity increases. In Region C, where $1 < \gamma$ and $\gamma < \beta$, consumption of v is crowded out by y, and deadweight loss is unambiguously decreasing with labour productivity. In Region B, where $\beta < \gamma < 1$, at low values of w s_v increases with affluence at the expense of leisure. However, as affluence through w grows, allocations of resource to y dominate and s_v decreases. Likewise in Region D, where $1 < \gamma < \beta$, at low values of w s_v increases with affluence at the expense of y. However, as affluence through w grows, allocations of resource to leisure dominate and s_v decreases.

6 Conclusion

In this paper we analyze a simple general equilibrium model in which individuals allocate their resources between a standard consumption good, leisure and a pure Veblen good. As utility is derived from the Veblen good only based on relative consumption, in equilibrium any resources devoted to the Veblen good are squandered. Using this model, we ask: Under what preference conditions will greater affluence, in the form of greater resource or human productivity, lead to an increase in the proportion of resources dedicated to the Veblen good? In the relatively general preference framework presented in Section 2, we derive a sufficient condition under which the Veblen good crowds out standard forms of consumption and leisure, resulting in an inverse relationship between affluence and utility. With additional structure on the model we are able to fully characterize the behavior of deadweight loss and utility as an economy becomes more affluent.

The findings of this paper suggest that estimating measures of relative curvature—$r_x(x)$, $r_y(y)$ and $r_v(v)$—should be of some empirical interest. Understanding these measures will guide policy by improving our understanding of consumption and deadweight loss as our economies continue to grow. Several studies have tested the prevalence of Veblen-type consumption by looking at the consumption of visible versus non-visible goods. Heffetz (2004, 2010) constructs and index of consumption visibility, based on the degree to which individuals notice differences between their own consumption and the consumption of others. Heffetz looks at the relationship between visibility and income elasticities, finding that income elasticity is considerably higher for visible goods (suggesting that $r_v(v) < r_y(y)$.) Further, a third of the difference in income elasticity between high-income and low-income individuals can be explained by the visibility of their consumption. Charles et al. (2010) find that controlling for the average income of ones reference group explains most of the differences in conspicuous expenditures between Blacks and Whites in the United States[9].

Consumption referencing offers an explanation for puzzling empirical relationships between income and well-being. If it is the case that the effects of consumption referencing are strong then there is a serious case to be made for rethinking the use aggregate measures of per-capita income and productivity as reflecting population well-being. Even in cases

[9]This conspicuous expenditure gap is nontrivial. Charles et al. (2010) estimate that Blacks spend about 35 percent more of their income on conspicuous goods than Whites.

when well-being monotonically increases with affluence, there may still be room for a Pareto improving re-allocation of resources, and the potential for improvement may be increasing as our economies continue to grow.

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A Conditions for Interior Stability

An interior equilibrium is stable if and only if the following conditions hold.

$$H''(v^\star) < 1 - F''(x^\star) \tag{11}$$

$$F''(x^\star)G''(y^\star) + H''(v^\star)\left[F''(x^\star) + w^2 G''(y^\star)\right] > 0 \tag{12}$$

Conditions (12) and (12) are derived below.

In equilibrium all individuals choose the same consumption bundles $(x^\star, y^\star, v^\star)$. Equilibrium consumption of the Veblen good will increase only if doing so will increase utility for each individual. Letting $\dot{x}$, $\dot{v}$ and $\dot{y}$ be the uniform instantaneous change of the consumption goods and leisure for all individuals:

$$\begin{aligned}\dot{x} &= F'(x) - wG'(y) \\ \dot{v} &= H'(v) - G'(y) \\ \dot{y} &= a + (1-x)w - y - v\end{aligned}$$

If the marginal utility of Veblen consumption exceeds that of consuming the standard good, all individuals increase their consumption of the Veblen good, $\dot{v} > 0$. Conversely, if consumption of the standard good yields a higher marginal utility, individuals reduce their consumption of the Veblen good, $\dot{v} < 0$. If an equilibrium is stable, small deviations in consumption will always tend back to the equilibrium. Performing a linear Taylor series approximation about the equilibrium allocation, $(x^\star, v^\star, y^\star)$ yields the system:

$$\begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} F''(x^\star) & 0 & -wG''(y^\star) \\ 0 & H''(v^\star) & -G''(y^\star) \\ -w & -1 & -1 \end{bmatrix} \begin{bmatrix} x - x^\star \\ v - v^\star \\ y - y^\star \end{bmatrix} \tag{13}$$

To ensure that a given equilibria is stable, the coefficient matrix should have eigenvalues with negative real parts (Dixit 1986). This will be the case if the trace and determinant are negative, which implies conditions (12) and (12).

Lemma 1 (Existence)**.** *Assumptions 1–3 (see main text) are sufficient to ensure the existence of at least one locally stable equilibrium.*

A.1 Proof [Existence]

This proof will proceed in two parts. First we show that an equilibrium where $H'' < G''$ is always stable. Next, we show that if no corner solution exists then at least one stable equilibrium exists. Finally, we show that if a corner solution exists, then it is stable.

Consider an equilibrium consumption bundle $(x^\star, v^\star, y^\star)$ which is at an interior equilibrium such that (referring to Figure 1 in text):

$$H'(v^\star) = L(v^\star, a, w) \quad \text{and} \quad H''(v^\star) < L_1(v^\star, a, w)$$

we know that $L(v, a, w)$ is an implicit function defined by:

$$\begin{aligned} L(v, a, w) &= G'(a + w(1 - x^\star) - v) \\ \frac{F'(x^\star)}{w} &= G'(a + w(1 - x^\star) - v) \end{aligned}$$

From this system of equations we can derive $L_1(v^\star, a, w) = -G''F''/(F'' + w^2G'')$. It follows that:

$$H''(v^\star) < \frac{-G''(y^\star)F''(x^\star)}{F''(x^\star) + w^2G''(y^\star)}$$

which is sufficient for Condition 2 to hold. It follows from this that:

$$\begin{aligned}\frac{-G''(y^\star)F''(x^\star)}{F''(x^\star) + w^2G''(y^\star)} &< -\frac{F''(x^\star)}{w}\\ H''(v^\star) &< -\frac{F''(x^\star)}{w}\end{aligned}$$

A stable interior equilibrium will always exist if $H'(0) > L(0, a, w)$. Assumption 2 states that when v tends to $w + a$, $L(v, a, w)$ tends to infinity and $H'(v)$ tends to a finite constant. As $H'(v)$ and $L(v, a, w)$ are continuous over $v \in (0, w+a)$, it follows that there exists a value $\tilde{v} < a + w$ where $H'(\tilde{v}) = L_1(\tilde{v}, a, w)$ and $H''(\tilde{v}) < L_1(\tilde{v}, a, w)$. Therefore, it follows that a stable equilibrium interior equilibrium exists.

If $H'(0) < L(0, a, w)$, a stable equilibrium always exists at $v^\star = 0$. □

B Comparative Static Equations

The total change in equilibrium consumption for a change in our exogenous variables can be captured by the following system of equations, written in matrix form:

$$\begin{bmatrix} F''(x^\star) + w^2G''(y^\star) & wG''(y^\star) \\ wG''(y^\star) & G''(y^\star) + H'(v^\star) \end{bmatrix} \begin{bmatrix} dx^\star \\ dv^\star \end{bmatrix} = \begin{bmatrix} G' + w(1-x^\star)G''(y^\star) & wG''(y^\star) \\ (1-x^\star)G''(y^\star) & G''(y^\star) \end{bmatrix} \begin{bmatrix} dw \\ da \end{bmatrix}$$

where $y^\star = (1 - x^\star)w + a - v^\star$. Given the stability conditions, we can sign the comparative statics of interest for the general Veblen model.

We can derive the comparative statics for a change in the exogenous endowment:

$$\frac{dv^\star}{da} = \frac{F''(x^\star)G''(y^\star)}{F''(x^\star)G''(y^\star) + H''(v^\star)\left[F''(x^\star) + w^2G''(y^\star)\right]} \geq 0 \tag{14}$$

$$\frac{dx^\star}{da} = \frac{wH''(v^\star)G''(y^\star)}{F''(x^\star)G''(y^\star) + H''(v^\star)\left[F''(x^\star) + w^2G''(y^\star)\right]} \gtreqless 0 \tag{15}$$

$$\frac{dy^\star}{da} = \frac{H''(v^\star)\left[F''(x^\star) + w(w-1)G''(y^\star)\right]}{F''(x^\star)G''(y^\star) + H''(v^\star)\left[F''(x^\star) + w^2G''(y^\star)\right]} \gtreqless 0 \tag{16}$$

An exogenous productivity change have a similar effect on consumption:

$$\frac{dv^\star}{dw} = \frac{G''(y^\star)\left[(1-x^\star)F''(x^\star) - wG'(y^\star)\right]}{F''(x^\star)G''(y^\star) + H''(v^\star)\left[F''(x^\star) + w^2G''(y^\star)\right]} \geq 0 \tag{17}$$

$$\frac{dx^\star}{dw} = \frac{G'(y^\star)G''(y^\star) + H''(v^\star)\left[G'(y^\star) + (1-x^\star)G''(y^\star)\right]}{F''(x^\star)G''(y^\star) + H''(v^\star)\left[F''(x^\star) + w^2G''(y^\star)\right]} \gtreqless 0 \tag{18}$$

$$\frac{dy^\star}{dw} = \frac{H''(v^\star)\left[(1-x^\star)F''(x^\star) + (1-x^\star)(w-1)wG''(y^\star) - wG'(y^\star)\right]}{F''(x^\star)G''(y^\star) + H''(v^\star)\left[F''(x^\star) + w^2G''(y^\star)\right]} \gtreqless 0 \tag{19}$$

B.1 Constant Arrow-Pratt Curvature

Comparative Statics:

$$\frac{dx}{dw} = \frac{x\left[(\beta-1)\gamma w^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}} + (\gamma-1)\beta w^{\frac{1}{\gamma}}x^{\frac{\alpha}{\gamma}}\right]}{w\left[xw\beta\gamma + \alpha\left[\gamma w^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}} + \beta w^{\frac{1}{\gamma}}x^{\frac{\alpha}{\gamma}}\right]\right]} \tag{20}$$

$$\frac{dy}{dw} = \frac{\gamma w^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}}\left[x + \alpha(1-x)\right]}{w\left[xw\beta\gamma + \alpha\left[\gamma w^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}} + \beta w^{\frac{1}{\gamma}}x^{\frac{\alpha}{\gamma}}\right]\right]} \tag{21}$$

$$\frac{dv}{dw} = \frac{\beta w^{\frac{1}{\gamma}}x^{\frac{\alpha}{\gamma}}\left[x + \alpha(1-x)\right]}{w\left[xw\beta\gamma + \alpha\left[\gamma w^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}} + \beta w^{\frac{1}{\gamma}}x^{\frac{\alpha}{\gamma}}\right]\right]} \tag{22}$$

$$\frac{d\frac{v}{w}}{w} = \frac{w^{\frac{1}{\gamma}-1}x^{\frac{\alpha}{\gamma}}\left[wx\beta(1-\gamma) + w^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}}\alpha(\beta-\gamma)\right]}{xw\beta\gamma + \alpha\left[\gamma w^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}} + \beta w^{\frac{1}{\gamma}}x^{\frac{\alpha}{\gamma}}\right]} \tag{23}$$

C Proof of Propositions

C.1 Proof [Proposition 1]

The signs of comparative statics for $x^\star$, $y^\star$ and $v^\star$ follow directly from Comparative Statics 14–16 The marginal change for utility follows directly as well, as it depends only on the value of $dv^\star/da$ relative to unity:

$$sign\left[\frac{dU(x^\star, y^\star, v^\star)}{da}\right] = sign\left[1 - \frac{dv^\star}{da}\right]$$

Finally, the sign of ds_v/da is determined by the value of $dv^\star/da$ relative to the initial value of s_v:

$$\frac{ds_v}{da} = \frac{1}{w+a}\left(\frac{dv^\star}{da} - s_v\right)$$

We know that $s_v < 1$, therefore, if $H'' \geq 0$ it follows from the comparative statics on $v^\star$ that $ds_v/da > 0$. However, the magnitude of $dv^\star/da$ relative to s_v is indeterminant for $H'' < 0$, and therefore the sign for ds_v/da is ambiguous. □

C.2 Proof [Proposition 2]

The sign of ds_v/da is determined by the value of $dv^\star/da$ relative to the initial value of s_v:

$$sign\left[\frac{ds_v}{da}\right] = sign\left[\frac{dv^\star}{da} - s_v\right]$$

