Biased Technological Change and the Relative Abundance of Natural Resources

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Abstract

This paper documents that natural resources that are more abundant have higher production, lower prices, higher primary industry revenues, and higher R&D. These empirical facts are explained by a model of biased technological change in which relatively more abundant resources attract greater R&D because the return from obtaining a patent is higher in larger markets. Resource specific R&D may be targeted either towards upstream extraction technologies or towards downstream production technologies, and R&D is subject to diminishing knowledge spillovers and diminishing productivity of labor. The estimated elasticity of substitution between natural resources is greater than one, implying that natural resources are substitutes in production. Declining real resource prices in the face of rising resource production are explained by the increasing productivity of labor as knowledge stocks grow.

Key Words: Biased technological change, natural resource abundance, natural resource prices and production

JEL Codes: O33, Q32, L71

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"It is the stars, The stars above us, govern our conditions"
King Lear, Act IV, Scene 3

1 Introduction

The economic theory of mineral resources developed by Hotelling (1931) explains how markets allocate a fixed stock of a single resource over time. This theory, however, treats each resource in isolation from all other resources. But the allure of a diamond versus a lump of coal suggests that relative abundance across resources may be at least as important to understanding the economics of natural resources as the impending exhaustion of one resource. This paper develops an empirically founded theory of how markets utilize a wide variety of natural resources when what is important is the relative abundance across natural resources, not their impending exhaustion.

This paper makes three contributions. First, for the 48 chemical elements for which economic data exists, it documents that a 1% increase in a measure of relative abundance in the Earth's crust is associated with a 0.72% increase in world production, a -0.45% decrease in the real price, hence a 0.27% = 0.72% - 0.45% increase in primary industry revenues, and a 0.12% increase in the number of patents.[1] Second, these relationships are explained using a model of biased endogenous technological change (e.g., Hicks 1932, Kennedy 1964, Acemoglu 1998, 2002, 2007, 2012, Acemoglu, Aghion, Bursztyn, and Hemous 2012). The economy is characterized by a number of vertically aligned sectors each producing from a particular natural resource, where the relative abundance varies across resources. Both the upstream sectors which produce raw materials and the downstream sectors which process these materials into intermediate goods use varieties of specialized machines designed to extract raw materials from the resource stock or to transform the raw materials into intermediate goods. These machines use designs created by R&D using labor inputs and the fruits of past knowledge. The relative shares of R&D depend upon expectations of profitability, which are traced ultimately to the relative abundance of the resources. The third contribution is to use the relative production, price and R&D elasticities with respect to abundance to estimate the elasticity of substitution between raw materials, the natural resource rental share of income in primary production, and the elasticity of R&D with respect to primary industry revenues. External validation of these parameter estimates is made by comparing model predictions with observed production and real price growth.

The central argument is that an increase in the relative abundance of a resource causes the economy to discover more uses for and more ways to find, extract and

[1] These elasticities are from Table **??**, discussed below.

refine that resource. Rosenberg (1973), for example, argued that the abundance of timber in America relative to Europe induced Americans to use wood for heating, home construction and even plank roads long after these uses had disappeared in Europe. While our distant ancestors had no way of knowing which resources were most useful, they could identify those which were most abundant. Beginning with stone and wood, and after many hundreds of thousands of years eventually working their way to the metals, early humans learned how to use, find, extract and refine many resources. But the advantage for more abundant resources has continued into modern times. The economic mechanism identified here is that, all else equal, because primary industry revenues are higher for more abundant resources, it is more profitable to obtain a patent for a new use or for a new means of finding, extracting or refining a more abundant resource than it is for a less abundant resource. Thus, more abundant resources attract greater R&D and as a consequence, we know more about how to use, find, extract, and refine those resources.

The elasticity of substitution between intermediate goods produced from the different resources plays a crucial role in determining the relative R&D across sectors. This elasticity is related to two other derived elasticities of substitution, one for raw materials and one for primary resources. Econometric estimates of these elasticities show each to be greater than one in value, indicating that resources are substitutes in production.[2] This is encouraging, because an elasticity of substitution greater than one between exhaustible natural resources and a broad form of capital is a necessary condition for per capita consumption to be non-declining (Dasgupta and Heal 1974). The elasticity of substitution, however, is not too much larger than one in value. Thus, resources are substitutable, but not too substitutable. Unlike relative wages for high-skilled labor, which have been rising as the relative supply of high-skilled labor has increased (Acemoglu 1998), the technology-adjusted relative demand for raw materials is decreasing in relative prices.

An important feature of the model is that R&D occurs at both the downstream use of raw materials stage as well as at upstream resource production stage. Upstream R&D finds new ways to find, extract and refine raw materials from natural resources, which shifts out the supply curve (e.g., Slade 1982), while downstream R&D finds new uses for raw materials, which shifts out the demand curve (e.g., Acemoglu et al. 2012). Thus, while both types of R&D increase production, their effects on raw

[2]The natural resource literature has focused on substitution between energy resources, e.g., Chakravorty, Roumasset, and Tse (1997) and Chakravorty, Moreaux, and Tidball (2008). Like the metals considered here, oil, coal and natural gas are undoubtedly substitutes, as which has been used to produce electricity or to power transportation has depended largely on price. Furthermore, like the ordering of stone to bronze to iron to more exotic metals, the ordering of extraction from wood to coal to oil and natural gas has used the most abundant first, given the state of technology (see Fouquet and Pearson 1999).

materials prices work in opposition. For both up- and downstream R&D, however, the mechanism driving technical change is the size of the market, which is increasing in relative abundance.[3] Equilibrium growth in raw materials price relative to growth in wages is shown to depend upon the product of the elasticity of R&D with respect to market size and the rental share of resources in primary industry income. The parameter estimates are used to explain observed declining real resource prices and rising production over the last century. Intuitively, these occur because R&D shifts out both demand and supply, and it makes labor more productive, causing real prices to decline.

In addition to the production, price and R&D correlations with abundance, two other features of the data are important. One is that positive levels of R&D are observed for all chemical elements.[4] Therefore, following Jones (1995) it is assumed that R&D is subject to diminishing returns both in the variable input of scientists engaged in R&D and in the spillovers from past R&D, and the elasticity of R&D with respect to primary industry revenues is shown to be increasing in these parameters. In contrast, when returns to R&D are linear in the number of scientists involved in R&D, as in Acemoglu *et al* (2012), an advanced relative state of knowledge in one sector yields a corner solution with all R&D occurring in that sector.

Second, cumulative discoveries over the last century are on the order of 10^{-8} of what physicists believe to exist in the upper crust of the Earth. This fact is interpreted to imply that changes in relative scarcity are sufficiently small that they may be ignored. Thus, unlike a Hotelling exhaustible resource model in which price and production vary over time due to rising scarcity rents as a particular resource is exhausted,[5] this paper attempts to determine how much of the variation in production, prices, and R&D in the cross-section can be explained by a model in which the relative abundance across resources, not the impending exhaustion of resources, is the key determinant. Previous researchers who have examined data from a number of natural resources, such as Barnett and Morse (1963), Smith (1979), Slade (1982), and Lee, List, and Strazicich (2006), emphasized the intertemporal properties of the price paths, not the cross-resource variations.[6] The cross-resource variations are shown to be empirically

[3]In contrast, the literature has focused almost exclusively on price effects on R&D: Newell, Jaffe, and Stavins (1999) found that air conditioners became more energy efficient in response to increases in oil prices and Popp (2002) found that an increase in energy prices increases the number of patents filed for eleven energy-saving technologies.

[4]Indeed, even the heaviest twenty elements, Einsteinium (Es_{99}) through Ununoctium (Uuo_{118}), observed only in physics laboratories, averaged 35 patents each from 1976-2012. See Table **??** for the other elements.

[5]Hotelling's central prediction that price minus marginal cost should be rising over time remains controversial (e.g., see Hamilton 2009).

[6]An exception is Pindyck and Rotemberg (1990), who investigated whether commodity prices move together. However, there is no role for relative abundance or R&D in their explanation.

supported by simple cross-section correlations and, unlike Hotelling predictions of rising prices, are robust across sample periods.

The remainder of the paper is organized as follows. Section **??** explains the physical basis for the relative abundance of the chemical elements, and Section **??** documents the relationship between relative abundance and production, prices and innovation, as measured by patents. Section **??** develops the assumptions of the economic model and Section **??** derives the conditions that must hold in equilibrium. Section **??** derives the equations governing how the equilibrium is affected by relative abundance. Section **??** estimates the underlying economic parameters implied by the correlations between relative abundance and relative price, production and innovation and uses these to predict price and quantity growth of resources. Section **??** concludes.

2 Relative Abundance

This section explains the origin of the relative abundance of the nearly 90 naturally occurring chemical elements and their isotopes through the physics of nucleosynthesis, the transformation of elements into other elements by nuclear fusion or radioactive decay. This theory arose when physicists recognized that relative abundances of the elements "bore signs of representing the ash of a cosmic nuclear fire in which [they were] created" (Suess and Urey 1956, p. 53).

Nuclear fusion, in which lighter elements are fused into heavier elements, is the primary mechanism by which the elements were created. Temperatures on the order of millions degrees Kelvin are required to fuse hydrogen (H_1^1) into helium (He_2^4) and much greater temperatures (on the order of a hundred billion degrees) to create the heaviest elements.[7] These conditions were present in the Big Bang, and so it might be thought that all elements arose then. But two factors prevented this from occurring. The first was that temperatures were so high that the first step in the fusion of hydrogen into helium, the creation of deuterium (H_1^2) by the "proton-proton chain" process of a "neutron capture," was impossible because neutrons and protons collided with too great of intensity to form a stable atom. Second, because of the rapid expansion of the universe in the Big Bang, by the time it cooled sufficiently to allow deuterium to form, less than twenty minutes remained before temperatures dropped below where fusion was possible. Thus, out of the Big Bang, only hydrogen and helium and their

[7] The superscript is the atomic mass, A, the number of protons plus neutrons in the nucleus of an atom; the subscript is the atomic number, Z, the number of positive charged protons. All isotopes of an element have the same atomic number, but differ in their atomic mass by having more or fewer neutral charged neutrons. Although Z and the element name are redundant, they are each listed here as Figures **??** and **??** report abundance by atomic number.

isotopes were formed with a 3:1 mass ratio (twelve hydrogen atoms for each helium atom) (Weinberg 1984).[8]

All of the other elements are created by nuclear fusion in stars. All stars begin by "burning" hydrogen into helium. Depending upon its mass, a star may, by a series of gravitational core contractions, generate sufficient heat to eventually produce all elements up to the atomic mass of iron (Hansen et al 2004). Stars smaller than about eight times the mass of the sun end their lives after billions of years when the remaining white dwarf star of helium or carbon (C_6^{12}) is insufficient in mass to generate further nuclear fusion.[9] Nevertheless, these stars contribute about half of the elements higher than iron through a process called the "slow" *s*-process (neutron capture is less than the rate of radioactive decay), which occurs in the red giant phase of the star's life as heavy elements in the core are thrown to the surface by "helium flashes," nuclear explosions at the shell of the core caused as the core contracts, where they gain mass through neutron capture and are expelled by the solar winds of the helium flashes. Stars with a mass between eight and forty to fifty times the mass of the sun, however, end their lives in a core collapse Type II supernova.[10] This is caused when the iron-nickel (Ni_{28}^{56}) core of the star, composed of the heaviest elements that can be created by stellar nuclear fusion, reaches its Chandrasekhar limit of about 1.4 solar masses, at which point the "electron degeneracy" which held the atoms in the core apart can no longer withstand the gravitational pressure. The core then collapses at speeds reaching nearly a quarter the speed of light until the atoms in the core are compressed into a dense neutron star. As the core collapses, the outer layers of the star fall inwards until they bounce off the neutron star. During this process, temperatures approaching 10^{10} K $^\circ$ are reached, creating about half of the elements between iron and uranium (U_{92}^{238}), through processes such as the 'rapid' *r* process (neutron capture is faster than the rate of radioactive decay), the *p*-process (proton capture), and the α-freeze out process (Hansen *et al.* 2004 pp. 78-83).[11] Because elements heavier than iron are created either by supernovae explosions, which occur over a mater of seconds and only in massive stars, or by the *s*-process, which occur over a matter of thousands of years,

[8] Trace amounts of lithium (Li_3^7), beryllium (Be_4^9), and and boron (B_5^{10}) (on the order of 10^{-10} percent of mass) were created in the Big Bang, but because these atoms are unstable, most lithium, beryllium and boron on Earth were created by "cosmic ray spallation" (Arnett 1996), in which radiation causes heavier elements to decay into these elements.

