Energy Markets Volatility Modelling using GARCH*

Olga Efimova and Apostolos Serletis†
Department of Economics
University of Calgary
Canada

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†Corresponding author. Phone: (403) 220-4092; Fax: (403) 282-5262; E-mail: Serletis@ucalgary.ca; Web: http://econ.ucalgary.ca/serletis.htm.

Abstract:

This paper investigates the empirical properties of oil, natural gas, and electricity price volatilities using a range of univariate and multivariate GARCH models and daily data from wholesale markets in the United States for the period from 2001 to 2013. The key contribution to the literature is the estimation of trivariate BEKK and DCC models that allow us to observe spillovers and interactions among energy markets. We evaluate and compare the performance of univariate and multivariate models with a range of diagnostic and forecast performance tests, and assess forecasting performance and conditional correlation dynamics.

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1 Introduction

Energy markets are taking an increasingly pre-eminent place in the global economy. Energy is not only one of the most important consumer commodities, but also a major input for almost every industry. The U.S. Energy Information Administration's long-term projections suggest that total energy consumption and energy commodity prices will grow steadily over the next several decades. The price of natural gas, which reached its minimum in 2012, is expected to increase by 60%, the price of electricity by 7%, and the price of crude oil by 62%, reaching $145 per barrel by 2035 — see Energy Information Administration (2012).

Oil, natural gas, and increasingly, electricity, are traded in competitive wholesale markets, which experience dynamics similar to those of financial markets. Prices fluctuate significantly by day and even hour, and periods of relative tranquility are interspersed with those of extreme volatility, which can last for days or weeks and are triggered by supply and demand shocks, or events in other energy markets, derivative markets, and the macroeconomy. Factors like political instability in the oil-producing Middle East region and the expansion of wind electricity generation are likely to keep energy markets volatile into the foreseeable future.

Energy commodity price volatility is of great concern to oil, natural gas and electricity market participants, as well as policymakers. Being able to accurately forecast this volatility carries direct implications for hedging and derivatives trading. Moreover, failing to account for changing volatility (heteroskedasticity in the data) results in biased standard error estimates, invalidating inference and tests of statistical significance.

Since the development of the ARCH and GARCH models by Engle (1982) and Bollerslev (1986), respectively, a significant body of literature has focused on using these to model the volatility of energy commodity prices. The 1990's and early 2000's saw several empirical studies using univariate GARCH models with energy data — the most notable include Morana (2001) and Lin and Tamvakis (2001). More recently, the standard of practice has shifted towards multivariate GARCH, a class of vector autoregressive models that was first proposed by Bollerslev *et. al.* (1988) and became much more widely used after the popularization of its simplified BEKK variant, proposed by Engle and Kroner (1995). Despite the explosion of new types of multivariate GARCH models in recent years, including fractionally integrated GARCH [see Baillie *et. al.* (1996)], nonparametric GARCH [see Bühlmann and McNeil (2002)], and multiplicative component GARCH [see Engle and Sokalska (2012)], simple models of the GARCH(1,1) type remain very useful because they converge much faster to a local maximum in quasi-maximum likelihood estimation, while delivering very competitive forecasting performance [see Andersen and Bollerslev (1998) and Wang and Wu (2012)].

This paper contributes to the literature on energy price volatility modelling in several ways. First, it fills the gap in univariate GARCH modelling of energy commodity volatility — there have been very few such studies published since 2005. We estimate univariate GARCH models for wholesale oil, natural gas, and electricity prices, using daily U.S. data that is as recent as April 2013, and present two alternative specifications for each commodity (Section 3). Secondly, we estimate trivariate BEKK and DCC models that explore the interdependence of wholesale oil, natural gas, and electricity market prices and volatilities using recent U.S. data (Section 4). Existing studies have explored the relationships among several electricity markets [see Goto and Karolyi (2004) and Worthington *et al.* (2005)], between oil and natural gas markets [see Ewing *et al.* (2002) and Serletis and Shahmoradi (2006)], and between oil markets and financial or macroeconomic indicators [see Lee *et al.* (1995), Sadorsky (2012), Elder and Serletis (2010), and Rahman and Serletis (2012)], and among separate crude oil markets [Jin *et al.* (2012)]. However, no study has used multivariate GARCH to model oil, natural gas, and electricity markets as a system, to the best of our knowledge. As an additional contribution to the literature, the use of both univariate and multivariate models over the same data set allows us to compare the performance of these models, including forecasting performance.

2 The Data

We use daily crude oil, natural gas, and electricity wholesale price data for the period from January 2, 2001 to April 26, 2013, obtained from the U.S. Energy Information Administration (EIA). Specifically, we use the West Texas Intermediate crude oil price at Cushing, Oklahoma; the wholesale natural gas price at Henry Hub; and an electricity price that is a weighted average of prices at six of the largest wholesale markets (Nepool Mass Hub in New England, PJM West in Pennsylvania, Entergy in Louisiana, Mid Columbia hub, SP15 EZ and SP15 hubs in California, and the ERCOT South hub in Texas). Due to the extremely high degree of correlation among prices at these hubs, averaging hardly attenuates the degree of price volatility in electricity markets. Table 1 presents summary statistics for the log-levels, $\ln o_t$, $\ln g_t$, $\ln e_t$, and logarithmic first-differences, $\Delta \ln o_t$, $\Delta \ln g_t$, and $\Delta \ln e_t$ of the three price series. Note that all first-differenced series are scaled up by a factor of 100. Figure 1 shows the log levels of the series and their growth rates.

For several models, we use additional data, including the nationwide natural gas storage inventory v_t from the EIA, the S&P 500 index data from Yahoo! Finance, and temperature data for California and Texas from the National Climatic Data Center (2012). We originally worked with temperature data for other states (such as, for example, New York), and with regional averages, but found that the former did not have much predictive power in models estimated, and the latter attenuated local extreme temperature events.

Natural gas storage data was converted from weekly into daily frequency in the following way. Assuming a constant rate of inventory change during each week, we calculate the average daily rate of change as $(v_w - v_{w-1})/5$, where v_w represents the observed storage level on Wednesday of week w, then use this value to fill in the four missing observations in each week. Further, we take a logarithm of each daily inventory level to obtain the $\ln v_t$ series.

Temperature data is transformed into Degree Days (DD), a common measure used in the empirical finance literature — see Mu (2007). Total degree days (DD) is a sum of heating degree days (HDD) and cooling degree days (CDD), as follows

$$\begin{aligned} DD_t &= CDD_t + HDD_t \\ CDD_t &= \max(0, T - 65^o F) \\ HDD_t &= \max(0, 65^o F - T). \end{aligned}$$

Panels A and B of Table 2 report the results of unit root and stationarity tests in log levels, $\ln o_t$, $\ln g_t$, and $\ln e_t$, and logarithmic first differences, $\Delta \ln o_t$, $\Delta \ln g_t$, and $\Delta \ln e_t$. The Augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)] and the Phillips-Perron (PP) test [see Phillips and Perron (1988)] evaluate the null hypothesis of a unit root against an alternative of stationarity, while the Kwiatkowski *et al.* (1992) tests assume a null hypothesis of stationarity (around a constant for test statistic $\widehat{\eta}_\mu$ and around a trend for $\widehat{\eta}_\tau$) and an alternative of a unit root. Due to the presence of unit roots in the log levels, in what follows we estimate all autoregressive GARCH models using logarithmic first differences. We choose a GARCH (1,1) formulation for all univariate models, because it has been found to yield the best performance compared to other GARCH lag configurations, under the most general conditions [see Hansen and Lunde (2005)].