We can rewrite Comparative Static (14) in terms of Arrow-Pratt curvature. To do this divide the numerator and denominator of the righthand side of (14) twice by $G'(y^\star) = H'(v^\star) = F'(x^\star)/w$. Then multiply each instance of $G''(y^\star)/G'(y^\star)$, $H''(v^\star)/H'(v^\star)$ and $F''(x^\star)/F'(x^\star)$, by $-y^\star/y^\star$, $-v^\star/v^\star$ or $-x^\star/x^\star$ respectively. The resulting derivative is written as:

$$\frac{dv^\star}{da} = \frac{v^\star r_x(x^\star) r_y(y^\star)}{v^\star r_x(x^\star) r_y(y^\star) + r_v(v^\star)(y^\star r_x(x^\star) + w x^\star r_y(y^\star))}$$

For $ds_v/da > 0$ we require that $dv^\star/da > s_v$, a condition which can be written as:

$$r_v(v^\star) < \frac{(s_y + s_{wx}) r_x(x^\star) r_y(y^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

For $ds_v/da = 0$ we require that $dv^\star/da = s_v$:

$$r_v(v^\star) = \frac{(s_y + s_{wx}) r_x(x^\star) r_y(y^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

For $ds_v/da < 0$ we require that $dv^\star/da < s_v$:

$$r_v(v^\star) > \frac{(s_y + s_{wx}) r_x(x^\star) r_y(y^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

□

C.3 Proof [Proposition 3]

The signs of comparative statics for $x^\star$, $y^\star$ and $v^\star$ follow directly from Comparative Statics (17)–(19). The marginal change for utility depends on the value of $dv^\star/dw$ relative to $1-x^\star$:

$$sign\left[\frac{dU(x^\star, y^\star, v^\star)}{dw}\right] = sign\left[1 - x^\star - \frac{dv^\star}{dw}\right]$$

Therefore when $H''(v^\star) \geq 0$ utility is strictly decreasing in w. As $H''(v^\star) \geq 0$ is sufficient but not necessary for $dv^\star/dw > 1 - x^\star$, $H''(v^\star) \leq 0$ the sign of dU/dw is ambiguous.

Finally, the sign of ds_v/da is determined by the value of $dv^\star/dw$ relative to the initial value of s_v:

$$\frac{ds_v}{dw} > 0 \Leftrightarrow \frac{dv^\star}{dw} > s_v$$

$dv^\star/dw > s_v$ can be written as:

$$r_x(x^\star) > \frac{((1 - s_v - s_y)r_v(v^\star) - x^\star)r_y(y^\star)}{(1 - x - s_v)r_y(y^\star) - s_y r_v(v^\star)}$$

from which it is straightforward to show that when $r_v(v^\star) \leq 0$ the right-hand-side of this condition is negative. As $r_x(x^\star)$ is strictly positive (this follows from the curvature restrictions placed on $F()$) the condition is strictly satisfied when $H''(v^\star) \geq 0$ (which implies $r_v(v^\star) < 0$). □

C.4 Proof [Proposition 4]

The sign of ds_v/dw is determined by the value of $dv^\star/dw$ relative to the initial value of s_v:

$$sign\left[\frac{ds_v}{dw}\right] = sign\left[\frac{dv^\star}{dw} - s_v\right]$$

We can rewrite Comparative Static (17) in terms of Arrow-Pratt curvature. To do this divide the numerator and denominator of the righthand side of (17) twice by $G'(y^\star) = H'(v^\star) = F'(x^\star)/w$. Then multiply each instance of $G''(y^\star)/G'(y^\star)$, $H''(v^\star)/H'(v^\star)$ and $F''(x^\star)/F'(x^\star)$, by $-y^\star/y^\star$, $-v^\star/v^\star$ or $-x^\star/x^\star$ respectively. The resulting derivative is written as:

$$\frac{dv^\star}{dw} = \frac{v^\star r_y(y^\star)((1-x^\star)r_x(x^\star) - x^\star)}{v^\star r_x(x^\star)r_y(y^\star) + r_v(v^\star)(y^\star r_x(x^\star) + wx^\star r_y(y^\star))}$$

For $ds_v/da > 0$ we require that $dv^\star/da > s_v$, a condition which can be written as:

$$\hat{r}_v(v^\star) < \frac{x^\star(1-s_y-s_{wx})(1-r_x(x^\star))r_y(y^\star) + (s_y+s_{wx})r_y(y^\star)r_x(x^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

For $ds_v/da = 0$ we require that $dv^\star/da = s_v$:

$$\hat{r}_v(v^\star) = \frac{x^\star(1-s_y-s_{wx})(1-r_x(x^\star))r_y(y^\star) + (s_y+s_{wx})r_y(y^\star)r_x(x^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

For $ds_v/da < 0$ we require that $dv^\star/da < s_v$:

$$\hat{r}_v(v^\star) > \frac{x^\star(1-s_y-s_{wx})(1-r_x(x^\star))r_y(y^\star) + (s_y+s_{wx})r_y(y^\star)r_x(x^\star)}{s_y r_x(x^\star) + s_{wx} r_y(y^\star)}$$

□

C.5 Proof [Proposition 5]

For $x^\star = 1$ and $v^\star = 0$ to be an equilibrium the following first order conditions must be satisfied:

$$\frac{F'(1)}{w} \geq G'(y^\star)$$
$$H'(v^\star) \leq G'(y^\star)$$
$$y^\star = a$$

It follows that marginal changes in w have no effect on consumption or utility. Further, all changes in a are allocated directly to the consumption of y. Therefore, utility is strictly increasing in a, and $s_v = 0$. □

C.6 Proof [Proposition 6]

For $x^\star < 1$ and $v^\star = 0$ to be an equilibrium the following first order conditions must be satisfied:

$$\frac{F'(x^\star)}{w} = G'(y^\star)$$
$$H'(v^\star) \leq G'(y^\star)$$
$$y^\star + wx^\star = a + w$$

That utility is strictly increasing in w and a follows from $v^\star = 0$. The effect of a productivity increase for $x^\star$ and $y^\star$ are given by:

$$\frac{dx^\star}{dw} = \frac{(1 - x^\star)wr_y(y^\star) - y^\star}{(y^\star r_x(x^\star) + wx^\star r_y(y^\star))} \gtreqless 0 \qquad \frac{dy^\star}{dw} = \frac{(1 - x^\star)yr_x(x^\star) - y^\star x^\star}{(y^\star r_x(x^\star) + wx^\star r_y(y^\star))} > 0$$

From inspection of the above comparative statics it follows that $dx^\star/dw > 0$ only of $r_y(y^\star) < y^\star/(1-x^\star)w$. □

C.7 Proof [Proposition 7]

For $x^\star = 1$ and $v^\star > 0$ to be an equilibrium the following first order conditions must be satisfied:

$$\frac{F'(x^\star)}{w} \geq G'(y^\star)$$
$$H'(v^\star) = G'(y^\star)$$
$$y^\star + v^\star = a$$

It follows that a marginal change in w will have no impact on the equilibrium. The comparative statics for a marginal change in a are given by:

$$\frac{dy^\star}{da} = \frac{y^\star r_v(v^\star)}{y^\star r_v(v^\star) + v^\star r_y(y^\star)} \qquad \frac{dv^\star}{da} = \frac{v^\star r_y(y^\star)}{y^\star r_v(v^\star) + v^\star r_y(y^\star)}$$

If $H'' < 0$, then $r_v(v^\star) > 0$, and the comparative statics in i) hold. For $ds_v/da > 0$ we require that $dv^\star/da > s_v$. Using the above value for $dv^\star/da$, we find that $ds_v/da > 0$ if and only if $r_v(v^\star) < r_y(y^\star)$. □

C.8 Proof [Proposition 8]

i) If $\beta < \gamma < 1$ then $\frac{dx}{dw} < 0$ for all $w > 0$ and there exists a $\hat{w} \in [0, \infty)$ such that $\frac{ds_v}{dw} > 0$ when $w < \hat{w}$ and $\frac{ds_v}{dw} \leq 0$ when $w \geq \hat{w}$.

The sign for $\frac{dx}{dw}$ follows directly from Comparative Static (20). From Comparative Static (23) we know that $d\frac{v}{w}/dw < 0$ if $(1-\gamma)\beta w x^\star + (\beta - \gamma)\alpha y^\star < 0$. The left-hand term is positive and the right-hand term is negative. At low levels of w, $wx^\star > y^\star$ and the full term is positive. As w increases, $wx^\star$ increases less rapidly than $y^\star$, until the magnitude of the right-hand term dominates, and $(1-\gamma)\beta wx^\star + (\beta-\gamma)\alpha y^\star < 0$. After this critical w has been reached, $wx^\star$ continues to increase at a slower rate than $y^\star$. Therefore, $d\frac{v}{w}/dw$ is always negative. □

ii) If $\gamma \leq \beta < 1$ then $\frac{dx}{dw} < 0$ and $\frac{ds_v}{dw} > 0$ for all $w > 0$.

The sign for $\frac{dx}{dw}$ follows directly from Comparative Static (20), the sign for $\frac{ds_v}{dw}$ follows directly from Comparative Static (23). □

iii) If $\gamma < 1 \leq \beta$ then there exists a $\tilde{w} \in (0,\infty)$ such that $\frac{dx}{dw} > 0$ when $w < \tilde{w}$, and $\frac{dx}{dw} \leq 0$ when $w \geq \tilde{w}$ and $\frac{ds_v}{dw} > 0$ for all $w > 0$.

The sign for $\frac{ds_v}{dw}$ follows directly from Comparative Static (23). From Comparative Static (20) we know that $dx/dw < 0$ if $(\beta-1)\gamma y^\star + (\gamma-1)\beta v^\star < 0$. The left-hand term is positive and the right-hand term is negative. At low levels of w, $y^\star > v^\star$ and this term is positive. As w increases, $v^\star$ increases more rapidly than $y^\star$, until the magnitude of the right-hand term dominates, and $(\beta-1)\gamma y^\star + (\gamma-1)\beta v^\star < 0$. After this critical w has been reached, $v^\star$ continues to increase at a greater rate than $y^\star$. Therefore, dx/dw is always negative. □

C.9 Proof [Proposition 9]

i) If $1 < \beta \leq \gamma$ then $\frac{dx}{dw} < 0$ and $\frac{ds_v}{dw} < 0$ for all $w > 0$.

The sign of $\frac{dx}{dw}$ follows directly from Comparative Static (20), the sign of $\frac{ds_v}{dw}$ follows directly from (23). □

ii) If $\beta \leq 1 < \gamma$ then there exists a $\breve{w} \in (0,\infty)$ such that $\frac{dx}{dw} > 0$ when $w < \breve{w}$, and $\frac{dx}{dw} \leq 0$ when $w \geq \breve{w}$ and $\frac{ds_v}{dw} < 0$ for all $w > 0$.

The sign of $\frac{ds_v}{dw}$ follows directly from (23). From Comparative Static (20) we know that $dx/dw > 0$ if $(\beta - 1)\gamma y^\star + (\gamma - 1)\beta v^\star > 0$. From the values of γ and β we know the left-hand term is negative and the right-hand term is positive. At low levels of w, $y^\star < v^\star$ and the sum of the two terms is positive. As w increases, $v^\star$ increases less rapidly than $y^\star$, until the magnitude of the left-hand term dominates, and $(\beta - 1)\gamma y^\star + (\gamma - 1)\beta v^\star > 0$. After this critical w has been reached, $v^\star$ continues to increase at a greater rate than $y^\star$. Therefore, dx/dw is always negative. □

iii) If $1 < \gamma \leq \beta$ then $\frac{dx}{dw} > 0$ and there exists a $\bar{w} \in [0, \infty)$ such that $\frac{ds_v}{dw} > 0$ when $w < \bar{w}$, and $\frac{ds_v}{dw} \leq 0$ when $w \geq \bar{w}$.

The sign of $\frac{dx}{dw}$ follows directly from (20). From Comparative Static (23) we know that $d\frac{v}{w}/dw < 0$ if $(1 - \gamma)\beta w x^\star + (\beta - \gamma)\alpha y^\star < 0$. The left-hand term is negative and the right-hand term is positive. At low levels of w, $y^\star > wx^\star$ and the term is positive. As w increases, $wx^\star$ increases more rapidly than $y^\star$, until the magnitude of the left-hand term dominates, and $(1 - \gamma)\beta w x^\star + (\beta - \gamma)\alpha y^\star < 0$. After this critical w has been reached, $wx^\star$ continues to increase at a greater rate than $y^\star$. Therefore, $d\frac{v}{w}/dw$ is always negative. □