Fiscal Integration with Internal Trade: Quantifying the Effects of Federal Transfers in Canada†

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Abstract

Fiscal transfers between regions exist within many countries. Explicit transfers, such as Canada's equalization program, redistribute funds directly. Countless federal revenue and spending programs do so indirectly. Like capital flows between countries, such transfers interact with trade and affect the distribution of economic activity within and between sub-national jurisdictions. Previous research has largely abstracted from trade considerations; we fill this gap. With the aid of a rich quantitative model and detailed data on within-country trade and financial flows, we uncover important effects of fiscal transfers on provincial income, migration, and national GDP in Canada. The effects are large. Transfers lower Alberta's real income by over 8 per cent and its population by over 12 per cent, and increase PEI's real income by 30 per cent and its population by 50 per cent. As employment shifts to lower productivity regions, we find transfers shrink Canada's real GDP by 0.8 per cent and income-sensitive transfers do so by as much as 1.2 per cent — equal to $19-28 billion today. Finally, fiscal transfers affect the size and distribution of gains from internal trade liberalization and spread gains across all regions, even if policy (like the New West Partnership) liberalizes trade only among some.

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1 Introduction

Financial transfers between sub-national regions are ubiquitous. Explicit programs to equalize per capita incomes exist within many countries, including in Australia, Belgium, Canada, China, France, Germany, Switzerland, the United Kingdom, and others.[1] In the United States, while no explicit program exists, many federal revenue or spending programs respond to income (for example, Medicaid and unemployment insurance) and federal income taxes naturally raise more revenue per person from higher-income states. Though such transfers benefit certain regions, they can lower a country's overall level of economic activity. If workers migrate in response to fiscal benefits rather than more fundamental considerations like productivity, then aggregate GDP may suffer.[2] This is the subject of a large literature (Buchanan, 1950, 1952; Boadway and Flatters, 1982; Watson, 1986; Wilson, 2003; Albouy, 2012), but one that largely abstracts from trade. This matters: capital flows and trade imbalances between countries affect resource allocation, incomes and productivity, and there are similar effects within countries. With the aid of a rich quantitative model and detailed data on within-country trade and financial flows, we uncover important effects of fiscal transfers on trade flows, specialization patterns, and the gains from trade. We also show internal trade costs, by amplifying regional income inequality, increase the magnitude of fiscal transfers above what they would otherwise be. Finally, we demonstrate transfers shift employment towards lower-income and lower-productivity regions, which shrinks Canada's aggregate GDP.

The intuition behind the interaction between trade and fiscal transfers is as follows. Imagine a region in autarky, without any imports or exports. Financial transfers into this region are nothing more than "helicopter money" affecting nominal variables, not real. Incomes and prices rise proportionally and real incomes are left unchanged. For gains to exist at all, prices must rise less than incomes, and openness to trade will deliver just that. The degree of trade openness will also matter as incomes and wages will respond differently to transfers depending on the level of trade costs. Consider a world of frictionless trade. If preferences are similar across regions, then consumers will allocate their spending in the same way (that is, to the lowest cost producers) regardless of where they live. Any redistribution of income will not affect demand for any region's output. As a result, wages and prices are left unaffected, though the distribution of income and consumption are. In a world with costly trade, a net transfer into a region will differentially increase demand for locally-produced goods relative to imports. This increases wages and prices, though wages respond more as the presence of imports dampens the price change. Thus, depending on the level of trade costs, financial transfers will affect incomes, wages, and prices in different ways, which will in turn affect migration and aggregate real GDP differently.

We build on this intuition with a rich model of trade and transfers and present the first quantitative estimates of the effects on real income, real wages, trade, and migration from federal fiscal

[1] For a comprehensive examination of fiscal transfer systems around the world, see Boadway and Shah (2007).

[2] In principle, transfers may offset other factors behind inefficient migration — such as provincial resource rents. In such a situation, transfers could (if they are well calibrated) improve efficiency and increase aggregate economic activity. Canada's system of fiscal transfers appears poorly suited to that goal (Albouy, 2012).

transfers in Canada. Specifically, we augment a modern, quantitative trade model featuring multiple regions, multiple interconnected sectors, mobile labour across provinces, and an endogenous fiscal transfer regime. Our model is, at its core, an Eaton and Kortum (2002) trade model with multiple sectors linked through rich input-output relationships as in Caliendo and Parro (2015). Workers are mobile across regions, though not perfectly so. Each optimally chooses where to live and work based on a province's real incomes and their individual (heterogeneous) willingness to move there. On top of this core, we introduce a fiscal transfer system. Instead of exogenous lump-sum transfers typical in the literature, we incorporate endogenous transfers through federal revenue and spending programs that respond to changes in provincial income levels. Essentially, transfers add to labour income and help sustain trade imbalances.

We fit the model to high-quality and detailed data on internal trade and fiscal redistribution between provinces in Canada. Canada provides an ideal setting to study fiscal integration and trade. Not only is high quality data on both readily available, but federally-facilitated financial flows are large and strongly correlate with provincial income. Poor regions receive very large net inflows — on the order of 10-20 per cent of their GDP — and overall transfers are equivalent to roughly 1.9 per cent of Canada's GDP. Not all transfers are due to redistributive federal programs, but a significant portion are. We estimate that transfers that endogenously respond to provincial incomes — which we call "income-sensitive transfers" — are equivalent to 1.4 per cent of GDP. Canada's trade data is also uniquely detailed, and provides a full matrix of trade flows between provinces for multiple sectors. Properly calibrated, the model matches observed trade flows and fiscal transfers. For trade costs, we rely on recent evidence from Albrecht and Tombe (2016), who provide a variety of internal trade cost estimates for multiple sectors and each of Canada's provinces. Internal trade costs are high, and have strong asymmetries: export costs are higher in poorer regions (as Waugh, 2010, found between countries). These asymmetries matter for the effects of fiscal integration.

With this model we perform a variety of counterfactual simulations. First, we quantify the effect of fiscal integration in Canada. We find substantial effects, with real income gains ranging between 15 to 30 per cent for lower-income Maritime provinces and losses of 8 per cent for the highest-income province of Alberta. Migration responses are similarly large, with transfers increasing employment in the Maritime provinces by between 24 to 47 per cent, and decreasing employment in Alberta by over 12 per cent. Overall, we estimate 3.2 per cent of the population shifts away from relatively higher-income provinces of Alberta, Ontario, British Columbia and (to a lesser extent) Saskatchewan towards the other provinces. As higher-income provinces have higher labour productivity, this shift shrinks Canada's overall GDP. In Section 2, we show precisely how this happens with the aid of a simple model and approximate the aggregate costs at 0.3 to 0.4 per cent. With our full model, we find a larger effect: transfers lower real GDP by 0.84 per cent — equivalent to nearly $19 billion annually today. But while transfers may involve aggregate efficiency costs, this does not necessarily imply aggregate welfare is reduced. A more normative perspective considers individual preferences over locations and the potential for transfers to increase welfare, as in Fajgelbaum and Gaubert (2018). If the average person living in a low-income region has a stronger preference for their choice of province than the average person in a high-income region does, then there may be scope for welfare enhancing redistribution across provinces. Though our primary focus is aggregate real incomes and real GDP, we show in Section 4.5 that observed fiscal transfers in Canada may increase aggregate welfare even if real GDP declines.

Aside from these aggregate estimates, our model presents several interesting and important results. First, we show fiscal transfers change the way in which trade and trade costs affect economic activity. Moving to autarky not only eliminates the gains from trade, but also completely eliminates any effect that fiscal transfers have on real income or real wages. Without trade, there can be no trade imbalance and therefore no effect of transfers on real income. In this sense, trade and fiscal integration complement each other. In addition, different types of trade cost changes lead to different effects on provinces. Federal government income taxes are levied on nominal, not real, incomes. Changes in households' income has tax implications while changes in prices do not; the source of the gains from trade therefore matters. Typically, if import costs fall, then so do wages and prices (though prices fall more). If fiscal transfers shift income towards lower-wage areas, then a region with falling import costs will gain from trade *and* from increased transfers. On the other hand, if export costs fall, then wages and prices rise (though wages rise more). In this case, a region with falling export costs will gain from trade *but lose* from reduced transfers. We show these effects are quantitatively important, with large differences between regions and between sectors. We find that when internal asymmetries are eliminated, real income gains are far smaller than the real wage gains for poor regions — less than half as large as in a standard model.

Second, transfers spread the gains from trade even if only a subset of provinces liberalize. Bilateral agreements to liberalize internal trade are a growing trend in Canada. BC, Alberta, and Saskatchewan established the New West Partnership Agreement, for example, which Manitoba recently joined. The Ontario-Quebec Trade and Cooperation Agreement is similar. The international trade literature has conclusively established that bilateral deals can create trade diversion effects which can harm non-members. The same basic logic applies within a country, but in contrast, we find fiscal integration spreads gains to everyone. We show all regions experience gains when only certain provinces liberalize trade. Trade diversion effects still exist, and real wages (typically) fall in regions outside the agreement, but incomes in these regions increase as fiscal transfers more than compensate for these wage losses.

Finally, there are implications for the spatial distribution of economic activity within provinces. As some sectors supply inputs to others, shocks to one part of the economy cascade through, and are multiplied by, these linkages. In addition, fiscal transfers will have differential effects across sectors. A region receiving transfers will see an expansion of sectors close to final consumers (the downstream sectors) relative to input suppliers (the upstream sectors). The source of this effect is intuitive. Financial inflows raise household income in a region, increasing their demand for final goods. Wages also increase in these regions, lowering the competitiveness of upstream sectors that do not see the same increased demand as downstream sectors do. The opposite occurs in regions with financial outflows. This intuition is identical to Acemoglu et al. (2016), who show demand shocks propagate upstream. Measuring upstreamness as in Antras et al. (2012) — that is, as the average number of production stages producers are away from consumers — we find that over 10 per cent of the variation in observed upstreamness across provinces can be accounted for by Canada's system of fiscal integration.

Our work contributes to a number of literatures. First, research investigating the magnitude and consequences of within-country trade costs is a growing area of research, as new data and methods become available (Allen and Arkolakis, 2014; Agnosteva et al., 2014; Atkin and Donaldson, 2015; Redding, 2016; Cosar and Fajgelbaum, 2016). While not directly related to the literature measuring the internal costs of trade, in the appendix we demonstrate that trade cost asymmetries are as important within countries as they are between countries. Following Waugh (2010), we combine trade data with spatial price data to show that poor regions typically face larger costs of exporting than rich regions. Our primary contribution to this literature is to highlight the quantitatively important interactions between internal trade and fiscal integration. The distribution of gains from trade and of national employment depend crucially on inter-regional fiscal transfers.

Second, our results that link fiscal transfers to the composition of economic activity within a province is related to those exploring financial transfers and the structure of trade across countries. Epifani and Crino (2014), for example, find trade surpluses may increase or decrease labour demand in skill-intensive sectors, depending on whether a country is skill abundant or not. Epifani and Gancia (2017) find a strong and robust link between trade balances and industrial composition, empirically and theoretically, and show trade surpluses affect different sectors differently, shifting labour towards tradable sectors. In our analysis, all sectors are tradable and we show different transfers have different effects across sectors that depend on each sector's location along the supply chain. In particular, negative transfers create a trade surplus and shifts activity towards *upstream* sectors in our framework. The transfers in our framework are also endogenous and depend on a region's relative income levels.

Most importantly, we contribute to a large literature investigating the efficiency consequences of potential spatial distortions from fiscal policy within countries. Most recently, Fajgelbaum et al. (2019) show variation in state taxes in the United States is an important source of spatial misallocation. They estimate U.S. aggregate welfare would increase between 0.6 to 1.2 per cent if state taxes were harmonized. Our analysis complements theirs by focusing on spatial differences in federal revenue and spending, rather than state-level policy, and we demonstrate similarly large aggregate implications. Prior research on federal transfers has long focused on worker migration (Buchanan, 1950; Boadway and Flatters, 1982; Watson, 1986; Albouy, 2012); we build on this by allowing for trade and production responses. This is novel and, as we demonstrate, important. Albouy (2012) is perhaps closest to our work. And while he estimates the migration and economic consequences of net fiscal benefit differentials, we focus on federal transfers — a related, though distinct, concept. In addition, this previous literature has explored the aggregate consequences of explicit inter-provincial transfer programs like (and especially) equalization. We go beyond this and consider both explicit and implicit inter-provincial transfers from all federal revenue and spending programs. We also quantify how federal transfers may endogenously respond to changes in internal trade costs, and show they matter for the gains from trade liberalization.

2 Fiscal Integration and Trade Costs Within Canada

We begin our analysis by outlining key features of integration in Canada. We present measures of equalizing transfers between regions, and estimates of the internal trade costs faced by each region and sector. Canada provides a unique setting to jointly examine internal trade and fiscal integration. Not only does detailed trade data exist across provinces and sectors, but federal revenue and expenditures are reported by province. We can therefore precisely measure between-province financial transfers facilitated by the federal government. In this section, we present these data and relate fiscal transfers to provincial incomes and trade imbalances. We end the section with a brief review of existing internal trade cost estimates.

2.1 Fiscal Integration

The federal government transfers funds between regions in many ways. Most prominently, through a specific system of equalization payments, the federal government transfers funds to poorer provincial governments according to a preset formula. The purpose, enshrined in the Constitution, is to "ensure that provincial governments have sufficient revenues to provide reasonably comparable levels of public services at reasonably comparable levels of taxation" (Subsection 36(2) of the *Constitution Act, 1982*). But most transfer programs are not explicitly redistributive. Indeed, the two other major programs — the Canada Health Transfer and the Canada Social Transfer — are together nearly triple the size of equalization but are distributed on an equal per-capita basis across all provinces. Most inter-provincial transfers are instead the side-effect of countless federal programs. Regions with higher employment rates or with disproportionately more high-income households will tend to pay more federal personal income taxes. High-income regions also tend to have higher levels of consumer spending, and therefore pay more federal GST per capita than elsewhere. On the spending side, regions with greater employment and higher incomes will also see lower federal spending on employment insurance payments. And regions with a higher share of elderly individuals, such as in the lower-income Atlantic provinces, receive more Old Age Security and Canada Pension Plan payments.

In Table 1, we quantify transfers as the aggregate of all federal revenue and spending per capita relative to the national average.[3] Specifically, we measure per capita federal spending s_i and revenue r_i relative with the national average values $\bar{s}$ and $\bar{r}$ and define per capita implicit transfers for each province i as the difference between the two: $(s_i - \bar{s}) - (r_i - \bar{r})$. Federal programs are a net inflow due to higher federal spending, lower federal revenue, or some combination of the two.

[3]For a broader review of transfers and redistributive federal revenue and spending programs, see Tombe (2018). The transfer data and decompositions we provide in this section are not novel observations.

Table 1: Measuring Fiscal Transfers in Canada (2007-2016)

	Dollars per Capita Relative to the National Average				*Share of National*	
Province	Federal Expenditure	Federal Revenue	Net Transfers	GDP per Capita	Population (%)	Economy (%)
BC	-619	246	-865	49,253	13.1	12.5
AB	-1,761	4,524	-6,285	78,785	11.2	17.0
SK	616	781	-165	66,512	3.1	4.0
MB	2,471	-894	3,365	46,766	3.6	3.3
ON	85	525	-440	50,631	38.7	37.9
QC	-987	-2,990	2,003	43,375	23.3	19.5
NB	4,518	-1,359	5,877	41,280	2.2	1.8
NS	6,070	-951	7,021	40,069	2.7	2.1
PE	6,358	-1,668	8,026	38,014	0.4	0.3
NL	5,046	1,539	3,507	59,414	1.5	1.7

Summarizes the magnitude and distribution of between-province transfers in Canada. Population data are from Statistics Canada data table 17-10-0005-01 and federal revenue and spending by province data are from table 36-10-0450-01. The data are averaged across the ten years from 2007 to 2016.

Net outflows are the reverse. Across all provinces, the aggregate transfer is half the absolute value of $T_i = ((s_i - \bar{s}) - (r_i - \bar{r})) \times P_i$, where P_i is the population of province i; that is, $T = \frac{1}{2}\sum_{i=1}^{N}|T_i|$. Overall, transfers through federal revenue and spending programs are large. In aggregate, they total nearly two per cent of GDP.

Fiscal transfers are also highly redistributive. For the relatively lower-income Maritime provinces, net inflows are large. Transfers to Prince Edward Island, Nova Scotia, and New Brunswick, for example, are equivalent to 21, 18, and 14 per cent of their respective GDP. For higher-income provinces, transfers are typical outflows and approach -8 per cent of Alberta's GDP. Overall, the correlation between net transfers and real GDP per capita is -0.75. Given this strong negative relationship and the overall magnitude of transfers, they are effective at lowering regional income differences in Canada. The variance of log observed relative per capita GDP, for example, is nearly half the variance of per capita GDP minus transfers. The reason transfers equalize is because federal revenue and spending are often (at least implicitly) a function of income. To illustrate this, we decompose transfers by specific federal program. In Alberta, the highest-income province, personal income tax payments averaged $5,786 per capita between 2007 and 2016, which is nearly $2,168 higher than the national average. Personal income taxes therefore account for one-third of the nearly $6,300 per person total net outflow from Alberta. Decomposing overall national transfers by source is not such a simple comparison, though, since (unlike Alberta) many provinces experience a net inflow due to some components but net outflows due to others. For instance, Ontario is the destination of significant federal spending, as it is the home of the capital city of Ottawa, but also pays disproportionately more in federal income taxes. The contribution of any particular revenue or spending program to aggregate transfers therefore depends on the values of

Table 2: Decomposing the Sources of Fiscal Transfers in Canada (2007-2016)

Component	Share of National GDP (%)	Share of Transfers (%)	Correlation with GDP/Capita
Personal Income Taxes	0.60	31.1	0.88
Equalization / Stabilization	0.44	22.7	-0.77
Corporate Income Taxes	0.20	10.2	0.94
CPP Net Contributions	0.15	8.0	0.80
Non-Defense Purchases	0.11	5.8	-0.68
EI Payments less Receipts	0.11	5.7	0.36
OAS Benefits	0.10	5.4	-0.66
GST and Excise Taxes	0.09	4.6	0.90
Defense Purchases	0.05	2.8	-0.42
Other Items	0.07	3.6	0.05

Decomposes the aggregate between-province transfers by component. Transfers here are defined as half the absolute value of deviations from average per-capita values. Column two displays the share of total transfers T accounted for by each component. See text for details. Column three reports the correlation between each component's implicit transfer across provinces with provincial GDP per capita.

other revenue or spending programs.[4] We therefore quantify the marginal contribution of transfers T_i^j from component j to or from province i on total transfers $T = \frac{1}{2}\sum_i |T_i| = \frac{1}{2}\sum_i |\sum_j T_i^j|$ by adding each in sequence. We average these marginal contributions across all 3.6 million possible orderings, as in Tombe (2018), and report the results in Table 2.

The bulk of transfers are due to components that are sensitive to a province's GDP per capita. Over 40 per cent of all transfers are due to personal and corporate income taxes. Adding explicit transfer programs from the federal government, along with GST payments and net employment insurance (EI) contributions, means roughly three-quarters of transfers automatically respond to economic conditions. To show this more clearly, we report their correlation to GDP per capita in the last column of Table 2. All are intuitive. Higher income regions disproportionately contribute to federal revenue through higher personal and corporate income taxes, and greater EI and Canada Pension Plan (CPP) payments relative to their receipts. Government-to-government transfers are also negatively related to income — primarily due to the equalization program. To be sure, not all components may be related to a region's economic conditions. Defence expenditures per capita are particularly large in Nova Scotia, home of Canada's Atlantic fleet at CFB Halifax. But a large majority of inter-provincial transfers respond endogenously to provincial economic conditions.

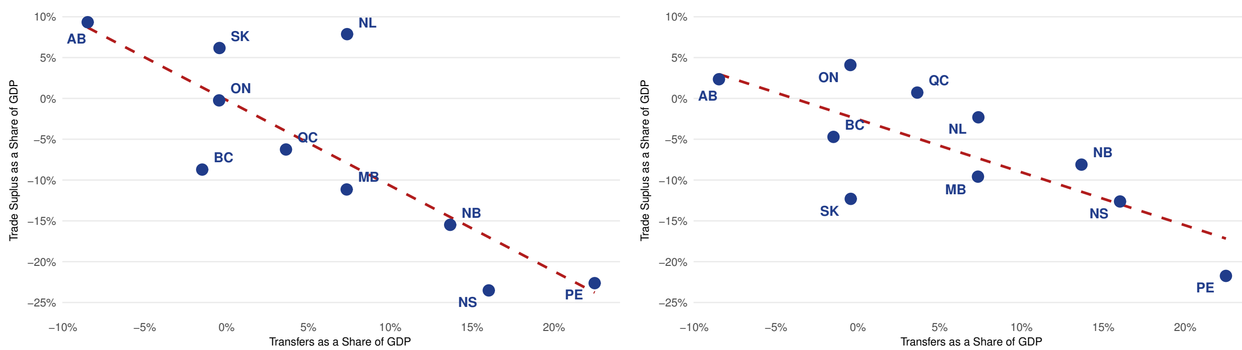
2.2 Aggregate Effects of Fiscal Transfers

Federal fiscal transfers matter not only because they reallocate financial resources across governments, businesses and individuals across Canada. They also affect the distribution of production and expenditures — through their effect on trade flows — and the distribution of population and

[4]More formally, Jensen's inequality implies $\frac{1}{2}\sum_i |T_i| = \frac{1}{2}\sum_i |\sum_j T_i^j| \leq \frac{1}{2}\sum_i \sum_j |T_i^j|$ and holds with equality only if T_i^j are the same sign across all components j within each province i.

Figure 1: Fiscal Transfers and Trade Imbalances (2010)

(a) Overall Trade Surplus vs Transfers   (b) Internal Trade Surplus vs Transfers



Panel (a) displays the relationship between net transfers and trade imbalances. Panel (b) displays the relationship between net transfers and internal trade imbalances (that is, excluding international trade flows). All data is for year 2010.

employment — through migration. Fiscal transfers affect inter-provincial trade flows in particular by sustaining trade imbalances. A region's own income comes from purchases of the goods and services it produces, including exports. It spends this income on other goods and services, including imports. If there are no other sources of income for a region, then its exports and imports will balance. Positive transfers, however, allow the region to purchase more, including more imports, creating a trade deficit. The strong relationship between federal fiscal transfers and trade imbalances is evident in the data. In Panel (a) of Figure 1, we plot provincial trade surpluses against net transfers, both as a share of GDP. In Panel (b), we plot trade surpluses based on inter-provincial trade flows only.[5] In addition, as in Dekle et al. (2007), regions with a trade surplus have lower wages and production costs than would otherwise be the case. In our full model to come, we formalize this idea but this simple intuition suffices for now.

Fiscal transfers also affect the distribution of employment across provinces through their effect on migration incentives. Inflows will draw workers in from elsewhere and outflows will do the reverse. To the extent that such fiscal-induced migration causes some workers to locate in regions with lower labour productivity, transfers can affect Canada's aggregate productivity. To illustrate, consider a simple model where a region's output is a function of productivity A_n and employment L_n according to $Y_n = A_n L_n^{1-\lambda}$, where $\lambda \in (0, 1)$ governs the strength of diminishing returns to labour. Diminishing returns could result from a variety of sources, such as the presence of fixed factors or, as in the full model to come, the dispersion in productivity across different producers. In any case, total national output $Y = \sum_n Y_n$ is maximized when the marginal products of labour are equalized across regions; that is, when $L_n^* \propto A_n^{1/\lambda}$. This also implies the share of employment in any given region equals that region's share of national output. This is not found in the data. As seen in the last two columns of Table 1, differences in population and GDP shares suggest there

[5]We use 2010 since we adopt the trade cost estimates from Albrecht and Tombe (2016), which are for 2010. These trade cost estimates are robust to other years between 2007 and 2014.

are gains from reallocating labour from low productivity regions to high.

This simple model allows us to quantify the magnitude of aggregate losses from an inefficient allocation of labour. Let t_n represent transfers that change per capita income in a region from its underlying wage w_n to post-transfer income $t_n w_n$. If $t_n > 1$ then a region is a net recipient, while if $t_n < 1$ then it is a net contributor. If workers are freely mobile, they will respond to such income differences until post-transfer per capita incomes equalize, such that $t_n w_n = \bar{w}$ for all n. Differences in transfers t_n therefore create a wedge between worker incomes and their marginal products. In particular, a competitive equilibrium here implies $(1-\lambda)A_n L_n^{-\lambda} \propto t_n^{-1}$ or, equivalently, $L_n \propto (t_n A_n)^{1/\lambda}$. Thus, regions that receive positive transfers have lower marginal products of labour (and higher employment), relative to the first-best allocations without transfers. Contributor regions have the reverse. This lowers national output. Indeed, so long as there are diminishing returns to labour ($\lambda < 1$) any variation in t_n across regions *necessarily* lowers aggregate output. As we show in the appendix, one can quantify the magnitude of the aggregate loss in terms of observables using a ratio of means

$$\hat{Y} = \frac{\left(\sum_n l_n t_n^{-1/\lambda}\right)^{-\lambda}}{\left(\sum_n l_n t_n^{-1}\right)^{-1}}, \tag{1}$$

where $\hat{Y} = Y'/Y^*$ is aggregate output, given distortions t_n and labour allocations l_n, relative to the first-best aggregate output Y^*.

This expression is useful both qualitatively and quantitatively. The numerator is the (employment weighted) power mean of t_n, with parameter $-1/\lambda$, while the denominator is the harmonic mean of the same. If $\lambda < 1$ and $var(t_n) > 0$ then, by the Power Mean Inequality, $\hat{Y} < 1$ and differences in transfer rates lowers aggregate output. For a sense of magnitude, if $\lambda = 0.3$ then our data on l_n and t_n in the above expression yields $\ln(\hat{Y}) \approx -0.3$ per cent.[6] To be sure, this simple exercise abstracts from many complications, such as multiple sectors, input-output links, imperfect labour mobility, and other sources of inefficiencies. In the quantitative analysis to come, we use a richer trade model that incorporate these factors. This simple exercise is nonetheless useful both to provide a sense of scale and to transparently highlight the aggregate implications of t_n variation across provinces. It also clarifies how transfers can misallocate labour across provinces and lower aggregate productivity. Transfers are not the only source of differences in marginal products of labour, however, nor are such differences the only way in which transfers matter. As we will demonstrate, the presence of transfers also changes the distribution of gains from interprovincial trade. The next section demonstrates the scope for internal trade liberalization is large.

[6]This is similar to the Harberger triangle created by distortions t_n. The percent change in employment is approximately $(t_n - 1)/\lambda$. The percent change in income is $(t_n - 1)$. Thus the deadweight loss is approximately $\frac{1}{2}\sum_n \omega_n (t_n - 1)^2/\lambda$, where ω_n is region n's share of national output. Given our data, this is -0.6 per cent. This expression also implies the deadweight loss is proportional to the variance of t_n across regions. This expression is also found in the related work of Albouy (2012), who finds the aggregate loss from provincial Net Fiscal Benefit differences (a related but narrower concept from federal transfers) is -0.4 per cent. We choose $\lambda = 0.3$ here for comparability with Albouy (2012). There, the reduced-form income-elasticity of employment is 3.2, which is corresponds to $\lambda = 0.31$ in our simple model.

2.3 Internal Trade Costs

Barriers to internal trade rarely take the form of explicit taxes or tariffs. Although examples exist — the Octroi in Ethiopia or the former Local Body Tax in various Indian municipalities — barriers are typically non-tariff and, therefore, difficult to quantify. Consider sales taxes levied on goods purchased from another state without an offset for sales taxes paid in that other state, taxation of nonresident commercial vehicles, discriminatory liquor laws, local government procurement procedures that favour local suppliers, or restrictions on professional certifications. For Canada, Beaulieu et al. (2003) provide an anecdotal review of a wide variety of inter-provincial trade barriers, covering province-specific occupational licenses, home-biased government procurement, and local marketing boards for agricultural goods.

These examples are illustrative. In our quantitative analysis to come, we use recent and systematic evidence on the magnitude of internal trade costs in Canada from Albrecht and Tombe (2016). We reprint their main results in Table 3. The data we use is the same as in their setting and while our models differ slightly, fiscal transfers do not affect the trade cost estimates. We leave the details to their paper and our supplementary analysis in the appendix, and discuss here only the broad interpretation of their various measures.

The first column of Table 3 reports a summary measure of overall trade costs in Canada based on the Head-Ries Index (Head and Ries, 2001). The Index estimates the average trade cost between two regions, regardless of the direction of trade, relative to the cost of trading within each region (say, between cities). These estimates are unique to each importer-exporter-sector triple, and we aggregate to single summary measures in the table using trade-volume weights. Overall, this measure shows average internal trade costs of nearly 68 per cent in Canada, with larger costs in poorer regions and substantial variation across sectors. This measure is symmetric, in the sense that it does not distinguish between trade costs facing goods moving from Ontario to Quebec, say, with trade costs for goods moving from Quebec to Ontario.

But asymmetric trade costs matter in the data, and will matter in our quantitative analysis. To distinguish exporter- from importer-specific trade costs, one requires data on prices and trade flows because the two types of trade cost asymmetries have different implications for prices, but either can be consistent with observed trade flows alone. If prices in a country are high but imports are low, for example, then the cost of importing must be high. After all, in the absence of high import costs, consumers in a high-price country would import more from elsewhere. Similarly, if prices in a country are low but export flows are too, then the cost of exporting must be high. Between countries, Waugh (2010) demonstrates that export costs fit the data best, and also finds that poor countries tend to have particularly high export costs. In the appendix, we demonstrate that exporter-specific trade costs are also a key feature of internal trade between provinces in Canada. Given these results, we use exporter-specific trade costs in our model. Formally, with a slight abuse of notation, if symmetric costs are $\tilde{\tau}_{ni}^{j}$ and export costs are τ_i^j then the cost of importing into region n from region i is $\tau_{ni}^j = \tilde{\tau}_{ni}^j \tau_i^j$. The second column of Table 3 reports the average values of τ_i^j that Albrecht and Tombe (2016) measure. In general, poor regions tend to have higher export

Table 3: Average Trade Costs Within Canada from Albrecht and Tombe (2016)

(a) By Exporting Province

	Trade-Weighted Averages (Per Cent, Tariff-Equivalent)			
	Average Symmetric Costs	Average Export Costs	Contribution of Asymmetric Trade Costs	Contribution of Non-Distance Trade Costs
British Columbia	78.5	-11.4	6.0	10.5
Alberta	56.1	-15.5	4.1	7.2
Saskatchewan	62.8	11.9	34.1	14.3
Manitoba	74.0	-4.8	11.9	9.8
Ontario	73.5	-15.8	1.3	17.1
Quebec	62.5	-1.4	13.4	17.4
New Brunswick	66.4	7.3	16.4	24.4
Nova Scotia	85.4	14.3	19.0	31.5
Prince Edward Island	106.1	22.2	30.3	32.1
Newfoundland	47.4	-6.8	14.4	3.2
Canada	67.8	0.0	7.8	14.5

(b) By Industry

	Trade-Weighted Averages (Per Cent, Tariff-Equivalent)			
	Average Symmetric Costs	Average Export Costs	Contribution of Asymmetric Trade Costs	Contribution of Non-Distance Trade Costs
Agriculture, Mining	24.4	-25.7	6.3	-8.3
Food, Textiles	42.0	-21.0	5.8	-4.4
Wood	24.9	-14.4	2.1	3.6
Paper	25.7	-17.8	3.4	0.6
Chemicals, Rubber	12.5	-16.7	1.9	1.6
Metals	63.2	-2.8	9.8	11.8
Equipment, Vehicles	37.4	-17.0	4.3	3.1
Manufacturing, n.e.c.	60.2	-9.5	4.8	9.2
Utilities	-	-	-	-
Construction	-	-	-	-
Wholesale and Retail	101.9	-14.8	6.6	14.8
Hotels and Restaurants	97.0	3.4	9.8	29.4
Transport	83.5	-8.6	10.8	16.9
Communication	84.8	19.6	12.2	55.3
Finance	91.7	-5.0	12.4	36.2
Real Estate	192.4	8.4	12.1	57.8
Software	132.3	18.4	19.7	54.6
Other Business Services	90.6	-7.4	8.1	18.7
Public Admin.	-	-	-	-
Education	230.0	66.5	15.5	105.3
Health and Social	245.8	40.1	16.7	82.8
Other Services	134.0	17.1	10.7	44.5

Reports trade cost measures found by Albrecht and Tombe (2016). Details are in section 2.3, and the appendix.

costs than rich regions. This fact will matter when we explore the gains from lower trade costs in the presence of fiscal transfers.

Of course, not all trade costs are under the control of policy makers. It will always cost more to ship from Vancouver to Toronto than from Vancouver to Calgary, for example. To quantify the magnitude of policy-relevant trade costs, Albrecht and Tombe (2016) use two measures. The first estimates by how much trade costs would fall if all asymmetries were removed. Intuitively, consider this exercise as the effect of harmonizing regulations across provinces. With no regulatory differences, trade costs shouldn't differ when moving from Alberta to BC versus BC to Alberta. The second estimates by how much trade costs would fall if only distance mattered. We provide more detail in the appendix, but report these two measures in the third and fourth columns of Table 3. We will use these two policy-relevant measures in the quantitative analysis to come.

3 A Model of Internal Trade, Migration, and Taxes

To quantify the consequences of fiscal integration in Canada, and examine how it interacts with internal trade costs, we develop a model that builds on a recent multi-sector models of trade featuring realistic input-output relationships — specifically, the model of Caliendo and Parro (2015). We augment the model in two ways. First, we explicitly distinguish internal from international trade flows. Second, federal government revenue and spending programs redistribute fiscal resources across provinces to mitigate income differences. The latter component of the model is original to this paper. The core trade components of the model are standard.

3.1 Core Components of the Model

There are $N + 1$ regions; $N = 10$ provinces of Canada plus the rest of the world aggregated as one entity. Each region has L_n workers, who are imperfectly mobile between regions but perfectly mobile between sectors. All labour and product markets are perfectly competitive.

There are J sectors each producing a composite non-tradable final good using a CES technology

$$Y_n^j = \left(\int_0^1 y_n^j(\nu)^{\frac{\sigma^j - 1}{\sigma^j}} \mathrm{d}\nu \right)^{\frac{\sigma^j}{\sigma^j - 1}}, \tag{2}$$

where $y_n^j(\nu)$ are individual product varieties and σ^j is the elasticity of substitution within sector j. The final good is either consumed or used as an intermediate input within region n. Households derive utility from these final goods through

$$U_n = \prod_{j=1}^{J} \left(C_n^j \right)^{\beta^j}, \tag{3}$$

where C_n^j is the amount of the sector-j good consumed out of Y_n^j. Households earn income from inelastically supplying labour to each sector, earning a wage w_n. A government may supplement this income through inter-provincial fiscal transfers (described later).

Goods not consumed are used as intermediates by producers of individual product varieties within region n. There is a continuum of individual product varieties within each sector, produced with labour and material inputs. Production technologies are identical within a sector except for differences in total factor productivity. Across sectors, the importance of various inputs can differ. With wages w_n and the price of sector j goods P_n^j, the cost of an input bundle is

$$c_n^j \quad \propto \quad w_n^{\phi^j} \prod_{k=1}^{J} \left(P_n^k\right)^{\gamma^{jk}\left(1-\phi^j\right)}, \tag{4}$$

where ϕ^j is labour's share and γ^{jk} is the share of intermediates purchased by sector j from sector k. A producer with productivity φ will therefore have marginal costs c_n^j/φ.

Final goods producers will source each product variety from the lowest cost source, either at home or from another region. Products shipped between regions incur an iceberg trade cost $\tau_{ni}^j \geq 1$, where τ_{ni}^j goods must be shipped from region i in order for one unit to arrive at region n. The resulting price paid by buyers in region n for a product of sector j from region i will therefore be $\tau_{ni}^j c_i^j/\varphi$, where φ is the producer's productivity. If productivity across varieties within each sector is identically and independently distributed Frechet, with CDF $F_n^j(\varphi) = \mathrm{e}^{-\left(\varphi/A_n^j\right)^{-\theta^j}}$, then well known results from Eaton-Kortum models follow: the share of region n's total spending on goods from region i in sector j is

$$\pi_{ni}^j = \frac{\left(\tau_{ni}^j c_i^j/A_i^j\right)^{-\theta^j}}{\sum_{k=1}^{N+1}\left(\tau_{nk}^j c_k^j/A_k^j\right)^{-\theta^j}}, \tag{5}$$

and the sector j price index in region n is

$$P_n^j \propto \left[\sum_{i=1}^{N+1}\left(\tau_{ni}^j c_i^j/A_i^j\right)^{-\theta^j}\right]^{-1/\theta^j}. \tag{6}$$

The proportionality constant in the price index is completely irrelevant for our purposes. The parameter θ^j governs the variation of productivity across varieties and A_n^j is the location parameter corresponding to overall average productivity for region n and sector j. Given prices P_n^j of each sector's output, the overall price index for region n is

$$P_n = \prod_{j=1}^{J}\left(P_n^j\right)^{\beta^j}. \tag{7}$$

Given trade shares π_{ni}^j, total sales of sector j goods by region n is $R_n^j = \sum_i \pi_{in}^j X_i^j$, where X_i^j is region i's total expenditures on sector j goods. These expenditures originate from producers buying inputs and from consumer final demand. In the appendix, we detail how input-output linkages and intermediate input spending combine with final demand to solve for equilibrium sales R_n^j. Here, we focus only on the factors influencing household income I_n. First, labour earnings from production is total value-added $w_n L_n = \sum_j \phi^j R_n^j$. If labour income were the only source of income, trade would balance. But other sources of income can sustain trade imbalances S_n and therefore total income is $I_n = w_n L_n - S_n$. If $S_n > 0$ then region n has a trade surplus, while if $S_n < 0$ it has a deficit.

A variety of factors contribute to trade imbalances. First, there may be exogenous transfers $\bar{S}_n$ (perhaps due to private investment flows, for example). Given our data on trade imbalances S_n and government fiscal transfers T_n, exogenous sources of trade imbalances are $\bar{S}_n = S_n + T_n$. Second, a central government taxes and spends in each region. The net effect of such revenue and spending measures may be positive or negative, depending on whether central government revenue from, or spending in, a province is larger. We distinguish between exogenous fiscal transfers $\bar{T}_n$ and income-sensitive. The latter augment labour earnings by a factor t_n such that post-transfer labour earnings are $t_n w_n L_n$. In particular, this income-sensitive transfer is redistributive and governed by

$$t_n \propto w_n^{\zeta}, \tag{8}$$

where $\zeta < 0$ governs the strength of income-sensitive transfers and the constant of proportionality ensures a balanced federal budget.[7] If $t_n > 1$ then a region is a net recipient and if $t_n < 1$ then it is a net contributor. Note also that there are no international fiscal transfers, so $t_{1+N} = 1$ always holds. Given this, and data on fiscal transfers T_n, exogenous fiscal transfers are $\bar{T}_n = T_n - (t_n - 1) w_n L_n$ and total income is $I_n = t_n w_n L_n + \bar{T}_n - \bar{S}_n$, or equivalently, $I_n = w_n L_n + T_n - \bar{S}_n$.

Finally, workers are perfectly mobile across sectors but imperfectly mobile across provinces. We model regional migration in a flexible yet tractable way. We assume workers are heterogeneous in their willingness to move into any given province. That is, some are more willing to live in Prince Edward Island than others. Workers also care about real incomes in each province. They select where to live to maximize real incomes, subject to their individual willingness to move; specifically, $m_n z_n / P_n$, where z_n is captures their willingness to move into province n and $m_n = I_n / L_n$ is per capita income there. Conveniently, a particular distribution of z_n across workers implies the following equilibrium allocation of employment across provinces.

Proposition 1 *If z_n are distributed Frechet with CDF $F_n(x) = \mathrm{e}^{-(x/\delta_n)^{-\kappa}}$, then the share of workers that choose province n is*

$$l_n = \frac{(m_n \delta_n / P_n)^{\kappa}}{\sum_{i=1}^{N} (m_i \delta_i / P_i)^{\kappa}} \tag{9}$$

where $l_n = (L_n / \sum_{i=1}^{N} L_i)$ is province n's share of national employment.

[7] In the appendix, we derive the exact expression for t_n such that $\sum_n t_n w_n L_n = \sum_n w_n L_n$.

Proof: See appendix.

This implies the real income elasticity of a region's employment is constant and equal to κ. If $\kappa = 0$ then workers are immobile. As κ increases, workers' sensitivity to real income differences increases. One can view z_n as a preference parameter, a migration cost, or merely a convenient device to incorporate a reduced-form migration elasticity into the model. We adopt the latter interpretation for most of our results, and therefore individual welfare remains determined by equation 3. As in the simple model of Section 2, the first-best labour allocations equalize real wages (labour productivity) across provinces.[8] Beyond these aggregate efficiency considerations, however, there may be an incentive to redistribute across workers with different locational preferences. Intuitively, the average individual in a lower income region has a stronger preference for their home region than the average individual in a high income region has for theirs.[9] We consider this case explicitly in Section 4.5.

3.2 Counterfactual Real Income, Real Wages, and Employment

Though this is a rich model with many regions, sectors, and input-output linkages, solving for the initial equilibrium and counterfactual changes is straightforward. And much of the model mirrors work by others (Costinot and Rodriguez-Clare, 2014; Caliendo and Parro, 2015; Albrecht and Tombe, 2016). In the appendix, we provide full details of solving the model and here only highlight the three main outcomes of interest: changes in real income, real wages, and employment by province and in aggregate.

The real wage in sector j and province n is simply w_n/P_n^j and therefore the real wage for province n overall is w_n/P_n. We express changes in a variable between its initial value x and its counterfactual equilibrium value x' as $\hat{x} = x'/x$. Under this notation, as we show in the appendix, real wages are affected by trade flows through changes in π_{nn}^j (province n's home-share of spending in each sector j). Specifically,

$$\frac{\hat{w}_n}{\hat{P}_n} = \prod_{j=1}^{J} \left(\hat{\pi}_{nn}^j\right)^{-g^j/\theta^j} \tag{10}$$

where g^j is a term that summarizes the "influence" of sector j on the overall economy through its input-output linkages and its use in final demand. Changes in provincial real wages and employment map naturally to changes in Canada's aggregate real GDP. Real wages equal labour productivity, after all, and aggregate labour productivity in this model is real GDP. Thus, given provincial real wage changes $\hat{y}_n = \hat{w}_n/\hat{P}_n$ and employment changes $\hat{l}_n$, the change in national real GDP is

$$\hat{Y} = \sum_{i=1}^{N} \omega_n \hat{y}_n \hat{l}_n, \tag{11}$$

[8] Analytically, in a one-sector frictionless trade version of our model, one can show $l_n^* \propto A_n^{\theta}$.

[9] This insight is from Fajgelbaum and Gaubert (2018), who provide extensive analysis of optimal spatial policies in a more general setting.

where ω_n is province n's initial share of national real GDP.

Real income changes differ from real wage changes due to trade imbalances and fiscal transfers. Counterfactual real per capita income in province n — which, from equation 3, is also individual welfare — changes according to

$$\hat{U}_n = \left(\hat{m}_n / \hat{P}_n\right), \tag{12}$$

where $\hat{m}_n = \hat{I}_n / \hat{L}_n$ and $\hat{I}_n = (w'_n L'_n - S'_n)/(w_n L_n - S_n)$. Aggregating across provinces, national real per capita income changes according to

$$\hat{U} = \sum_{i=1}^{N} \tilde{\omega}_n \hat{m}_n \hat{l}_n, \tag{13}$$

where $\tilde{\omega}_n$ is province n's initial share of national real income. Both ω_n and $\tilde{\omega}_n$ are related, but differ due to imbalances. Specifically, $\tilde{\omega}_n \propto \omega_n \cdot (1 - S_n / w_n L_n)$.

Finally, equilibrium changes in the distribution of employment from equation 9, given changes in per capita real income $\hat{m}_n / \hat{P}_n$, are

$$l'_n = \frac{l_n \left(\hat{m}_n / \hat{P}_n\right)^{\kappa}}{\sum_{i=1}^{N} l_i \left(\hat{m}_i / \hat{P}_i\right)^{\kappa}}. \tag{14}$$

3.3 Calibrating Model Parameters and Setting Initial Equilibrium Values

To perform our quantitative analysis, we must calibrate parameters $\{\phi^j, \beta^j, \gamma^{jk}, \theta^j, \kappa, \zeta\}$ and set initial equilibrium values for trade and employment shares $\{\pi^j_{ni}, L_n\}$ to match data. For trade shares π^j_{ni}, we use 2010 data from Statistics Canada Table 12-10-0088-01, which provides internal and international trade, production, and expenditure data for each of Canada's provinces and for a variety of commodities and years. We map commodities in the trade and production data to ISIC Rev. 3 and aggregate them to 22 sectors for which positive production exists in all provinces. It is straightforward to calculate π^j_{ni} as the ratio of trade flows in sector j from region i to region n relative to region n's total spending on sector j goods. For employment, we use the Labour Force Survey data for 2010 from Statistics Canada Table 14-10-0090-01.

Many of the model parameters are straightforward to calibrate, as there are readily available counterparts in the data. For production and preference parameters ϕ^j and β^j, we turn to the OECD structural analysis database (OECD STAN). The value-added-to-output ratio of each sector is ϕ^j and the shares of final demand shares allocated to each sector is β^j. We report ISIC codes, value-added to output ratios, final demand shares, and other industry characteristics in the appendix. The inter-sectoral input shares γ^{jk} are also from the OECD STAN, though we do not report them individually.

Next, we calibrate the two Frechet parameters θ^j and κ. First, from equation 5, θ^j is the cost-elasticity of trade. Countless papers estimate these elasticities (Head and Mayer, 2014), though there are no within-country sector-specific estimates that we are aware of. Between countries, however, Caliendo and Parro (2015) estimate elasticities at a similar level of aggregation. As their model is the base upon which ours is built, we adopt their estimates. We follow Costinot and Rodriguez-Clare (2014) for sectors where Caliendo and Parro (2015) do not have estimates, and set $\theta^j = 5$. Second, from equation 9, κ is the real income elasticity of migration. We do not estimate this directly, but others find an income elasticity of migration in the range of 1.5. For example, Helliwell (1996) finds an elasticity with respect to per capita GDP and real disposable income of slightly over 1.5 for Canada. For the United States, Fajgelbaum et al. (2019) estimate $\kappa = 1.39$ within a parameterization of their model that most closely resembles ours. For China, Tombe and Zhu (2018) find elasticities in the range of 1.2 and 1.6. We set $\kappa = 1.5$. In the appendix, we explore the sensitivity of our results to alternative values for both θ^j and κ.

Finally, the parameter ζ governs the strength of fiscal integration. It determines a province's income-sensitive fiscal transfers as a function of a province's underlying wage w_n and affects the size of a province's trade imbalance. We use data on fiscal transfers and trade imbalances in 2010, each relative to GDP, to calibrate ζ. In the model's initial equilibrium, $I_n = w_n L_n - S_n$ and we therefore infer $w_n L_n$ from this initial equilibrium. If all transfers are due to income differences, then $T_n = (t_n - 1) w_n L_n$, and, given equation 8, we have $\log(1 + T_n / w_n L_n) \propto \zeta \cdot \log(w_n)$ where the constant term (which depends on $\bar{w}^{\zeta}$) is common across regions. That is, the cross-province elasticity of transfers (relative to total factor incomes, $w_n L_n$) with respect to wages pins down ζ. We estimate $\zeta = -0.33$. In the appendix, we explore alternative values for this parameter and the sensitivity of our results. Of course, as mentioned earlier, not all transfers are sensitive to income and not all of provincial trade imbalances are due to transfers. We infer exogenous trade surpluses $\bar{S}_n$ and exogenous transfers $\bar{T}_n$ by solving $T_n = (t_n - 1) w_n L_n + \bar{T}_n$ followed by $S_n = \bar{S}_n - T_n$. By construction, these exogenous terms sum to zero across all regions.

4 Quantitative Exercises

With the full model now established, we proceed to our quantitative analysis where we change either fiscal integration or trade costs. We begin with a simple experiment to gauge the effect of current fiscal transfers on incomes, wages, and migration. We then examine how trade and fiscal transfers interact.

4.1 The Aggregate Real Income and GDP Effect of Transfers

To quantify how transfers affect economic activity, we simulate moving from the initial equilibrium to one where there are no fiscal transfers. This is similar to Dekle et al. (2007)'s analysis between countries, but we hold the portion of trade imbalances unexplained by transfers constant. In what follows, we hold trade costs unchanged, so $\hat{\tau}_{ni}^{j} = 1$ for all (n, i, j). Since the simulations involve moving from an initial equilibrium with transfers to a counterfactual one without, the inverse of the changes are the effect of fiscal transfers.

Our first experiment quantifies the effect of Canada's observed between-region fiscal transfers.

Table 4: Effect of Inter-Provincial Transfers (Per Cent Changes)

	All Fiscal Transfers			Equalization Only			Income-Sensitive Transfers		
	Real Income	Real Wages	Emp.	Real Income	Real Wages	Emp.	Real Income	Real Wages	Emp.
BC	-1.7	-0.1	-2.8	-0.9	0.0	-1.3	5.0	-0.1	6.9
AB	-8.3	0.0	-12.5	-1.1	0.0	-1.7	-10.1	0.0	-15.3
SK	-0.9	0.0	-1.6	-1.0	0.0	-1.6	-7.2	0.1	-11.2
MB	7.2	-0.1	10.6	2.3	0.0	3.5	5.6	-0.1	7.8
ON	-0.8	0.0	-1.5	-0.8	0.0	-1.2	-0.3	0.0	-1.2
QC	3.4	0.1	4.8	1.6	0.0	2.5	5.7	0.1	7.9
NB	15.6	0.1	23.8	3.9	0.0	5.9	0.8	0.1	0.5
NS	17.1	0.6	26.3	1.7	0.1	2.5	5.3	0.2	7.2
PE	29.8	0.1	47.3	4.6	0.1	7.0	16.3	0.1	24.5
NL	10.4	-0.2	15.6	-1.0	0.0	-1.5	-2.1	0.1	-3.8
CAN	-0.7	-0.8	–	-0.1	-0.2	–	-1.1	-1.2	–

Displays the per cent change in real incomes (a narrow measure of welfare), real wages (labour productivity), and employment as a result of (1) observed inter-provincial fiscal transfers, (2) Canada's equalization program, and (3) income-sensitive transfers in general. See text for details.

Specifically, we simulate a counterfactual equilibrium where $T_n = 0$, and therefore $S_n = \bar{S}_n$ for all provinces. We find such transfers have large effects on real incomes and migration, though limited effects on real wages. Our main estimates are in the first three columns of Table 4. Real income gains in poor regions are large and losses in rich regions are equally so. This fits our earlier intuition well. Trade deficits are the source of income gains, and recipient provinces (provinces with positive net fiscal transfers) have large deficits and contributor provinces have large surpluses. We find relatively small effects on real wages, with changes ranging from a 0.6 per cent gain in Nova Scotia to a 0.2 per cent loss in Newfoundland and Labrador. These limited changes are due to migration. While financial inflows increase wages more than prices in recipient regions, the resulting migration inflow mostly offsets this effect.

In addition to income changes, we find transfers shift employment across provinces. In New Brunswick and Nova Scotia, for example, employment increases by roughly one quarter and as much as nearly 50 per cent in Prince Edward Island. Alberta, meanwhile, has employment that is over 12 per cent smaller due to fiscal transfers. This reallocation of labour has large implications for Canada's aggregate real GDP, which aggregates provincial real wage changes according to equation 11. Overall, observed transfers reduces national real GDP by 0.84 per cent — equivalent to nearly \$19 billion today. This is larger than the decline found in our simple illustrative model of Section 2, but the underlying intuition is the same: transfers induce workers to shift towards regions with lower labour productivity. To see this, consider an approximation of $\hat{Y}$. As provincial real GDP per capita changes are small, we have $\hat{Y} = \sum_{i=1}^{N} \omega_n \hat{y}_n \hat{l}_n \approx (\sum_n y_n l'_n)/(\sum_n y_n l_n)$. In the counterfactual equilibrium without transfers, employment shifts towards regions with higher real GDP per worker: the correlation between provincial real wage changes y_n and $\hat{l}_n$ is 0.59. This implies $\hat{Y} > 1$ and the inverse of this is approximates the aggregate real GDP losses from observed transfers. We find this approximation is -0.85 per cent, which is nearly identical to our main result of -0.84 per cent. Shifting labour towards regions with lower labour productivity is therefore the key source of the aggregate consequences of observed fiscal transfers.[10]

Specific components of fiscal transfers can also affect aggregate outcomes. One that is particularly salient politically in Canada is the federal equalization program. We explore the implications of such payments within the model by removing them from the initial equilibrium. Specifically, we simulate $S'_n = \bar{S}_n - T_n + \tilde{e}_n$, where $\tilde{e}_n = e_n I_n - \sum_i e_i I_i \left(\frac{I_i}{\sum_m I_m}\right)$ is the net change in fiscal transfers resulting from (1) eliminating equalization and (2) returning the savings to each province in a manner proportional to their GDP. The parameter e_n is actual equalization received in 2010 as a share of provincial GDP, which ranged from 0.2 per cent for Ontario to 6.3 per cent for PEI. We display the results in the middle three columns of Table 4. As with other transfers, recipient provinces benefit from higher real incomes and real wages, with one exception. Ontario received a relatively small payment in 2010 compared to its overall contribution to federal government revenue, and its real income would increase if equalization were eliminated. In terms of migration, just under 1 per cent of Canada's employment shifts across provinces as a result of equalization, with the largest flows between Ontario and Quebec. The former's population share fell 0.6 percentage points while the latter gained a similar proportion. Proportionally, employment increases the most in Maritime provinces, increasing by as much as 7 per cent in Prince Edward Island. The aggregate effect of the equalization program is not trivial. We find national real GDP is 0.16 per cent lower — equivalent to roughly \$3.5 billion today, or approximately one-fifth of the total size of the equalization program.[11] Real income, meanwhile, declines by nearly 0.13 per cent.

Finally, we estimate the effect of income-sensitive transfers only — that is, the subset of federal revenue and spending programs that respond endogenously to a province's average income. This exercise holds fixed those transfers unrelated to a province's underlying economic strength ($\bar{T}_n$) and sets $\zeta = 0$. We display the results in last three columns of Table 4. The overall pattern of real income and real wage changes are similar to the effect of transfers overall. Income shrinks in contributor regions — by as much as 10 per cent in Alberta — and employment migrates towards lower income regions. As with transfers overall, migration almost fully mitigates the effect of transfers on provincial real wages but lowers Canada's aggregate real income by 1.1 per cent and real GDP by 1.2 per cent.

Overall, the changes in real incomes implied by the above simulations are similar to the ratio of transfers to provincial income in the data. For Alberta, T_n/I_n is -8.8 per cent compared to the -8.3 per cent we find in Table 4. Across all provinces, we find the average difference between

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[10]For additional context, the allocation of labour that maximizes aggregate real GDP is one that equalizes real wages across provinces, as in the simple model. Though not explored here, we find such an allocation implies aggregate gains of over 26 per cent relative to the observed allocation of labour. Fiscal transfers are therefore a relatively small factor in understanding the spatial distribution of labour in Canada.

[11]In the appendix, we explore the aggregate implications of equalization in a model that explicitly incorporates source-based provincial tax revenues. This is typically the focus of quantitative studies of equalization, and we find our results change only slightly. As equalization is not our primary focus, abstracting from source-based revenues simplifies the model without overly biasing the results.

Table 5: Gains from Observed Trade Relative to Autarky

	Per Cent Change in		
Region	Real Income	Real Wages	Employment
BC	29.9	21.4	15.4
AB	6.0	18.3	-14.9
SK	19.0	29.0	1.3
MB	36.4	24.2	24.2
ON	11.3	13.5	-8.4
QC	21.1	16.2	3.9
NB	63.4	41.7	62.9
NS	56.9	23.5	53.3
PE	66.5	32.5	67.5
NL	35.7	49.4	23.3
CAN	16.4	16.2	–

Displays per cent change in real incomes (a narrow measure of welfare), labour productivity (real wages), and employment for each province relative to a counterfactual equilibrium of no trade (autarky).

our estimated real income change $\hat{y}_n$ and a simple measure T_n/I_n is 1.6 percentage points. A rough approximation of the effect of transfers therefore gets one most of the way to our estimate of provincial real income changes. There is value in our model nonetheless. In particular, as we'll see in the following sections, how endogenous fiscal transfers and interprovincial trade interact.

4.2 Fiscal Transfers and Interprovincial Trade Costs

We begin with a simple counterfactual experiment often found in the trade literature: estimating the gains from trade. Specifically, we compare an initial equilibrium with observed trade volumes to a counterfactual where trade is zero (that is, trade costs are infinite). Transfers affect the distribution of these gains, because if trade costs are infinite there can be no effect from fiscal transfers. Consider moving all provinces to autarky by simulating $\hat{\tau}^j_{ni} \to \infty$ for all trading pairs $n \neq i$ and sectors j. Comparing counterfactual real incomes, real wages, and employment to the initial equilibrium reveals the gains associated with the observed level of interprovincial and international trade. We display these results in Table 5. Real income gains and real wage gains are often substantially different. In lower income regions, fiscal transfers amplify income gains from trade over and above their real wage gains. In higher income regions, the reverse is true. This is because moving to autarky not only eliminates the gains from trade, but also eliminates any effect fiscal transfers have on real income or real wages. Without trade, there can be no trade imbalance and therefore no scope for net financial inflows to raise incomes more than prices. For rich regions, the opposite is the case. In this sense, trade and fiscal integration complement each other.

Next, we quantify the gains from internal trade liberalization. This is a more policy-relevant counterfactual than quantifying observed trade relative to autarky. Lowering policy-relevant trade

Table 6: Gains from Lower Internal Trade Costs (Per Cent)

	Asymmetric Costs			Non-Distance Costs			All Internal Costs		
	Real Income	Real Wages	Emp.	Real Income	Real Wages	Emp.	Real Income	Real Wages	Emp.
BC	2.8	2.7	-0.6	4.9	4.6	-2.4	59.3	60.2	8.2
AB	3.1	2.5	-0.1	6.3	6.0	-0.3	56.8	54.4	5.6
SK	3.9	8.4	1.1	15.9	16.1	13.4	74.2	85.3	23.6
MB	3.6	4.9	0.6	7.9	7.7	1.9	81.5	93.7	31.5
ON	4.1	2.9	1.3	3.9	3.8	-3.8	38.4	32.1	-12.5
QC	1.9	2.5	-1.8	7.0	7.0	0.6	44.9	45.8	-6.2
NB	2.8	7.2	-0.6	19.2	24.5	18.3	79.8	109.8	29.7
NS	2.1	6.3	-1.6	14.4	19.6	11.2	77.2	110.6	26.9
PE	6.5	15.3	4.8	23.9	29.0	25.4	144.9	199.8	106.1
NL	3.2	5.0	0.0	24.3	22.5	26.0	102.4	109.4	54.8
CAN	3.2	3.3	–	7.3	7.3	–	55.9	56.4	–

Displays the effect on real income (a narrow measure of welfare), labour productivity (real wages) and employment from lowering various measures of internal trade costs. The difference between real income and real wages is due to fiscal transfers.

costs involves eliminating trade cost asymmetries or trade costs unrelated to distance, which we reported earlier in Table 3. We also report the effect of eliminating all internal trade costs, which is less policy-relevant but quantifies how far the economy is from a frictionless benchmark. In each experiment, we find gains are large. Aggregate real GDP increases range from 3.3 per cent from removing trade cost asymmetries to 7.3 per cent from removing non-distance costs. We estimate large employment responses as workers tend to migrate towards lower-income regions. Overall, removing trade cost asymmetries shifts 0.6 per cent of Canada's work force to another province. Removing non-distance trade costs leads 1.8 per cent to move. Importantly, we find large differences between changes in a province's real income and changes in its real wages.

In general, income gains in poor regions are systematically smaller than wage gains because fiscal inflows to those regions decline as wages rise. Put another way, internal trade liberalization lowers cross-province income differences and therefore federal revenue and spending programs that are sensitive to income become more evenly distributed across regions. This also demonstrates that liberalizing trade would decrease the extent of interprovincial transfers. After removing trade cost asymmetries, fiscal transfers fall from their initial level of 1.9 per cent of GDP to 1.6. And after removing non-distance trade costs, fiscal transfers decline to less than 1.8 per cent of GDP. In an important sense, policy-relevant internal trade costs account for between 4 to 12 per cent of observed fiscal transfers. Today, that is roughly equivalent to between $1 and $4 billion per year. This is important. Policy makers in Canada are striving to lower internal trade costs — through various initiatives, but in particular the new *Canadian Free Trade Agreement* — and inter-provincial fiscal transfers are also a pressing political issue for some, especially in Alberta. These results demonstrate that internal trade liberalization can help contribute to lower inter-provincial transfers by reducing regional income inequality.

These results also demonstrate that models of internal trade that abstract from fiscal integration

Table 7: Unilateral Reduction of Import and Export Costs (Per Cent)

Province	Gains from 10 Per Cent Lower Import Costs			Gains from 10 Per Cent Lower Export Costs		
	Real Income	Real Wages	Transfers	Real Income	Real Wages	Transfers
BC	4.3	4.2	0.1	1.0	3.7	-2.6
AB	4.6	4.4	0.2	1.6	4.0	-2.3
SK	5.4	5.6	-0.2	1.8	5.1	-3.1
MB	4.8	5.1	-0.3	1.1	4.5	-3.2
ON	4.4	4.2	0.2	2.1	3.7	-1.5
QC	4.4	4.3	0.1	1.4	3.8	-2.3
NB	5.0	7.7	-2.6	1.2	7.1	-5.5
NS	4.1	5.3	-1.2	0.3	4.7	-4.2
PE	4.9	5.0	0.0	1.1	4.3	-3.0
NL	6.4	6.0	0.4	2.8	5.5	-2.6

Displays the per cent change in real incomes, real wages, and fiscal transfers if each province unilateral lowers its import or export costs by 10 per cent. That is, the "BC" row reports the results of a single experiment where the cost of importing into (or exporting from) BC falls by 10 per cent, and trade costs between all other pairs remain unchanged. Similar, and separate, experiments are run for each province. The collected results are reported here.

will tend to overestimate gains to lower income regions and underestimate gains to higher income ones. This is particularly true for asymmetric trade cost changes, as lower income regions tend to have higher exporter-specific trade costs. The difference between asymmetric trade costs and non-distance trade costs is notable and reveals an important way in which trade cost changes interact with fiscal transfers. Lowering export costs tends to increase wages, whereas lowering import costs tends to decrease prices. And asymmetric trade costs are such that it is typically more costly to export from poor regions than it is from rich ones. Eliminating asymmetries therefore increases wages in poor region, which lowers their transfer payments. The reverse holds in rich regions. This is a previously unexplored and quantitatively significant effect.

To cleanly illustrate the difference between import cost reductions and export cost reductions, we report in Table 7 the effect of lowering one or the other for each province. That is, we simulate reductions in the cost of importing into (or exporting out of) each province, in sequence, one at a time. Lowering import costs in Prince Edward Island, for example, results in real income and real wage gains of 5 per cent. And since the source of these gains are predominately from lower prices, transfers into the province are largely unaffected. If we instead lower the cost of exporting from PEI, real income increases only 1.1 per cent while real wages increase 4.3 per cent. In this case, most of the gains are the result of higher wages, which results in lower transfers from income-sensitive federal programs. There is nothing special about PEI in this case; the pattern holds across provinces. Income-sensitive federal revenue and spending programs shrink the gains from lower export costs but have only minor effects on gains from lower import costs.

4.3 Bilateral Internal Trade Liberalization and Fiscal Integration

Although the intent of the 2017 Canadian Free Trade Agreement is to lower internal trade barriers throughout the country, there is a growing trend for various subsets of provinces to make more detailed and comprehensive deals amongst themselves. BC, Alberta, and Saskatchewan established the New West Partnership Agreement, for example, which Manitoba joined in 2017. This seeks to harmonize regulations and lower barriers to trade in goods and workers between the provinces and improve procurement rules. The Ontario-Quebec Trade and Cooperation Agreement seeks to do the same. An agreement among Maritime provinces to harmonize trucking regulations, which lowers inter-provincial trade costs, is another example. In the international trade literature, it is well known that bilateral deals can create trade diversion effects that can harm non-members. The same basic logic applies within a country, but does fiscal integration allow all regions to benefit from bilateral deals? Benefits to some provinces, after all, will increase their implicit contributions to others, which spreads some of the gains. We quantitatively explore this possibility.

We simulate lowering trade costs between certain groups of provinces by 10 per cent. Specifically, we set $\hat{\tau}^{j}_{ni} = 0.9$ if n and i are both within the group of provinces liberalizing, and $\hat{\tau}^{j}_{ni} = 1$ otherwise. As before, the strength of fiscal integration ζ is held constant and income-sensitive transfers respond endogenously to changing economic circumstances in each province. We choose the sets of provinces to correspond to the three examples of selective agreements just mentioned, though this is by no means an analysis of those agreements. In Table 8, we report the effect on each province's real income and employment from this hypothetical trade liberalization.

Fiscal transfers create gains for all provinces in each of the scenarios we explore. In the first panel of Table 8, we report the effect on each region's real income and employment. In the second, we report alternative results in a model without fiscal transfers. Comparing the two, we see that all regions experience real income gains when only certain provinces liberalize trade. As regions liberalizing trade see incomes rise, they contribute more to federal revenue and receive less in federal spending. As fiscal resources are redistributed to other regions, their real incomes increase too. Fiscal transfers spread the gains from selective trade liberalization. Without fiscal transfers, however, regions not included in the deal to liberalize trade see their real incomes fall.

Workers also migrate towards regions that liberalize trade, away from those that do not, though fiscal transfers dampen the magnitude of such movements. For the three original provinces within the New West Partnership Agreement, for example, employment rises by as much as 2.6 per cent in Saskatchewan and just over 0.8 per cent in Alberta if trade costs decline by 10 per cent. Without fiscal transfers, however, the resulting migration flows would be larger. We estimate a 4.1 per cent increase in employment in Saskatchewan, 1.6 per cent increase in British Columbia, and a 1.4 per cent increase in Alberta, for example, when those three provinces liberalize. In effect, fiscal transfers reduce the incentive to migrate into liberalizing provinces because some of the gains are being spread to all provinces.

The aggregate effect of liberalization, even among a subset of provinces, is to increase Canada's overall real GDP. We estimate 0.6 per cent higher real GDP if trade costs decline by 10 per cent be-

Table 8: Lowering Trade Costs Between Selected Provinces (Per Cent)

	Effect of 10 Per Cent Lower Trade Costs Between					
	BC-Alberta-Sask		Ontario-Quebec		Maritimes (NB-NS-PE)	
Province	Real Income	Emp.	Real Income	Emp.	Real Income	Emp.
BC	1.26	1.10	0.34	-0.94	0.02	-0.04
AB	1.07	0.82	0.31	-0.98	0.02	-0.04
SK	2.24	2.58	0.30	-0.99	0.02	-0.04
MB	0.22	-0.45	0.33	-0.96	0.02	-0.04
ON	0.22	-0.44	1.11	0.21	0.02	-0.04
QC	0.23	-0.44	1.82	1.26	0.02	-0.05
NB	0.24	-0.42	0.31	-0.99	0.47	0.62
NS	0.24	-0.41	0.35	-0.93	0.44	0.59
PE	0.24	-0.42	0.31	-0.99	1.80	2.64
NL	0.20	-0.48	0.26	-1.06	0.01	-0.06
With No Fiscal Transfers ($S_n = \bar{S}_n$)						
BC	1.53	1.61	0.11	-1.35	0.01	-0.09
AB	1.37	1.37	-0.07	-1.61	0.00	-0.11
SK	3.17	4.07	-0.15	-1.74	-0.01	-0.11
MB	0.00	-0.68	0.03	-1.47	0.00	-0.10
ON	0.01	-0.67	1.22	0.30	0.00	-0.10
QC	0.00	-0.68	2.39	2.04	0.00	-0.10
NB	-0.03	-0.73	-0.13	-1.71	1.07	1.51
NS	0.03	-0.65	0.05	-1.44	1.01	1.42
PE	-0.03	-0.74	-0.04	-1.58	3.19	4.72
NL	-0.22	-1.01	-0.28	-1.92	0.01	-0.09

Displays the per cent change in real income and employment in all regions resulting from a liberalization between only certain regions lowering trade costs. These are three separate experiments, where the provinces indicated in the header lower bilateral trade costs by 10 per cent; that is, $\hat{\tau}^j_{ni} = 0.9$ for all sectors and all provinces within the set indicated. All other trade costs are unchanged. Real wage effects are negligible, and therefore not reported.

tween the three western provinces. Aggregate gains exceed 0.9 per cent for the same liberalization between Ontario and Quebec. Given their relatively small size, however, liberalization among the three Maritime provinces has only a minor aggregate effect of less than 0.1 per cent, though their own individual gains are meaningful.

4.4 Effect of Fiscal Transfers on Within-Province Industrial Structure

Beyond the provincial and national aggregates, transfers also change the within-province composition of economic activity and patterns of specialization. These within-province effects are driven by the differential effect that transfers have on individual sectors. Fiscal transfers change a province's level of household income, after all, and if income rises then sectors that produce goods mainly for final consumption will expand, bidding up wages. Sectors producing mainly intermediate inputs will see rising costs but do not experience as large an increase in demand. The reverse will hold in provinces whose incomes decline due to fiscal transfers. To investigate this formally, we must define a notion of how far a sector is from final consumers. Sectors that are "far" from final consumers are "upstream" sectors.

How can we measure upstreamness? Let's start with the classic notion of forward linkages. Consider a matrix $\tilde{\mathbf{B}}$ collecting the share of each sector's output going to each other sector as inputs.[12] That is, the element in the i^{th} row and j^{th} column is the share of sector i's output used by sector j as inputs. The row-sum of this matrix is a measure of each sector's direct forward linkages. There are also indirect forward linkages, as supplying inputs to sector j is indirectly supplying inputs to any sector supplied by sector j. The total direct and indirect forward linkages is the row-sum of the Ghosh Inverse Matrix $(\mathbf{I} - \tilde{\mathbf{B}})^{-1}$. Sectors with many forward linkages are considered upstream.

This forward linkage measure has a very long history, though more recently Fally (2012) develops another measure of a sector's "distance" to final consumers. If a sector sells output to relatively upstream sectors, then Fally (2012) posits that this sector is also upstream. He defines upstreamness of sector j as $u^j = 1 + \sum_k b^{jk} u^j$, where b^{jk} are the elements of the output matrix $\tilde{\mathbf{B}}$ just described. Solving this equation yields $\mathbf{u} = (\mathbf{I} - \tilde{\mathbf{B}})^{-1}\boldsymbol{\iota}$, where $\boldsymbol{\iota}$ is a $J \times 1$ vector of ones, which is the classic total forward linkage measure described in the previous paragraph. Antras et al. (2012) show that this measure is also equivalent to the average number of production stages each sector's output is from final consumers. We proceed with this intuition in mind.

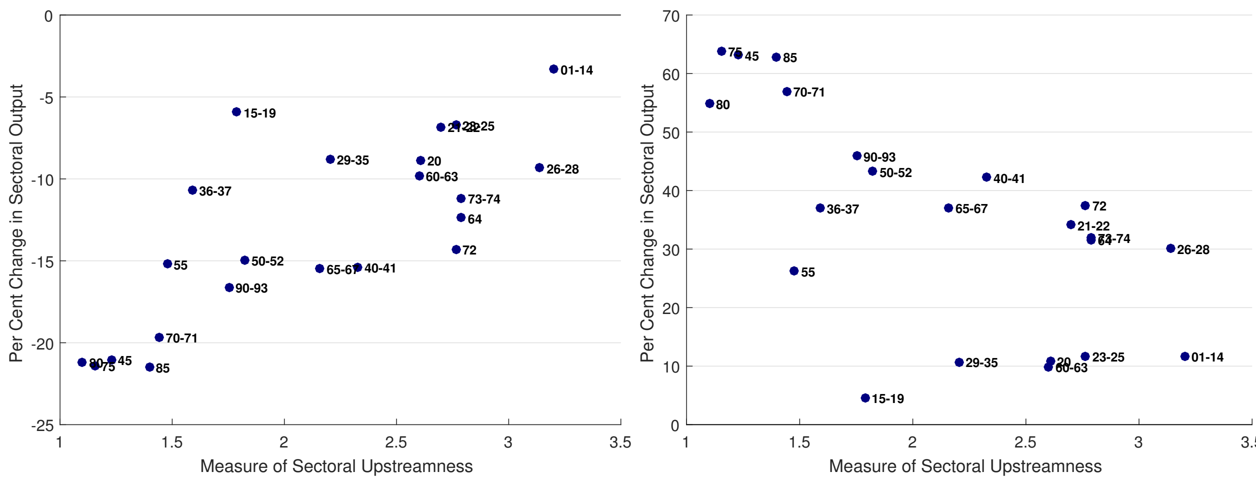
In autarky, the average upstreamness of all provinces is the same, as household preferences and production technologies are the same across all regions. Without trade, the distribution of economic activity across sectors will therefore be the same. With trade, specialization takes place and different regions expand output of different sectors relative to others — comparative advantage at work. In the data, higher-income regions tend to concentrate relatively more in upstream sectors. This is consistent with recent evidence that upstreamness in international exports increases with a country's income (Antras et al., 2012).

Fiscal transfers, however, also affect the pattern of specialization. We discussed the intuition at the beginning of this section and display our quantitative results in Figure 2. We plot the effect of fiscal transfers on each sector for two provinces, Alberta and Prince Edward Island. That is, we compare each industry's size in the initial equilibrium with a counterfactual equilibrium with $T_n = 0$ and therefore $S_n = \bar{S}_n$. For Alberta, there is a clear positive relationship between a sector's upstreamness and the change in its output. Relatively upstream sectors do better than downstream ones. For PEI, the effect is reversed. This confirms the intuition with which we opened this section. Provinces that receive net transfers see downstream sectors expand

[12] This is a direct corollary of the standard direct requirements matrix, denoted $\tilde{\mathbf{A}}$. In a single-region closed-economy version of our model, sectoral output would be exactly $\mathbf{R} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1}\boldsymbol{\beta}$. The elements of the output matrix $\tilde{\mathbf{B}}$ would then be $\gamma^{kj}(1 - \phi^k)R^j/R^k$ for the k^{th} row and j^{th} column. It is straightforward to show $\tilde{\mathbf{B}} = \hat{\mathbf{R}}^{-1}\tilde{\mathbf{A}}\hat{\mathbf{R}}$, where (with an abuse of our hat-notation) $\hat{\mathbf{R}}$ is a diagonal matrix of the vector $\mathbf{R}$.

Figure 2: Within-Province Effect of Fiscal Transfers

(a) Alberta (A Net Contributor) (b) Prince Edward Island (A Net Recipient)



Displays the change in industry output (equivalently real value-added) in Alberta and PEI. Industry codes are ISIC Rev. 3 two-digit codes. These patterns are representative of all net contributors and net receipients of fiscal transfers. The horizontal axis is each industry's upstreamness, as measured by the average number of production stages away from final consumption. It also corresponds to the classic total forward linkage measure as the row-sum of a Ghosh Inverse Matrix. See section 4.4 for details.

relative to upstream sectors. Interestingly, while provinces that are net contributors experience real income and real wage losses, different sectors are affected differently. Upstream sectors within net contributing provinces, for example, see share of a province's overall economy increase due to transfers while downstrem sectors see their shares decrease. Equivalently, there is a spatial reallocation of economic activity: upstream sectors shift towards higher income provinces. Across all provinces, the effect varies. Alberta's upstreamness measure is two per cent larger than it would be without fiscal transfers, while Nova Scotia and PEI see their upstreamness decline by 3.6 and 4.6 per cent, respectively. This is quantitatively important, as the variation across provinces is relatively small. Average upstreamness is just over 2, with a standard deviation of 0.12. The increase in Alberta's upstreamness is therefore nearly half of a standard deviation whereas the decline in PEI is equivalent to nearly 80 per cent of a standard deviation. Finally, as higher income provinces tend to be more upstream than lower income provinces, transfers increase the variation in upstreamness across provinces. We find the standard deviation of upstreamness increases by 11 per cent due to transfers.

4.5 Extension: Aggregate Welfare and Location-Specific Tastes

We conclude our analysis with a slight extension to our baseline model. We previously presumed individuals derive utility from consumption alone. In effect, this restricted our analysis to aggregate efficiency considerations. A more normative perspective would also consider individual preferences over locations. In this section, we explicitly include worker-specific location preferences z_n by changing equation 3 to

$$U_n = z_n \cdot \prod_{j=1}^{J} \left(C_n^j\right)^{\beta^j}. \tag{15}$$

If, as before, z_n is identically and independently distributed Frechet across workers then one can show the average welfare of workers in all provinces is

$$U = \left(\sum_n (m_n \delta_n / P_n)^\kappa\right)^{1/\kappa}, \tag{16}$$

which is (almost) the denominator of equation 9. Given optimal location choices by workers, this implies the average welfare among residents of all regions are equal. That is, $U = \delta_n l_n^{-1/\kappa} m_n P_n$ for any provinces n. In this expression, the average taste for region n among those that optimally choose to live there is $\delta_n l_n^{-1/\kappa}$, which is inversely related to its real income. This matters. Given some change in real incomes $\hat{m}_n / \hat{P}_n$ and the optimal reallocation of labour $\hat{l}_n$, changes in aggregate welfare are

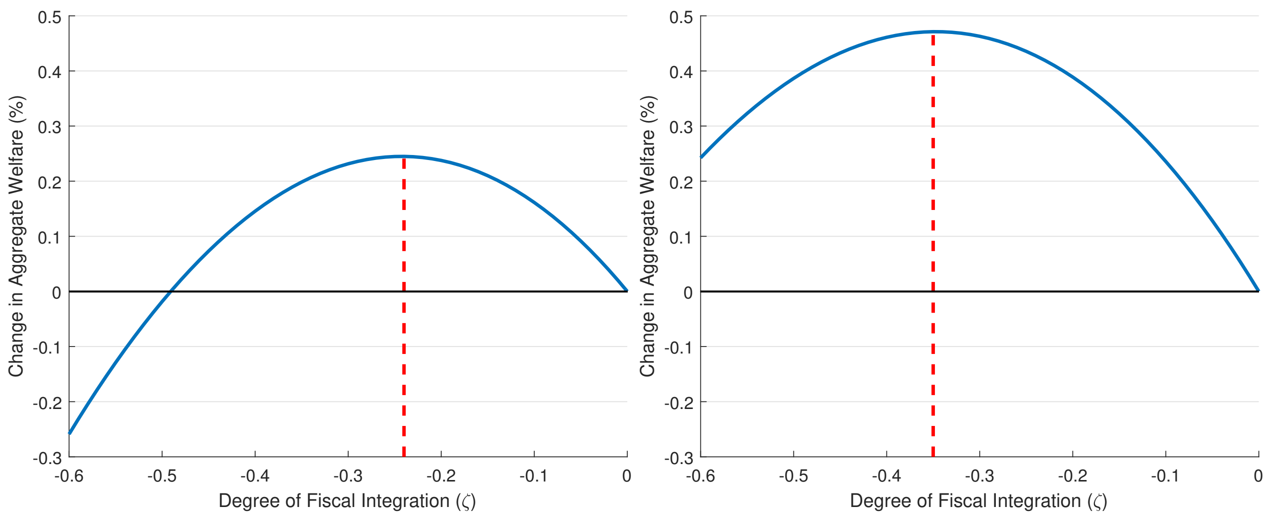
$$\hat{U} = \hat{l}_n^{-1/\kappa} \hat{m}_n / \hat{P}_n, \tag{17}$$

for any province n. The term $\hat{l}_n^{-1/\kappa}$ implies that as a province's population increases, the average resident's taste for that region declines as individuals who value living there less move in. This effect is perfectly balanced, in every region, by changes in real income.

In this setting, there is scope for inter-provincial transfers to increase welfare — even if they lower aggregate real incomes and real wages. The intuition is straightforward. Individuals who live in low-income regions will tend to have stronger tastes for that region. If location preferences complement consumption, as in equation 15, then individuals in low income regions will have a higher marginal utility of income than individuals who live elsewhere. For a social planner that places equal weight all individuals, there is an incentive to engage in (at least some) redistribution despite the aggregate production inefficiencies such transfers entail. This is not new insight but is explored extensively in Fajgelbaum and Gaubert (2018), who examine optimal spatial policies (including fiscal transfers) in the United States. Our formulation here corresponds closely to their special case of no spillovers from local amenities, though in their formulation the central government taxes income and redistributes the revenue on an equal per capita basis. They demonstrate that the optimal income tax rate in this case is, using our notation, $\kappa^{-1}/(1+\kappa^{-1})$. Our fiscal transfer regime, governed by $t_n \propto w_n^\zeta$, is not identical to theirs but is similar in the sense that per capita income and wages are log-linearly related in our model, but linearly related in a tax-and-rebate approach.

With this augmented model, we explore how U depends on varying degrees of fiscal integration through ζ. To accomplish this, we simulate a counterfactual where $T_n = 0$ for all n and therefore $S_n = \bar{S}_n$. Then, starting from that counterfactual equilibrium, we simulate ever greater degrees

Figure 3: Optimal Fiscal Integration with Location-Specific Tastes



Displays the change in aggregate welfare from various degrees of fiscal integration, relative to an initial equilibrium with either no fiscal transfers and only exogenous initial trade imbalances $\bar{S}_n$ (panel (a)) or only exogenous transfers and trade imbalances $\bar{S}_n - \bar{T}_n$ (panel (b)). This differs from aggregate real income explored in the main text and includes location-specific tastes z_n into the utility of individuals.

of fiscal integration with ζ ranging from 0 to -0.6. We illustrate the implied aggregate welfare changes, relative to the no-transfers equilibrium, in panel (a) of Figure 3. In this case, the utility-maximizing ζ, accounting for location-specific taste differences, is -0.24. This is less than the observed relationship between t_n and per capita incomes that we estimate from data. But the optimal degree of income-sensitive transfers also depends on the exogenous transfers through $\bar{T}_n$. Starting from an initial equilibrium with imbalances at $S_n = \bar{S}_n - \bar{T}_n$, the welfare maximizing level of ζ is -0.35. We display this in panel (b) of Figure 3. Relative to our estimated value of -0.33, the observed degree of income-sensitive transfers may be close to welfare maximizing if one considers the potential gains from redistribution across individuals with differing location preferences. For completeness, starting from a balanced-trade initial equilibrium where $S_n = 0$ for all n, we find the welfare maximizing ζ at -0.39 — nearly identical to the optimal income tax rate $\kappa^{-1}/(1+\kappa^{-1}) = 0.4$ implied by Fajgelbaum and Gaubert (2018)'s model. As our main focus was on aggregate efficiency, a richer exploration of optimal transfers and the nature of location-specific tastes, amenities, trade imbalances, and so on, is ripe for future work.

5 Conclusion

Fiscal transfers that ease income disparities are very common. Even in countries that do not explicitly transfer between rich and poor regions, many national revenue or spending programs implicitly do so. While a large body of work studies the effects of fiscal integration, and its effect on regional migration, the literature typically abstracts from trade. We show that fiscal transfers and internal trade interact in important ways. To do this, we expand an otherwise standard quantitative trade model to feature both endogenous fiscal transfers and imperfect worker mobility, which we calibrate to match detailed data on trade and financial flows between Canadian provinces.

Through various counterfactual simulations, we find transfers increase income and employment in lower-income regions, sometimes dramatically so. The reverse is true in contributor (rich) regions. Real income gains to poor provinces are on the order of 15 to 30 per cent while real income losses in higher income provinces can exceed 8 per cent. Employment in poor regions also increases, by between 25 and 50 per cent, while employment in Alberta declines more than 12 per cent. This migration shrinks Canada's economy as workers move to lower income regions. We find real GDP in Canada is 0.8 smaller due to fiscal transfers. But while there are negative efficiency consequences, we present suggestive analysis that if one considers individual preferences over locations then observed fiscal transfers may improve welfare.

Beyond the effect of transfers on GDP and migration, we uncover quantitatively important effects of fiscal transfers on trade flows, specialization patterns, gains from trade, and the effect of trade policy. Transfers represent a demand shock to downstream industries. In recipient regions, transfers expand downstream sectors more than upstream ones. In contributor regions, the reverse is true. Finally, we find fiscal integration increases the dispersion of gains from trade across Canada's provinces — amplifying gains for lower income provinces and dampening them elsewhere. Gains from trade policy changes are also affected. Gains to poor regions from asymmetric trade cost reductions are reduced as fiscal transfers are effectively clawed-back in regions where wages rise. And bilateral trade deals, which can harm non-members, actually benefit everyone when fiscal integration is strong. Overall, this research uncovers novel results of the effects of fiscal integration on the distribution of employment and economic activity in Canada.

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Appendix

Supplementary Material and Analysis

Table 9: Provincial Data and Initial Equilibrium Values

Province	Emp. Share	Transfer to Income Ratio T_n/I_n	Surplus to Income Ratio S_n/I_n	Share of National Factor Income w_nL_n	Share of National Total Income I_n	Relative Real Per Capita Income	Average Upstream-ness
BC	12.9	-1.9	-6.5	11.1	11.9	0.91	1.97
AB	12.0	-8.8	11.6	16.7	15.0	1.34	2.18
SK	3.0	-0.7	8.4	3.8	3.5	1.32	2.17
MB	3.8	7.0	-8.9	3.2	3.5	0.89	1.97
ON	38.4	-0.8	2.0	39.2	38.4	0.92	1.99
QC	23.1	3.3	-4.0	19.9	20.7	0.88	2.00
NB	2.2	13.4	-13.3	2.1	2.5	0.87	2.05
NS	2.8	15.7	-21.3	2.2	2.8	0.79	1.90
PE	0.4	22.0	-20.4	0.3	0.3	0.75	1.86
NL	1.4	7.0	10.1	1.5	1.3	1.33	2.25

Displays the value of selected provincial variables and initial equilibrium values. Data are for years 2010. Initial equilibrium values are used in the quantitative simulations of the model in relative changes. Real income adjusts per capita nominal GDP by the spatial price index (all-items) reported in Statistics Canada Data Table 18-10-0003-01, relative to the national average.

Table 10: Industry Data from OECD-STAN

Industry	ISIC Rev. 3 Codes	Value-Added Share, ϕ^j	Final Goods Share, β^j	Input-Output Mult., μ^j	Upstream Measure, u^j	Trade Elasticity, θ^j
Agriculture, Mining	01-14	0.63	0.014	0.128	3.202	11.92
Food, Textiles	15-19	0.33	0.050	0.089	1.789	4.56
Wood	20	0.35	0.001	0.014	2.608	10.83
Paper	21-22	0.43	0.009	0.044	2.698	9.07
Chemicals, Rubber	23-25	0.21	0.027	0.120	2.764	19.16
Metals	26-28	0.34	0.005	0.090	3.139	5.02
Equipment, Vehicles	29-35	0.26	0.086	0.197	2.204	6.19
Manufacturing, n.e.c.	36-37	0.45	0.015	0.023	1.590	5.00
Utilities	40-41	0.73	0.013	0.033	2.327	5.00
Construction	45	0.40	0.133	0.154	1.228	5.00
Wholesale and Retail	50-52	0.61	0.110	0.185	1.824	5.00
Hotels and Restaurants	55	0.49	0.037	0.048	1.477	5.00
Transport	60-63	0.50	0.017	0.060	2.601	5.00
Communication	64	0.59	0.001	0.009	2.788	5.00
Finance	65-67	0.55	0.058	0.135	2.159	5.00
Real Estate	70-71	0.78	0.114	0.147	1.442	5.00
Software	72	0.57	0.006	0.035	2.764	5.00
Other Business Services	73-74	0.66	0.005	0.072	2.789	5.00
Public Admin.	75	0.51	0.140	0.154	1.154	5.00
Education	80	0.79	0.057	0.061	1.100	5.00
Health and Social	85	0.71	0.048	0.071	1.397	5.00
Other Services	90-93	0.61	0.057	0.095	1.753	5.00

Industry data from the OECD Structural Analysis Database. The Input-Output Multiplier μ^j is the j^{th} element of $(\boldsymbol{I}-\tilde{\boldsymbol{A}})^{-1}\boldsymbol{\beta}$, where $(\boldsymbol{I}-\tilde{\boldsymbol{A}})^{-1}$ is the Leontief Inverse Matrix and $\boldsymbol{\beta}$ if the vector of final goods shares β^j. The trade elasticity is from the Caliendo and Parro (2015) estimates, averaged up to a slightly higher level of aggregation. Sectors 40 and above have elasticities of 5, consistent with Costinot and Rodriguez-Clare (2014). The measure of upstreamness is the average number of production stages output from each sector is from final consumers; it is the row-sum of the Ghosh Inverse Matrix $(\boldsymbol{I}-\tilde{\boldsymbol{B}})^{-1}$ described in section 4.4.

Measuring Trade Costs in Canada

We adopt the Albrecht and Tombe (2016) measure of trade costs within Canada. Their results are replicated in Table 3. For added clarity, we expand upon their results to illustrate the importance of asymmetries. We also provide further evidence that asymmetries are best characterized by exporter-specific trade costs within Canada.

How large are trade costs in Canada? For a broad class of models, one can infer barriers to trade from observable data on trade flows and production, conditional on an assumption for the cost-elasticity of trade (Head and Ries, 2001; Novy, 2013). This estimate is known as a Head-Ries Index and takes the form,

$$\bar{\tau}_{ni} = \left(\frac{\tau_{ni}\tau_{in}}{\tau_{nn}\tau_{ii}}\right)^{\frac{1}{2}} = \left(\frac{x_{nn}x_{ii}}{x_{ni}x_{in}}\right)^{\frac{1}{2\theta}}, \tag{18}$$

where $\bar{\tau}_{ni}$ is the geometric-average of actual trade costs, x_{ni} is the trade flows imported by region n that originate from region i, x_{nn} is the output of region n consumed locally, and θ is the cost-elasticity of trade. Interpreting this measure is simple, as it represents an iceberg trade cost: a producer in region i must ship τ_{ni} units of a good for one unit to arrive at the destination region n. However, the τ_{nn} terms in the denominator of equation 18 make clear that we can only measure trade costs *relative to within-region* trade costs. A value of $\bar{\tau}_{ni} > 1$ therefore implies inter-regional trade is more costly than trade within a region (say, between cities). Finally, this measure is valid whether a country's total trade balances or not. The model we develop in Section 3 features endogenous trade imbalances and the above expression holds.

To measure $\bar{\tau}_{ni}$, we require data on trade flows x_{ni} and gross output consumed locally x_{nn}. We use sectoral data on inter-provincial trade, international trade, and gross output in 2010 from Statistics Canada's CANSIM Table 12-10-0088-01. In this section, we work only with aggregated data. In the quantitative analysis in the main paper, we work with 22 sectors for which each province has at least some production. Finally, we require a value for the cost-elasticity of trade θ. We review the evidence in Section 3.3 but here we simply set $\theta = 5$. Any particular trade cost measure we present can be easily rescaled to other values.

What about trade cost asymmetries? The Head-Ries measure is symmetric by construction. There are two ways to measure trade cost asymmetry. First, we can use price differences between regions along with data on trade flows to infer trade costs. Second, we can infer them from fixed-effects within a standard gravity regression. Let's begin with the price-based measure. As Waugh (2010) demonstrates, from the same large class of trade models for which equation 18 holds, we have

$$\tau_{ni} = \frac{P_n}{P_i}\left(\frac{\pi_{ni}}{\pi_{ii}}\right)^{-\frac{1}{\theta}}, \tag{19}$$

where τ_{ni} is the cost for region n to import from region i, P_n is the aggregate price index in region n, and π_{ni} is the fraction of region n expenditures allocated to goods from region i. We have spatial price data for Canadian provinces through the inter-city price index constructed by Statistics Canada.[13] These are price *level* comparisons, not standard CPI price indexes. Using these data and the trade data outlined earlier, we can calculate τ_{ni} using this expression. In Table 11, we provide our estimates of $\bar{\tau}_{ni}$ and τ_{ni} for all regional pairs within Canada. For example, British Columbia incurs a 183 per cent tariff-equivalent cost of trade when it imports from Manitoba; the reverse flow, Manitoba's imports from British Columbia, incur only a 111 per cent cost. Overall, poorer regions, such as the Maritime provinces, tend to display higher trade costs costs in general, and higher costs of exporting in particular, than richer regions of Canada. Importantly, the trade costs reported in Table 11 are not particular to 2010. We re-estimated these trade costs for the period 2007 through 2014 and find similar values.

An alternative way to estimate asymmetric trade costs involves a fixed-effect regression. Consider the case where trade cost asymmetries are due to additional export costs — region-specific costs that are incurred regardless of the eventual destination. To measure export costs, we follow Waugh (2010). It is straightforward to show $\ln(\pi_{ni}/\pi_{nn}) = S_i - S_n - \theta \ln(\tau_{ni})$, where the S terms capture region-specific factors such as productivity and factor prices. If $\tau_{ni} = D_{ni}^{\delta}\tau_i$ then

$$\ln\left(\frac{\pi_{ni}}{\pi_{nn}}\right) = \delta \ln(D_{ni}) + \iota_n + \eta_i + \epsilon_{ni}.$$

where $\eta_i = S_i - \theta \ln(\tau_i)$ and $\iota_n = -S_n$. We infer the exporter specific trade costs from fixed-effect estimates $\hat{\tau}_i = e^{-(\hat{\eta}_i+\hat{\iota}_i)/\theta}$ and adjust the symmetric trade cost measure $\bar{\tau}_{ni}$ with

$$\tau_{ni} = \bar{\tau}_{ni}\sqrt{\hat{\tau}_i/\hat{\tau}_n}. \tag{20}$$

If trade cost asymmetries were the result of region-specific *import* costs, we identify them in the same way but instead $\tau_{ni} = \bar{\tau}_{ni}\sqrt{\hat{\tau}_n/\hat{\tau}_i}$.

We plot both sets of τ_{ni} estimates in Figure 4. This exercise is important, as we do not have sectoral price level data across all regions. Based on the aggregate results, the export-cost specification is a good match to the price-based estimates. We therefore use the export-cost specification to estimate trade costs for all sectors.

Incorporating Source-Based Taxes

In the main text, we quantified the aggregate productivity cost of Canada's equalization program but abstracted from the presence of provincial revenue sources not paid by residents. Source-based taxes, such as corporate taxes, resource revenues, investment income, and so on, allow a provincial government to provide public services or lower taxes to residents. And since provinces differ

[13] We consider P_i as the simple average of the spatial price index across the following goods: alcoholic beverages, bakery and other cereal products, clothing and footwear, dairy products and eggs, fruit and vegetables, gasoline, household furnishings and equipment, meat, poultry and fish, other food, personal care supplies and equipment, purchase of passenger vehicles, and tobacco products. Our results hold very closely if we only look at the All-Items index.

Table 11: Aggregate Internal Trade Costs in Canada (2010)

(a) Symmetric Trade Costs (Head-Ries Index)

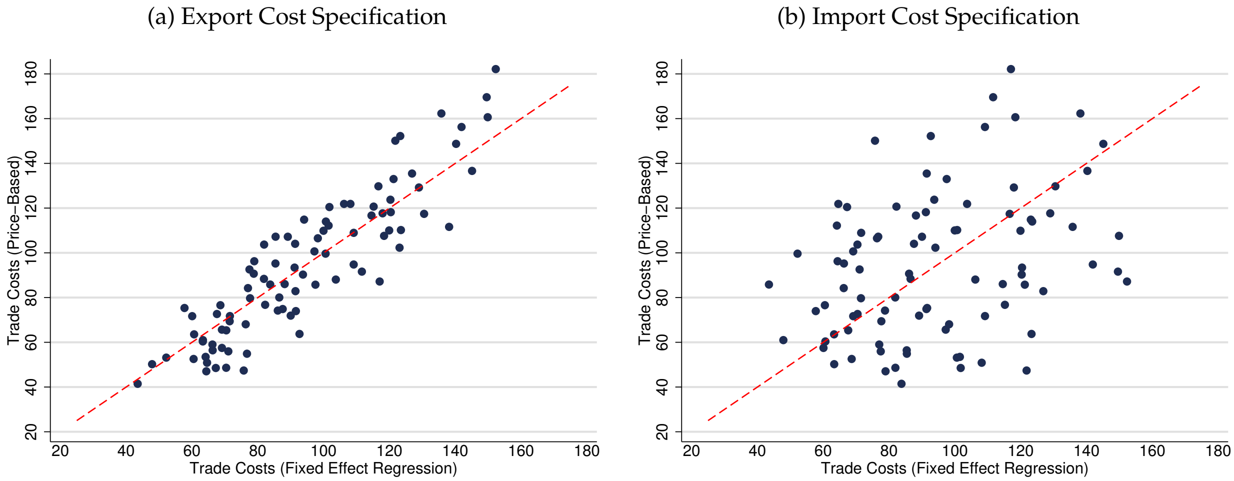
	Exporter									
Importer	AB	BC	MB	NB	NL	NS	ON	PE	QC	SK
AB		86	110	176	198	188	91	269	128	89
BC	86		145	199	251	192	105	284	132	147
MB	110	145		200	196	207	105	282	145	111
NB	176	199	200		91	99	133	114	113	244
NL	198	251	196	91		111	137	186	130	317
NS	188	192	207	99	111		129	125	139	244
ON	91	105	105	133	137	129		168	74	115
PE	269	284	282	114	186	125	168		194	293
QC	128	132	145	113	130	139	74	194		168
SK	89	147	111	244	317	244	115	293	168	

(b) Asymmetric Trade Costs (Price Adjusted)

	Exporter									
Importer	AB	BC	MB	NB	NL	NS	ON	PE	QC	SK
AB		83	152	221	288	226	69	445	110	136
BC	89		183	278	377	245	80	518	123	197
MB	75	111		200	178	201	60	398	105	110
NB	136	136	200		78	92	62	182	76	224
NL	129	158	215	105		100	66	238	93	292
NS	154	147	213	107	122		71	192	98	264
ON	115	132	162	234	238	207		349	84	171
PE	150	139	193	63	141	73	61		89	205
QC	148	142	192	159	174	190	65	358		234
SK	51	105	112	266	343	224	70	407	115	

Our measure of symmetric tariff-equivalent internal trade costs between all Canadian provinces, based on the Head-Ries Index of Head and Ries (2001). Our measures of asymmetric tariff-equivalent internal trade costs between all Canadian provinces. We follow Waugh (2010) and use additional price data to distinguish between the direction of trade for a given pair.

Figure 4: Comparing Two Methods to Estimate Trade Costs



Displays price-based trade cost estimates from equation 19 with the fixed-effect regression estimates from equation 20. Panel (a) interprets fixed-effects results as a province-specific export cost while panel (b) interprets the fixed-effects as an import cost.

dramatically in their access to source-based taxes, there is an incentive for workers to migrate to capture such fiscal benefits, rather than for more fundamental considerations like productivity or amenities. As we demonstrated in the main paper, and the simple model in particular, this misallocates labour across provinces and lowers productivity. Equalization payments to provinces with less access to such revenue sources can, at least partially, offset this rent-seeking migration This idea was first explored by Buchanan (1950), and later in the Canadian context by Boadway and Flatters (1982) and others. Though this theoretical motivation for equalization is sound, in practice it may not improve efficiency outcomes given the many other policies that distort labour allocations. Indeed, Albouy (2012) finds just that.

To ensure our main results are not dramatically overestimating the aggregate productivity cost of equalization, we augment our main model to feature source-based taxes. Specifically, we incorporate economic rents F_n^j as an additive term within each sector's total revenue $R_n^j = \frac{w_n L_n^j}{\phi^j} + F_n^j$. This would result from a production technology that features an additive productivity term, such that the marginal product of labour is no longer proportional to the average product of labour as in a typical Cobb-Douglas technology. Aggregating across sectors within a region results in total income $I_n = w_n L_n - S_n + F_n$, where $F_n = \sum^j F_n^j$. Workers optimally choose to work in each province such that $l_n \propto \left(\left(w_n - s_n + f_n\right)\delta_n / P_n\right)^{\kappa}$, where s_n and f_n are the per worker values of S_n and F_n. The presence of source-based tax revenue therefore distorts the allocation of labour in the same way that other fiscal flows s_n can. In principal, f_n can offset distortions caused by s_n. To explore this possibility, we calibrate the initial value of f_n to match the sum of provincial resource revenues, investment income, and corporate income tax revenue. We then simulate the effect of eliminating equalization payments on aggregate real GDP, as in the main text. We find that instead of lowering aggregate real GDP by 0.16 per cent, it falls by 0.14 per cent. Thus, we conclude abstracting from

Table 12: Effect of Interprovincial Transfers (Per Cent), Alternative κ Values

Region	No Migration ($\kappa = 0$)			Higher Migration Elasticity ($\kappa = 3$)		
	Real Income	Real Wages	Emp.	Real Income	Real Wages	Emp.
BC	-1.9	-0.2	0.0	-1.5	-0.1	-4.7
AB	-8.5	-0.4	0.0	-8.2	0.4	-22.7
SK	-0.7	0.0	0.0	-1.0	0.0	-3.2
MB	8.1	0.6	0.0	6.4	-0.6	20.2
ON	-0.8	-0.1	0.0	-0.7	0.1	-2.4
QC	3.6	0.3	0.0	3.2	-0.1	9.5
NB	16.6	1.0	0.0	14.6	-0.8	50.3
NS	20.2	1.6	0.0	14.7	-0.2	50.4
PE	31.3	2.3	0.0	28.5	-2.0	111.5
NL	7.8	0.1	0.0	25.5	-2.2	97.2

Displays the effect of fiscal transfers on real income, real wages, and employment for different values of the real income elasticity of migration.

source-based revenue sources in our primary analysis does not overly bias our results. To be sure, while exploring the interactions between trade and all the various transfer and tax arrangements is beyond the scope of this paper, it is an area potentially ripe for further investigation.

Alternative Parameter Values

Our main results are robust to alternative parameter values. First, we consider two alternatives where workers cannot migrate between provinces ($\kappa = 0$) or where they are more sensitive to income differences than our baseline model supposes ($\kappa = 3$). We display the results of the gains from transfers analysis in Table 12. There is little difference between the two in terms of real income changes, although it is clear that more elastic migration amplifies the employment change due to fiscal transfers. They become more negative for contributor regions, and more positive for recipient regions. This matters for aggregate changes. With highly elastic migration, aggregate real GDP declines 1.5 per cent. But without a migration response, the real GDP change is nil.

Next consider alternative values for the trade elasticity θ^j. Lower values of θ^j mean a greater dispersion of productivity across varieties, and therefore larger gains from trade. We report the effect of observed fiscal transfers in Table 13. Between a small trade elasticity and large, the real income, real wage, and employment changes due to observed fiscal transfers are largely similar. The migration induced by transfers is also slightly larger with smaller values of θ^j as the real income changes are slightly larger. We also report the effect of removing asymmetric trade costs in the last three columns of Table 13. This requires we re-estimate the trade costs and changes due to trade cost asymmetries that we reported in the main text. We do not report those estimates here. Simulating the gains from removing asymmetric trade costs, we find the real wage change results depend more on the specific values for θ^j, as this parameter directly governs the gains from trade.

Changes in real income and employment are less sensitive. The aggregate real GDP gains from removing trade cost asymmetries ranges from a low of 2.6 per cent if $\theta = 8$ to a high of 6.9 per cent if $\theta = 3$.

Our final robustness exercise concerns the degree of fiscal integration, ζ. This parameter does not affect the results concerning the effect of observed transfers. When we remove both income-sensitive and exogenous transfers, we simply set $T_n = 0$ and ζ plays no role. In the experiments involving removal of income-sensitive transfers, the effect on each province's real income will be tied directly to ζ by construction. Here, we explore how the results of changing trade costs depend on whether ζ is higher or lower than our preferred calibration. Changing trade costs affects income, and therefore affects transfers. In particular, given the discussion in the main text, the stronger the sensitivity of transfers to income, the smaller the gains from lowering trade cost asymmetries will be for poor regions. In Table 14 we report our results for both a lower and a higher value for ζ.

Table 13: Effect of Interprovincial Transfers and Trade Costs (Per Cent), Alternative θ Values

	Effect of Observed Transfers			Removing Asymmetric Trade Costs		
Region	Real Income	Real Wages	Employment	Real Income	Real Wages	Employment
Uniform Trade Elasticity ($\theta^j = 5$ for all j)						
BC	-1.7	-0.1	-2.9	3.5	3.3	-0.7
AB	-8.3	0.1	-12.5	3.9	3.0	-0.2
SK	-0.9	0.0	-1.7	5.9	12.6	2.8
MB	7.2	-0.1	10.6	5.0	6.7	1.5
ON	-0.8	0.0	-1.5	4.8	3.3	1.2
QC	3.4	0.1	4.8	2.6	3.3	-2.0
NB	15.6	0.1	23.9	3.6	9.3	-0.6
NS	17.3	0.8	26.6	2.6	7.8	-2.0
PE	29.8	0.2	47.3	7.7	17.7	5.4
NL	10.1	-0.3	15.2	4.7	7.2	1.0
High Trade Elasticity ($\theta^j = 8$ for all j)						
BC	-1.7	-0.1	-2.8	2.2	2.1	-0.5
AB	-8.3	0.0	-12.5	2.4	1.8	-0.1
SK	-0.9	0.0	-1.6	4.0	8.5	2.2
MB	7.2	-0.1	10.6	3.2	4.3	1.0
ON	-0.8	0.0	-1.5	3.0	2.0	0.7
QC	3.4	0.1	4.7	1.7	2.1	-1.2
NB	15.6	0.1	23.8	2.3	6.2	-0.3
NS	17.1	0.6	26.3	1.7	5.1	-1.2
PE	29.7	0.1	47.2	5.1	11.9	3.9
NL	10.4	-0.2	15.5	3.2	4.8	1.0
Low Trade Elasticity ($\theta^j = 3$ for all j)						
BC	-1.7	-0.2	-3.0	5.8	5.5	-1.3
AB	-8.3	0.1	-12.5	6.6	5.2	-0.2
BC	-1.7	-0.2	-3.0	5.8	5.5	-1.3
MB	7.2	-0.1	10.5	8.4	10.8	2.3
ON	-0.8	0.0	-1.5	8.3	5.9	2.3
QC	3.4	0.1	4.8	4.3	5.3	-3.4
NB	15.7	0.2	23.9	5.9	14.3	-1.2
NS	17.6	1.2	27.1	4.3	11.9	-3.3
PE	29.9	0.3	47.5	11.9	26.8	7.4
NL	9.9	-0.5	14.7	7.1	11.0	0.6

Displays the effect of fiscal transfers and asymmetric trade costs on real income, real wages, and employment.

Table 14: Gains from Lower Internal Trade Costs (Per Cent)

	Asymmetric Costs			Non-Distance Costs		
	Real Income	Real Wages	Emp.	Real Income	Real Wages	Emp.
Lower Degree of Fiscal Integration ($\zeta = -0.2$)						
BC	2.7	2.7	-0.7	4.8	4.6	-2.5
AB	2.9	2.5	-0.3	6.3	6.0	-0.3
SK	6.1	8.2	4.3	16.6	16.1	14.4
MB	3.8	4.9	1.0	7.7	7.7	1.6
ON	3.7	2.9	0.7	3.9	3.8	-3.8
QC	2.1	2.5	-1.5	6.9	7.0	0.5
NB	3.7	7.2	0.8	19.7	24.5	19.1
NS	2.7	6.3	-0.6	14.6	19.6	11.5
PE	7.6	15.2	6.5	23.1	29.2	24.2
NL	4.4	4.9	1.8	24.8	22.5	26.8
Higher Degree of Fiscal Integration ($\zeta = -0.4$)						
BC	2.8	2.7	-0.6	4.9	4.6	-2.3
AB	3.2	2.5	-0.1	6.4	6.0	-0.3
SK	2.8	8.4	-0.6	15.5	16.2	12.9
MB	3.4	4.9	0.3	8.0	7.7	2.1
ON	4.4	2.9	1.7	3.9	3.8	-3.7
QC	1.8	2.5	-2.0	7.0	7.0	0.6
NB	2.3	7.3	-1.3	18.9	24.6	17.9
NS	1.7	6.3	-2.2	14.2	19.6	11.0
PE	5.7	15.3	3.6	24.4	28.9	26.2
NL	2.5	5.1	-1.0	24.0	22.5	25.5

Displays the effect on real income, real wages and employment from lowering various measures of internal trade costs using different levels of the fiscal integration parameter ζ.

Supplementary Derivations and Proofs of Propositions

Deriving the Aggregate Effect of Transfers in a Simple Model

In Section 2 we present a simple model of distortions to show the aggregate efficiency cost of transfers is given by

$$\hat{Y} = \frac{\left(\sum_n l_n(1+t_n)^{-1/\lambda}\right)^{-\lambda}}{\left(\sum_n l_n(1+t_n)^{-1}\right)^{-1}}. \tag{21}$$

We begin this appendix section by providing the derivation.

National output is the sum of regional output $Y = \sum_n A_n L_n^{1-\lambda}$. Labour allocations that equalize marginal products of labour $(1-\lambda)A_n L_n^{-\lambda}$ are given by $L_n^* \propto A_n^{1/\lambda}$, and therefore $l_n^* = A_n^{1/\lambda}/\sum_i A_i^{1/\lambda}$. These are the aggregate output maximizing allocations. Taking observed labour allocations in the data as given, and normalizing the aggregate labour supply to $\bar{L} = 1$, the ratio of Y with those allocations relative to optimal aggregate output Y^* with labour allocations given by l_n^* is

$$\begin{aligned} \hat{Y} &= \frac{\sum_n A_n l_n^{1-\lambda}}{\sum_n A_n \left(l_n^*\right)^{1-\lambda}}, \\ &= \frac{\sum_n A_n l_n^{1-\lambda}}{\sum_n A_n \left(\frac{A_n^{1/\lambda}}{\sum_i A_i^{1/\lambda}}\right)^{1-\lambda}}, \\ &= \frac{\sum_n A_n l_n^{1-\lambda}}{\left(\sum_n A_n^{1/\lambda}\right)^{\lambda}}. \end{aligned} \tag{22}$$

Without loss of generality, if we normalize initial aggregate wages to $\bar{w} = 1-\lambda$, then distorted labour allocations are $l_n = ((1+t_n)A_n)^{1/\lambda}$ and therefore $A_n = l_n^{\lambda}/(1+t_n)$ and $A_n l_n^{1-\lambda} = l_n/(1+t_n)$. Inserting these into in equation 22,

$$\begin{aligned} \hat{Y} &= \frac{\sum_n l_n(1+t_n)^{-1}}{\left(\sum_n l_n(1+t_n)^{-1/\lambda}\right)^{\lambda}}, \\ &= \frac{\left(\sum_n l_n(1+t_n)^{-1/\lambda}\right)^{-\lambda}}{\left(\sum_n l_n(1+t_n)^{-1}\right)^{-1}}. \end{aligned} \tag{23}$$

The power mean inequality establishes that $\hat{Y} < 1$ for $\{t_n\}$ with positive variance if $\lambda < 1$. This demonstrates that variation in t_n lowers aggregate productivity by misallocating labour across regions by sustaining wedges in marginal products of labour.

Deriving the Balanced Budget Income-Sensitive Transfer Rate t_n

Define $t_n = Bw_n^{\zeta}$ and find the term B such that the central government budget balances. The government budget balances if $\sum_{n=1}^{N}(t_n - 1)w_n L_n = 0$ and therefore

$$\begin{aligned} \sum_{n=1}^{N} w_n L_n &= \sum_{n=1}^{N} t_n w_n L_n, \\ &= B\sum_{n=1}^{N} w_n^{\zeta} w_n L_n, \\ \Rightarrow \quad B^{-1} &= \sum_{n=1}^{N} w_n^{\zeta}\left(\frac{w_n L_n}{\sum_{n=1}^{N} w_n L_n}\right). \end{aligned}$$

Raising each side to the power $1/\zeta$, we have $B^{-1/\zeta} = \left[\sum_{n=1}^{N} w_n^{\zeta}\left(\frac{w_n L_n}{\sum_{n=1}^{N} w_n L_n}\right)\right]^{1/\zeta} \equiv \bar{w}$. We substitute this into the initial definition of t_n to yield our result

$$\begin{aligned} t_n &= Bw_n^{\zeta}, \\ &= \left(w_n / B^{-1/\zeta}\right)^{\zeta}, \\ &= \left(w_n/\bar{w}\right)^{\zeta}. \blacksquare \end{aligned}$$

Solving for the Initial and Counterfactual Equilibria

To solve the model, it is helpful conceptually (and computationally) to build on the long history of input-output economics. Following these models, one can derive a simple way to solve for equilibrium wages, given trade shares.

Total sales in region n and sector j equals total spending from all other regions, $R_n^j = \sum_{i=1}^{N} \pi_{in}^j X_i^j$. Total spending includes spending on both final goods and intermediate inputs, $X_n^j = \beta^j I_n + \sum_{k=1}^{J} \gamma^{kj}(1-\phi^k)R_n^k$. Together, we have $R_n^j = \sum_{i=1}^{N} \pi_{in}^j \beta^j I_i + \sum_{i=1}^{N}\sum_{k=1}^{J} \pi_{in}^j \gamma^{kj}(1-\phi^k)R_i^k$. It is helpful to write these expressions in matrix form. Define $\mathbf{A_{ni}}$ as the $J \times J$ input coefficient matrix, with elements $\gamma^{kj}(1-\phi^k)\pi_{ni}^j$ for the j^{th} row and k^{th} column. Now, form the $NJ \times NJ$ matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_{11}} & \cdots & \mathbf{A_{N1}} \\ \vdots & \ddots & \vdots \\ \mathbf{A_{1N}} & \cdots & \mathbf{A_{NN}} \end{bmatrix}. \tag{24}$$

Call this matrix the *global input coefficient matrix*.

Next, define global demand as the $NJ \times 1$ vector $\mathbf{F}$ with elements $\sum_i \beta^j I_i \pi_{in}^j$ for the row indexed $j + J \times (n-1)$. This represents global spending on final goods from each region and sector. In particular, $\mathbf{F}$ stacks the $J \times 1$ vectors $\mathbf{F_n}$ with elements $\sum_i \beta^j I_i \pi^j_{in}$, which is global demand for final goods from each of region n's sectors. With given wages, employment, and trade flows we know income from $I_n = w_n L_n - S_n$. Given the matrix of global input coefficients $\mathbf{A}$ and the vector of final demand $\mathbf{F}$, total revenue for each region and sector is the $NJ \times 1$ vector

$$\mathbf{R} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{F}. \tag{25}$$

Given the vector of region-sector sales $\mathbf{R}$, we know implied wages in each region, since $w_n = \sum_j \phi^j R^j_n / L_n$. The set of N equilibrium wages is the solution to these equations.

Equation 25 is a familiar expression in any input-output model. The key difference is that in our setup the input coefficients are endogenous and solved in full general equilibrium. They react to changes in productivity, trade costs, input prices, or fiscal transfers. Importantly, the above allows us to solve for equilibrium wages as a function of trade shares. Importantly, the vector of sales $\mathbf{R}$ implies wages in each region, since $w_n = \sum_j \phi^j R^j_n / L_n$. All together, this is a system of equations that solves N equilibrium wages given trade shares π^j_{ni}.

If our goal was to estimate equilibrium wages and incomes consistent with observed trade shares from data, we would be done (and we would have learned little). Instead, our goal is to estimate counterfactual responses to policy changes — namely, fiscal transfers or changes in trade costs. Conveniently, there is a simple yet powerful method to solve these counterfactual responses. It is known as the Exact-Hat Algebra approach of Dekle et al. (2007). Specifically, consider moving from an initial equilibrium consistent with data to a new counterfactual equilibrium. Denoting the equilibrium change in all variables as $\hat{x} = x'/x$, we can write the changes in equations 4 to 6 as

$$\hat{c}^j_n = \hat{w}_n^{\phi^j} \prod_{k=1}^{J} \left(\hat{P}^k_n\right)^{\gamma^{jk}(1-\phi^j)}, \tag{26}$$

$$\hat{\pi}^j_{ni} = \left(\hat{\tau}^j_{ni}\hat{c}^j_i / \hat{P}^j_n\right)^{-\theta^j}, \tag{27}$$

$$\hat{P}^j_n = \left[\sum_{i=1}^{N+1} \pi^j_{ni}\left(\hat{\tau}^j_{ni}\hat{c}^j_i\right)^{-\theta^j}\right]^{-1/\theta^j}. \tag{28}$$

Equations 26 to 28 define a system $\hat{\boldsymbol{\pi}} = f(\hat{\mathbf{w}}; \hat{\boldsymbol{\tau}})$, which maps wage changes, given trade cost changes, to trade-share changes. With counterfactual trade shares $\pi^{j\,\prime}_{ni} = \pi^j_{ni}\hat{\pi}^j_{ni}$, equations 24 and 25 give counterfactual sales and wages. The equilibrium wage changes solve this system, taking the initial trade shares π^j_{ni} as given. So, with equations 24 to 28 we can solve the equilibrium response to any change to trade costs (τ^j_{ni}) or fiscal integration (ζ), all from an initial equilibrium that exactly matches trade data. Trade imbalances also change in any counterfactual equilibrium. Some of the imbalances are exogenous $\bar{S}_n$ while others are due to fiscal transfers, and therefore $S'_n = (1 - t'_n)w'_n L'_n - \bar{T}_n + \bar{S}_n$.

Equilibrium changes in the distribution of employment can be similarly solved in terms of relative changes. From equation 9, and given changes in per capita real income $\hat{m}_n/\hat{P}_n$, the counterfactual share of national employment in province n is

$$l'_n = \frac{l_n \left(\hat{m}_n/\hat{P}_n\right)^{\kappa}}{\sum_{i=1}^{N} l_i \left(\hat{m}_i/\hat{P}_i\right)^{\kappa}}. \tag{29}$$

Notice $\hat{l}_n \propto \left(\hat{m}_n/\hat{P}_n\right)^{\kappa}$, and therefore κ is simply the real income elasticity of migration. There are good estimates of κ available, which simplifies the model calibration.

Our key outcomes of interest are real income and real wages for all regions and sectors. For a given sector, real value-added is simply total value-added divided by the price index $w_n L_n^j/P_n^j$, so real wages (real value-added per worker) are simply w_n/P_n^j. For aggregate labour productivity, we look at overall real wages w_n/P_n in region n. This is simply a region's total value-added per worker $\sum_{j=1}^{n} w_n L_n^j/L_n = w_n$, deflated by the aggregate price index P_n. Finally, real income is $I_n/L_n P_n$.

The link between trade changes and real wage changes is particularly informative. Proposition 2 of Albrecht and Tombe (2016) provides a convenient and compact expression for equilibrium real wage changes. We do not reproduce the proof here, but the intuition is straightforward. In a standard Eaton-Kortum model without input-output relationships, log real wage changes depend on changes in the home-share of spending $\hat{\pi}_{nn}^j$. Specifically, equations 26 and 27 with $\phi^j = 1$ for all j implies $\log(\hat{w}_n/\hat{P}_n^j) = -\log(\hat{\pi}_{nn}^j)/\theta^j$. Collect these changes into a $J \times N$ matrix $\mathbf{G}$. With input-output relationships, we can simply transform $\mathbf{G}$ according to

$$\tilde{\mathbf{G}} = (\mathbf{I} - \tilde{\mathbf{A}}')^{-1}\mathbf{G},$$

where $(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$ is the $J \times J$ Leontief Inverse Matrix, where the input matrix $\tilde{\mathbf{A}}$ has elements $\gamma^{kj}(1 - \phi^k)$.[14] The matrix $\tilde{\mathbf{G}}$ is the $J \times N$ matrix of equilibrium real wage changes for all sectors given $\hat{\pi}_{nn}^j$. Next, with a slight abuse of notation, collect aggregate real wage changes in each province into a vector $\hat{\mathbf{y}}$ with elements $\hat{w}_n/\hat{P}_n$. This is simply

$$\hat{\mathbf{y}} = \mathbf{G}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1}\boldsymbol{\beta},$$

where $\boldsymbol{\beta}$ is a $J \times 1$ vector with elements β^j. With these aggregate real wage changes, the real income of a worker in region n is

$$\hat{U}_n = \left(\hat{m}_n/\hat{P}_n\right),$$

where $\hat{m}_n = \hat{I}_n/\hat{L}_n$ and $\hat{I}_n = \frac{w'_n L'_n - S'_n}{w_n L_n - S_n}$.

What do these expressions mean in plain language? The $J \times 1$ vector $(\mathbf{I} - \tilde{\mathbf{A}})^{-1}\boldsymbol{\beta}$ is a very straightforward measure of a sector's "influence" on an economy. A sector may be extremely valuable as an input supplier to many other sectors, so productivity shocks in that sector cascade throughout the economy. Research by Acemoglu et al. (2012), Jones (2013), and Carvalho and Gabaix (2013) all show in closed-economy settings that this vector collects the elasticities of aggregate output with

[14] The tilde distinguishes this matrix from the *global* input coefficient matrix **A** defined earlier.

respect to sectoral productivity. In our setting, this vector represents by how much standard gains from trade $\mathbf{G}$ are amplified by input-output linkages. We refer to $(\mathbf{I}-\tilde{\mathbf{A}})^{-1}\boldsymbol{\beta}$ as the vector of *input-output multipliers*, though they should not be confused with multipliers from classic input-output analysis. The linkages are very important for our quantitative results, and also for our qualitative results regarding the distribution of economic activity across sectors and regions.

Proof of Proposition 1

Workers choose their province of residence to maximize real income, adjusted for their "willingness" to migrate into any given region. Specifically, a worker chooses province n if $t_n w_n z_n / P_n > \max_{k \neq n} \{t_k w_k z_k / P_k\}$, where z_n captures their willingness to move there. If z_n are independently and identically distributed Frechet with CDF $F_n(x) = \mathrm{e}^{-(x/\delta_n)^{-\kappa}}$, then the share of workers choosing province n is

$$l_n \quad = \quad Pr\left(t_n w_n z_n / P_n > \max_{k \neq n} \{t_k w_k z_k / P_k\}\right).$$

Consider both sides of the inequality separately. The left side is distributed according to

$$\begin{aligned} Pr\left(\frac{t_n w_n z_n}{P_n} < x\right) &= Pr\left(\frac{t_n w_n z_n}{P_n} < x\right), \\ &= Pr\left(z_n < \frac{P_n}{t_n w_n} x\right), \\ &= exp\left\{-\left(\frac{P_n}{t_n w_n \delta_n} x\right)^{-\kappa}\right\}, \\ &= exp\left\{-\left(x/\phi_n\right)^{-\kappa}\right\}, \end{aligned}$$

where $\phi_n = t_n w_n \delta_n / P_n$. The third line follows from the Frechet distribution of z_n.

Turning to the right side, the probability that all other regions k have a value below x is

$$\begin{aligned} Pr\left(\max_{k \neq n} \{t_k w_k z_k / P_k\} < x\right) &= \prod_{k \neq n} Pr\left(t_k w_k z_k / P_k < x\right), \\ &= \prod_{k \neq n} Pr\left(z_k < \frac{P_k}{t_k w_k} x\right), \\ &= \prod_{k \neq n} exp\left\{-\left(\frac{P_k}{t_k w_k \delta_k} x\right)^{-\kappa}\right\}, \\ &= exp\left\{-x^{-\kappa} \sum_{k \neq n} \left(\frac{P_k}{t_k w_k \delta_k}\right)^{-\kappa}\right\}, \\ &= exp\left\{-\left(x/\Phi_n\right)^{-\kappa}\right\} \end{aligned}$$

which is also Frechet but with parameter $\Phi_n = \left(\sum_{k\neq n} (t_k w_k \delta_k / P_k)^{\kappa}\right)^{1/\kappa}$. The share of workers in province n is therefore just the probability that one Frechet-distributed random variable is smaller than another. This has a simple solution. If X and Y are both independently distributed Frechet random variables with parameters a and b, respectively, and variance parameter κ then $Pr(X \geq Y) = \frac{b^{\kappa}}{a^{\kappa}+b^{\kappa}}$. This follows from a standard property of exponential distributions, and the fact that if X is Frechet with parameter a then $X^{-1/\kappa}$ is exponential with parameter a^{κ}. Thus,

$$
\begin{aligned}
l_n &= 1 - \frac{\Phi_n^{\kappa}}{\phi_n^{\kappa} + \Phi_n^{\kappa}}, \\
&= \frac{(t_n w_n \delta_n / P_n)^{\kappa}}{\sum_{k=1}^{N} (t_k w_k \delta_k / P_k)^{\kappa}},
\end{aligned}
$$

which is our result. ■