3 Univariate GARCH Modelling

This section presents a range of univariate GARCH models for crude oil, natural gas, and electricity prices. As mentioned earlier, univariate GARCH models have been neglected by academic research in recent years despite their strong performance. Moreover, as we will argue in Section 5, univariate models produce accurate forecasts, converge much faster in maximum likelihood estimation, and allow for the inclusion of a significant number of additional parameters whereas multivariate systems quickly become overparameterized.

3.1 Crude Oil

Oil price volatility is of great interest to energy and financial market participants, as well as policymakers. In fact, the oil price and its volatility are widely used as leading macroeconomic indicators. At the same time, both are notoriously hard to forecast due to the complexity of the factors affecting outcomes in the oil markets.

In this section, we estimate two GARCH (1,1) models that differ in their mean equations to model the daily change in the oil price. The Schwarz Information Criterion (SIC) suggests the random walk (ARMA(0,0)) as the optimal specification; therefore, the first mean equation only contains an intercept. The second is augmented with additional regressors -

a GARCH-in-mean parameter, the daily return rate on the S&P 500 index Δlnx_t (scaled up by a factor of 100), and a set of three seasonal dummy variables s_t. We found that the latter has more predictive power than monthly dummies or weather-related variables. The two mean equations are represented as

$$\Delta lno_t = \alpha + \varepsilon_t \tag{1}$$

$$\Delta lno_t = \alpha + \beta_3 h_t + \beta_4 \Delta lnx_t + \sum_{j=1}^{4} \beta_{4+j} s_t + \varepsilon_t. \tag{2}$$

When estimated as Box-Jenkins equations using maximum likelihood methods, both (1) and (2) show strong evidence of heteroskedasticity (this result persists with the exclusion of all observations since the beginning of the Great Recession in 2008). Univariate GARCH corrects for the non-normal error distributions by dynamically adjusting the conditional variances to take account of variations in the magnitude of the error term, allowing us to obtain unbiased standard error estimates and forecast confidence bounds. We use a GARCH(1,1) specification with an assumed GED error distribution; the latter allows for "fat tails," and provides a better fit to the oil price data than the normal distribution. In addition, we include the GJR asymmetry coefficient of Glosten *et al.* (1993), $\varepsilon_{t-1}^2 \times I_{\varepsilon<0}(\varepsilon_{t-1})$, which captures the disproportionate response of a commodity's variance to unexpected price decreases. The resulting variance equation is

$$h_t = c_0 + a_1 \varepsilon_{t-1}^2 + b_1 h_{t-1} + d_1 \varepsilon_{t-1}^2 I_{\varepsilon<0}(\varepsilon_{t-1}). \tag{3}$$

The empirical estimates for both models, equations (1) and (3), and equations (2) and (3), are presented in panels A and B of Table 3. The positive intercept in both models indicates an upward trend in crude oil prices. The most striking feature in the extended model is the powerful effect of the contemporaneous return on the S&P 500 index on the proportional change in the oil price — a 10% increase in the S&P index is associated with a 1.2% increase in the oil price. On the other hand, seasonal and GARCH-in-Mean effects are not significant.

The variance equation estimates underscore the high persistence of oil price volatility with a very high "GARCH" coefficient on h_{t-1} (0.92 in both models), and very low "ARCH" coefficients on ε_{t-1}^2 (0.02 in both models). This is a common finding in the literature on oil price volatility. There is also a significant asymmetric effect: negative residuals (representing unexpected declines in the oil price) are associated with 7% lower variance than positive residuals of equal magnitude. Finally, the shape parameter estimate of 1.4 confirms the fat-tailed shape of the residual distribution.

Panel C of Table 3 reports the log-likelihood values and diagnostic test statistics for the standardized residuals

$$\hat{\varepsilon}_t = \frac{\varepsilon_t}{\sqrt{h_t}}$$

including descriptive statistics, the Jarque-Bera statistic, the Ljung-Box Q test for residual autocorrelation, and the McLeod-Li Q^2 test for squared residual autocorrelation. Both tests assume the null hypothesis that the data are independently distributed, and an alternative hypothesis of autocorrelation. Q and Q^2 statistics are reported for 30 lags, with p-values in parentheses. Ljung-Box and McLeod-Li tests pass at conventional significance levels, so there is no significant evidence of autocorrelation in the levels or squares of the standardized residuals. We also created histogram and Gaussian kernel estimator plots (not reported here), which showed approximately normal distributions for $\hat{\varepsilon}_t$.

3.2 Univariate Models of Natural Gas Prices

Volatility modelling of natural gas prices receives almost as much attention as that of oil prices, primarily because being able to construct accurate forecasts has tremendous implications for hedging and derivatives trading in financial markets. At the same time, the natural gas market is influenced in a much larger extent by fundamental factors, such as predictable fluctuations in demand driven by weather variables, storage and transportation conditions, and seasonal production and consumption patterns, which make it much easier to construct models with significant predictive power. A third of natural gas in the United States is delivered to residential and commercial consumers (32.4%); 31.2% is used to generate electricity; and 27.8% is used in industrial sectors — see Energy Information Administration (2012).

Like in the previous section, we estimate two univariate GARCH(1,1) models of natural gas prices and volatility, which differ only in the mean equations. Model (1) uses a baseline random walk mean equation (chosen by the SIC criterion) with no additional variables. Model (2) employs a mean equation augmented with: weekday and seasonal dummy variables w_t and s_t, respectively; nationwide storage inventory $ln\, v_t$; its interactions with seasonal dummies $ln\, v_t \times s_t$; and temperature in degree days for California and Texas d_{ct} and d_{xt} (see Section 2 for a discussion on these). The inclusion of interaction terms is motivated by a very clear seasonal pattern in natural gas storage inventories, which are built up during the summer season of off-shore drilling, and depleted during the winter. The two mean equations are presented below

$$\Delta ln g_t = \alpha + \varepsilon_t \tag{4}$$

$$\Delta ln g_t = \alpha + \beta_1 ln\, v_t + \beta_2 d_{ct} + \beta_3 d_{xt} + \sum_{i=1}^{4} \beta_{3+i} w_t + \sum_{j=1}^{3} \beta_{7+j} s_t + \sum_{k=1}^{3} \beta_{10+k} s_t \times ln\, v_t + \varepsilon_t. \tag{5}$$

The $\Delta ln g_t$ series displays strong evidence of heteroskedasticity, motivating the use of a GARCH(1,1) variance equation. In fitting a GARCH model, we found that the normal error distribution assumption yields the best fit to natural gas price data. Also, in contrast to univariate oil price GARCH models, we do not include any asymmetry coefficient after finding small and insignificant estimates. The result in the "classic" GARCH(1,1) variance specification

$$h_t = c_0 + a_1 \varepsilon_{t-1}^2 + b_1 h_{t-1}. \tag{6}$$

Empirical estimates for equations (4) and (6), and (5) and (6), are presented in Table 4, in the same fashion as those for crude oil in Table 3. The extended mean equation reveals distinct weekly and seasonal fluctuations in the natural gas price (pairs of seasonal variables and corresponding season-storage interaction terms are jointly significant). Specifically, the price tends to increase from Monday until Thursday, and then decrease on Friday. This effect might be caused by higher industrial demand for natural gas and electricity during weekdays. The price is higher in the winter and summer than in the transition seasons. The effect of the nation-wide storage inventory on the price is large and significant in the spring, when inventories are at the lowest level (a 10% increase in the inventory level is associated with a 5.3% increase in price). In other seasons, the inventory effect is insignificant, likely indicating an abundant and readily available short-term supply. DD measures of temperature in California and Texas have small but significant effects on the natural gas price, with opposite signs. This possibly reflects significant differences in electricity fuel mix of the two states, and seasonal interactions between hot-weather months and renewable electricity supply in California. California's electricity generation fuel mix currently includes 20% hydro and 20% other renewable sources, both of which are more productive in the spring/summer season (U.S. Department of Energy, 2012).

Estimates of the variance equation coefficients are reported in panel B of Table 4. These indicate that volatility in the natural gas market is less persistent than that in the crude oil market, with ε_{t-1}^2 and h_{t-1} coefficients of 0.2 and 0.8, respectively. Volatility is estimated to be slightly less persistent in the model which includes the extended mean equation, indicating that the extended model, (5) and (6), is successful in capturing a larger set of relevant information from the error term than the baseline model, (4) and (6).

Finally, panel C of Table 4 presents the log-likelihood values for models (1) and (2), as well a range of statistics and diagnostic tests applied to the standardized residuals $\hat{\varepsilon}_t$. The Ljung-Box test for residual autocorrelation does not pass at conventional significance levels; however, after creating a plot of autocorrelations at each lag length, we found that only a few of the autocorrelations exceeded 5%, indicating likely spurious effects. Overall, the diagnostic tests indicate that both GARCH models are correctly specified.

3.3 Univariate Models of Electricity Prices

Wholesale electricity is much more vulnerable to extreme price events than other energy commodities, because of its nonstorability, high transportation costs caused by physical losses and transmission constraints, and highly inelastic demand. In addition, electricity supply is also inelastic at high output levels. Every region's generation capacity is composed of a unique mix of technologies, which differ by marginal cost and by their ability to quickly change the level of output, and are alternately used for baseload, mid-peak and on-peak electricity supply. At times of abnormally high demand or restricted supply, a region's generators may be pushed to full capacity, bringing high-cost "peaker" natural gas turbines online, and resulting in periods of extreme price volatility marked by dramatic and frequent price spikes. Electricity price volatility is, therefore, possibly the best candidate for GARCH modelling.

Following the pattern established in the previous two subsections, we construct two GARCH models, starting with a baseline ARMA(2,1)-GARCH(1,1) framework for the first model

$$\Delta lne_t = \alpha + \beta_1 \Delta lne_{t-1} + \beta_2 \Delta lne_{t-2} + \beta_3 \varepsilon_{t-1} + \varepsilon_t. \tag{7}$$

We then augment equation (7) with additional variables to construct the second model. We include weekday and seasonal dummy variables, and temperature in degree days for California and Texas, d_{ct} and d_{xt}, as additional variables. The number of AR and MA lags was selected using SIC. The mean equation of the expanded model is

$$\begin{aligned} \Delta lne_t = \alpha + \beta_1 \Delta lne_{t-1} + \beta_2 \Delta lne_{t-2} + \beta_3 \varepsilon_{t-1} + \beta_4 d_{ct} + \beta_5 d_{xt} + \\ \sum_{i=1}^{4} \beta_{5+i} w_t + \sum_{j=1}^{3} \beta_{9+j} s_t + \varepsilon_t. \end{aligned} \tag{8}$$

The variance equation for each model is a "classic" GARCH(1,1) equation identical to the one used for the natural gas models

$$h_t = c_0 + a_1 \varepsilon_{t-1}^2 + b_1 h_{t-1}. \tag{9}$$

Empirical estimates for GARCH models formed by equations (7) and (9), and (8) and (9) are reported in Table 5. AR and MA coefficients indicate that changes in the electricity price reverse direction with a high frequency. Although seasonal effects are insignificant, the temperature effects for California and Texas mirror those estimated in the natural gas models. A 1^oF increase in temperature above, or a decrease below, the optimal temperature of 65^oF, is associated with a 2.2% electricity price increase in Texas, and a 2.5% decrease in California (both effects are highly significant). Variance equation estimates indicate that electricity price volatility is somewhat more "spiky" than that of natural gas prices, with ε_{t-1}^2 and h_{t-1} estimates of 0.23 and 0.77, respectively.

Panel C of Table 5 reports the log-likelihood values and a range of diagnostic statistics and tests applied to the standardized residuals $\hat{\varepsilon}_t$. The descriptive statistics for $\hat{\varepsilon}_t$ reveal a distribution that is very close to normal. The only issue is kurtosis, which is reduced with the inclusion of additional variables in model (8) and (9). The Ljung-Box and McLeod-Li tests do not pass at conventional significance levels; however, after creating correlation plots for 30- and 100-lag horizons, we found no significant correlations at any lag. We conclude that both models, (7) and (9), and (8) and (9), adequately control for heteroskedasticity in the data, with model (8) and (9) delivering slightly better performance.

4 Multivariate GARCH Modelling

First proposed by Bollerslev *et al.* (1988), multivariate GARCH (MGARCH) models are becoming standard in finance and energy economics. Combined with a vector autoregressive (VAR) or vector error-correction (VEC) model for the mean equation, they allow for rich dynamics in the variance-covariance structure of series, making it possible to model spillovers in both the values and the conditional variances of series under study.

In this section we estimate two trivariate models: a BEKK model of Engle and Kroner (1995) and a Dynamic Conditional Correlation (DCC) model of Engle (2002). BEKK is a more general specification, while the DCC is less computationally demanding and enables time-varying correlations among series with only two additional parameters. MGARCH is a valuable approach in our case because volatility spillovers are expected among oil, natural gas and electricity markets: not only are the three substitutes in consumption, but also, natural gas and oil are both used as inputs in electricity generation (mid-peak and on-peak, respectively); and natural gas and oil are complements in production. The chosen specifications allow us to model the transmission of price volatility from one energy commodity to another, and estimate the effects of volatility in any of the three markets on the price of each commodity.

4.1 Model Specification

The BEKK and DCC models estimated in this section share the same trivariate vector autoregressive moving average VARMA (1,1) specification, with logarithms of oil, gas and electricity prices forming the dependent variables. We include the daily return in the S&P 500 index $\Delta ln\, x_t$ (scaled up by a factor of 100), and a GARCH-in-mean term $\sqrt{h_t}$:

$$\begin{aligned} \boldsymbol{z}_t &= \phi + \boldsymbol{\Gamma} \boldsymbol{z}_{t-1} + \boldsymbol{\Psi}\sqrt{\boldsymbol{h}_t} + \boldsymbol{\Theta}\boldsymbol{\epsilon}_{t-1} + \gamma\Delta ln x_t + \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t &\mid \Omega_{t-1} \sim (\boldsymbol{0},\, \boldsymbol{H}_t) \end{aligned} \tag{10}$$

where Ω_{t-1} is the information set available in period $t-1$, and

$$\boldsymbol{z}_t = \begin{bmatrix} lno_t \\ lng_t \\ lne_t \end{bmatrix}; \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{11}^i & \gamma_{12}^i & \gamma_{13}^i \\ \gamma_{21}^i & \gamma_{22}^i & \gamma_{23}^i \\ \gamma_{31}^i & \gamma_{32}^i & \gamma_{33}^i \end{bmatrix};$$

$$\sqrt{\boldsymbol{h}_t} = \begin{bmatrix} \sqrt{h_{11,t}} & 0 & 0 \\ 0 & \sqrt{h_{22,t}} & 0 \\ 0 & 0 & \sqrt{h_{33,t}} \end{bmatrix}; \quad \boldsymbol{\Psi} = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix};$$

$$\boldsymbol{\Theta} = \begin{bmatrix} \theta_{11}^i & \theta_{12}^i & \theta_{13}^i \\ \theta_{21}^i & \theta_{22}^i & \theta_{23}^i \\ \theta_{31}^i & \theta_{32}^i & \theta_{33}^i \end{bmatrix}; \quad \gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}.$$

Normally, the presence of unit roots would suggest logarithmic first differences as the correct data representation in our model. However, we find evidence of cointegration among the three energy commodity prices, as can be seen in panels B and C of Table 2 where we present Engle and Granger (1987) and Johansen (1988) cointegration tests. A system of I(1) variables is cointegrated if there exists a linear combination of them that is stationary or I(0) — see Lütkepohl (2004). In a VAR or VARMA framework, cointegration encourages both maximum likelihood and OLS estimation methods to select parameters that correspond to this stationary combination, since parameters that eliminate the trends are always associated with the smallest deviations of actual observations from their predicted values. This result was formally proven by Davidson and MacKinnon (1993), who also showed that a VAR with cointegrated series produces estimates that are not only consistent, but superconsistent (converge to their true values faster than normal).

VEC models are often used with cointegrated series because they allow for an explicit analysis of cointegrating relations. However, a VAR in levels is sufficient if the latter are not the focus of study, as in our case. In fact, VAR and VEC are equivalent, as demonstrated by Lütkepohl (2004). A VAR system of order p, (VAR(p)) in general form can be expressed as

$$\boldsymbol{z}_t = \boldsymbol{A}_1 \boldsymbol{z}_{t-1} + ... + \boldsymbol{A}_p \boldsymbol{z}_{t-p} + \boldsymbol{\epsilon}_t \tag{11}$$

where $\boldsymbol{z}_t$ and $\boldsymbol{\epsilon}_t$ are n-dimensional vectors, and $\boldsymbol{A}_k$ $(1 \leq k \leq p)$ are parameter matrices. A VEC can be obtained from the above equation by subtracting $\boldsymbol{z}_{t-1}$ from both sides and rearranging the terms

$$\begin{aligned} \boldsymbol{z}_t - \boldsymbol{z}_{t-1} &= \boldsymbol{A}_1 \boldsymbol{z}_{t-1} - \boldsymbol{z}_{t-1} + \boldsymbol{A}_2 \boldsymbol{z}_{t-2} + ... + \boldsymbol{A}_p \boldsymbol{z}_{t-p} + \boldsymbol{\epsilon}_t \\ \Delta \boldsymbol{z}_t &= \boldsymbol{\Pi} \boldsymbol{z}_{t-1} + \boldsymbol{\Gamma}_1 \Delta \boldsymbol{z}_{t-1} + ... + \boldsymbol{\Gamma}_{p-1} \Delta \boldsymbol{z}_{t-p+1} + \boldsymbol{\epsilon}_t \end{aligned} \tag{12}$$

where $\mathbf{\Pi} = -(\boldsymbol{I}_n - \boldsymbol{A}_1 - ... - \boldsymbol{A}_p)$, $\mathbf{\Gamma}_i = -(\boldsymbol{A}_{i+1} + ... + \boldsymbol{A}_p)$ for $i = 1, ..., p-1$, and $\boldsymbol{I}_n$ is an n-dimensioned identity matrix — see Lütkepohl (2004).

We test for cointegration using the maximum likelihood method, developed independently by Johansen (1988) and Stock and Watson (1988). The method attempts to detect the implied restrictions on an otherwise unrestricted VAR involving the series in question. An implied restriction suggests that there exists a VEC model that is equivalent to the VAR, something that can only happen in an integrated system. The trace version of the test evaluates the null hypothesis of r or fewer linearly independent cointegrating vectors. The eigenvalue version tests the null of exactly r cointegrating vectors. In our case, the trace test suggests a cointegration order of no more than two, and the eigenvalue test suggests an order of one (see panel C of Table 2). We conclude that the series are cointegrated, which motivates us to use the VARMA in log-levels formulation for multivariate GARCH models in this section.

Our first model contains a variance equation that is an asymmetric form of BEKK(1,1,1) introduced by Grier et. al. (2004)

$$\boldsymbol{H}_t = \boldsymbol{C}'\boldsymbol{C} + \boldsymbol{B}'\boldsymbol{H}_{t-1}\boldsymbol{B} + \boldsymbol{A}'_{t-1}\boldsymbol{\epsilon}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{A} + \boldsymbol{D}'\boldsymbol{u}_{t-1}\boldsymbol{u}'_{t-1}\boldsymbol{D} \tag{13}$$

where $\boldsymbol{C}'\boldsymbol{C}$, $\boldsymbol{A}'\boldsymbol{A}$, $\boldsymbol{B}'\boldsymbol{B}$ and $\boldsymbol{D}'\boldsymbol{D}$ are 3×3 matrices with $\boldsymbol{C}$ being a triangular matrix to ensure positive definiteness of $\boldsymbol{H}$. This specification allows past volatilities, $\boldsymbol{H}_{t-1}$, as well as lagged values of $\boldsymbol{\epsilon}\boldsymbol{\epsilon}'$ and $\boldsymbol{u}\boldsymbol{u}'$, to show up in estimating current volatilities of crude oil, natural gas, and electricity. The asymmetry vector is denoted as $\boldsymbol{u}_{t-1} = \boldsymbol{\epsilon}_{t-1} \circ I_{\boldsymbol{\epsilon}<0}\boldsymbol{\epsilon}_{t-1}$ where $\circ$ denotes the elementwise product of vectors. Assuming matrix $\boldsymbol{H}$ is symmetric, the model produces six unique equations modeling the dynamic variances of oil, gas and electricity prices, as well the covariances between them.

The second model uses the "VARMA" DCC specification. The first step in estimating a DCC model is to obtain conditional correlations from the covariance matrix $\boldsymbol{Q}_t$, which is typically estimated with a "GARCH (1,1)" equation governed by two scalar parameters a and b

$$\boldsymbol{Q}_t = (1 - a - b)\boldsymbol{Q}_0 + a\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1} + b\boldsymbol{Q}_{t-1} \tag{14}$$

where $\boldsymbol{Q}_0$ is the unconditional covariance matrix [see Engle (2002)]. The matrix $\boldsymbol{Q}_t$ does not replace $\boldsymbol{H}_t$; its sole purpose is to provide conditional correlations $\sqrt{Q_{ij,t}}$, $i \neq j$. The $\boldsymbol{H}_t$ matrix is generated by fitting univariate GARCH models to estimate the variances, and combining these variances with $\sqrt{Q_{ij,t}}$ to estimate the covariances. The process is summarized as

$$H_{ij,t} = \frac{Q_{ij,t}\sqrt{H_{ii,t}}\sqrt{H_{jj,t}}}{\sqrt{Q_{ii,t}Q_{jj,t}}}. \tag{15}$$

When we estimate the DCC, we use a "VARMA" specification for the variances in $\boldsymbol{H}_t$

$$H_{ii,t} = c_{ii} + \sum_{j=1}^{3} a_{ij}\varepsilon^2_{j,t-1} + \sum_{j=1}^{3} b_{ij}H_{jj,t-1} + d_{ii}u^2_{t-1} \tag{16}$$

where the last term represents the asymmetry coefficient. This specification allows for spillovers among the variances of the three series, and also makes the form almost identical to that used for the BEKK model, allowing for direct comparisons of model performance.

4.2 Empirical Estimates

The trivariate BEKK and DCC models described above were estimated in Estima RATS using quasi-Maximum Likelihood. We used the BFGS (Broyden, Fletcher, Goldfarb & Shanno) estimation algorithm, which is recommended for GARCH models, combined with the derivative-free Simplex pre-estimation method. Tables 6 and 7 report the coefficients obtained (significance levels in parentheses), as well as key diagnostics for standardized residuals

$$\hat{z}_{jt} = \frac{e_{jt}}{\sqrt{\hat{h}_{jt}}}$$

for $j = o,\ g,\ e$.

The mean equation estimates are similar for the two models. Specifically, the AR(1) coefficients in matrix $\mathbf{\Gamma}_1$ are very close to one along the main diagonal, while the MA(1) coefficients in matrix $\mathbf{\Theta}_1$ are small and insignificant (although the Schwarz Information Criterion still chooses ARMA(1,1) over various AR specifications). The off-diagonal elements in $\mathbf{\Gamma}_1$ suggest significant price spillover effects affecting the natural gas and especially the electricity markets, but not the oil market. For example, both BEKK and DCC estimates indicate that a 10% increase in the oil price would cause a 0.28% increase in electricity price and a 1.3% decrease in the gas price in the next period, while a 10% in the gas price would raise the next period's electricity price by 1.3-1.4%. In fact, the off-diagonal estimates for $\mathbf{\Gamma}_1$ suggest a hierarchy of influence from the oil market (which is affected by neither of the other two markets), to gas and electricity markets. Vector γ shows that the impact of the S&P 500 index is small and only significant in the oil price equation.

BEKK estimates show high GARCH coefficients along the main diagonal of $\boldsymbol{B}'\boldsymbol{B}$ and low corresponding ARCH coefficients in $\boldsymbol{A}'\boldsymbol{A}$, suggesting that volatility is persistent, especially in the oil and natural gas markets (this finding is mirrored in the DCC estimates). BEKK matrix $\boldsymbol{A}'\boldsymbol{A}$ and DCC matrix $\boldsymbol{A}$ reveal powerful short-term spillovers effects, with DCC clearly showing the hierarchy of influence from oil to natural gas and electricity markets. The greatest difference between BEKK and DCC estimates is in the asymmetry coefficients. BEKK shows strong asymmetric ARCH effects in all three markets, with significant spillovers. However, DCC shows an asymmetric effect only in the oil market.

Overall, the VARMA-BEKK and DCC models allow us to observe significant interactions between the three wholesale energy commodity markets, including spillovers from a price change in one asset to the volatility of another asset. Thanks to the large dataset and the powerful multivariate model structures, we are able to not only detect these spillover effects, but also estimate their magnitude. Choosing models that are direct multivariate extensions of the asymmetric GARCH(1,1) models presented in Section 3 allows us to compare the forecasting performance of univariate versus multivariate models in the following section.

5 Forecasting and Conditional Correlations

Volatility forecasting is arguably the most important application of GARCH models in oil, natural gas, and electricity markets. In this section we evaluate the forecasting performance of the univariate models and the multivariate BEKK model by constructing a series of rolling dynamic one-day forecasts (there is no simple extrapolative formula for the DCC model). There has been considerable debate over which GARCH model type delivers the best forecasting performance, with no consensus to date. Andersen and Bollerslev (1998) find that univariate GARCH models generally outperform multivariate GARCH models. Wang and Wu (2012) find that the optimal choice of the model is sensitive to the series under study (for example, the crude oil price versus the crack spread), but that univariate models yield more precise estimates for asymmetry coefficients. Several studies compare 'classic' multivariate GARCH specifications, such as the BEKK and CCC, with new multivariate models, including nonparametric GARCH [see Sadorsky (2006)] and Markov regime-switching GARCH [see Nomikos and Pouliasis (2011)], to name a few. Generally, only the very recent multivariate model types seem to consistently outperform simple univariate models in forecasting. However, many of these models are either difficult to estimate, or specialized to accommodate large systems of financial series at the cost of imposing restrictions on interactions among series, which are of interest in our study of interrelated energy markets.

Table 8 presents an assessment of the quality of mean-model forecasts using a range of forecast performance statistics, including Mean Error (ME), Mean Absolute Error (MAE), Root Mean Squared Error (RMSE) and Theil's U statistics [see Theil (1971)]. Theil's U statistic is the ratio of RMSE from the specified model to the RMSE of a "no-change" forecast that results from a random walk model. A value of $U < 1$ indicates that the model in question outperforms the random walk. In order to make the forecasts from the univariate and multivariate GARCH models used in this paper directly comparable, we convert the $\Delta \ln p_t$ forecasts from univariate models to the lnp_t form. The RMSE and Theil's U statistics clearly indicate that univariate models outperform BEKK in forecasting. In fact, forecasts produced by BEKK do not outperform a random walk. For example, the RMSE for oil price is $0.21 - 0.30$ for univariate models, compared to 0.40 for BEKK.

The latter finding is likely due to the higher precision in key estimates achieved by using a less computationally demanding model. The univariate models used in this paper contain a total of $11 - 16$ parameters, while the BEKK employs 69 parameters; even with a data set of over 4000 observations, this poses significant estimation challenges. It is also worth noting that 'extended' univariate models augmented with additional variables deliver slightly better forecasting performance for natural gas and electricity prices, but not for the oil price, supporting the notion that the crude oil price is difficult to forecast and is close to the random walk in its movement. However, multivariate GARCH remains useful in the study of energy markets because it allows us to observe complex interactions between markets.

Unfortunately, no formal statistics exist that would allow us to formally compare the quality of variance forecasts produced by the models examined (an extremely valuable contribution to the body of research in volatility forecasting would be to develop such statistics.) After constructing sample forecasts graphs (not presented here), we found that the conditional variances produced by all models estimated correctly identify periods of high and low volatility, and yield accurate 95% confidence bounds around the forecasted price levels. However, the mechanism tends to be rather 'adaptive' in the sense that all GARCH models estimated fail to anticipate a change in the direction of price movement (a characteristic feature of autoregressive models).

Although the DCC model cannot be used for forecasting, it is an invaluable tool in studying correlation dynamics between energy commodity prices. Figure 2 presents the DCC conditional correlations between the returns on each commodity pair for the period from January 2001 to April 2013. Both figures show the oil-gas and oil-electricity correlations decreasing during times of recession or slow economic growth, specifically, 2003-2005 and 2009-2010. The correlation between natural gas and electricity increased dramatically during the same periods. It is possible that abnormal dynamics in the wholesale oil market, as well as increased trade in oil futures during times of uncertainty and recession weakened the link between the oil price and energy market fundamentals. At the same time, natural gas and electricity markets, being more responsive to fundamental factors, may have adapted in a concerted manner to recessionary demand conditions. A second interesting feature is the decrease in the correlation between all pairs of commodities since 2011. An interesting direction for future research would be to investigate what caused this recent weakening of the links between energy markets.

6 Conclusion

Globalization, growing energy demand, increasing intensity of extreme weather events and geopolitical tensions, as well as the deregulation of electricity markets, all mean that price volatility will remain a central feature of oil, natural gas and electricity markets for decades to come. Originally developed in finance, GARCH models have become indispensable in short-term volatility modelling of energy commodity prices, largely because they are very efficient at accommodating irregular periods of price volatility and tranquility that are characteristic of energy markets. This paper presented an empirical application of a range of univariate and multivariate GARCH models to daily oil, natural gas and electricity price data from U.S. wholesale markets, for the period from 2001 to 2013.

We found that univariate and multivariate models yield similar estimates, although univariate models produce more accurate forecasts. The optimal choice of the model would depend on the research question (e.g. forecasting versus price and volatility spillovers), the energy commodity of greater interest, and the availability of high-quality data on exogenous factors affecting the energy commodity price and volatility. Many other variables can potentially be included to reflect relevant political events, financial market conditions, extreme weather events like hurricanes, facility breakdowns causing supply shocks, and oil storage inventories. Also, with wind energy generation growing at a rapid rate, future studies on electricity volatility could benefit from incorporating wind speed data for key producing regions. With multivariate GARCH models, on the other hand, our choice of additional regressors is extremely limited, since each one adds an entire vector of parameters. However, one advantage of using multivariate GARCH in our case is the opportunity to forego first-differencing in the vector autoregressive mean equation due to the presence of cointegration among the three series, leading to information preservation and more robust estimates.

The greatest strength of multivariate GARCH is the potential to investigate the interactions among all three commodity prices and their volatilities, which makes it possible for us to discover surprising and significant spillover effects. We find that price spillovers are rather unidirectional, suggesting the existence of a hierarchy of influence from oil to gas and electricity markets. These findings underline the importance of oil in the U.S. economy today, and the far-reaching implications of events in wholesale oil markets.

By applying several types of GARCH models to oil, natural gas and electricity price data, we contribute to the understanding of price volatility in wholesale energy markets, and suggest several effective models that would be of use to energy market participants, derivatives market participants, large energy consumers interested in hedging strategies, and policymakers.

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Table 1. Summary Statistics

Variable	Mean	Variance	Skewness	Excess kurtosis	J-B normality
		A. Log levels			
$\ln o_t$	4.031	0.250	-0.405	-1.035	219.5
$\ln g_t$	1.594	0.176	0.035	-0.348	15.97
$\ln e_t$	3.874	0.148	-0.143	0.760	83.79
		B. Logarithmic first differences			
$\Delta \ln o_t$	0.040	6.353	-0.074	5.294	3561
$\Delta \ln g_t$	-0.029	21.69	0.545	21.88	60954
$\Delta \ln e_t$	-0.021	191.3	-0.053	11.97	18186

Figure 1: Descriptive Graphs for Wholesale Oil, Natural Gas and Electricity Prices.

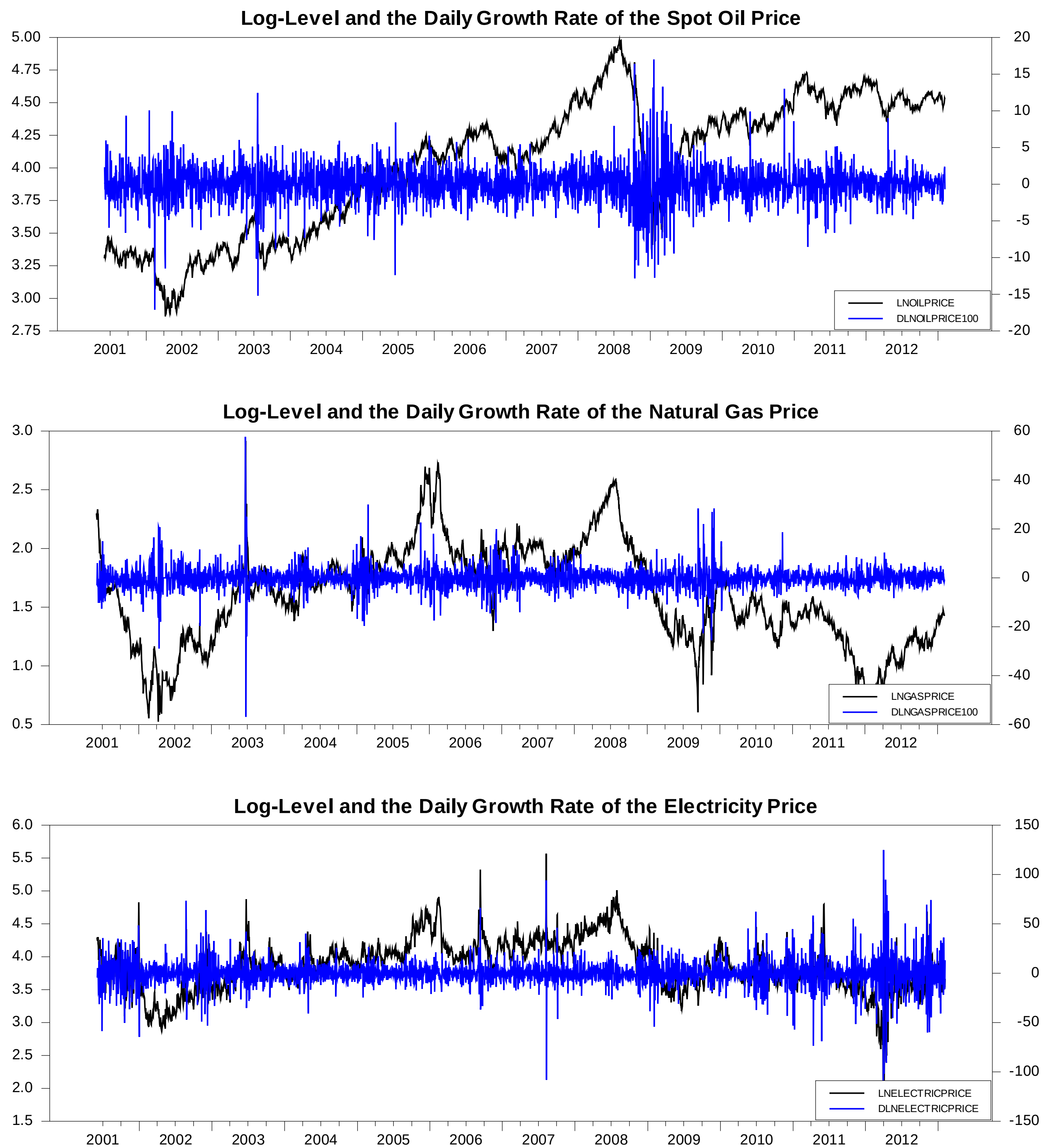

Table 2. Unit Root, Stationarity And Cointegration Tests

A. Unit root and stationarity tests

Variable	Unit root tests: ADF	Unit root tests: PP	KPSS stationarity tests: $\widehat{\eta}_{\mu}$	KPSS stationarity tests: $\widehat{\eta}_{\tau}$
		Log levels		
$\ln o_t$	−1.454	−1.506	48.27	5.864
$\ln g_t$	−2.169	−2.875	11.00	9.361
$\ln e_t$	−3.129	−10.08	7.991	7.923
		Logarithmic first differences		
$\Delta \ln o_t$	−9.508	56.85	0.043	0.030
$\Delta \ln g_t$	−11.45	−55.24	0.051	0.053
$\Delta \ln e_t$	−12.45	−92.18	0.007	0.007
5% cv	−2.863	−2.863	0.463	0.146
1% cv	−3.436	−3.436	0.739	0.216

B. Engle-Granger cointegration tests

Dependent variable	ADF with trend	5% cv with trend	ADF no trend	5% cv no trend
$\ln o_t$	−4.132	−4.12	−4.190	−3.75
$\ln g_t$	−5.540	−4.12	−6.175	−3.75
$\ln e_t$	−8.178	−4.12	−8.044	−3.75

C. Johansen ML cointegration tests

Cointegration order	Eigenvalue	Trace	5% cv (trace)	5% cv (eigenvalue)
At most 1	0.0222	74.88	29.80	67.88
At most 2	0.0016	7.003	15.41	4.814
At most 3	0.0001	2.188	3.84	2.188

Table 3. Univariate GARCH Crude Oil Models

	GARCH model	
Coefficient	Baseline	Extended
A. Conditional mean equation		
Constant	0.088 (0.009)	0.018 (0.056)
h_{t-1}		−0.001 (0.786)
$\Delta \ln x_t$		0.126 (0.000)
Winter		−0.141 (0.161)
Summer		−0.049 (0.605)
Fall		0.160 (0.116)
B. Conditional variance equation		
Constant	0.102 (0.008)	0.109 (0.008)
ε_{t-1}^2	0.024 (0.022)	0.026 (0.017)
h_t	0.924 (0.000)	0.921 (0.000)
$\varepsilon_{t-1}^2 I_{\varepsilon<0}(\varepsilon_{t-1})$	0.065 (0.000)	0.066 (0.001)
Shape	1.427 (0.000)	1.431 (0.000)
C. Standardized residual diagnostics		
$\widehat{\varepsilon}$ mean	−0.021	−0.020
$\widehat{\varepsilon}$ standard error	1.005	1.005
$\widehat{\varepsilon}$ variance	1.011	1.011
$\widehat{\varepsilon}$ skeweness	−0.241	−0.234
$\widehat{\varepsilon}$ kurtosis	2.870	2.920
Jarque-Bera	1075	1110
$Q(30)$ p-value	0.772	0.758
$Q^2(30)$ p-value	0.201	0.277
Log likelihood	−6733	−6727

Note: Numbers in parentheses are p-values.

Table 4. Univariate GARCH Natural Gas Models

Coefficient	GARCH model Baseline	Extended
A. Conditional mean equation		
Constant	−0.075 (0.214)	−4.601 (0.005)
Winter		6.529 (0.232)
Summer		6.565 (0.125)
Fall		1.706 (0.839)
$\ln v_t$		0.527 (0.015)
$\ln v_t \times$Winter		−0.846 (0.225)
$\ln v_t \times$Summer		−0.840 (0.134)
$\ln v_t \times$Fall		−0.267 (0.799)
d_{ct}		−0.023 (0.009)
d_{xt}		0.015 (0.089)
Monday		0.848 (0.000)
Tuesday		0.813 (0.000)
Wednesday		0.700 (0.000)
Thursday		0.523 (0.008)
B. Conditional variance equation		
Constant	0.336 (0.000)	0.378 (0.000)
ε^2_{t-1}	0.151 (0.000)	0.167 (0.000)
h_t	0.849 (0.000)	0.832 (0.000)
C. Standardized residual diagnostics		
$\widehat{\varepsilon}$ mean	0.011	0.015
$\widehat{\varepsilon}$ standard error	1.000	1.000
$\widehat{\varepsilon}$ variance	1.001	1.000
$\widehat{\varepsilon}$ skeweness	0.914	0.785
$\widehat{\varepsilon}$ kurtosis	11.40	9.160
Jarque-Bera	16914	10965
$Q(30)$ p-value	0.000	0.000
$Q^2(30)$ p-value	0.996	0.996
Log likelihood	−8420	−8398

Note: Numbers in parentheses are p-values.

Table 5. Univariate GARCH Electricity Models

Coefficient	GARCH model: Baseline	Extended
A. Conditional mean equation		
Constant	−0.018 (0.708)	1.002 (0.008)
$\Delta \ln e_{t-1}$	0.513 (0.000)	0.519 (0.000)
$\Delta \ln e_{t-2}$	−0.128 (0.000)	−0.127 (0.000)
ε_{t-1}	−0.686 (0.000)	−0.696 (0.000)
Winter		0.216 (0.214)
Summer		0.221 (0.231)
Fall		−0.077 (0.610)
d_{ct}		−0.025 (0.016)
d_{xt}		0.022 (0.032)
Monday		−0.821 (0.187)
Tuesday		−1.989 (0.000)
Wednesday		−0.710 (0.223)
Thursday		−2.104 (0.001)
B. Conditional variance equation		
Constant	4.927 (0.000)	5.161 (0.000)
ε^2_{t-1}	0.227 (0.000)	0.231 (0.000)
h_t	0.771 (0.000)	0.765 (0.000)
C. Standardized residual diagnostics		
$\widehat{\varepsilon}$ mean	0.014	0.008
$\widehat{\varepsilon}$ standard error	1.000	1.000
$\widehat{\varepsilon}$ variance	1.000	1.000
$\widehat{\varepsilon}$ skeweness	0.512	0.480
$\widehat{\varepsilon}$ kurtosis	3.920	3.737
Jarque-Bera	2085	1890
$Q(30)$ p-value	0.000	0.000
$Q^2(30)$ p-value	0.025	0.020
Log likelihood	−11509	−11496

Note: Numbers in parentheses are p-values.

Table 6. The Trivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With Daily Crude Oil, Natural Gas, and Electricity Prices

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} 0.001\ (0.916) \\ 0.037\ (0.000) \\ 0.380\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma} = \begin{bmatrix} 0.999\ (0.000) & -0.001\ (0.783) & 0.001\ (0.538) \\ 0.000\ (0.737) & 1.007\ (0.000) & -0.013\ (0.000) \\ 0.028\ (0.000) & 0.134\ (0.000) & 0.816\ (0.000) \end{bmatrix}; \boldsymbol{\gamma} = \begin{bmatrix} 0.001\ (0.001) \\ -0.001\ (0.166) \\ 0.000\ (0.595) \end{bmatrix};$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.105\ (0.170) & -0.048\ (0.007) & -0.001\ (0.799) \\ -0.043\ (0.608) & -0.033\ (0.511) & 0.002\ (0.794) \\ -0.279\ (0.165) & 0.072\ (0.295) & 0.052\ (0.198) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} -0.032\ (0.056) & 0.016\ (0.049) & -0.004\ (0.262) \\ 0.219\ (0.000) & -0.007\ (0.734) & 0.030\ (0.000) \\ 0.176\ (0.001) & 0.010\ (0.764) & -0.077\ (0.005) \end{bmatrix}.$$

B. Conditional variance-covariance structure

$$\boldsymbol{C}'\boldsymbol{C} = \begin{bmatrix} 0.003\ (0.000) & 0.000\ (0.692) & 0.004\ (0.153) \\ & 0.006\ (0.000) & 0.002\ (0.370) \\ & & 0.021\ (0.000) \end{bmatrix}; \boldsymbol{B}'\boldsymbol{B} = \begin{bmatrix} 0.966\ (0.000) & 0.002\ (0.784) & -0.048\ (0.073) \\ 0.002\ (0.377) & 0.934\ (0.000) & 0.056\ (0.000) \\ -0.001\ (0.870) & -0.003\ (0.084) & 0.870\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A}'\boldsymbol{A} = \begin{bmatrix} -0.013\ (0.604) & -0.063\ (0.005) & -0.074\ (0.286) \\ -0.010\ (0.055) & 0.312\ (0.000) & -0.126\ (0.000) \\ 0.001\ (0.626) & 0.011\ (0.000) & 0.461\ (0.000) \end{bmatrix}; \boldsymbol{D}'\boldsymbol{D} = \begin{bmatrix} 0.282\ (0.000) & -0.061\ (0.076) & 0.235\ (0.024) \\ 0.033\ (0.045) & 0.245\ (0.000) & 0.356\ (0.000) \\ -0.007\ (0.036) & -0.004\ (0.636) & -0.297\ (0.000) \end{bmatrix}.$$

C. Residual diagnostics

	Mean	Std. Error	Skewness	Kurtosis	$Q(40)$	$Q^2(40)$
$\hat{z}_{o_t}$	−0.003	0.996	−0.242	3.201	42.88 (0.348)	63.71 (0.010)
$\hat{z}_{g_t}$	0.015	0.982	0.816	10.30	92.43 (0.000)	23.63 (0.981)
$\hat{z}_{e_t}$	0.036	0.989	0.501	3.753	256.2 (0.000)	66.42 (0.005)

Note : Sample period, daily data: January 2, 2001 to April 26, 2013. Numbers in parentheses are tail areas of tests.

Table 7. The Trivariate Varma, Garch-in-Mean, Asymmetric DCC Model With Daily Crude Oil, Natural Gas, and Electricity Prices

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} 0.002\ (0.762) \\ 0.030\ (0.007) \\ 0.403\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma} = \begin{bmatrix} 0.998\ (0.000) & -0.001\ (0.812) & 0.002\ (0.527) \\ 0.002\ (0.073) & 1.007\ (0.000) & -0.013\ (0.001) \\ 0.028\ (0.000) & 0.140\ (0.000) & 0.807\ (0.000) \end{bmatrix}; \boldsymbol{\gamma} = \begin{bmatrix} 0.001\ (0.001) \\ -0.001\ (0.177) \\ -0.001\ (0.910) \end{bmatrix};$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.044\ (0.541) & -0.027\ (0.131) & -0.003\ (0.559) \\ -0.010\ (0.888) & -0.006\ (0.913) & 0.002\ (0.862) \\ -0.174\ (0.343) & 0.095\ (0.231) & 0.056\ (0.239) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} -0.033\ (0.065) & 0.012\ (0.131) & -0.003\ (0.401) \\ 0.218\ (0.000) & -0.010\ (0.637) & 0.030\ (0.000) \\ 0.171\ (0.004) & -0.004\ (0.915) & -0.059\ (0.054) \end{bmatrix}.$$

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.000\ (0.002) \\ 0.000\ (0.000) \\ 0.001\ (0.002) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.874\ (0.000) & -0.025\ (0.382) & -0.005\ (0.688) \\ 0.442\ (0.003) & 0.807\ (0.000) & 0.005\ (0.705) \\ 0.824\ (0.117) & -0.133\ (0.083) & 0.716\ (0.000) \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} 0.103\ (0.000) \\ 0.002\ (0.925) \\ 0.002\ (0.944) \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 0.035\ (0.004) & -0.006\ (0.410) & -0.004\ (0.058) \\ -0.091\ (0.000) & 0.150\ (0.000) & 0.015\ (0.008) \\ -0.186\ (0.007) & -0.109\ (0.000) & 0.272\ (0.000) \end{bmatrix}; \quad a = 0.05; \quad b = 0.995;$$

C. Residual diagnostics

	Mean	Std. Error	Skewness	Kurtosis	$Q(40)$	$Q^2(40)$
$\hat{z}_{o_t}$	−0.014	0.999	−0.241	2.624	43.60(0.321)	49.85 (0.137)
$\hat{z}_{g_t}$	0.007	0.997	0.679	8.310	89.55(0.000)	19.52 (0.997)
$\hat{z}_{e_t}$	0.022	0.998	0.463	3.555	244.6(0.000)	69.20 (0.002)

Note : Sample period, daily data: January 2, 2001 to April 26, 2013. Numbers in parentheses are tail areas of tests.

Table 8. Forecast Performance Statistics

	ME (in dollars)	MAE (in dollars)	RMSE (in dollars)	Theil's U (1-step)
	A. Univariate baseline model			
$ln\, o_t$	0.094	0.225	0.291	0.963
$ln\, g_t$	0.001	0.311	0.460	0.998
$ln\, e_t$	0.002	0.084	0.128	0.932
	B. Univariate extended model			
$ln\, o_t$	0.082	0.234	0.299	0.999
$ln\, g_t$	0.002	0.310	0.416	0.700
$ln\, e_t$	0.001	0.084	0.125	0.915
	C. VARMA BEKK model			
o_t	−0.021	0.342	0.401	1.592
g_t	0.174	0.438	0.538	1.133
e_t	0.149	0.373	0.465	1.367
	D. Random walk			
$ln\, o_t$	0.094	0.225	0.291	0.963
$ln\, g_t$	0.001	0.311	0.460	0.998
$ln\, e_t$	−0.015	0.119	0.239	1.000

Figure 2: Dynamic Conditional Correlations Between Oil, Natural Gas and Electricity Returns.