[9] If the white dwarf star has a binary companion star (which occurs for about 1/3 of stars), it is possible that a complex gravitational interaction can cause the white dwarf to "steal" mass from its companion until the additional mass causes the shell of the star to heat sufficiently that a hydrogen nuclear explosion occurs, causing a Type I supernova, but this mostly releases hydrogen.

[10] Stars of greater mass collapse into a black hole, and so contribute no new elements.

[11] Table **??** in the Appendix summarizes the physical processes responsible for each of the different elements and their isotopes.

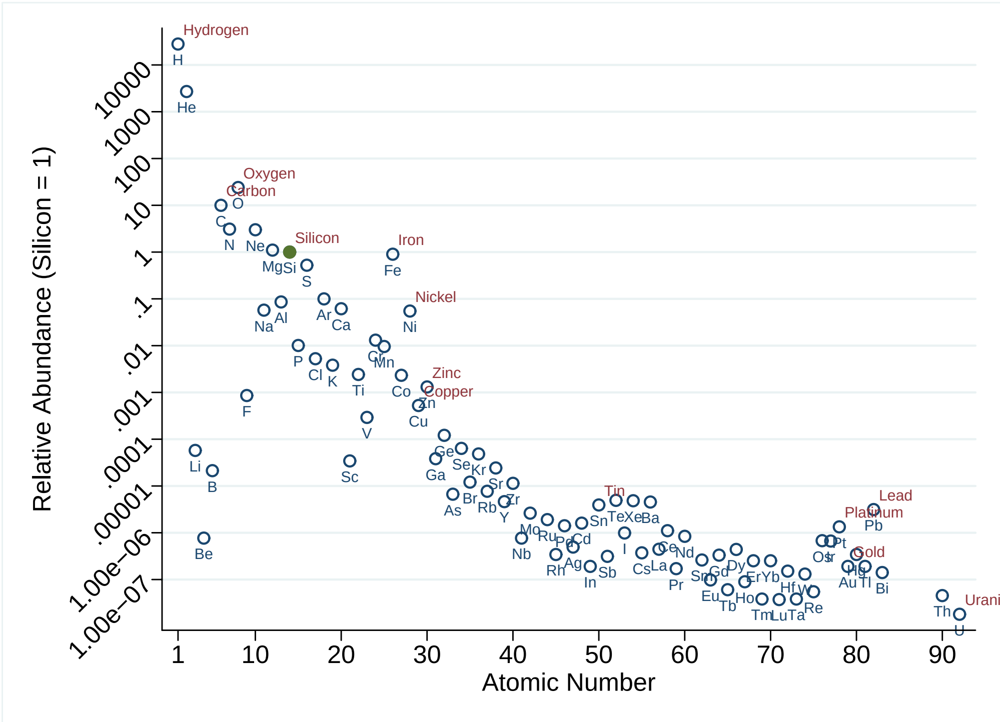

Figure 1: Relative Abundance of Chemical Elements by Atomic Number in the Solar System (log scale, Silicon=1).

rather than over the billions of years of which elements lighter than iron, these elements are many orders of magnitude less abundant than elements lighter than iron.

The solar system formed about 4.5 billion years ago when of the dust from some previous supernova began spinning, causing it to gravitationaly collpase (McDonough 2000). Figure **??** shows the modern estimate of relative mean abundance of the chemical elements in the solar system.[12] (See Table **??** in the Appendix for the chemical element names associated with each symbol in Figure **??**, although several important elements are indicated by name.) The horizontal axis is the atomic number Z while the vertical axis is the mean estimate of abundance relative to silicon (S^{28}_{14}), which has been normalized to 1. As the variation in scale of abundance is on the order of 10^{12}, the abundance axis is in $\log_{10}$ scale.

Hydrogen and helium are greater than 10,000 and 1,000 times more abundant than

[12]This data is from Newsome (1995, 159-89). Since the Sun comprises 99.8% of the solar system's mass, this is essentially the abundance of elements in the Sun. The estimated errors are higher for heavier elements, but are thought to be smaller than an order of magnitude for all elements.

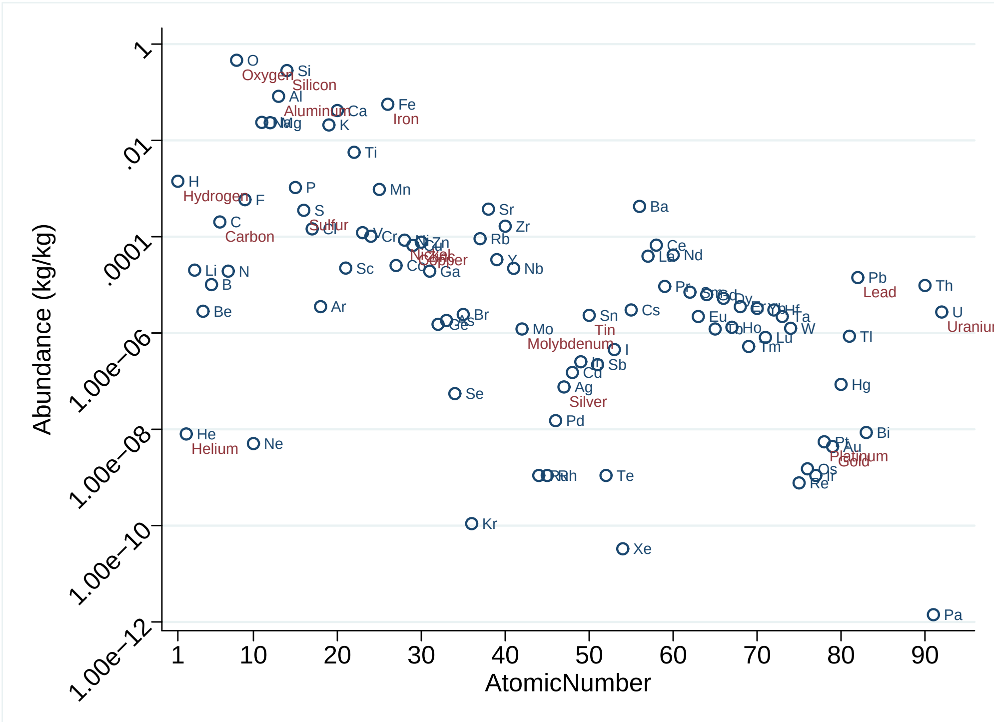

Figure 2: Relative Abundance (kilograms / kilogram) of Chemical Elements by Atomic Number on Earth (log scale).

silicon, respectively. There is low abundance of three of the lightest elements, lithium, beryllium and boron, because these were not easily formed in the either the Big Bang nor in stellar nucleosynthesis. Two other elements, technetium (Tc_{43}) and promethium (Pm_{61}) are only formed by decay of heavier elements and are thus extremely rare.[13] In addition, six elements, polonium (Po_{84}), astatine (At_{85}), francium (Fr_{87}), radium (Ra_{88}), actinium (Ac_{89}), and protactinium (Pa_{91}) are radioactive with half-lives sufficiently short that they have all but disappeared. There is also a distinct drop in abundance for each odd numbered element (a result known as Harkin's rule). There are also spikes occurring at iron and lead (Pb^{208}_{82}), corresponding to the highest stable elements that can be created by normal fusion burning and by supernovae explosions, respectively (Burbidge et al. 1957). The simple correlation between the natural log of solar system abundance and the atomic number is $r = -0.82$ ($p < 0.01$).

[13] The discovery of technetium in a "red giant" star of much greater age than the half-life of technetium (4.2×10^6 years for Tc^{99}_{43}) is direct evidence for the theory of nucleosynthesis (Hansen *et al.* 2004, p. 69).

But what about the abundance of the elements on Earth? The Earth has a nickel/iron core, surrounded by a liquid silicate mantel and a solid crust. The upper crust (from 0-5 km deep), which is what is of economic interest, comprises only 0.4% of Earth's mass. Figure **??** documents the relationship between the atomic number and abundance of the chemical elements found in the Earth's crust using data from Haynes and Lide (2011). The horizontal axis is again the atomic number; but the vertical axis is the estimate of mass fraction of abundance, measured in kilograms of the element per random kilogram of the crust of the Earth (plotted again using a $\log_{10}$ scale). As in the solar system, on average the more abundant chemical elements are those with lower atomic numbers. The simple correlation between natural log of abundance and the atomic number is $r = -0.59$ ($p < .01$). The exceptions noted for the solar system also hold for the Earth, although the crust (which excludes the oceans and atmosphere) exhibits lower levels of most of the gaseous elements and is relatively rich in non-metals such as silicon, carbon and oxygen.

3 Economics of Relative Abundance

Now, let us turn to the central question of the paper, which is how relative abundance in the Earth's crust correlates with various economic measures.

Figure **??** displays the correlation between the mean annual world production between 1970-2008 in metric tons (1000 kilograms) for the 48 chemical elements for which production data is available, and the abundance in the Earth's crust, measured in kilograms of mass per kilogram of Earth's crust.[14] As is apparent, there is a strong positive correlation between annual world production and abundance, with order of magnitude differences in relative abundance resulting in orders of magnitude differences in production. The correlation between the natural log of mean annual production and the natural log of abundance is $r = 0.72$ ($p < 0.01$).

But what about prices? Prices are determined by Marshall's scissors of the intersection of demand and supply, which in turn depend upon the states of knowledge in how to use and how to find, extract and refine resources. Thus, *a priori*, it is not clear what to expect—the price and abundance correlation could show an uncorrelated "cloud." Figure **??**, however, shows that this is not the case. There is a strong negative correlation between the natural log of the mean real price per metric ton between 1970-2008 for the same 48 chemical elements from Figure **??** and the measure of abundance. The simple correlation between the natural log of price and the natural log of abundance in the Earth's crust is $r = -0.66$ ($p < 0.01$).

[14] World production and price data is from Kelly and Matos (2012), who have compiled production and

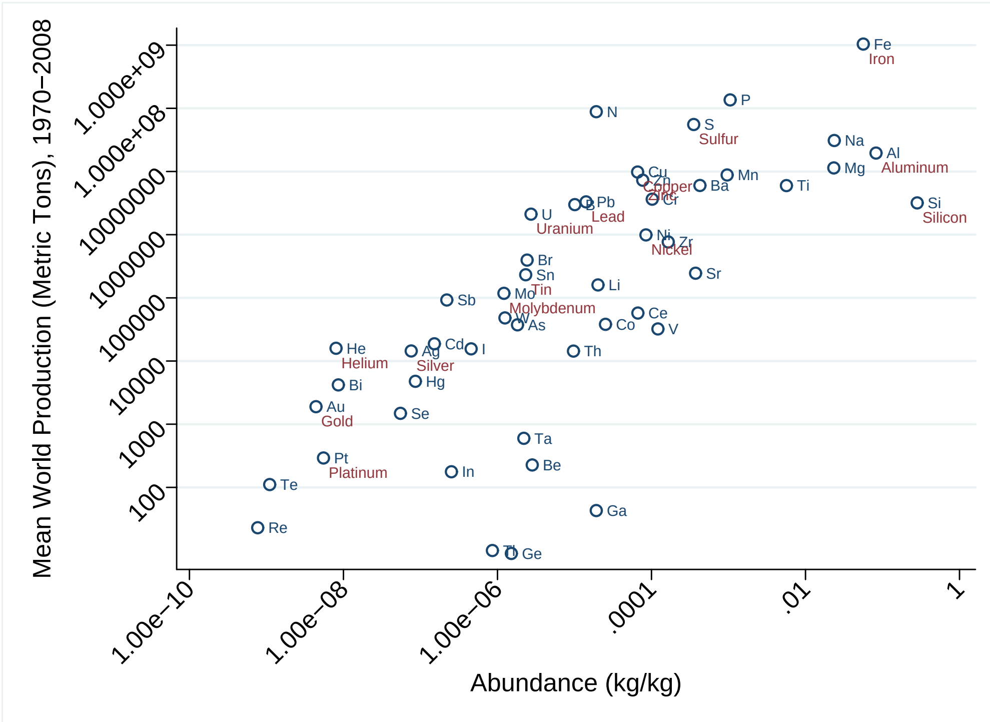

Figure 3: Mean Annual World Production by Abundance, 1970-2008 (log-log scale).

Next, consider the relationship between relative abundance and innovation. A simple measure of innovation is a count of the number of U.S. patents filed between 1976-2010 in which each chemical elements' name appears.[15] This is a flow variable, measuring the change in the state of knowledge. It is imperfect since many innovations are not patented since once a patent is filed, competitors might use the information in the patent application to engineer alternatives (Popp 2002). Also, common usage of certain elements' names (e.g., gold, silver and platinum) may be reflected in the numbers. Nevertheless, Figure **??** shows that for the 48 elements for which other economic data exist, there is a positive correlation between the natural log of the number of U.S. patents filed between 1976-2010 which name each chemical element and the measure of abundance of that element. The correlation between the natural log of the number of patents in this interval and the natural log of abundance is $r = 0.44$

price data for a number of minerals from 1900 to present. Data before 1970 is discussed below.

[15]The patent data was collected in December 2010 using the U.S. patent system on-line search engine (http://patft.uspto.gov/).

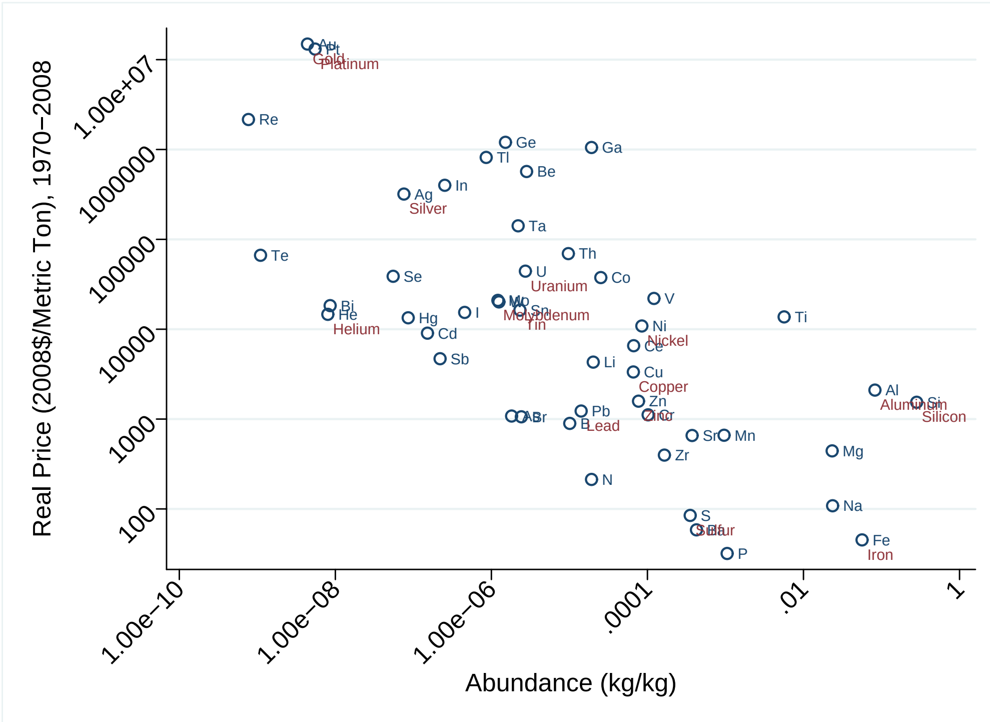

Figure 4: Mean Annual Real Price by Abundance, 1970-2008 (log-log scale).

($p < 0.01$). Thus, the number of patents associated with an element is increasing in abundance.

The theory developed below argues that the reason there are more patents for more abundant resources is because the size of the market is larger for more abundant resources and large markets attract more innovation because the value of a patent is higher in a larger market. One measure of the size of the market is the total primary industry revenues, i.e., price times world production, for each element. The correlation between the natural log of total primary industry revenues and the natural log of abundance is positive and significant ($r = 0.52$, $p < 0.01$). More importantly, the relationship between the number of patents and primary industry revenues is plotted in Figure **??** for the 48 chemical elements. The correlation between the natural log of the number of patents from 1976-2010 and natural log of annual real industry revenues from 1970-2008 is $r = 0.61$ ($p < 0.01$). While there are important exceptions involving three of the radioactive elements, barium (Ba_{56}^{138}), thorium (Th_{90}^{232}) and uranium, it is clear that the relationship between total primary industry revenues and innovation is

Figure 5: U.S. Patents by Abundance, 1976-2010 (log-log scale).

positive and significant.

Figure **??** shows the relationship between absolute abundance (measured as relative abundance times the mass of the Earth's crust, $0.4\% \times 5.97 \times 10^{24}$ kilograms $= 2.38 \times 10^{19}$ metric tons) and the cumulative discoveries, measured as the sum of cumulative production 1900-2008 plus proved reserves of a mineral in 2010. The data show that cumulative discoveries are higher for more abundant chemical elements ($r = 0.79$, $p < 0.01$).[16] But perhaps the most interesting fact to be observed in Figure **??** is that the ratio of cumulative discoveries to abundance in the Earth's crust average on the order of 10^{-8} of absolute abundance. Thus, even if only one percent of the crust is ever exploitable, and even though society has produced and discovered more in the last century than ever before, we have still exploited at most about a million$^{\text{th}}$ of the potential.

[16]Since absolute abundance is simply abundance times the estimated mass of the Earth's crust, which is constant, this correlation also holds for relative abundance. The correlation between proved reserves in 2010 and relative abundance is also positive and highly significant ($r = 0.77$, $p < 0.01$).

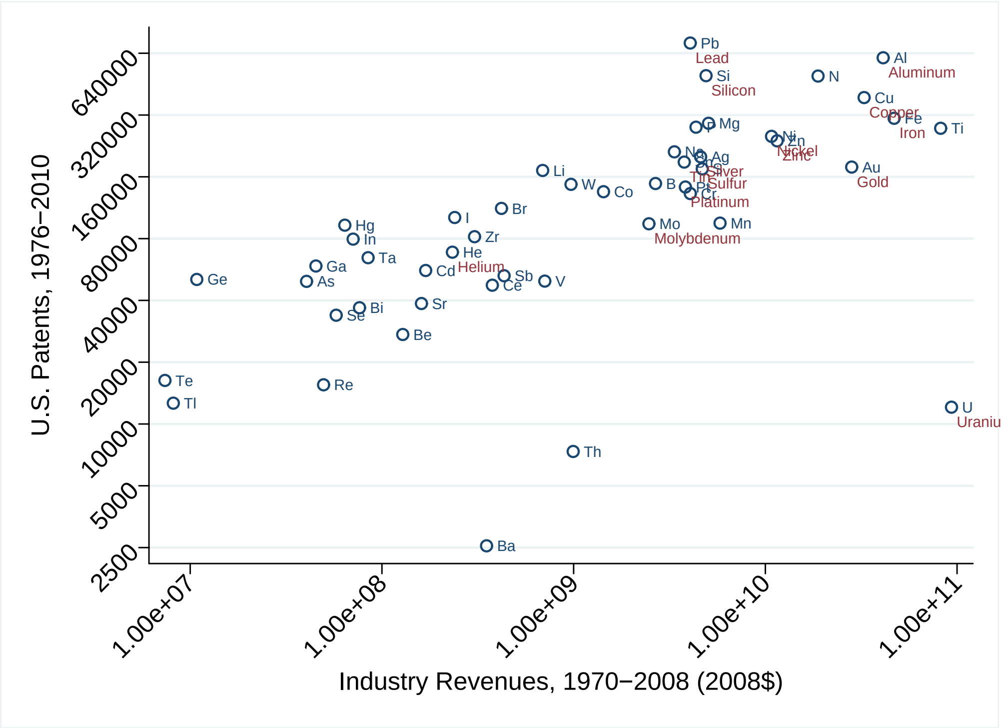

Figure 6: U.S. Patents, 1976-2010, by Mean Primary Industry Revenues, 1970-2008 (log-log scale).

The correlations in Figures **??** - **??** use price and production data from 1970-2008.[17] Table **??**, Panel A shows that these relationships are robust across different eras, even though production of most of the chemical elements has been rising over the last century. The correlations in Table **??**, Panel B show how the number of patents in the 1976-2010 period is related to mean primary industry revenues and its constituent parts of annual production and mean real prices across different time periods. Again, these relationships are robust across different sample periods. Given that the literature has focused on price effects (e.g., Newell, Jaffe and Stavis [1989] and Popp [2002]) it is interesting that price-innovation correlation is the weakest.

[17] The elasticities in the introduction are related to the correlations in Table **??** according to $\epsilon_{XA} = (s_A/s_X)r_{XA}$, where s_X and s_A are estimated standard deviations and where ϵ_{XA} and r_{XA} are the elasticity and the correlation, respectively, between A = abundance and X = prices, production, total revenues, and patents (all in logs), where $s_A = 4.8$, $s_P = 3.2$, $s_Q = 4.7$, $s_{P\times Q} = 2.5$ and $s_{\text{Patent}} = 1.2$.

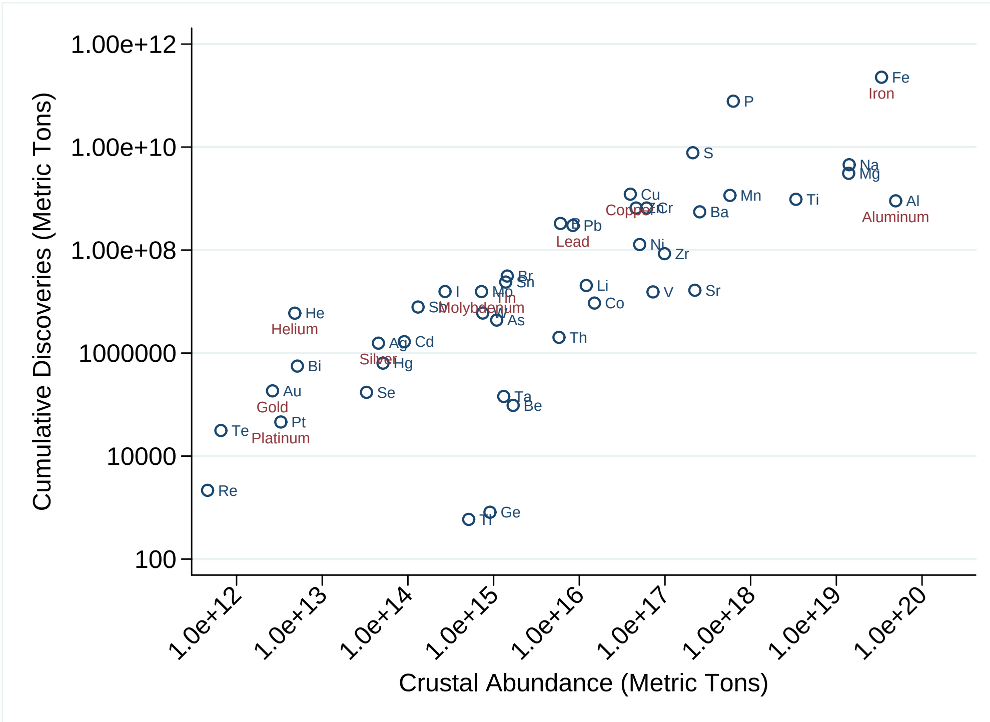

Figure 7: Cumulative Discoveries by Absolute Abundance, 2010 (log-log scale).

4 Theoretical Model

Now let us develop a theory to explain the observed correlations in production, prices and innovation. Consider an economy in which a single final good is produced using inputs derived from N intermediate goods sectors, indexed $i = 1, \ldots, N$, where each sector which is identified with a particular natural resource input upon which it depends. Each natural resource sector is further characterized by an upstream raw materials producing sector and a downstream intermediate goods producing sector. In addition, in each upstream and downstream sector, there exist a number of producers of machines specific to that sector, and where each machine is characterized by a particular variety of use of raw materials (at the downstream level) or a particular variety of methods for extracting raw materials from natural resources (at the upstream level), and where each variety is protected by an infinitely lived patent.

Table 1: Correlations between Abundance and Economic Variables for Different Time Periods

		A: Correlation between Crustal Abundance vs.		
Period	Observations	Annual Production	Real Price	Primary Industry Revenues
1970-2010	48	$r = 0.72$, $p < 0.01$	$r = -0.66$, $p < 0.01$	$r = 0.52$, $p < 0.01$
1950-1969	43	$r = 0.74$, $p < 0.01$	$r = -0.66$, $p < 0.01$	$r = 0.46$, $p < 0.01$
1900-1949	35	$r = 0.78$, $p < 0.01$	$r = -0.61$, $p < 0.01$	$r = 0.47$, $p < 0.01$

		B: Correlation between Number of Patents, 1976-2010, vs.		
Period	Observations	Annual Production	Real Price	Primary Industry Revenues
1970-2010	48	$r = 0.53$, $p < 0.01$	$r = -0.30$, $p = 0.04$	$r = 0.61$, $p < 0.01$
1950-1969	43	$r = 0.52$, $p < 0.01$	$r = -0.28$, $p = 0.06$	$r = 0.67$, $p < 0.01$
1900-1949	35	$r = 0.47$, $p < 0.01$	$r = -0.19$, $p = 0.22$	$r = 0.69$, $p < 0.01$

Notes: Reported correlations are for natural logs of period sample means. Production is in metric tons; real prices and primary industry revenues are in 2008 dollars per metric ton; patents are total U.S. patents over 1976-2010; and crustal abundance is measured in kilograms per kilogram.

4.1 Goods Production

Final good production is assumed to be constant elasticity of substitution (CES) between the N intermediate inputs:[18]

$$Y = \left[\sum_{i=1}^{N} y_i^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where $\epsilon \geq 0$ is the elasticity of substitution between intermediate goods inputs. When $\epsilon < 1$, the goods are complements and when $\epsilon > 1$, the goods are substitutes. Final good production is constant-returns-to-scale.

Each intermediate input is produced using raw materials, q_i, and production machines embodied with various technologies, x_{ij}, indexed over $j \in [0, A_i]$, where A_i is the state of knowledge in sector i according to a Cobb-Douglas (CD) technology:

$$y_i = \frac{1}{1-\alpha} (q_i)^{\alpha} \int_0^{A_i} (x_{ij})^{1-\alpha} \, \mathrm{d}j, \qquad i = 1, \ldots, N, \tag{2}$$

where $0 < \alpha < 1$ is the primary industry share of intermediate goods income. This production function is constant-returns-to-scale in x_{ij} and q_i, the variable inputs at any point in time.

Raw materials production in the upstream sectors depend upon the resource abundance R_i, and extraction machines embodied with technologies indexed $j \in [0, B_i]$, where B_i is the state of knowledge in extraction machines according to a Cobb-Douglas

[18] As most of the focus is on cross-sectional correlations, time subscripts for variables are omitted.

technology as follows:

$$q_i = \frac{1}{1-\beta}(R_i)^\beta \int_0^{B_i} (z_{ij})^{1-\beta}\,\mathrm{d}j, \qquad i = 1,\ldots,N, \tag{3}$$

where $0 < \beta < 1$ is the resource rents share of primary industry income. The key assumption is that an increase in R_i reduces the amount of work required to produce a unit of raw materials. This production function is constant-returns-to-scale in the inputs R_i and z_{ij}. Extraction at any point in time is sufficiently small that resource abundance is unaffected by extraction, except through the effect of having previously extracted all of the raw materials that can be extracted at less than the current price.

4.2 Knowledge Production

Therefore, at any point in time there are two stocks of knowledge, downstream knowledge A_i and upstream knowledge B_i, for each resource $i = 1,\ldots,N$. These stocks of knowledge are increased by research and development activities which uses labor as the sole variable input. The growth in these stocks is governed by the following:

$$\dot{A}_i \equiv dA_i/dt = \eta_i \left(L_A^i\right)^\delta (A_i)^\gamma, \qquad A_i(0) \geq 1 \qquad i = 1,\ldots,N, \tag{4}$$

and

$$\dot{B}_i \equiv dB_i/dt = \eta_i \left(L_B^i\right)^\delta (B_i)^\gamma, \qquad B_i(0) \geq 1 \qquad i = 1,\ldots,N, \tag{5}$$

where $0 < \delta, \gamma < 1$. Thus research and development is subject to positive but diminishing spillovers from past R&D and the production of ideas there are diminishing returns to the variable input labor. These assumptions follow Jones (1995). The diminishing returns to labor may occur be because as the number of scientists increase the odds of two or more scientists discovering the same idea increase. The diminishing returns to past knowledge imply that the low-hanging fruit are the first discovered.

The parameters η_i represent differences in the rates at which blueprints for machines are discovered for different resources at the upstream and downstream levels, for given labor and states of knowledge. These may differ because iron may be such that its usefulness in making varieties of products is higher than for vanadium (V_{23}^{51}), yielding a higher η_i for iron relative to vanadium. Conversely, minerals such as the rare earth mineral cerium (Ce_{58}^{140}) are inherently more difficult to extract as their dispersion is less lumpy than for minerals such as copper (Cu_{29}^{63}), which has similar abundance, so that η_i may be less for cerium than for copper.

Knowledge production occurs because once a blueprint is discovered its owner is entitled to earn monopoly profits for the rental of machines based on that blueprint.

The nature of these profits is discussed below.

4.3 Welfare and Constraints

Welfare in this economy is given by a constant intertemporal substitution elasticity utility function in consumption c

$$U = \int_0^\infty L(0)e^{-(\rho-n)t}\left[\frac{c(t)^{1-\theta}-1}{1-\theta}\right]\mathrm{d}t, \tag{6}$$

where $\rho > 0$ is the discount rate, where n, such that $0 < n < \rho$, is the population growth rate, where $L(0)$ is the initial population, and where $\theta > 0$ is the intertemporal elasticity of substitution.

Total output in the economy is governed by

$$Y \geq cL + (1-\alpha)X + (1-\beta)Z, \tag{7}$$

where $X = \sum_{i=1}^{N}\int_0^{A_i} x_{ij}\,\mathrm{d}j$ and $Z = \sum_{i=1}^{N}\int_0^{B_i} z_{ij}\,\mathrm{d}j$. The costs incurred by production of machines follows Acemoglu (2012, Chapter 15), in that the cost relative to final goods production of downstream intermediate goods producing machines is $1-\alpha$ and the cost of upstream resource extraction machines is $1-\beta$.

Labor is thus allocated across the $2N$ R&D sectors, with the constraint that total labor supply is fixed:

$$\sum_{i=1}^{N} L_A^i + L_B^i \leq L. \tag{8}$$

This model differs from the baseline directed technical change model of Acemoglu (2012) in several respects. First, the raw materials in the downstream intermediate goods production are endogenous variables. It is the resource abundance R_i in the upstream raw materials production that is exogenous. Second, because there are two production processes feeding into final goods production, for each resource there are two knowledge state variables, A_i, and B_i, rather than just one, A_i. This allows consideration of both cost-reducing and demand-enhancing R&D. The model also differs from Acemoglu *et al.* (2012) in two important ways. As in Acemoglu's baseline model, here R&D is of the expanding varieties form where the number of machines capable of producing the intermediate good or the natural resource simply expands as the knowledge set expands. Thus R&D produces a new machine. In Acemoglu *et al.*, in contrast, R&D produces an improved machine so that in the quality-ladder approach, a machine's patent is thus of uncertain life, while in the expanding-varieties approach, a patent is of infinite length. Second, I follow Jones (1995) in specifying that the number of new varieties of machines in each sector is subject both to diminishing returns to labor employed in each sector and to diminishing returns to past knowledge.

5 The Market Economy

This section derives the equilibrium conditions that hold in a market economy.

5.1 Final Goods Production

Let the price of the final good be normalized to unity, i.e., $P_Y = 1$. Facing prices p_y^i for each intermediate good, the final good sector maximizes

$$\pi_Y = \left[\sum_{i=1}^{N} y_i^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} - \sum_{i=1}^{N} p_y^i y_i.$$

Thus, the price-taking final goods producer chooses intermediate goods inputs y_i to satisfy

$$\frac{\partial \pi_Y}{\partial y_i} = \left[\sum_{j=1}^{N} y_j^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}} y_i^{-1/\epsilon} - p_y^i = 0, \qquad i = 1, \ldots, N. \tag{9}$$

This implies that the price index for intermediate goods satisfies

$$\left[\sum_{i=1}^{N} (p_y^i)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1. \tag{10}$$

5.2 Downstream Intermediate Goods Production

A price-taking downstream intermediate goods sector producer for resource i faces prices p_q^i for the raw materials it purchases from the upstream sector and p_x^{ij} for each variety j of machines it rents. Thus profits are

$$\pi_y^i = p_y^i \frac{1}{1-\alpha} (q_i)^\alpha \int_0^{A_i} (x_{ij})^{1-\alpha} \, \mathrm{d}j - p_q^i q_i - \int_0^{A_i} p_x^{ij} x_{ij} \, \mathrm{d}j, \qquad i = 1, \ldots, N.$$

Thus, the firm's choices satisfy

$$\frac{\partial \pi_y^i}{\partial q_i} = \alpha p_y^i \frac{y_i}{q_i} - p_q^i = 0, \qquad i = 1, \ldots, N. \tag{11}$$

and

$$\frac{\partial \pi_y^i}{\partial x_{ij}} = (q_i)^\alpha (x_{ij})^{-\alpha} p_y^i - p_x^{ij} = 0, \qquad \forall j \in [0, A_i],\ i = 1, \ldots, N. \tag{12}$$

5.3 Downstream Machines Production

Each intermediate goods machine producer is endowed with an infinitely lived patent for the blueprint on its machine. Each machine is produced using $1-\alpha$ units of the final good and and lasts one period. From (**??**), the demand for a machine of type j is given by $p_x^{ij} = (q_i)^\alpha (x_{ij})^{-\alpha} p_y^i$. Thus profits in each period are

$$\pi_x^{ij} = (q_i)^\alpha (x_{ij})^{1-\alpha} p_y^i - (1-\alpha) x_{ij}, \qquad \forall j \in [0, A_i],\ i = 1, \ldots, N.$$

Each machine sector solves a standard monopoly problem with constant-elasticity of demand and constant marginal costs, implying

$$p_x^{ij} = 1, \qquad x_{ij} = \left(p_y^i\right)^{1/\alpha} q_i, \qquad \pi_x^{ij} = \alpha \left(p_y^i\right)^{1/\alpha} q_i, \qquad \forall j \in [0, A_i],\ i = 1, \ldots, N. \tag{13}$$

Thus machine sector profits and production is increasing in raw materials supply q_i and in the intermediate goods price p_y^i. Since each x_{ij} is identical for all j, intermediate goods production may be written as

$$y_i = \frac{1}{1-\alpha} \left(p_y^i\right)^{(1-\alpha)/\alpha} q_i A_i, \qquad i = 1, \ldots, N. \tag{14}$$

Thus supply of intermediate good i is increasing in raw materials q_i, the state of technology A_i, and the intermediate good price p_y^i.

5.4 Upstream Raw-Materials Production

Each price-taking upstream raw materials sector producer produces raw materials for the intermediate goods production using machines and the resource stock according to (**??**), taking prices p_q^i, p_R^i, p_z^{ij}, for output, natural resources, and machines as given:

$$\pi_q^i = p_q^i \frac{1}{1-\beta} (R_i)^\beta \int_0^{B_i} (z_{ij})^{1-\beta}\, \mathrm{d}j - p_R^i R_i - \int_0^{B_i} p_z^{ij} z_{ij}\, \mathrm{d}j, \qquad i = 1, \ldots, N.$$

Thus, the price taking, profit maximizing firm chooses machines and resources to satisfy

$$\frac{\partial \pi_q^i}{\partial R_i} = \beta p_q^i \frac{q_i}{R_i} - p_R^i = 0, \qquad i = 1, \ldots, N, \tag{15}$$

and

$$\frac{\partial \pi_q^i}{\partial z_{ij}} = (R_i)^\beta (z_{ij})^{-\beta} p_q^i - p_z^{ij} = 0, \qquad \forall j \in [0, B_i],\ i = 1, \ldots, N. \tag{16}$$

5.5 Upstream Machines Production

Each resource extraction machine producer is endowed with an infinitely lived patent on the blueprint for its machine. Each machine is produced using $1-\beta$ units of the final good and lasts one period. From (**??**), the demand for a machine of type j is given by $p_z^{ij} = (R_i)^\beta (z_{ij})^{-\beta} p_q^i$. Thus profits in each period are of the form

$$\pi_z^{ij} = (R_i)^\beta (z_{ij})^{1-\beta} p_q^i - (1-\beta) z_{ij}, \qquad \forall j \in [0, B_i],\ i = 1, \ldots, N,$$

and the profit maximizing solution implies

$$p_z^{ij} = 1, \qquad z_{ij} = \left(p_q^i\right)^{1/\beta} R_i, \qquad \pi_z^{ij} = \beta \left(p_q^i\right)^{1/\beta} R_i \qquad \forall j \in [0, B_i],\ i = 1, \ldots, N. \tag{17}$$

Thus upstream machine profits and production are increasing in the resource abundance R_i and in the raw materials price p_q^i. Since z_{ij} is the same for all j, production of the i^{th} resource is given by

$$q_i = \frac{1}{1-\beta} \left(p_q^i\right)^{(1-\beta)/\beta} R_i B_i \qquad i = 1, \ldots, N. \tag{18}$$

Thus raw materials supply is increasing in the resource abundance R_i, the state of knowledge B_i, and the price of raw materials p_q^i.

5.6 R&D Production

A successful R&D enterprise discovers a machine useful either to producing intermediate goods from raw materials or to producing raw materials from natural resources. Thus, a R&D entrepreneur expects to earn $V_A^i \equiv \pi_x / r$ or $V_B^i \equiv \pi_z / r$ if successful, where r is the equilibrium interest rate (which will be constant along a balanced growth path). Suppose each R&D entrepreneur takes the number of other potential entrepreneurs, l_j^i, as given, where in equilibrium $l_j^i = L_j^i$, for $j = A, B$. Then entry into sector i of R&D type $j = A, B$, continues until the value of the marginal product of labor is equated with its wage rate, w, where the price labor is paid is the present value (at interest rate r) of the stream of profits that can be earned by having a monopoly on a new type of machine in each sector:

$$\frac{\alpha \delta \eta_i}{r} \left(L_A^i\right)^{\delta-1} (A_i)^\gamma \left(p_y^i\right)^{1/\alpha} q_i = w, \qquad i = 1, \ldots, N, \tag{19}$$

$$\frac{\beta \delta \eta_i}{r} \left(L_B^i\right)^{\delta-1} (B_i)^\gamma \left(p_q^i\right)^{1/\beta} R_i = w, \qquad i = 1, \ldots, N. \tag{20}$$

These show that R&D is increasing in market size (as measured by the inputs q_i and R_i, respectively) and in the output prices p_y^i and p_q^i of what is being produced.

5.7 Consumption and Transversality Conditions

Per capita consumption grows according to:

$$g_c \equiv \frac{\dot{c}}{c} = \frac{1}{\theta}\left(r + n - \rho\right). \tag{21}$$

For utility to be bounded, substituting (**??**) into (**??**) reveals that

$$r < \frac{\rho - n}{1 - \theta}. \tag{22}$$

Since the only form of capital in this model is in the knowledge variables, the transversality condition (written in terms of the balanced growth path constant r) is

$$\lim_{t\to\infty} e^{-rt} \sum_{i=1}^{N} \left[A_i V_A^i + B_i V_B^i\right] = 0. \tag{23}$$

6 Equilibrium

This section derives the equilibrium relative prices, production and R&D as functions of relative abundance.

6.1 Raw Materials Equilibrium

First, consider the correlations implied by the downstream demand side of the market. From (**??**), relative production of intermediate goods $i, j = 1, \ldots, N$, is given by

$$\frac{y_i}{y_j} = \left(\frac{q_i}{q_j}\right)\left(\frac{A_i}{A_j}\right)\left(\frac{p_y^i}{p_y^j}\right)^{1/\alpha}. \tag{24}$$

The final goods firm's relative demand is found from (**??**):

$$\frac{y_i}{y_j} = \left(\frac{p_y^i}{p_y^j}\right)^{-\epsilon}. \tag{25}$$

Similarly, the intermediate goods producers' demand for raw materials in sector i (**??**) implies

$$\left(\frac{p_q^i}{p_q^j}\right)\left(\frac{q_i}{q_j}\right) = \left(\frac{y_i}{y_j}\right)\left(\frac{p_y^i}{p_y^j}\right) = \left(\frac{p_y^i}{p_y^j}\right)^{1-\epsilon}. \tag{26}$$

Substituting for p_y^i/p_y^j from (**??**), and for y_i/y_j from (**??**) into (**??**) yields the inverse relative demand function:

$$\frac{p_q^i}{p_q^j} = \left(\frac{A_i}{A_j}\right)^{(\sigma-1)/\sigma} \left(\frac{q_i}{q_j}\right)^{-1/\sigma}, \tag{27}$$

where $\sigma \equiv 1 + \alpha(\epsilon - 1) > 0$ is the derived elasticity of substitution between raw materials, holding technology constant. Thus, the relative demand for natural resources is decreasing in the relative quantity of raw materials q_i/q_j and, when $\sigma > 1$, is also increasing in the relative state of knowledge A_i/A_j. Note that $\epsilon > 1$ implies that $\sigma > 1$.

Now consider the upstream supply side of the market. From (**??**), the relative (inverse) supply of resource i to resource j can be written as

$$\frac{p_q^i}{p_q^j} = \left(\frac{B_i}{B_j}\right)^{-\beta/(1-\beta)} \left(\frac{R_i}{R_j}\right)^{-\beta/(1-\beta)} \left(\frac{q_i}{q_j}\right)^{\beta/(1-\beta)}. \tag{28}$$

Thus, the relative supply price is decreasing in the relative abundance of natural resources R_i/R_j and the relative state of extraction knowledge B_i/B_j, and is increasing in the relative supply q_i/q_j. If resource i is relative more abundant than resource j, then $R_i/R_j > 1$. Thus, all else constant, resources that are relatively more abundant have lower relative prices. The relative state of knowledge B_i/B_j depends upon past investments in R&D. For now it suffices to note that if the state of knowledge is increasing with relative abundance, then this effect augments the effect of relative abundance.

Equating relative supply with relative demand yields expressions for the relative raw materials prices and quantities as functions only of the relative abundance and relative knowledge stocks. These are:

$$\frac{p_q^i}{p_q^j} = \left(\frac{A_i}{A_j}\right)^{\beta(\sigma-1)/\psi} \left(\frac{B_i}{B_j}\right)^{-\beta/\psi} \left(\frac{R_i}{R_j}\right)^{-\beta/\psi} \tag{29}$$

and

$$\frac{q_i}{q_j} = \left(\frac{A_i}{A_j}\right)^{(1-\beta)(\sigma-1)/\psi} \left(\frac{B_i}{B_j}\right)^{\beta\sigma/\psi} \left(\frac{R_i}{R_j}\right)^{\beta\sigma/\psi}, \tag{30}$$

where $\psi \equiv 1 + \beta(\sigma - 1)$ implies that $\psi - 1 = \beta(\sigma - 1) = \alpha\beta(\epsilon - 1)$. Thus $\psi > 1$ if and only if $\sigma > 1$, which we saw above is greater than one if and only if $\epsilon > 1$. The partial correlations between relative abundance and relative prices and relative production agree with the observed correlations if $\sigma > 1$. When $\sigma > 1$, then relative production is positively correlated with both states of relative knowledge, and relative prices are positively correlated with downstream relative knowledge but negatively correlated with upstream relative knowledge. Intuitively, upstream relative knowledge shifts down the relative supply while downstream relative knowledge shifts up the relative demand.

Substituting for relative raw materials quantity and prices from (**??**) and (**??**), respectively, into (**??**) yields an expression for the relative intermediate goods prices in terms of relative knowledge states and relative abundance:

$$\frac{p_y^i}{p_y^j} = \left(\frac{A_i}{A_j}\right)^{-\alpha/\psi} \left(\frac{B_i}{B_j}\right)^{-\alpha\beta/\psi} \left(\frac{R_i}{R_j}\right)^{-\alpha\beta/\psi}. \tag{31}$$

Thus, for $\sigma > 1$, the downstream price of intermediate goods is decreasing in relative abundance and both relative states of knowledge.

Writing the resource demand equation in relative terms and using (**??**) to substitute for p_q^i/p_q^j yields an expression for relative resource prices in terms of relative knowledge states and relative abundance:

$$\frac{p_R^i}{p_R^j} = \left(\frac{A_i}{A_j}\right)^{(\sigma-1)/\psi} \left(\frac{B_i}{B_j}\right)^{(\sigma-1)\beta/\psi} \left(\frac{R_i}{R_j}\right)^{-1/\psi} \tag{32}$$

From this equation, we see that ψ is the derived elasticity of substitution for resources, holding technology constant. Thus, relative resource prices are decreasing in relative resource abundance and (when $\sigma > 1$) increasing in the relative states of knowledge.

6.2 Research and Development Equilibrium

Along a balanced growth path (BGP) we may write equations describing both growth in R&D and equations describing how labor is allocated across sectors. The equations describing the BGP growth rates and the equilibrium interest rate are derived in the Appendix. Here, the focus is on the equilibrium relative R&D labor allocation and the equilibrium relative states of knowledge, each of which are constant along a BGP.

Along a balanced growth path, knowledge in all sectors grows at the same rate, g, which depends on the rate of growth in labor, n,

$$g = \frac{\delta n}{1-\gamma}. \tag{33}$$

Thus, in terms of relative growth, (**??**) and (**??**) imply

$$\frac{L_A^i}{L_A^j} = \left(\frac{\eta_i}{\eta_j}\right)^{-1/\delta} \left(\frac{A_i}{A_j}\right)^{(1-\gamma)/\delta}, \tag{34}$$

and

$$\frac{L_B^i}{L_B^j} = \left(\frac{\eta_i}{\eta_j}\right)^{-1/\delta} \left(\frac{B_i}{B_j}\right)^{(1-\gamma)/\delta}. \tag{35}$$

From (**??**) and (**??**), and substituting for L_A^i/L_A^j and L_B^i/L_B^j from (**??**) and (**??**), the equations describing relative labor supply to R&D in developing new down- and upstream machines may be written in terms of the states of knowledge and market size effects in those markets. Doing so yields,

$$\left(\frac{A_i}{A_j}\right)^{-(1-\gamma-\delta)/\delta} \left(\frac{\eta_i}{\eta_j}\right)^{1/\delta} \left(\frac{p_y^i}{p_y^j}\right)^{1/\alpha} \left(\frac{q_i}{q_j}\right) = 1, \tag{36}$$

and

$$\left(\frac{B_i}{B_j}\right)^{-(1-\gamma-\delta)/\delta} \left(\frac{\eta_i}{\eta_j}\right)^{1/\delta} \left(\frac{p_q^i}{p_q^j}\right)^{1/\beta} \left(\frac{R_i}{R_j}\right) = 1. \tag{37}$$

The left-hand-side of each of these expressions is the relative return to labor engaged in each type of R&D in each resource sector. The sign of the coefficient on relative knowledge states depends upon the sign of the expression $1-\gamma-\delta$. In Acemoglu's (2012) baseline model, this expression is zero ($\gamma = 0$ and $\delta = 1$). But when there is congestion in ideas production (so $\delta < 1$) and declining marginal spillovers from past knowledge (so $\gamma < 1$), this expression may be non-zero. When $0 < \gamma + \delta < 1$ and $\delta > 0$, the relative value of R&D in each sector is positively correlated with the relative market size and prices in each output market.

Then substituting from (**??**) and (**??**) into (**??**) and from (**??**) into (**??**) and solving for the equilibrium BGP states of knowledge yields

$$\left(\frac{A_i}{A_j}\right) = \left(\frac{B_i}{B_j}\right) = \left(\frac{R_i}{R_j}\right)^{-\delta(\psi-1)/\Omega} \left(\frac{\eta_i}{\eta_j}\right)^{-\psi/\Omega}, \tag{38}$$

where

$$\Omega \equiv [\delta(1+\beta) - \beta(1-\gamma)](\sigma-1) - (1-\gamma).$$

Substituting these results into the raw materials supply and raw materials prices equations yields the relationship between relative raw materials production and prices and relative abundance:

$$\left(\frac{q_i}{q_j}\right) = \left(\frac{R_i}{R_j}\right)^{\beta[\delta(\sigma-1)-(1-\gamma)\sigma]/\Omega} \left(\frac{\eta_i}{\eta_j}\right)^{(1+\beta-\sigma)/\Omega}, \tag{39}$$

$$\left(\frac{p_q^i}{p_q^j}\right) = \left(\frac{R_i}{R_j}\right)^{\beta[(1-\gamma)-\delta(\sigma-1)]/\Omega} \left(\frac{\eta_i}{\eta_j}\right)^{-\beta(\sigma-2)/\Omega}. \tag{40}$$

Substituting from (**??**) into (**??**) and (**??**) yields

$$\left(\frac{L_A^i}{L_A^j}\right) = \left(\frac{L_B^i}{L_B^j}\right) = \left(\frac{R_i}{R_j}\right)^{-(1-\gamma)(\psi-1)/\Omega} \left(\frac{\eta_i}{\eta_j}\right)^{-(1+\beta)(\sigma-1)/\Omega}. \tag{41}$$

From this equation, relative knowledge growth is found by substituting for relative knowledge from (**??**) and for relative R&D labor from (**??**) into (**??**) and (**??**) to yield

$$\left(\frac{\dot{A}_i}{\dot{A}_j}\right) = \left(\frac{\dot{B}_i}{\dot{B}_j}\right) = \left(\frac{R_i}{R_j}\right)^{-\delta(\psi-1)/\Omega} \left(\frac{\eta_i}{\eta_j}\right)^{-(\psi-\gamma)/\Omega}. \tag{42}$$

Finally, substituting from (**??**) into (**??**) yields the correlation between relative resource rental prices and relative abundance

$$\left(\frac{p_R^i}{p_R^j}\right) = \left(\frac{R_i}{R_j}\right)^{[(1+\beta)(\sigma-1)(1-\delta)+(1-\gamma)\psi]/\psi\Omega} \left(\frac{\eta_i}{\eta_j}\right)^{(1+\beta)(\sigma-1)/\Omega}. \tag{43}$$

Equations (**??**), (**??**) and (**??**) form the basis of the empirical analysis.

7 Estimation of Economic Parameters

The observables are prices and production, 1900-2008, and knowledge growth, as measured by the number of patents filed over 1976-2010, and relative abundance. Thus, the data yields three correlations with relative abundance. This section estimates these correlations, derives the underlying economic parameters of interest, and uses these parameters to predict raw materials real price and world production growth.

7.1 Estimation of Relative Abundance Elasticities

The correlations are obtained by regressing relative price, relative quantity, and relative innovation on relative abundance. In each case, iron is the numeraire mineral. Because the relative abundance measure, R_i/R_j, varies across minerals but not across time, fixed-effects differences in η_i/η_j cannot be identified. Therefore, identification of the correlations in relative abundances rests upon the assumption that $\eta_i = \eta_j$. Second, while relative price and production data exist both across minerals and across time, the relative patent data is the sum of patents over the 1976-2010 period, which varies only across mineral. Therefore, the regressions reported in Table **??** use a random effects panel estimator for the relative price and relative production regressions and an ordinary least squares estimator (with robust standard errors) for the relative patents regression.

The estimated regressions are of the form:

$$\begin{aligned}
\ln\left(q_{it}/q_{jt}\right) &= \phi_{0Q} + \phi_Q \ln\left(R_i/R_j\right) + e_{Qit}, \qquad i = 1, \ldots, N,\ t = 1900, \ldots, 2008, \\
\ln\left(p_q^{it}/p_q^{jt}\right) &= \phi_{0P} + \phi_P \ln\left(R_i/R_j\right) + e_{Pit}, \qquad i = 1, \ldots, N,\ t = 1900, \ldots, 2008, \\
\ln\left(\dot{A}_i/\dot{A}_j\right) &= \phi_{0A} + \phi_A \ln\left(R_i/R_j\right) + e_{Ai}, \qquad i = 1, \ldots, N,
\end{aligned} \tag{44}$$

where the relative downstream R&D growth $\dot{A}_i/\dot{A}_j$ is approximated by the relative number of patents filed between 1976-2010, where q_{it}/q_{jt} and p_q^{it}/p_q^{jt} are mean annual relative production and prices in year t, and where R_i/R_j is the estimated relative crustal abundance. The errors e_{Qit} and e_{Pit} are assumed to have different variances for each mineral i, and the errors e_{Ai} are white noise.

The regression results are presented in Table **??**. The relative price and relative production regressions in Panels A and B, respectively, include a specification both with and without year effects. The estimated elasticities with respect to relative abundance are statistically different from zero in all samples and show only small variation across the different sample periods and whether year effects are included or not. The relative prices to relative abundance elasticity estimates range between -0.428 and -0.443; the relative production to relative abundance elasticity estimates range between 0.648 and 0.728; and the relative patents to relative abundance elasticity is 0.12.

7.2 Predicted Economic Paramters

The estimates in Table **??** are used to recover the underlying economic parameters. Using (**??**), (**??**), and (**??**), the primitive parameters β, σ, and ψ can be shown to equal the following:

$$\sigma = \frac{\phi_A + \phi_Q}{\phi_A - \phi_P}, \qquad \beta = \frac{\phi_P}{\phi_Q + \phi_P - \phi_A - 1},$$
$$\text{and} \qquad \psi = 1 + \frac{\phi_P(\phi_P + \phi_Q)}{(\phi_A - \phi_P)(\phi_Q + \phi_P - \phi_A - 1)}. \tag{45}$$

The parameters γ and δ, however, are under-identified. But using (**??**), (**??**), and (**??**), the relationship between γ and δ can be shown to be

$$\delta = \zeta\left(1 - \gamma\right), \qquad \text{where} \qquad \zeta = \frac{\phi_A}{\phi_Q + \phi_P} = \frac{d\ln(\dot{A}_i/\dot{A}_j)}{d\ln(p_q^i q_i / p_q^j q_j)}. \tag{46}$$

Table 2: Parameter Estimates from Correlations with Relative Abundance

(A) Panel Estimates for Relative Prices

Sample	1900-2008		1970-2008		1950-1969		1900-1949	
Intercept	1.63	1.37	1.36	1.38	1.59	1.44	2.35	2.16
	(.78)	(.63)	(.78)	(.73)	(.75)	(.76)	(.86)	(.65)
Relative Abundance, $\widehat{\phi_P}$	-.431	-.435	-.430	-.430	-.442	-.443	-.428	-.429
	(.078)	(.063)	(.078)	(.073)	(.075)	(.076)	(.088)	(.066)
Observations	4321	4321	1815	1815	889	889	1617	1617
R^2	.444	.444	.406	.409	.412	.414	.511	.511
Year Effects χ^2		1754		242		352		264
Degrees of Freedom		108		38		18		48

(B) Panel Estimates for Relative Production

Sample	1900-2008		1970-2008		1950-1969		1900-1949	
Intercept	-3.41	-3.48	-2.96	-3.25	-3.38	3.29	-3.11	-2.45
	(.97)	(.71)	(.96)	(.85)	(.88)	(.66)	(.91)	(.86)
Relative Abundance, $\widehat{\phi_Q}$	.699	.701	.728	.728	.649	.649	.648	.648
	(.097)	(.070)	(.095)	(.084)	(.091)	(.068)	(.092)	(.086)
Observations	3802	3802	1811	1811	741	741	1250	1250
R^2	.571	.560	.557	.558	.594	.593	.597	.593
Year Effects χ^2		720		227		23.5		562
Degrees of Freedom		104		38		18		44

(C) OLS Estimates for Relative Patents, 1976-2010

Dependent Variable	Relative Abundance, $\widehat{\phi_A}$		Intercept		$\bar{R}^2$	N
Relative Patents, 1976-2010	0.12	(.021)	-.011	(.20)	.30	47

Notes: The regressions are of the equations given in (**??**). In each case, the numeraire mineral is iron. The standard errors (in parentheses) in the price and production regressions are calculated from a random-effects panel regressor. The standard errors in the patents regression are calculated as robust standard errors.

This restricts δ and γ to a line through the point $\{\delta, \gamma\} = \{0, 1\}$ with slope $-\zeta$. Thus a necessary condition for $0 < \delta, \gamma < 1$ is that $\zeta > 0$. But ζ can be seen to be the elasticity of R&D with respect to total primary industry revenues. Because of its importance both for what it implies about the relationship between the parameters δ and γ and for its causal implications for R&D, estimates of ζ are also presented.

The standard errors of the primitive parameter estimates are calculated using the "delta method" (e.g., Greene 1997, pp. 278-280) assuming that the estimated covariance matrix for the parameters ϕ_P, ϕ_Q and ϕ_A has diagonal elements equal to the estimated standard errors of each parameter and off-diagonal elements all zero.[19]

Table **??** reports the estimates of the primitive parameters. The estimated value of the natural resource share of primary industry income is $\hat{\beta} \approx 0.5$, and are slightly lower for earlier samples. The estimated elasticity of substitution between raw materials is

[19] Estimates using bootstrap errors on estimates of the seemingly unrelated system of equations using period sample means produce very similar estimates and standard errors.

Table 3: Derived Estimates of Primitive Parameters

Panel Sample	1900-2008	1970-2008	1950-1969	1900-1949
$\widehat{\beta}$	.506	.520	.482	.478
	(.003)	(.004)	(.003)	(.003)
$\widehat{\sigma}$	1.48	1.54	1.36	1.41
	(.043)	(.064)	(.047)	(.052)
$\widehat{\psi}$	1.24	1.28	1.17	1.20
	(.012)	(.019)	(.010)	(.013)
$\widehat{\zeta}$	.470	.419	.607	.551
	(.034)	(.029)	(.101)	(.078)

Notes: The standard errors (in parentheses) for parameters β, σ, ψ and ζ are calculated from the panel estimates with year effects using the delta method assuming that the errors on ϕ_P, ϕ_Q and ϕ_A are uncorrelated.

$\hat{\sigma} \approx 1.5$, with lower values for the 1950-1969 sample than for the 1900-1949 and for the 1970-2008 samples. The alternative hypothesis that $\sigma = 1$ is rejected in every sample. Thus raw materials are substitutes, not complements in production. Similarly, the elasticity of substitution between natural resources is $\hat{\psi} \approx 1.2$. Thus, natural resources are also substitutes, not complements. The estimated elasticity of patents with respect to primary industry revenues ranges from $\hat{\zeta}$ equal to 0.42 to 0.61 in value, and has been declining over time. Because $0 < \delta, \gamma < 1$, the estimate of $\hat{\zeta}$ places an upper bound on the parameter δ. That is, $\hat{\delta} \leq \hat{\zeta}$. Thus, the assumption of diminishing marginal productivity of labor in R&D production is supported by the data.

These elasticity of substitution estimates can be compared to those in the literature. Broda and Weinstein (2006) calculated elasticity of substitution parameters for U.S. 3-digit industry sectors for various ores, raw materials, and a selection of intermediate goods. Their median estimated elasticity of substitution at the ores level is $\hat{\psi} = 3.71$, and their median estimated elasticity of substitution at the raw materials level is $\hat{\sigma} = 3.78$.[20] Broda and Weinstein's estimates, both individually and in aggregate, are also each greater than one in value, although their estimates of ψ and σ are about three times as high as the estimates obtained in Table **??**. Furthermore, using the theory developed above, their implied estimate of β is $\hat{\beta} = (\hat{\psi} - 1)/(\hat{\sigma} - 1) = 0.97$, which is about double the estimates of β in Table **??**. Broda and Weinstein's elasticity of substitution

[20] Broda and Weinsten's 3-digit SITC codes for ores (and elasticity of substitution estimate) are: 273 stone, sand, & gravel (2.27), 274 sufur (12.6), 277 abrasives (1.98), 278 other crude materials (4.76), 281 iron ore (5.75), 2.82 ferrous waste & scrap (7.30), 283 copper (2.26), 284 nickel (1.62), 285 aluminum (2.66), 287 base metals (33.6), 288 nonferrous waste & scrap (5.26), 289 precious metals (1.40). The 3-digit SITC codes for raw materials are: 671 pig iron (25.0), 672 steel ingots (2.60), 673 rolled steel (25.0), 675 alloy rolled (5.43), 677 steel railway (3.28), 678 steel wire (1.32), 679 steel tubes (3.73), silver & platinum group (1.25), 682 copper (1.58), 683 nickel (4.04), 684 aluminum (5.94), 685 lead (5.74), 686 zinc (3.78), 687 tin (3.65), 689 Other metals (3.07).

estimates, however, differ in two respects. First, their estimates are based on U.S. imports rather than world production. Second, their relative upstream and downstream elasticity of substitution estimates are unrestricted, while the relative differences in the upstream and downstream estimates here are restricted by the relationship that $\psi = \beta(\sigma - 1) + 1$.

Finally, the parameters tell us something about whether a strong equilibrium bias exits. Since $\hat{\psi} > 1$, the positive correlation between relative R&D and relative resource abundance implies that $\hat{\Omega} < 0$. Thus, from (**??**), it can be seen that the coefficient on relative resource abundance is negative in the relative resource prices equation. Thus, unlike wages for high skilled labor over the last 40 years, there does not exist a strong equilibrium bias in natural resource prices.

7.3 Predicting Real Price and Production Growth Rates

Two important external validation tests can be made using the parameter estimates. The growth in production of raw materials, g_q, and the growth in real raw materials prices, $g_{p_q} - g_w$, the difference in the growth in prices less the growth in wages, are derived in the appendix [see equations (**??**) and (**??**)]. These may be expressed in terms of the estimated parameters β and ζ, and the mean population growth rate, n, using the relationship that $\zeta = \delta/(1-\gamma)$ from (**??**):

$$g_q = \frac{n\zeta}{\beta}, \qquad \text{and} \qquad g_{p_q} - g_w = -n\left(\frac{1-\beta\zeta}{\beta\zeta}\right). \tag{47}$$

Production is predicted to be rising and if $\beta\zeta < 1$ real prices are predicted to be falling.

Figure **??** shows the mean quantity growth and mean real price growth over 1900-2008 for each chemical element. The mean production and price growth data uses the U.S.G.S. data utilized in the other figures. The mean growth in wages is approximated by the mean annual growth in real per capita income in the 29 countries for which Maddison has continuous real per capita income data since 1900, resulting in $g_w = 0.017$.[21] All but two elements, thallium (Tl) and molybdenum (Mo_{42}), have experienced negative real price growth. The mean annual real price growth is $g_{p_q} - g_w = -0.026$. All but three elements have had positive mean production growth, with the exceptions being thorium, mercury (Hg_{80}), and vanadium. An additional three (tin Sn_{50}, silver Ag_{47} and gold) have experienced positive production growth, but negative per capita production growth, using $n = 0.016$ from Maddison's 29 countries. The mean annual

[21] The countries are: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Columbia, Denmark, Finland, France, Germany, Indonesia, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sri Lanka, Sweden, United Kingdom, United States, Uruguay, and Venezuela.

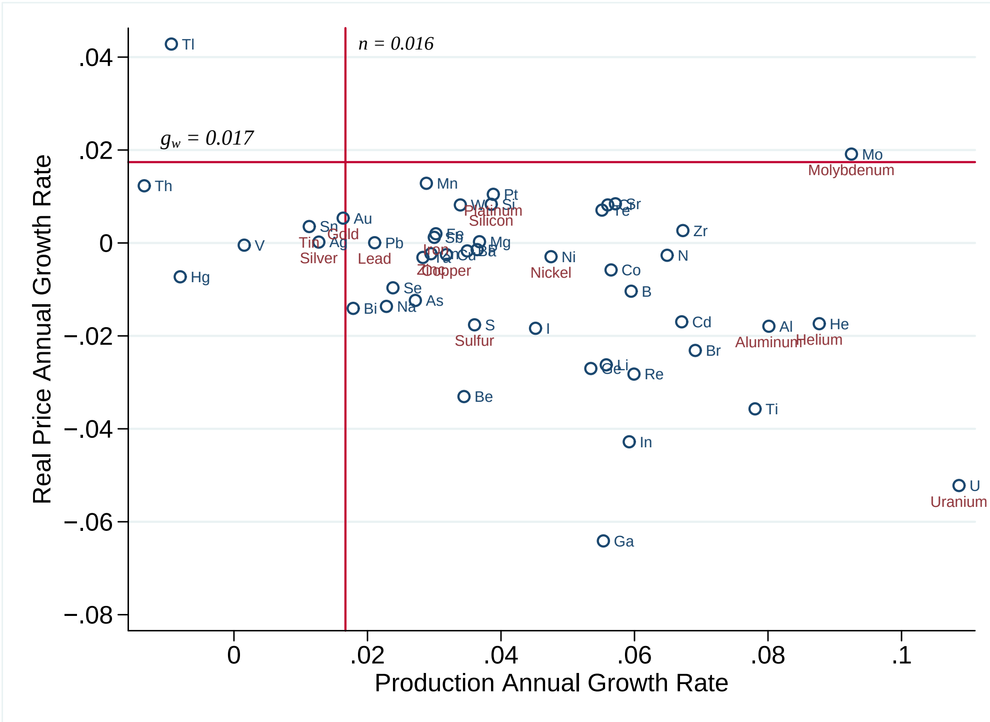

Figure 8: Net Price Growth and Production Growth, 1900-2008.

production growth rate across the 48 chemical elements is $g_q = 0.042$.

Table **??** presents the estimates of real price growth and real production growth with standard errors calculated by the delta method. The estimates are in agreement in sign with the mean price real growth and mean production growth for the 48 elements. While the predicted real price growth is about twice the observed mean real price growth and the predicted production growth is about half the observed mean real production growth, these are remarkably close to the observed estimates, considering that the parameter estimates were derived from completely different equations.

8 Conclusions

This paper documents that, in a sample of the 48 chemical elements for which economic data are available, more abundant resources have higher world production, higher real primary industry revenues, and higher innovation, as measured by the number of U.S. patents filed per chemical element, and that more abundant resources have lower real

Table 4: Derived Estimates of Real Price and Production Growth

Panel Sample	1900-2008	1970-2008	1950-1969	1900-1949
$\widehat{g_{p_q} - g_w}$	-0.053	-0.06	-0.04	-0.047
	(.001)	(.001)	(.001)	(.001)
$\widehat{g_q}$	0.015	0.013	0.021	0.019
	(.0004)	(.0004)	(.0001)	(.0001)

Notes: The standard errors (in parentheses) for parameters g_q and $g_{p_q} - g_w$ are calculated from the panel estimates with year effects using the delta method assuming that the errors on ϕ_P, ϕ_Q and ϕ_A are uncorrelated.

prices. These correlations are found to hold throughout the last century.

These correlations are explained by a model of biased technical change in a multi-resource model of an economy in which the key characteristic of a natural resource is its relative abundance, not its impending exhaustion. For each natural resource, an upstream sector uses specialized machines to produce raw materials from the resource and a downstream sector uses specialized machines to turn the raw material into an intermediate good which can be used in final good production. The number of varieties of machines used in upstream and downstream sectors for each resource varies across resources depending upon the relative intensity of R&D applied to each sector, which, in turn, depends upon the relative profitability of obtaining a patent in each sector. The causal link from relative abundance to relative production, prices and R&D identified in this paper that of biased (endogenous) technical change. More uses for and more ways to find, extract and refine the resource have developed for relatively abundant resources because finding an additional use or method of extraction for such resources yields greater returns to inventors and scientists, since the potential size of the market is larger for more abundant resources.

The observed correlations imply elasticities of substitution between intermediate goods, between raw materials, and between natural resources which are each greater than one in value, though are bounded from above. Thus, in the event that some resources become scarce, substitution towards more abundant resources is possible. Also estimated from the correlations are two other parameters of economic interest: the resource rental share of primary industry income, and the elasticity of R&D with respect to primary industry revenues. Based on these parameter estimates, the model predicts negative real price growth and positive production growth for raw materials, both of which are consistent with observed behavior over the past century. Intuitively, the increase in production occurs because R&D shifts out both demand and supply, while the decrease in real prices occurs because R&D makes labor more productive.

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Appendix

A. Balanced Growth Path

Along a BGP, the relative states of knowledge are in constant proportion, which implies that g satisfies (**??**) in the text. Thus, from the necessary conditions for labor allocation in R&D,

$$-(1-\delta)n + \frac{\gamma\delta n}{1-\gamma} + g_q + \frac{1}{\alpha} g_{p_y} = g_w, \tag{A.1}$$

and

$$-(1-\delta)n + \frac{\gamma\delta n}{1-\gamma} + g_R + \frac{1}{\beta} g_{p_q} = g_w, \tag{A.2}$$

where g_q is the growth in raw materials production, g_{p_q} is the growth in raw materials prices, g_{p_y} is the growth in intermediate goods prices, and g_w is the growth in wages paid to labor. From (**??**), $g_{p_y} = 0$ is required. Assuming $g_R = 0$, equating (**??**) and (**??**) yields

$$g_{p_q} = \beta g_q. \tag{A.3}$$

From the equilibrium production of raw materials, (**??**), we obtain

$$g_q = g + \frac{1-\beta}{\beta} g_{p_q}. \tag{A.4}$$

Substituting from (**??**) and (**??**), we obtain that

$$g_q = \frac{\delta n}{\beta(1-\gamma)} \qquad \text{and} \qquad g_{p_q} = \frac{\delta n}{1-\gamma}. \tag{A.5}$$

Therefore, both raw materials production and raw materials prices are growing over time.

Substituting for g_q and g_{p_q} from (**??**) into (**??**) yields the growth in real wages:

$$g_w = \frac{[\delta(1+\beta) - \beta(1-\gamma)]\, n}{\beta(1-\gamma)}. \tag{A.6}$$

Even though resource prices are rising over time, real resource prices, p_q/w, are rising only if $g_{p_q} > g_w$. In order for this to be true, it must be that $\delta < \beta(1-\gamma)$.

Then from (**??**), growth in intermediate production, g_y, is given by

$$g_y = g + g_q + \frac{1-\alpha}{\alpha} g_{p_y} = \left(\frac{1+\beta}{\beta}\right) \frac{\delta n}{1-\gamma}. \tag{A.7}$$

Then from (**??**), we obtain the growth in final good production g_Y to be

$$g_Y = g_y = \left(\frac{1+\beta}{\beta}\right) \frac{\delta n}{1-\gamma}. \tag{A.8}$$

Total consumption, $C = cL$, grows at rate $g_c + n$, where g_c is given by (**??**). Equation (**??**) requires equating the growth rate in total consumption with the growth rate in production. Thus, $g_c + n = g_Y$. From (**??**), this yields the BGP equilibrium interest rate:

$$r = \rho - n + \frac{[\delta(1+\beta) - \beta(1-\gamma)]\, \theta n}{\beta(1-\gamma)}. \tag{A.9}$$

This implies that per capita consumption growth is

$$g_c = \frac{[\delta(1+\beta) - \beta(1-\gamma)]\, n}{\theta\beta(1-\gamma)}. \tag{A.10}$$

From these it follows that along a BGP that X/Y and Z/Y are each growing at the same rate.

Finally, for utility to be bounded, (**??**) must hold, which implies that

$$(1-\theta)\left[\frac{\delta(1+\beta) - \beta(1-\gamma)}{\beta(1-\gamma)}\right] < \frac{\rho - n}{n}. \tag{A.11}$$

Using (**??**), the transversality condition (**??**) implies that

$$\frac{(1-\theta)\delta(1+\beta) + \theta\beta(1-\gamma)}{\beta(1-\gamma)} < \frac{\rho - n}{n}. \tag{A.12}$$

When the conditions (**??**) and (**??**) are satisfied, a balanced growth path exists.

B. Summary Statistics

Table 5: Summary Statistics by Chemical Element

(A) Chemical Elements for Which Production and Price Data Exists

Atomic Number Z	Element	Atomic Symbol	Crustal Abundance (Kg/Kg)	Solar System Abundance (Silicon = 1)	1970-2008 Mean World Production (Metric Tons)	1970-2008 Mean Real Price (1998 $ / Metric Ton)	Number of Patents 1976-2010	Nuclear Source
2	Helium	He	8.01E-09	2.70E+03	1.59E+04	14,652	68711	BB,H
3	Lithium	Li	2.02E-05	5.80E-05	1.60E+05	4,307	171834	BB,*x*
4	Beryllium	Be	2.83E-06	7.70E-07	2.26E+02	568,538	27288	BB,*x*
5	Boron	B	1.01E-05	2.10E-05	2.99E+06	894	148404	*x*
7	Nitrogen	N	1.92E-05	3.10E+00	8.83E+07	213	495338	H
11	Sodium	Na	2.36E-02	5.80E-02	3.09E+07	108	517044	C
12	Magnesium	Mg	2.33E-02	1.10E+00	1.14E+07	443	291796	Ne,C,He
13	Aluminium	Al	8.24E-02	8.60E-02	1.96E+07	2,100	608104	C,Ne
14	Silicon	Si	2.82E-01	1.00E+00	3.19E+06	1,537	497414	O,Ne
15	Phosphorus	P	1.05E-03	1.00E-02	1.36E+08	32	279145	Ne
16	Sulfur	S	3.54E-04	5.30E-01	5.54E+07	85	174927	O,C,Ne
22	Titanium	Ti	5.66E-03	2.40E-03	5.99E+06	13,714	275760	O,Si
23	Vanadium	V	1.21E-04	2.90E-04	3.22E+04	21,985	49639	Si
24	Chromium	Cr	1.02E-04	1.30E-02	3.64E+06	1,114	132524	O,Si
25	Manganese	Mn	9.60E-04	9.60E-03	8.80E+06	660	95139	Si
26	Iron	Fe	5.64E-02	9.00E-01	1.04E+09	45	308658	Si,α-f,He,Ne
27	Cobalt	Co	2.52E-05	2.30E-03	3.81E+04	37,574	135393	α-f,He
28	Nickel	Ni	8.48E-05	5.50E-02	9.93E+05	10,854	251825	Si,α-f,C,Ne,He
29	Copper	Cu	6.60E-05	5.30E-04	9.81E+06	3,337	389046	C,Ne,He
30	Zinc	Zn	7.70E-05	1.30E-03	7.29E+06	1,581	239450	He,C,Ne
31	Gallium	Ga	1.92E-05	3.80E-05	4.34E+01	1,052,692	58815	C,Ne,He
32	Germanium	Ge	1.52E-06	1.20E-04	8.76E+00	1,202,892	50600	C,Ne,He
33	Arsenic	As	1.82E-06	6.70E-06	3.74E+04	1,081	49534	*s*,*r*,Ne,He
34	Selenium	Se	5.50E-08	6.40E-05	1.49E+03	38,776	33868	*p*,*s*,*r*,C,Ne,He
35	Bromine	Br	2.42E-06	1.20E-05	3.96E+05	1,061	112203	*s*,*r*,Ne,He
38	Strontium	Sr	3.74E-04	2.40E-05	2.45E+05	656	38621	*p*,*s*,*r*
40	Zirconium	Zr	1.65E-04	1.10E-05	7.66E+05	397	81780	*s*,*r*
42	Molybdenum	Mo	1.21E-06	2.60E-06	1.18E+05	20,937	94518	*p*,*s*,*r*
47	Silver	Ag	7.58E-08	4.90E-07	1.44E+04	318,982	200283	*s*,*r*
48	Cadmium	Cd	1.52E-07	1.60E-06	1.87E+04	9,029	55973	*p*,*s*,*r*
49	Indium	In	2.53E-07	1.90E-07	1.77E+02	399,883	79579	*p*,*s*,*r*
50	Tin	Sn	2.32E-06	3.90E-06	2.32E+05	16,287	188859	*p*,*s*,*r*
51	Antimony	Sb	2.20E-07	3.10E-07	9.24E+04	4,695	52730	*p*,*s*
52	Tellurium	Te	1.10E-09	4.90E-06	1.11E+02	66,500	16291	*p*,*s*,*r*
53	Iodine	I	4.54E-07	9.90E-07	1.56E+04	15,383	101333	*s*,*r*
56	Barium	Ba	4.25E-04	4.50E-06	6.01E+06	58	99250	*p*,*s*,*r*
73	Tantalum	Ta	2.20E-06	3.80E-08	5.99E+02	141,335	64566	*p*,*s*,*r*
74	Tungsten	W	1.25E-06	1.30E-07	4.81E+04	20,136	147007	*p*,*s*,*r*
75	Rhenium	Re	7.70E-10	5.50E-08	2.35E+01	2,156,667	15520	*s*,*r*
78	Platinum	Pt	5.50E-09	1.30E-06	2.92E+02	13,058,462	142697	*p*,*s*,*r*
79	Gold	Au	4.40E-09	1.90E-07	1.89E+03	14,855,385	178599	*s*,*r*
80	Mercury	Hg	8.58E-08	3.40E-07	4.78E+03	13,408	93006	*p*,*s*,*r*
81	Thallium	Tl	8.59E-07	1.90E-07	1.03E+01	816,226	12635	*s*,*r*
82	Lead	Pb	1.41E-05	3.10E-06	3.30E+06	1,227	717522	*s*,*r*
83	Bismuth	Bi	8.59E-09	1.40E-07	4.18E+03	18,272	36842	*s*,*r*
90	Thorium	Th	9.70E-06	4.50E-08	1.43E+04	69,401	7344	*r*
92	Uranium	U	2.73E-06	1.80E-08	2.12E+06	44,195	12068	*r*

(B) Chemical Elements with No Production or Price Data

Atomic Number Z	Element	Atomic Symbol	Crustal Abundance (Kg/Kg)	Solar System Abundance (Silicon = 1)	Number of Patents 1976-2010	Nuclear Source
1	Hydrogen	H	1.41E-03	2.80E+04	510826	BB
6	Carbon	C	2.02E-04	1.00E+01	700561	H,He,C
8	Oxygen	O	4.61E-01	2.40E+01	443187	H,He,C,Ne
9	Fluorine	F	5.86E-04	8.60E-04	2062	H
10	Neon	Ne	5.05E-09	3.00E+00	23781	C,He,Ne
17	Chlorine	Cl	1.45E-04	5.30E-03	173427	O
18	Argon	Ar	3.53E-06	1.00E-01	147082	O,C,Ne,He
19	Potassium	K	2.09E-02	3.80E-03	323284	O,C,Ne,He,Si
20	Calcium	Ca	4.15E-02	6.20E-02	313045	O,Si,α-f
21	Scandium	Sc	2.22E-05	3.40E-05	9062	C
36	Krypton	Kr	1.10E-10	4.80E-05	11810	*p*,*s*,*r*,C,Ne,He
37	Rubidium	Rb	9.09E-05	7.70E-06	12944	*s*,*r*,C,Ne,He
39	Yttrium	Y	3.33E-05	4.60E-06	28912	*s*,*r*
41	Niobium	Nb	2.20E-05	7.70E-07	29974	*s*,*r*
43	Technetium	Tc	0	0	5279	N/A
44	Ruthenium	Ru	1.10E-09	1.90E-06	38826	*p*,*s*,*r*
45	Rhodium	Rh	1.10E-09	3.40E-07	42574	*s*,*r*
46	Palladium	Pd	1.52E-08	1.40E-06	114738	*s*,*r*
54	Xenon	Xe	3.30E-11	4.80E-06	37239	*s*,*r*
55	Cesium	Cs	3.00E-06	3.70E-07	36655	*s*,*r*
57	Lanthanum	La	3.94E-05	4.40E-07	23569	*p*,*s*,*r*
58	Cerium	Ce	6.66E-05	1.10E-06	34948	*p*,*s*,*r*
59	Praseodymium	Pr	9.29E-06	1.70E-07	8315	*s*,*r*
60	Neodymium	Nd	4.15E-05	8.40E-07	22051	*s*,*r*
61	Promethium	Pm	0	0	1764	N/A
62	Samarium	Sm	7.06E-06	2.60E-07	13909	*p*,*s*,*r*
63	Europium	Eu	2.20E-06	9.80E-08	12261	*s*,*r*
64	Gadolinium	Gd	6.26E-06	3.30E-07	13732	*p*,*s*,*r*
65	Terbium	Tb	1.21E-06	6.10E-08	8977	*s*,*r*
66	Dysprosium	Dy	5.25E-06	4.40E-07	7378	*p*,*s*,*r*
67	Holmium	Ho	1.31E-06	9.00E-08	6008	*s*,*r*
68	Erbium	Er	3.53E-06	2.50E-07	15656	*s*,*r*
69	Thulium	Tm	5.25E-07	3.80E-08	5030	*s*,*r*
70	Ytterbium	Yb	3.23E-06	2.50E-07	9284	*p*,*s*,*r*
71	Lutetium	Lu	8.08E-07	3.70E-08	4791	*s*,*r*
72	Hafnium	Hf	3.00E-06	1.50E-07	28546	*p*,*s*,*r*
76	Osmium	Os	1.52E-09	6.80E-07	21604	*p*,*s*,*r*
77	Iridium	Ir	1.10E-09	6.70E-07	40874	*s*,*r*
84	Polonium	Po	2.20E-16	0	972	N/A
85	Astatine	At	0	0	791	N/A
86	Radon	Rn	4.40E-19	0	3330	N/A
87	Francium	Fr	0	0	838	N/A
88	Radium	Ra	9.90E-13	0	3374	N/A
89	Actinium	Ac	5.55E-16	0	916	N/A
91	Protactinium	Pa	1.41E-12	0	336	N/A
93	Neptunium	Np	0	0	519	N/A
94	Plutonium	Pu	0	0	2818	N/A

Notes: Source: Mean annual world production and real prices are based on data from 1970-2008: Kelly and Matos (2012). Prices are in 2008 U.S. dollars per metric ton. