Existence Advertising, Price Competition, and Asymmetric Market Structure

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Abstract

We examine a duopoly pricing game where some customers know of no firms, others know of only one firm, and some know of both firms. Firms have constant and identical marginal costs, sell homogenous goods and choose prices simultaneously. Customers observe the prices of the firms that are known to them. We show that there is no equilibrium in pure price strategies for this game. We find a mixed strategy equilibrium, and show that it has intuitively appealing comparative static properties. We then examine the two stage game in which firms advertise their existence in stage 1 to create their customer bases, and in stage 2 play the pricing game described above. The equilibrium to the two stage game is asymmetric, and far from the Bertrand equilibrium.

Keywords: Existence advertising, price dispersion, Bertrand paradox, information, duopoly

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1 Introduction

Before one can contemplate the decision to buy a good from a particular firm, one must be aware that the firm exists. Indeed, one of the more important functions of a firm's advertising effort would seem to be to inform potential customers of its existence. If it is costly for firms to advertise and/or it is costly for potential customers to search for firms, it seems unlikely that all potential customers will be aware of all firms. The ensuing partial overlap of customer bases across firms gives rise to the presence of captive consumers and these, in turn, give rise to market power. In this paper we examine two related problems facing firms when there is the potential for partial overlap of customer bases. First we determine the price equilibrium in a duopoly setting and second, given this price equilibrium, we determine how the firms determine the size of their customer bases.

We tackle the pricing problem in Section 2 in a simple duopoly model with identical goods and common constant marginal costs of production. Each consumer buys a quantity of the good from the lowest-priced firm that she knows of and all consumers have the same demand function. We show that when some potential customers are aware of both firms and some are aware of just one firm, no pure strategy price equilibrium exists. We find a mixed strategy equilibrium, where the upper bound on the price distribution is the monopoly price and the lower bound is a function of the monopoly price and the extent of the overlap between the two firms' customer bases. The equilibrium is therefore characterized by price dispersion. We also find that the expected equilibrium price is decreasing in the degree of overlap between customer bases and increasing in the degree of asymmetry between the sizes of the two firms' customer bases.

In Section 3 we go on to examine a two-stage game in which firms engage in costly advertising to make potential customers aware of their existence in stage 1 and choose prices in stage 2. The stage 2 price game is, of course, the game we analyze in Section 2. In the stage 1 game, firms' advertising efforts determine the size of their respective customer bases and, implicitly, the overlap between these customer bases. For a specific advertising technology that generates customer bases by random sampling with replacement from the entire customer population, the stage 1 equilibrium is asymmetric and one firm will have a customer base that is twice as large as the other. This leads to a mixed strategy equilibrium of the stage 2 game in which prices are, relative to the standard Bertrand model, very high. While both firms randomize between the same set of prices, the larger firm has a higher expected price than the smaller firm. Perhaps surprisingly, we show in the appendix that the subgame perfect equilibrium is efficient if consumer demand is perfectly inelastic up to the reservation price.

In Section 4, we derive the general result that the subgame perfect equilibrium is always asymmetric with respect to the size of customer bases if the advertising technology places positive probability on a consumer being contacted by both firms. Given that there will always be a large firm, this in turn implies that in the equilibrium the overlap of customer bases will be imperfect and the larger firm at least will have captive consumers.

In our model, firms make irreversible advertising decisions in stage 1 that influence the nature of price competition in stage 2. We find that the smaller firm has a strong incentive to, in effect, hide from some of the potential customers in order to limit the price competition in the stage 2 game. Others have demonstrated a similar incentive to take action in stage 1 to lessen price competition in stage 2. For example, in Kreps and Scheinkman (1983) firms choose capacities in stage 1 in order to credibly commit to smaller output and therefore above marginal cost prices in stage 2. In Gabszewicz and Thisse (1979, 1980, 1986) firms choose qualities before entering into price competition, which gives rise to maximum differentiation in qualities in stage 1 and above marginal cost prices in stage 2.

When firms sell a homogeneous product and there is less than perfect overlap between the firms' customer bases, only a mixed strategy price equilibrium exists and therefore the equilibrium is characterized by price dispersion. Several authors have documented the presence and persistence of price dispersion in retail markets. These papers range from early descriptive studies (see for example Stigler (1961) and Pratt, Wise and Zeckhauser (1979)) to more sophisticated studies that attempt to isolate the dispersion that is due to imperfect information as opposed to differences in production costs and quality differences (see for example Sorensen (2000), Lach (2002) and Baye, Morgan and Scholten (2004)). For example, Sorensen investigates the market for prescription drugs in two New York cities in 1998 and finds that, after controlling for pharmacy heterogeneity, on the average the highest posted price for a particular drug is 50 % higher than the lowest posted price.

In a pioneering paper that has not received the attention that it deserves, Ireland (1993) finds that when two firm who sell identical goods can costlessly advertise to a population of previously uninformed consumers, one firm will advertise to all consumers and the other firm will advertise to half of the consumers. This results in half of all consumers being captive to the large firm and a mixed strategy price equilibrium where firms put positive probability on all prices between $[\frac{R}{2}, R]$, where R is the reservation price common to all consumers. Ireland extends the duopoly model to a n-firm oligopoly and finds that in equilibrium there will be one large firm and $(n-1)$ equally sized smaller firms. The large firm will advertise to all consumers and the small firms will advertise to only part of the population. All firms' expected prices are strictly decreasing in n.

Our model differs from that of Ireland in a number of key ways. First, because we treat the size and overlap of customer bases as exogenous and independent in the stage 2 pricing analysis, we are able to gain more general insights about the price equilibrium that are applicable to any type of advertising technology. At one extreme we find that when advertising leads to zero overlap between customer bases, such as when firms are able to target different customer groups, the firms set monopoly prices. At the other extreme when the two firm's customer bases are identical in both size and composition, as they might be if both firms purchase the same contact list from a marketer, the pricing equilibrium corresponds to the standard Bertrand marginal-cost pricing result. More generally, as the overlap of the two firm's customer bases increases from zero to the smaller firm's customer base, prices and profits decrease linearly for both firms, but remain above zero as long as the firms' customer bases are different in size. Notice that our price equilibrium applies whether or not stage 1 is in equilibrium. As any advertising efforts to make yourself known cannot be easily reversed in the short run, it is plausible that the stage 1 outcome will not be an equilibrium when the firms play their pricing game, and our set-up can accommodate for this.

Second, while our stage 1 advertising analysis encompasses Ireland's setup, we are able to provide general insights into firms' advertising behavior. We get a strong result that the asymmetry of the advertising equilibrium prevails whenever the advertising technology places positive probability on the likelihood that a single consumer will be targeted by both firms. In the context of a specific advertising technology that is costly and random we examine how changes in the cost of advertising impacts on the size of customer bases and how, in turn, this affects the degree of market power enjoyed by each firm. We find that the size of a firm's customer base is inversely related to the ratio of the marginal cost of adding new customers into the customer base and the per customer monopoly revenue but, as long as the firms' advertising technologies use random sampling from a common population, the larger firm's customer base is always twice that of the smaller firm. Moreover, the lower bound of the equilibrium price distribution and the magnitude of the expected prices are positively related to this ratio. The larger firm always has a higher expected price and makes larger equilibrium profits than the smaller firm. Finally, we discuss the incentives of consumers to engage in search to break down the market power of firms and the incentives of the smaller firm to frustrate this search.

A seminal paper in advertising by Butters (1977) examines a situation where consumers become aware of the existence of the firms only through random advertising. If informed, consumers purchase at most one unit of the good from the firm advertising the lowest price. Butters looks at the cumulative distribution of advertised prices and sale prices in the market, the latter taking into account that each consumer chooses the lowest price advertised to her. Therefore, his focus is on the market and not on the behavior of each individual firm. Another main difference between our results and Butter's is that our equilibrium is asymmetric whereas his equilibrium is symmetric. This makes his equilibrium price support, when applied to two firms, quite different from ours. However, our comparative static results are similar in that we both find that the expected transaction price is decreasing in the cost of advertising and that, in the limit, all consumers buy the good as the marginal cost of informing an additional consumer approaches zero.

Butters also examines optimal non-sequential consumer search in conjunction with advertising, and Robert and Stahl (1993) extend the analysis to include optimal sequential search. Bester and Petrakis (1995) examine a model where consumers are spatially dispersed and know of their local store, but may also be informed of the price of the other firm through advertising. The equilibrium may involve random advertising where firms advertise and offer low prices at positive probability and do not advertise and have high prices at remaining probability. In the equilibrium, consumers are always aware of both firms' prices either because they receive an advertisement or because they do not receive one and deduce that the firm is charging a high price. As a result, our set-up with partial overlap of consumer bases cannot be derived from Bester and Petrakis model. Schmalensee (1983) uses the Butters advertising technology to examine whether an incumbent monopoly can deter entry through extensive advertising. If a potential entrant enters, the firms engage in Cournot competition, and therefore by definition Schmalensee does not examine price dispersion.

Varian (1980) examines temporal equilibrium price dispersion in a monopolistically competitive model with informed and uninformed consumers. He finds that firms randomize their prices over time, which results in price dispersion across the set of firms at any given time. Varian finds that all firms use the same pricing strategy in the equilibrium and enjoy zero profits due to free entry. Our paper differs from Varian in that we examine duopoly game where firms can maintain market power despite selling identical goods and where the firms always use asymmetric pricing strategies in the equilibrium. Baye and Morgan (2001) analyze firm advertising through a clearinghouse where consumers can observe the prices of many competitors by accessing one website. Both the firms and the consumers pay a fixed subscription fee only and therefore have a zero marginal cost of search. Price dispersion results because the clearinghouse owner maximizes profit by pricing access so as to achieve only partial participation of sellers. We differ from Baye and Morgan in that our firms must contact consumers directly, not through a clearinghouse.

In the search literature, there is usually a large number of firms setting prices and consumers have rational expectations on the price distribution but not the location of firms in the distribution. Consumers engage in either non-sequential search (Stigler, 1961, Burdett and Judd, 1983) or sequential search (McCall, 1965, Nelson 1970, Burdett and Judd, 1983) to locate a low price. The search costs of consumers determine the intensity of the search as well as the price equilibrium. Diamond (1970) shows that if consumers face the same non-zero cost for all searches beyond the first search, the only equilibrium in the market is one where all firms charge the monopoly price and where consumers do not search. Salop and Stiglitz (1977) show that when consumers differ in their cost of search and when a search makes consumers fully informed, it is possible to have an equilibrium where the high-cost consumers do not search and there are some firms who have above marginal cost prices and serve the uninformed consumers only. Stiglitz (1989) reviews and analyzes other search technologies that result in equilibrium price dispersion. It is also possible to have equilibrium price dispersion if consumers and firms optimally choose to remain somewhat ignorant of their economic environment (Rothschild and Yaari, in Rothschild 1973). Burdett and Judd (1983) show that we can get equilibrium price dispersion in a rational expectations model even without *ex ante* consumer heterogeneity if the model exhibits *ex post* heterogeneity in consumer information due to stochasticity in the acquisition of information. The commonality of all these papers is the assumption of rational expectations and therefore that consumers know the true distribution of prices in the market. This set-up takes as given that consumers are aware of the existence of all firms and all prices charged by the firms, but not which firm charges what. This is in contrast to our model where we assume that some consumers are only aware of the existence and price of one of the firms while others know the existence and price of both firms.

The rest of the paper is organized as follows. In Section 2 we analyze the duopoly pricing game that arises when customer bases are fixed and there is limited overlap between them. The price game analyzed in Section 2 then becomes, in Section 3, the second stage of a two stage game in which customer bases are chosen through existence advertising in stage 1 using a specific advertising technology. In Section 4 we generalize the advertising technology to demonstrate that the asymmetry result obtained in Section 3 holds for a wide range of advertising technologies. Section 5 concludes.

2 The Pricing Problem

There are two firms competing to sell a homogeneous good and many potential customers. Each of these potential customers demands a quantity $Q(p)$ from the firm offering the lowest price amongst the firms whose existence they know of. We assume that the associated per customer revenue function,

$$R(p) \equiv pQ(p),$$

is continuous and single peaked.[1] Then, letting $\bar{p}$ denote the price at which $R(p)$ attains its maximum value, we see that $R(p)$ is an increasing function of p on the interval $[0, \bar{p}]$.

It is convenient to think in terms of a larger and a smaller firm, so we index firms by L and S. The number of customers who know of firms L and S are N_L and N_S, respectively, and the number of customers who know of both firms is M. We assume that $N_L \geq N_S \geq M > 0$. The number of customers who know of firm S but not firm L is $N_S - M$ and the number of customers who know of firm L but not firm S is $N_L - M$. The M customers that know of both firms are, of course, *up for grabs*, but the willingness of firms to cut price to grab them is conditioned by the fact that they have captive customers who are unaware of the other firm.

Customers who know of just one firm patronize that firm. Customers who know of both firms patronize the firm with the lower price if prices differ, and if prices are identical they randomly choose one firm or the other, with equal probability. For convenience we assume that the marginal costs of both firms

[1] In fact neither continuity nor single peakedness are necessary for the main results we establish in this section, but for expositional purposes they are convenient.

are 0. Firms maximize expected profit, which is equal to aggregate expected revenue since marginal costs are 0.

Some additional notation is useful. The proportions of the larger and smaller firm's customers who are captive are

$$\lambda_L \equiv \frac{N_L - M}{N_L} \text{ and } \lambda_S \equiv \frac{N_S - M}{N_S}.$$

Define the maximized per customer monopoly revenue as

$$\bar{R} \equiv R(\bar{p}).$$

Define price $\underline{p}$ as the price that makes the larger firm indifferent between selling to all the consumers who know of it at price $\underline{p}$ and selling only to its captive consumers at price $\bar{p}$:

$$R(\underline{p}) \quad \equiv \quad \bar{R}\lambda_L.$$

The price $\underline{\underline{p}}$ is similarly defined for the smaller firm:

$$R(\underline{\underline{p}}) \quad \equiv \quad \bar{R}\lambda_S.$$

Clearly,

$$\underline{\underline{p}} \leq \underline{p} < \bar{p}.$$

The first inequality is strict when $N_L > N_S$.

2.1 No Equilibrium in Pure Price Strategies

Clearly, either firm's best response to the other's price is a price in the interval $[0, \bar{p}]$, so we can restrict attention to prices in this interval. We assume that the strategy spaces of the two firms are

$$S_i = \{p_i | 0 \leq p_i \leq \bar{p}\} \quad i \in \{S, L\}$$

Since the M customers who know of both firms patronize the firm with the lower price, the (expected) payoff function of firm i is

$$\begin{aligned}
\pi_i(p_i, p_j) &= R(p_i)N_i \text{ if } 0 \leq p_i < p_j \\
&= R(p_i)(N_i - \frac{M}{2}) \text{ if } p_i = p_j \\
&= R(p_i)(N_i - M) \text{ if } p_j < p_i \leq \bar{p}.
\end{aligned}$$

This can be rewritten as

$$\begin{aligned}
\pi_i(p_i, p_j) &= R(p_i)N_i \text{ if } 0 \leq p_i < p_j \\
&= R(p_i)\frac{1+\lambda_i}{2}N_i \text{ if } p_i = p_j \\
&= R(p_i)\lambda_i N_i \text{ if } p_j < p_i \leq \bar{p}.
\end{aligned}$$

Given that $p_j \leq \bar{p}$, firm i's best response is either to undercut firm j's price by an arbitrarily small amount, so as to capture the M customers who know of both firms, or it is price $\bar{p}$, so as to extract the largest possible profit from the $N_i - M$ customers who are unaware of firm j. The larger firm prefers the latter option when $0 \leq p_S \leq \underline{p}$ and the former when when $\underline{p} < p_S \leq \bar{p}$, and the smaller firm prefers the latter option when $0 \leq p_L \leq \underline{\underline{p}}$ and the former when when $\underline{\underline{p}} < p_L \leq \bar{p}$. So the best response functions are

$$\begin{aligned}
B_L(p_S) &= p_S - \epsilon \text{ if } \underline{p} < p_S \leq \bar{p} \\
&= \bar{p} \text{ if } 0 \leq p_S \leq \underline{p} \\
B_S(p_L) &= p_L - \epsilon \text{ if } \underline{\underline{p}} < p_L \leq \bar{p} \\
&= \bar{p} \text{ if } 0 \leq p_L \leq \underline{\underline{p}},
\end{aligned}$$

where ϵ is an arbitrarily small but positive real number.[2]

[2] Strictly speaking the larger (respectively, smaller) firm's best response is not well defined when $\underline{p} < p_S \leq \bar{p}$ (respectively $\underline{\underline{p}} < p_L \leq \bar{p}$) since there exists no highest price smaller than p_S (respectively, p_L), but for current purposes it is helpful to define the best response as $p_S - \epsilon$ (respectively, $p_L - \epsilon$).

The best response functions are illustrated in Figure 1, and it is apparent that there is no equilibrium in pure strategies. This would seem to be an interesting if somewhat awkward result since there is nothing bizarre about this pricing problem. The nonexistence result is driven by the fact that not all customers know of all firms, and given that information is costly this would seem to be a likely occurrence. With costly information, the improbable occurrence would be the case in which all customers know of all firms.

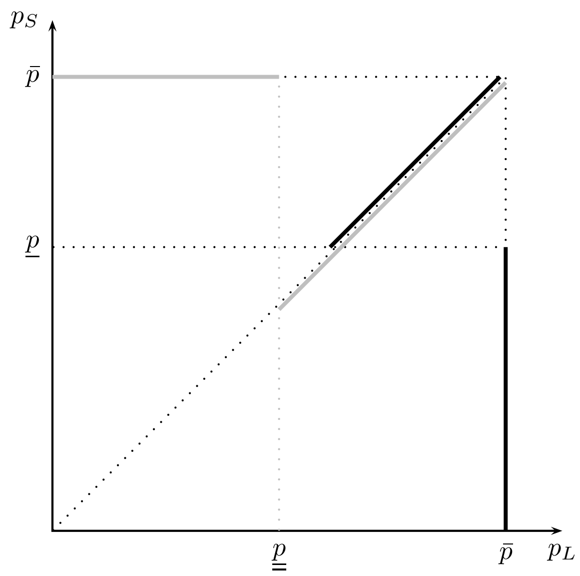

Figure 1: Best response functions for the smaller firm (gray) and the larger firm (black) for pure strategies and the non-existence of pure strategy equilibrium

2.2 A Mixed Strategy Equilibrium

We are looking for two price density functions (DFs), $f_L(p_L)$ for the larger firm and $f_S(p_S)$ for the smaller firm, or equivalently, for two cumulative price density functions (CDFs), $F_L(p_L)$ and $F_S(p_S)$.

Let $\Pi_i(p_i|F_j)$ $(i \in \{S, L\}, j \neq i)$ denote firm i's expected profit, given price p_i and the other firm's CDF, F_j, and let Π_i^* denote the expected equilibrium profit of firm i. A mixed strategy equilibrium is characterized by the following properties:

P1: if $f_i(p_i) > 0$, then $\Pi_i(p_i|F_j) = \Pi_i^*$

P2: if $f_i(p_i) = 0$, then $\Pi_i(p_i|F_j) \leq \Pi_i^*$

P3: if $\Pi_i(p_i|F_j) < \Pi_i^*$, then $f_i(p_i) = 0$.

In words: the prices that get positive probability in firm i's equilibrium density function all yield profit Π_i^*; all other prices yield an expected profit that is no larger than Π_i^*; and all prices that yield an expected profit that is less than Π_i^* get zero probability in firm i's equilibrium density function.

We first establish some useful results based on the assumption that a mixed strategy equilibrium exits, and then go on to find one. Notice that for any F_S, $\Pi_L(\bar{p}, F_S) \geq \lambda_L N_L \bar{R}$, because when $p_L = \bar{p}$ the number of customers who patronize the larger firm is no smaller that $\lambda_L N_L$ and revenue per customer is $\bar{R}$. This establishes a lower bound for the larger firm's profit in any mixed strategy equilibrium: $\Pi_L^* \geq \lambda_L N_L \bar{R}$.

Next we show that $\underline{p}$ is a lower bound on the support of the larger firm's DF. If $p_L < \underline{p}$, then for any F_S, $\Pi_L(p_L|F_S) < N_L R(\underline{p}) = \lambda_L N_L \bar{R} \leq \Pi_L^*$. The strict inequality follows because the number of customers who patronize the larger firm is no larger than N_L and revenue per customer is less than $R(\underline{p})$, the equality follows from the definition of $\underline{p}$, and the weak inequality was established in the previous paragraph. Property P3 then dictates that, in any mixed strategy equilibrium, $f_L(p_L) = 0$ for all $p_L < \underline{p}$.

To establish similar results for the smaller firm, assume that $f_L(p_L) = 0$ for all $p_L < \underline{p}$, as must be the case in any mixed strategy equilibrium. Given this assumption, if $p_S < \underline{p}$, then $\Pi_S(p_S|F_L) = R(p_S)N_S$, which is strictly increasing in p_S. Notice that the limit of $R(p_S)N_S$ as the p_S approaches $\underline{p}$ from below is

$R(\underline{p})N_S = \lambda_L N_S \bar{R}$, so $\Pi_S^* \geq \lambda_L N_S \bar{R}$. Then, from Property P3 we see that in any mixed strategy equilibrium, $f_S(p_S) = 0$ for all $p_S < \underline{p}$ (since for any such price $\Pi_S(p_S|F_L) < \lambda_L N_S \bar{R}$).

Result 1. *If a mixed strategy equilibrium exists, then*

$$\begin{aligned} \Pi_L^* &\geq \lambda_L N_L \bar{R} \textit{ and } f_L(p_L) = 0 \textit{ for all } p_L < \underline{p} \\ \Pi_S^* &\geq \lambda_L N_S \bar{R} \textit{ and } f_S(p_S) = 0 \textit{ for all } p_S < \underline{p}. \end{aligned}$$

This result suggests that there is a mixed strategy equilibrium in which $\Pi_L^* = \lambda_L \bar{R} N_L$, $\Pi_S^* = \lambda_L \bar{R} N_S$, and that the prices that get positive probability are in $[\underline{p}, \bar{p}]$. We will show that the following density functions in fact constitute such a mixed strategy equilibrium. The equilibrium DF for the smaller firm is[3]

$$f_S(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ \frac{\lambda_L}{1-\lambda_L}\left(\frac{\bar{R}R'(p)}{R(p)^2}\right) & \text{if } \underline{p} \leq p < \bar{p} \\ 0 & \text{if } p = \bar{p}. \end{cases} \tag{1}$$

Integrating (1) yields the associated CDF:

$$F_S(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ \frac{1}{1-\lambda_L}\left(1 - \frac{\lambda_L \bar{R}}{R(p)}\right) & \text{if } \underline{p} \leq p < \bar{p} \\ 1 & \text{if } p = \bar{p}. \end{cases}$$

The equilibrium DF for the larger firm is

$$\begin{aligned} f_L(p) &= \begin{cases} 0 & \text{if } p < \underline{p} \\ \frac{\lambda_L}{1-\lambda_S}\left(\frac{\bar{R}R'(p)}{R(p)^2}\right) & \text{if } \underline{p} \leq p < \bar{p} \end{cases} \\ m_L(\bar{p}) &= 1 - \frac{N_S}{N_L}. \end{aligned} \tag{2}$$

Notice that when $N_S < N_L$, there is a mass point at $\bar{p}$ in the larger firm's DF in (2) – the probability that $p_L = \bar{p}$ is $m_L(\bar{p}) = 1 - \frac{N_S}{N_L}$. The remainder of the

[3] As it turns out when $N_S < N_L$, the smaller firm's equilibrium price support is necessarily the half-open interval $[\underline{p}, \bar{p})$ because there is then a mass point in the larger firm's DF at $\bar{p}$. Something similar is seen in Narasimhan (1988) and Ireland (1993).

probability mass is distributed over the half open interval, $[\underline{p}, \bar{p})$. Integrating (2) yields the associated CDF:

$$F_L(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ \frac{1}{1-\lambda_S}\left(1 - \frac{\lambda_L \bar{R}}{R(p)}\right) & \text{if } \underline{p} \leq p < \bar{p} \\ 1 & \text{if } p = \bar{p}. \end{cases}$$

To prove that these price density functions do constitute a mixed strategy equilibrium, we must verify that properties P1, P2 and P3 set out above are satisfied for both firms. We begin with the larger firm. If $p_L > p_S$, the larger firm's profit is $R(p_L)\lambda_L N_L$ since the customers who know of both firms choose to buy from the smaller firm. The probability that $p_L > p_S$ is just $F_S(p_L)$. On the other hand, if $p_L < p_S$, the larger firm's profit is $R(p_L)N_L$ since the customers who know of both firms now choose to buy from the larger firm, and the probability that $p_L < p_S$ is just $1 - F_S(p_L)$. Since there are no mass points in $f_S(p_S)$ in (1), we can ignore the case where $p_L = p_S$. Then, the larger firm's expected profit is just

$$\Pi_L(p_L|F_S) = R(p_L)\lambda_L N_L F_S(p_L) + R(p_L)N_L(1 - F_S(p_L)) \text{ for all } 0 \leq p_L \leq \bar{p}.$$

It is straightforward to verify the following:

$$\begin{aligned} \Pi_L(p_L|F_S) &= \lambda_L \bar{R} N_L \text{ for all } \underline{p} \leq p_L \leq \bar{p} \\ \Pi_L(p_L|F_S) &< \lambda_L \bar{R} N_S \text{ for all } 0 \leq p_L < \underline{p}. \end{aligned}$$

It is then clear that properties P1, P2 and P3 are satisfied for the larger firm.

Similarly, the smaller firm's expected profit given p_S and F_L is

$$\Pi_S(p_S|F_L) = \begin{cases} R(p_S)\lambda_S N_S F_L(p_S) + R(p_S)N_S(1 - F_L(p_S)) & \text{for all } 0 \leq p_S < \bar{p} \\ R(p_S)\left(m_L(\bar{p})\left(\frac{1+\lambda_S}{2}N_S\right) + (1 - m_L(\bar{p}))N_S\right) & \text{for } p_S = \bar{p}. \end{cases}$$

It is straightforward to verify the following[4]:

$$\begin{aligned}
\Pi_S(p_S|F_L) &= \lambda_L \bar{R} N_S \text{ for all } \underline{p} \leq p_S < \bar{p} \\
\Pi_S(p_S|F_L) &< \lambda_L \bar{R} N_S \text{ for } p_S = \bar{p} \\
\Pi_S(p_S|F_L) &< \lambda_L \bar{R} N_S \text{ for all } 0 \leq p_S < \underline{p}.
\end{aligned}$$

It is then clear that properties P1, P2 and P3 are satisfied for the smaller firm.

Result 2. *The density functions $f_L(p)$ in (2) and $f_S(p)$ in (1) constitute a mixed strategy equilibrium in which $\Pi_L = \lambda_L \bar{R} N_L$ and $\Pi_S = \lambda_L \bar{R} N_S$.*

2.3 Comments on the Mixed Strategy Equilibrium

Since firms compete over the M customers who know of both firms, comparative statics with respect to M are of considerable interest. Naturally, as M increases, expected prices and profits decrease. In fact, expected profits decrease linearly at the rate $\bar{R}$ for the larger firm and at the rate $\bar{R}\frac{N_L}{N_S}$ for the smaller firm. In the limit as M approaches 0, we have two monopolists with prices equal to $\bar{p}$ enjoying full monopoly profits, $\pi_L^* = \bar{R} N_L$ and $\pi_S^* = \bar{R} N_S$. When M approaches its upper bound, N_S, equilibrium profits approach $\pi_L^* = (1 - \frac{N_S}{N_L})\bar{R} N_L$ and $\pi_S^* = (1 - \frac{N_S}{N_L}) R(\bar{p}) N_S$.

If the firms are of equal size, that is if $N_L = N_S = N$, equilibrium profits and prices go to zero as M approaches its upper bound, N. So in the symmetric case, as M transits the $[0, N]$ interval there is a smooth transition from the monopoly outcome where expected prices are $\bar{p}$ to the Bertrand outcome where expected prices are 0. This is a satisfying property of the model.

If the firms are of unequal size, however, as M approaches its upper limit N_S equilibrium profits persist and the model does not converge to the Bertrand outcome. For example, if the smaller firm's customer base is half that of the larger firm, equilibrium profits of the two firms converge to one half their monopoly

[4] The second condition holds with equality if $N_S = N_L$.

levels as M approaches its upper bound. In fact, $1-\frac{N_S}{N_L}$ can be regarded as the upper bound on the degree of competitiveness of this model, because the profits of the firms can be no less than $(1-\frac{N_S}{N_L})\%$ of full monopoly profits. This is an interesting property of the model.

Of course, in the larger picture both the sizes of the customer bases (N_L and N_S) and the degree of overlap (M) are endogenous and therefore ought to be modeled. We turn to this problem in the next section. It is clear, however, that foresighted firms will understand that both overlap and the degree of asymmetry in customer bases have significant impacts on equilibrium prices and will take this into account when choosing customer bases.

For future purposes, let us record some precise results with respect to equilibrium prices.

Result 3. *The smaller firm's expected price, $E(p_S)$, is*

$$E(p_S) = -\bar{p}\frac{\lambda_L}{1-\lambda_L}ln(\lambda_L).$$

The larger firm's expected price, $E(p_L)$, is

$$E(p_L) = \frac{\bar{p}}{1-\lambda_S}\left(\lambda_L - \lambda_S - \lambda_L ln(\lambda_L)\right).$$

The expected minimum price, $E\left(\min(p_L, p_S)\right)$, paid by consumers who know of both firms is

$$\begin{aligned} &E\left(\min(p_L, p_S)\right) = \\ &\quad \int_{p_S=\underline{p}}^{\bar{p}}\int_{p_L=\underline{p}}^{\bar{p}} \left(p_L f_L(p_L)\left(1-F_S(p_L)\right) + p_S f_S(p_S)(1-F_L(p_S))\right) dp_L dp_S \\ &= \bar{p}\frac{\lambda_L}{1-\lambda_S}\left(2 + \frac{\lambda_L+\lambda_S}{1-\lambda_L}ln(\lambda_L)\right). \end{aligned}$$

The expected transaction price, ETP, is

$$\begin{aligned} ETP &= \\ & \frac{N_L - M}{N_L + N_S - M} E(p_L) + \frac{N_S - M}{N_L + N_S - M} E(p_S) + \frac{M}{N_L + N_S - M} E\left(\min(p_L, p_S)\right) \\ &= \bar{p} \frac{\lambda_L (2 - \lambda_L - \lambda_S)}{1 - \lambda_L \lambda_S}. \end{aligned}$$

3 Choosing Customer Bases with a Random Advertising Technology

In this section we extend the game to analyze firms' choices with respect to the size of their customer bases and, implicitly, the degree of overlap. We have in mind an advertising technology that allows firms to manage their customer bases by varying their expenditure on advertising. We imagine a population of potential customers of size H. We assume that initially all the potential customers are ignorant of the firms and that they become aware of them only through the advertising efforts of the firms. We look at a two stage game: in stage 1 firms choose their customer bases and in stage 2 they choose prices. The mixed strategy equilibrium presented in the previous section is the equilibrium of the stage 2 price game.

3.1 The Advertising Technology

We denote the cost of making one's firm known to $N \leq H$ of the potential customers by the function $A(N)$. It is natural to suppose that A is increasing and convex in N and that marginal cost, $A'(N)$, is a decreasing function of population size, H. Given N_L and N_S, what should we assume about M, the overlap in the two customer bases? If both N_L and N_S are small relative to H it seems likely that there will be very little overlap, whereas if both are large relative to H there will inevitably be substantial overlap. It also seems sensible to suppose that the overlap is an increasing function of both N_L and N_S. The technology we lay out below exhibits these properties.

For concreteness, we assume that customer bases are generated by random drawings with replacement from the population of size H, and that each draw costs v. We assume that $v \in (0, \bar{R})$. If $v \geq \bar{R}$, then the cost of getting a customer would necessarily be greater than or equal to the maximum revenue the firm could generate from the customer.

Suppose a firm's customer base is initially N. Then, since new customers are generated by drawing with replacement from the population of size H, the probability that an additional draw from the distribution yields a customer that in not already in the firm's customer base is $\frac{H-N}{H}$, so the expected number of additional draws needed to get a new customer is $\frac{H}{H-N}$. Since each draw costs v, the marginal cost of an additional customer is $\frac{vH}{H-N}$. That is,

$$A'(N) = \frac{vH}{H-N}.$$

Then, integrating the marginal cost function yields the cost function

$$A(N) = vH[\ln(H) - \ln(H-N)].$$

Notice that $A(N)$ is increasing and strictly convex in N, and that $A'(N)$ is a decreasing function of the population size, H.

Given that customer bases are generated by random sampling with replacement, any particular person in the population is just as likely to be included in a firm's customer base as is any other person. So, the probability that any person in firm i's customer base is also included in firm j's customer base is $\frac{N_j}{H}$ and the expected overlap in customer bases is

$$M = \frac{N_L N_S}{H}.$$

If both N_L and N_S are small relative to H, there will be very little overlap, if both are large relative to H there will be substantial overlap, and the overlap is an increasing function of both N_L and N_S.

3.2 Stage 1 Objective Functions

In our two-stage game, firms simultaneously choose customer bases in stage 1, and price distributions in stage 2. A firm's stage 1 objective function is then the expected profit that it will earn in the stage 2 price game minus the costs of creating the customer base in stage 1.

Clearly, in the stage 2 mixed strategy equilibrium, a firm's expected profit is a function of the stage 1 choices of customer bases. Consulting Result 2, we see that expected profits of these two firms in the stage 2 price game are $\pi_L^* = \lambda_L \bar{R} N_L$ and $\pi_S^* = \lambda_L \bar{R} N_S$, where $\lambda_L = 1 - \frac{M}{N_L}$. But, given our advertising technology, $M = \frac{N_L N_S}{H}$, so $\lambda_L = 1 - \frac{N_S}{H}$. Of course, λ_L is the proportion of people in the larger firm's customer base who are captive – that is, who know nothing of the smaller firm. Notice that λ_L is completely determined by N_S and H. The expected profits of the two firms in the stage 2 price game are then

$$\begin{aligned} \pi_S^* &= \lambda_L \bar{R} N_S \\ &= \bar{R} N_S \left(1 - \frac{N_S}{H}\right) \end{aligned}$$

and

$$\begin{aligned} \pi_L^* &= \lambda_L \bar{R} N_L \\ &= \bar{R} N_L \left(1 - \frac{N_S}{H}\right). \end{aligned}$$

For the larger firm, the marginal value of an additional person in its customer base, $\bar{R}\left(1 - \frac{N_S}{H}\right)$, is independent of its choice variable, N_L, whereas for the smaller firm, the marginal value of an additional person in its customer base, $\bar{R}\left(1 - \frac{2N_S}{H}\right)$, is a decreasing function of its choice variable, N_S.

Using these results, and returning to the notation where firms are denoted by i and j allowing either firm to be the larger or the smaller firm, we see that the stage 1 objective function of firm i, $V_i(N_i, N_j)$, is the following:

$$V_i(N_i, N_j) = \begin{cases} \bar{R}N_i\left(1 - \frac{N_i}{H}\right) - A(N_i) & \text{if } N_i \leq N_j \\ \bar{R}N_i\left(1 - \frac{N_j}{H}\right) - A(N_i) & \text{if } N_i \geq N_j. \end{cases}$$

3.3 Concavity of the Stage 1 Objectives Functions

In choosing its customer base, firm i must contemplate two regimes, the LT regime where $N_i \leq N_j$, and the GT regime where $N_i \geq N_j$. In the LT regime,

$$\begin{aligned} \frac{\partial V_i(N_i, N_j)}{\partial N_i} &= \bar{R}\left(1 - \frac{2N_i}{H}\right) - \frac{vH}{H - N_i} \text{ if } N_i \leq N_j \\ \frac{\partial^2 V_i(N_i, N_j)}{\partial N_i^2} &= -\frac{2\bar{R}}{H} - \frac{vH}{(H - N_i)^2} < 0 \end{aligned}$$

and in the GT regime,

$$\begin{aligned} \frac{\partial V_i(N_i, N_j)}{\partial N_i} &= \bar{R}\left(1 - \frac{N_j}{H}\right) - \frac{vH}{H - N_i} \text{ if } N_i \geq N_j \\ \frac{\partial^2 V_i(N_i, N_j)}{\partial N_i^2} &= -\frac{vH}{(H - N_i)^2} < 0. \end{aligned}$$

So, *within* each of these regimes, firm i's objective function is strictly concave in N_i.

However, *across* the two regimes, firm i's objective function is not concave. This can be seen by comparing $\frac{\partial V_i(N_i, N_j)}{\partial N_i}$ for the two regimes, at the point where $N_i = N_j$. At this point, the marginal value of N_i in the GT regime is greater than its marginal value in the LT regime:

$$\bar{R}\left(1 - \frac{N_j}{H}\right) - \frac{vH}{H - N_j} > \bar{R}\left(1 - \frac{2N_j}{H}\right) - \frac{vH}{H - N_j}.$$

This, of course, means that over the two regimes, firm i's objective function is not concave. The objective function is illustrated in Figure 2 for four different values of N_j. Firm i's objective function is continuous in N_i, but it is kinked at $N_i = N_j$; in some neighborhood centered on the kink $\frac{\partial V_i(N_i, N_j)}{\partial N_i}$ is greater in the GT regime that it is in the LT regime.

It also means that there is no symmetric equilibrium. Suppose to the contrary that there was a symmetric equilibrium, $N_i^* = N_j^* = N^*$. For this to be an equilibrium, it must the case that N^* is a best response to N^*. Supposing that $N_j^* = N^*$, this requires that $\bar{R}\left(1 - \frac{N^*}{H}\right) \leq \frac{vH}{H-N^*}$, otherwise firm i's best response in GT regime would be some $N_i > N^*$, and it also requires that $\bar{R}\left(1 - \frac{2N^*}{H}\right) \geq \frac{vH}{H-N^*}$, otherwise firm i's best response in LT regime would be some $N_i < N^*$. Obviously, it is impossible to satisfy both inequalities. So we have a result.

Result 4. *There is no symmetric equilibrium in this model.*

3.4 The Best Response Functions

For clarity, we focus on firm i's best response function, $BR_i(N_j)$. We proceed as follows. First we arbitrarily restrict firm i to one of the two regimes, and find for each regime a *restricted* best response function, $BR_i^{LT}(N_j)$ for the LT and $BR_i^{GT}(N_j)$ for the GT regime. Then we splice these restricted best response functions to get the actual or unrestricted best response function, $BR_i(N_j)$.

In the LT regime,

$$\frac{\partial V_i(N_i, N_j)}{\partial N_i} = \bar{R}\left(1 - \frac{2N_i}{H}\right) - \frac{vH}{H - N_i}.$$

Notice that because $\bar{R} > v$, $\frac{\partial V_i(N_i=0,N_j)}{\partial N_i} > 0$, so firm i always chooses $N_i > 0$. Then, given the concavity of the objective function within the LT regime, if $\frac{\partial V_i(N_i=N_j,N_j)}{\partial N_i} \geq 0$ firm i's maximizing choice is $N_i = N_j$, and if $\frac{\partial V_i(N_i=N_j,N_j)}{\partial N_i} < 0$ firm i's maximizing choice is the N_i such that $\frac{\partial V_i(N_i,N_j)}{\partial N_i} = 0$. These observations lead directly to the best response function for the LT regime. But first it is helpful to define a composite parameter, Δ_1:

$$\Delta_1 \equiv H\left(\frac{3}{4} - \frac{1}{4}\sqrt{1 + 8\frac{v}{\bar{R}}}\right). \tag{3}$$

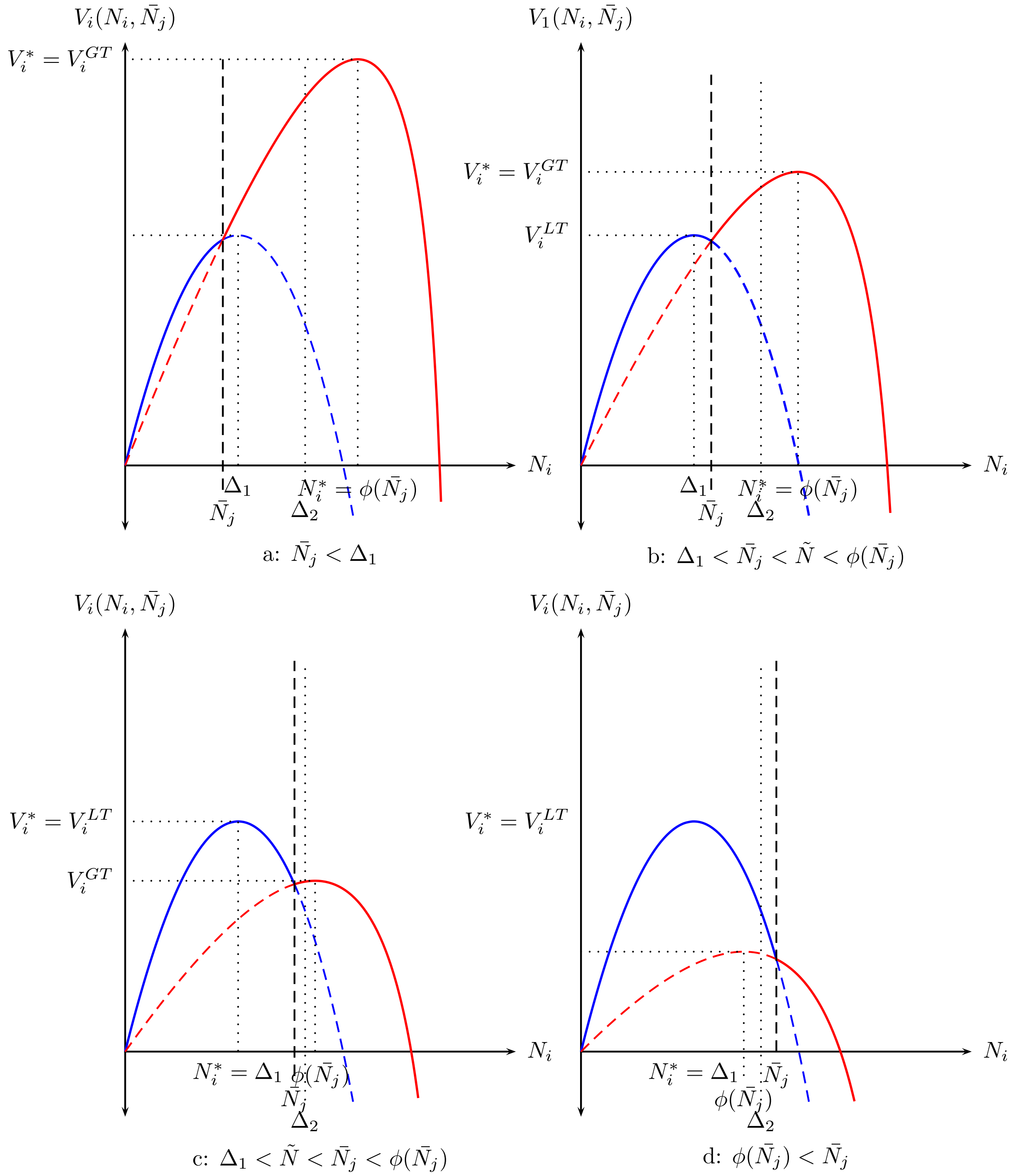

Figure 2: Stage 1 objective function for firm i for four different values of $\bar{N}_j$: The objective function is given by the solid line.

Δ_1 is the largest value of N_j such that $\frac{\partial V_i(N_i = N_j, N_j)}{\partial N_i} \geq 0$. The best response function for the LT regime is

$$\begin{aligned} BR_i^{LT}(N_j) &= N_j \text{ if } N_j \leq \Delta_1 \\ &= \Delta_1 \text{ if } N_j > \Delta_1. \end{aligned}$$

Notice that when $N_j > \Delta_1$, firm i's best response is $N_i = \Delta_1$, which is independent of N_j.

In the GT regime,

$$\frac{\partial V_i(N_i, N_j)}{\partial N_i} = \bar{R}\left(1 - \frac{N_j}{H}\right) - \frac{vH}{H - N_i}.$$

Because $v > 0$, $\frac{\partial V_i(N_i, N_j)}{\partial N_i}$ approaches negative infinity as N_i approaches H, so firm i always chooses $N_i < H$. Given the concavity of the GT objective function, if $\frac{\partial V_i(N_i = N_j, N_j)}{\partial N_i} \leq 0$ firm i's maximizing choice is $N_i = N_j$, and if $\frac{\partial V_i(N_i = N_j, N_j)}{\partial N_i} > 0$ firm i's maximizing choice is the N_i such that $\frac{\partial V_i(N_i, N_j)}{\partial N_i} = 0$. It is useful to define a composite parameter, Δ_2, and a function, $\Phi(N_j)$:

$$\begin{aligned} \Delta_2 &\equiv H\left(1 - \sqrt{\frac{v}{\bar{R}}}\right) \\ \Phi(N_j) &\equiv H\left(1 - \frac{v}{\bar{R}}\frac{H}{H - N_j}\right). \end{aligned} \tag{4}$$

Δ_2 is the smallest value of N_j such that $\frac{\partial V_i(N_i = N_j, N_j)}{\partial N_i} \leq 0$, and $\Phi(N_j)$ satisfies $\frac{\partial V_i(N_i = \Phi(N_j), N_j)}{\partial N_i} = 0$. Also, the size of Δ_2 relative to $\Phi(N_j)$ is easy to establish: when $N_j < \Delta_2$, $\Delta_2 < \phi(N_j)$, when $N_j > \Delta_2$, $\Delta_2 > \phi(N_j)$ and when $N_j = \Delta_2$, $\Delta_2 = \phi(N_j)$. As $\phi(N_j)$ has a more intuitive graphical interpretation than Δ_2, we will use the former in our illustrations. The best response function for the GT regime is

$$\begin{aligned} BR_i^{GT}(N_j) &= N_j \text{ if } N_j \geq \Delta_2 \text{ (or if } N_j \geq \phi(N_j)) \\ &= \Phi(N_j) \text{ if } N_j < \Delta_2 \text{ (or if } N_j \leq \phi(N_j)). \end{aligned}$$

A bit of algebra establishes the following useful inequalities:

$$0 < \Delta_1 < \Delta_2 < H.$$

Now let us splice the restricted best response functions to get the actual best response function, $BR_i(N_j)$. When $N_j < \Delta_1$, N_j is so small that if forced to be in the LT regime firm i would choose $N_i = N_j$. But it will not voluntarily choose the LT regime, because the non-concavity in $V_i(N_i, N_j)$ at $N_i = N_j$ means that it gets an even larger profit by choosing $N_i > N_j$ in the interior of the GT regime. Hence, when $N_j < \Delta_1$, $BR_i(N_j) = \Phi(N_j)$. This case is case a in Figure 2 where we have plotted $V_i(N_i, N_j)$ holding N_j fixed at a value less than Δ_1. Notice that $V_i(N_i, N_j)$ has a single local maximum, in the interior of the GT regime, where $N_i = \Phi(N_j)$.

When $N_j > \Delta_2$, the story is similar. In this case, N_j is so large that if forced to be in the GT regime firm i would choose $N_i = N_j$. But it will not voluntarily choose the GT regime, because the non-concavity in $V_i(N_i, N_j)$ at $N_i = N_j$ means that it gets an even larger profit by choosing $N_i = \Delta_1 < N_j$ in the interior of the LT regime. Hence, when $N_j > \Delta_2$, $BR_i(N_j) = \Delta_1$. This case is case d in Figure 2– notice that in this case there is a single local maximum in the interior of the LT regime.

The situation is a bit trickier when $\Delta_1 \leq N_j \leq \Delta_2$. In this case, illustrated in parts b and c of Figure 2, N_j is large enough so that $V_i(N_i, N_j)$ has a local maximum in the interior of the LT regime (at $N_i = \Delta_1$), and small enough so that it has a local maximum in the interior of the GT regime (at $N_i = \Phi(N_j)$). So $BR_i(N_j)$ is either Δ_1 in the LT regime, or $\Phi(N_j)$ in the GT regime, depending on which option yields the larger payoff. Intuitively, it would seem that there is an N_j in the open interval (Δ_1, Δ_2) where the two options yield the same payoff, and that for smaller values of N_j the best response is $\Phi(N_j)$ in the GT regime, and for larger values it is Δ_1 in the LT regime.

To confirm this intuition, we need to compare firm i's maximized payoff in the LT regime, which we denote by V_i^{LT}, with its maximized payoff in the GT regime, which we denote by $V_i^{GT}(N_j)$:

$$V_i^{LT} = \Delta_1 \bar{R}\left(1 - \frac{\Delta_1}{H}\right) - vH[\ln(H) - \ln(H - \Delta_1)]$$

and

$$V_i^{GT}(N_j) = \Phi(N_j)\bar{R}\left(1 - \frac{N_j}{H}\right) - vH[\ln(H) - \ln(H - \Phi(N_j))].$$

Observe that V_i^{LT} is independent of N_j, whereas $V_i^{GT}(N_j)$ is a function of N_j (as well as the parameters of the model). In fact, $V_i^{GT}(N_j)$ is a decreasing function of N_j, since in the GT regime the marginal value of an additional person in firm i's customer base, $\bar{R}[1 - \frac{N_j}{H}]$, is a decreasing function of N_j. Further, it is obviously the case that, when $N_j = \Delta_1$, $V_i^{GT}(N_j) > V_i^{LT}$, and when when $N_j = \Delta_2$, $V_i^{LT} > V_i^{GT}(N_j)$. Then, since $V_i^{GT}(N_j)$ is continuous in N_j, there exists a value of N_j such that $V_i^{GT}(N_j) = V_i^{LT}$. Denote the value by $\widetilde{N}$. It is implicitly defined by the following equality:

$$\Phi(\widetilde{N})\bar{R}\left(1 - \frac{\widetilde{N}}{H}\right) - vH[\ln(H) - \ln(H - \Phi(\widetilde{N}))] =$$
$$\Delta_1 \bar{R}\left(1 - \frac{\Delta_1}{H}\right) - vH[\ln(H) - \ln(H - \Delta_1)].$$

There is no closed form solution for $\widetilde{N}$. But to find the equilibria of the model it is sufficient to know that

$$\Delta_1 < \widetilde{N} < \Delta_2$$

This completes the splice for firm i: if $N_j \leq \widetilde{N}$, firm i's best response is $\Phi(N_j)$ in the GT regime, as in parts a and b of Figure 2, and if $N_j \geq \widetilde{N}$, firm i's best response is Δ_1 in the LT regime, as in parts c and d of Figure 2.

Let us record the best response functions:

Result 5. *The best response function of each firm is given by*

$$\begin{aligned} BR_i(N_j) &= \Phi(N_j) \text{ if } N_j \leq \widetilde{N} \\ BR_i(N_j) &= \Delta_1 \text{ if } N_j \geq \widetilde{N}, \end{aligned}$$

where $\Phi(N_j)$ *is given by (4),* Δ_1 *is given by (3) and* $\widetilde{N}$ *is implicitly defined by (5).*

3.5 Equilibria of The Customer Base Game

We have plotted the best response functions in Figure 3. Notice that there are two equilibria. In each equilibrium the smaller customer base is Δ_1 in (3) and the larger customer base is $\Phi(\Delta_1)$ in (4), which is equal to $2\Delta_1$. So, in equilibrium the larger customer base is twice the smaller customer base[5].

Result 6. *In the subgame perfect equilibrium of this model, the customer base of the smaller firm,* N_S^**, is*

$$N_S^* = \Delta_1 = H\left(\frac{3}{4} - \frac{1}{4}\sqrt{1 + 8\frac{v}{\bar{R}}}\right) = H\Omega,$$

and the customer base of the larger firm, $2\Delta_1$*, is*

$$N_L^* = 2\Delta_1 = 2H\Omega,$$

where

$$\Omega \equiv \frac{3}{4} - \frac{1}{4}\sqrt{1 + 8\frac{v}{\bar{R}}}.$$

The composite parameter Ω is the equilibrium proportion of the total population that is in the smaller firm's customer base. It is inversely related to the ratio $\frac{v}{\bar{R}}$, and given that $0 < \frac{v}{\bar{R}} < 1$, $0 < \Omega < \frac{1}{2}$. Of course, 2Ω is the equilibrium proportion of the total population that is in the larger firm's customer base. As the cost of sending a message to a customer, v, approaches $\bar{R}$, the ratio $\frac{v}{\bar{R}}$ approaches 1, Ω approaches 0, and both customer bases go to 0. As $\frac{v}{\bar{R}}$ approaches

[5]When advertising is costless, the larger firm targets the entire population and the smaller firm targets half of the population. This result was shown by Ireland (1993).

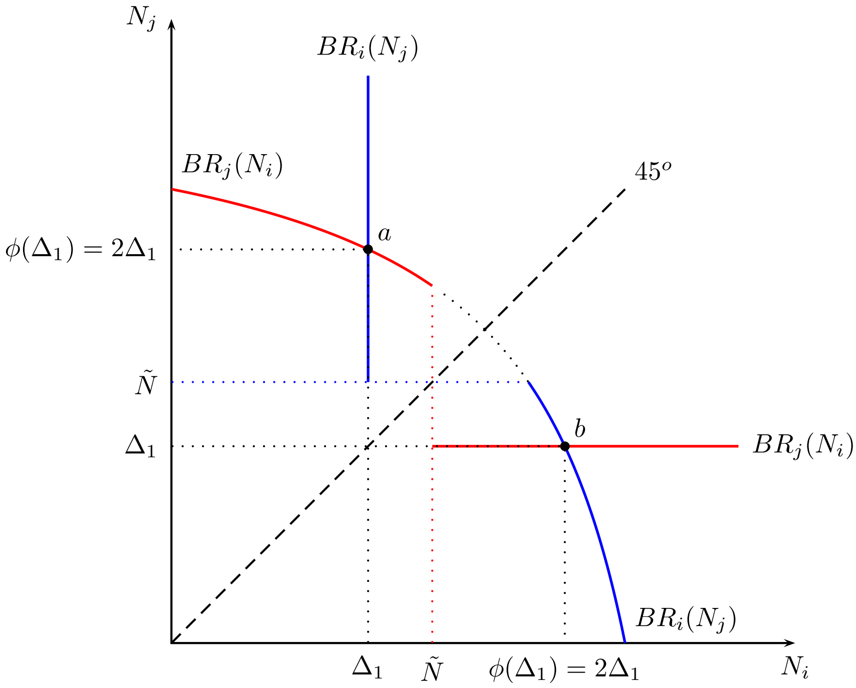

Figure 3: Best response functions and two Nash equilibria in customer bases

0, Ω approaches $\frac{1}{2}$, and in this limit the smaller firm targets half the population and the larger firm targets the entire population.[6]

3.6 Prices

Now let us explore the price equilibrium that emerges when firms choose their customer bases. For this purpose it is useful to choose a unit for the good such that the quantity demanded when price is $\bar{p}$ is 1. Then, $\bar{p} = \bar{R}$ and we can normalize prices by dividing them by $\bar{R}$.

Result 7. *In the subgame perfect equilibrium of this game, the normalized ex-*

[6]In Appendix A we show that in the special case where customer demand is completely price inelastic, the equilibrium is efficient in the sense that total surplus is maximized.

pected prices of the smaller and larger firms are

$$E\left(\frac{p_S^*}{\bar{R}}\right) = \frac{1-\Omega}{\Omega}\ln\left(\frac{1}{1-\Omega}\right)$$

and

$$E\left(\frac{p_L^*}{\bar{R}}\right) = \frac{1}{2\Omega}\left(\Omega + (1-\Omega)ln\left(\frac{1}{1-\Omega}\right)\right)$$

the normalized expected minimum price is

$$E\left(\min\left(\frac{p_L^*}{\bar{R}}, \frac{p_S^*}{\bar{R}}\right)\right) = \frac{1-\Omega}{2\Omega}\left(2 - \frac{2-3\Omega}{\Omega}\ln\left(\frac{1}{1-\Omega}\right)\right)$$

the normalized expected transaction price is

$$\frac{ETP^*}{\bar{R}} = \frac{3(1-\Omega)}{(3-2\Omega)};$$

the lower bound on normalized prices is

$$\lambda_L^* = 1 - \frac{N_S^*}{H} = 1 - \Omega.$$

The lower bound on normalized prices conveys the flavor of these results well and simply. Prices are never less than $1-\Omega$. Given that $0 < \Omega < \frac{1}{2}$, we see that $\frac{1}{2} < \lambda_L^* < 1$ and in equilibrium normalized prices are never less than $\frac{1}{2}$. As $\frac{v}{R}$ approaches 0, both Ω and the lower bound on the price support approach $\frac{1}{2}$. This seems to be an interesting result because in this limit both advertising and production are costless but yet the normalized equilibrium prices are never less than $\frac{1}{2}$. The contrast with the standard Bertrand model, where normalized price is 0 in equilibrium, is sharp.

In the Proposition, normalized prices are expressed as functions of the composite parameter Ω, but Ω is itself completely determined by the ratio $\frac{v}{R}$. Naturally, all of the normalized prices reported in the proposition are increasing functions of $\frac{v}{R}$. Table 1 conveys the nature of the dependence of various measures of normalized price on $\frac{v}{R}$.

Table 1: Normalized equilibrium prices, proportion of captive consumers and benefit from search as a percentage of $\bar{R}$ for different values of $\frac{v}{\bar{R}}$

$\frac{v}{\bar{R}}$	$E\left(\frac{p_S^*}{\bar{R}}\right)$	$E\left(\frac{p_L^*}{\bar{R}}\right)$	$\frac{ETP^*}{\bar{R}}$	$E\left(\min(\frac{p_L^*}{\bar{R}}, \frac{p_S^*}{\bar{R}})\right)$	λ_L^*	**benefit of search**
.001	.69	.85	.75	.65	.50	.25
.10	.76	.88	.81	.72	.59	.19
.20	.80	.90	.85	.77	.65	.15
.30	.84	.92	.88	.82	.71	.12
.40	.87	.94	.91	.85	.76	.09
.50	.90	.95	.93	.88	.81	.07
.60	.92	.96	.95	.91	.85	.05
.70	.94	.97	.96	.94	.89	.04
.80	.96	.98	.98	.96	.93	.02
.90	.98	.99	.99	.98	.97	.01
.99	1.00	1.00	1.00	1.00	1.00	.00

3.7 Search

Our model is motivated by the observation that in some circumstances not all customers know of the existence of all firms. Surely, this is not an uncommon occurrence and we have modeled the pricing problem that it raises. In the equilibrium of our model, some consumers have something to gain by learning about the existence of firms and so in this sense have an incentive to search. If they act on those incentives then the equilibrium will be upset. It is not at all clear, however, how to model search in this framework. How does a person go about finding a firm the very existence of which the person is not aware? How does one calculate the possible gains from finding such a firm? Further, from our analysis it is quite clear that beyond some point the smaller firm has an incentive to frustrate the attempts by customers to search it out because successful search generates overlap, and overlap leads to the dissipation of profit through price competition. A more complete model would perhaps include both consumer search and its frustration by smaller firms.

Although we are not in a position to formally model customer search, we can say a little bit about the incentives. For this purpose it is useful to consider the special case in which customer demand is perfectly price inelastic. In this special case, a customer demands 1 unit of the good for normalized prices in the interval $[0, 1]$ and 0 units for higher prices. In the equilibrium there are four categories of customers: those who know of the smaller firm but not the larger firm; those who know of the larger firm but not the smaller firm; those of who know of neither firm; and those of know of both firms. Customers in all but the last category have something to gain by search. Those who know of neither firm have the most to gain – a successful search that allowed them to identify one firm would yield a surplus equal to $\bar{R}(1 - \frac{ETP^*}{R})$ – from Table 1, we see that this can be as large as $.25\bar{R}$, but is a modest $.07\bar{R}$ when $\frac{v}{R} = .5$. Those who know of the larger firm but not the smaller one, have the second highest incentive to search – a successful search that allowed them to identify the smaller firm would yield a surplus equal to $\bar{R}\left(E\left(\frac{p_L^*}{R}\right) - E\left(\min(\frac{p_L^*}{R}, \frac{p_S^*}{R})\right)\right)$ – this could be as large as $.2\bar{R}$, but is only $.07\bar{R}$ when $\frac{v}{R} = .5$. The incentive to search for those who know only of the smaller firm is no more than $.04\bar{R}$. Clearly, as in all search models, if the cost of search is large relative to its expected benefit there will be no search to upset the equilibrium. This is most likely to be the case when $\frac{v}{R}$ is large.

4 Choosing Customer Bases with a General Advertising Technology

The incentive to avoid overlap in customer bases drives the asymmetry of the equilibrium we found in the previous section. The degree of overlap that results from a particular level of advertising is in turn determined by the advertising technology. This raises an obvious question: what can we say about the set of advertising technologies that generate the asymmetry result? Perhaps surprisingly, we will show here that the asymmetry in the customer base game is a very general result.

Let $M(N_1, N_2)$ denote the overlap associated with an arbitrary advertising technology. At the most general level, the only *a priori* restrictions on $M(N_1, N_2)$ would seem to be that overlap is non-negative ($M(N_1, N_2) \geq 0$) and that overlap is non-decreasing in the sizes of the customer bases ($M_1(N_1, N_2) \geq 0$, and $M_2(N_1, N_2) \geq 0$).

Consider the expected profit of firm 1 in the stage 2 price game. In the LT regime (where $N_1 < N_2$), firm 1 is the smaller firm so its profit in the stage 2 mixed strategy equilibrium is $\lambda_2 \bar{R} N_1$, and in the GT regime (where $N_1 > N_2$) it is the larger firm so so its profit in the stage 2 mixed strategy equilibrium $\lambda_1 \bar{R} N_1$. Of course, $\lambda_1 = 1 - \frac{M(N_1,N_2)}{N_1}$ and $\lambda_2 = 1 - \frac{M(N_1,N_2)}{N_2}$, so

$$\pi_1^* = \begin{cases} (1 - \frac{M(N_1,N_2)}{N_2})\bar{R}N_1 & \text{if } N_1 < N_2 \\ (1 - \frac{M(N_1,N_2)}{N_1})\bar{R}N_1 & \text{if } N_1 > N_2. \end{cases}$$

Then, differentiating with respect to N_1, we get

$$\frac{\partial \Pi_1^*}{\partial N_1} = \begin{cases} \bar{R} - \frac{\bar{R}M(N_1,N_2)}{N_2} - \frac{\bar{R}N_1 M_1(N_1,N_2)}{N_2} & \text{if } N_1 < N_2 \\ \bar{R} - \frac{\bar{R}N_1 M_1(N_1,N_2)}{N_1} & \text{if } N_1 > N_2. \end{cases}$$

When we evaluate these partial derivatives at the point $N_1 = N_2 = N$, we see that $\frac{\partial \Pi_1^*}{\partial N_1}$ for the LT regime is smaller than it is in the GT regime if and only if $\frac{\bar{R}M(N,N)}{N} > 0$. So, if there is any overlap in customer bases, except in very special circumstances there will be no symmetric equilibrium of the customer base game.

To be more precise, let $A_1(N_1)$ denote the firm 1's advertising cost for a customer base of size N_1, and assume that $A_1(N_1)$ is concave and differentiable. Then, if there is any overlap in customer bases, there can be no equilibrium in the customer base game where $N_1 = N_2 = N > 0$. Such an equilibrium would require that the following conditions be satisfied:

$$\begin{aligned} \bar{R} - \frac{\bar{R}M(N,N)}{N} - \bar{R}N\frac{M_1(N,N)}{N} &\geq A_1(N) \\ \bar{R} - \bar{R}N\frac{M_1(N,N)}{N} &\leq A_1(N). \end{aligned}$$

But, if $M(N,N) > 0$, it is impossible to satisfy both.

Result 8. *If the advertising technologies used by the firms generate positive overlap, and if the cost of generating a customer base in concave and differentiable, there is no symmetric equilibrium in the customer base game.*

5 Conclusions

We have investigated a two-stage game where firms advertise to manage their customer bases and then choose prices simultaneously to maximize profit. The analysis is limited to a two-firm, homogeneous good setting. We first focus on the stage 2 pricing game and find that, as long as one firm has some customers that know only of it and not of its competitor, there is no pure strategy equilibrium. When there is overlap between customer bases but the size of the customer bases is asymmetric, we find both firms use randomized pricing strictly above marginal cost even when all the customers of the smaller firm know of the larger firm. We therefore predict price dispersion and the failure of the law of one price in equilibrium whenever at least one of the firms has some captive consumers, a highly likely occurrence. As the overlap of the two firm's customer bases increases from zero to the smaller firm's customer base, prices and profits decrease linearly for both firms. At the extremes, when the customer bases have no overlap we have two monopolies charging monopoly prices and when both the firm's have identical customer bases we have marginal cost pricing and zero profits. So the model provides a simple solution to the Bertrand paradox that arises because of imperfect information regarding the existence of firms.

In the stage 1 game we find that as long as the advertising technology generates any overlap in customer bases there is no symmetric equilibrium. There are, however, two asymmetric equilibria in which one firm always chooses a smaller customer base than the other firm. The smaller firm has an incentive to limit its advertising in an effort to keep the overlap in the customer bases imperfect because the stage 2 price equilibrium profits are negatively affected by the size of the overlap. Given a specific advertising technology, we find that at the limit when the marginal cost of advertising is zero the large firm targets the entire population and the small firm targets half the population thereby limiting the overlap to half of the population. Both firms enjoy positive profits in equilibrium. We find that the equilibrium is efficient in the special case where consumers have unitary demand.

Finally, we are able to quantify and compare the incentive consumers have to search for the special case where consumers have unitary demand. We find that, at the very most, the consumers who have the most to gain from search benefit from search by an amount equal to one quarter of their reservation price. However, because search increases the overlap of the two firms customer bases thus reducing the firms' market power and equilibrium prices, the smaller firm has an incentive to find ways to discourage consumers from searching. Importantly, firms will lose out on all units of the good that they sell and so their incentive to frustrate search is many times greater than the incentives facing consumers.

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A Efficiency

Let us examine the special case in which the individual customer demands 1 unit of the good for prices in interval $[0, \bar{R}]$ and 0 units for higher prices. Then, since demand is perfectly inelastic with respect to price up to the reservation price $\bar{R}$, the only efficiency issue concerns the number of unique customers in the aggregate customer base. Call this number T (for total). The marginal cost of increasing the aggregate customer base is just $\frac{vT}{H-T}$, since the expected number of draws needed to uncover someone who is not already in the base is $\frac{H-T}{T}$. To maximize total surplus, T must be chosen so that this marginal cost is equal to the reservation price, $\bar{R}$, since $\bar{R}$ is the surplus that is generated when a new person enters the aggregate customer base. Solving this condition we get the optimal size of the aggregate customer base, T^*:

$$T^* = H\left(1 - \frac{v}{\bar{R}}\right).$$

Obviously, a monopolist would choose price $\bar{R}$ for everyone in its customer base. So to maximize its profit, a monopolist would choose its customer base N_m to equate the marginal cost $\frac{vN_m}{H-N_m}$ to $\bar{R}$. The resulting customer base equals $N_m^* = T^*$ and therefore the monopoly equilibrium is efficient. Given that the monopolist captures all of the consumer surplus and incurs all the costs of making customers aware of its product, this result is not surprising.

What about the duopoly equilibrium? When overlap in the customer bases is taken into account, we see that the number of people in at least one customer base is

$$N_S^* + N_L^* - \frac{N_S^* N_L^*}{H} = H\Omega + 2\Omega H - \frac{2\Omega^2 H^2}{H} = H\left(3\Omega - 2\Omega^2\right).$$

Then, a bit of algebra establishes, perhaps surprisingly, that the duopoly equilibrium is also efficient:

$$H(3\Omega - 2\Omega^2) = T^*.$$

Result 9. *Both the monopoly and duopoly equilibria are efficient, in the sense that total surplus is maximized.*

Of course, the monopolist captures all of the surplus as profit, whereas the duopolists dissipate some of the surplus in the competition to capture it.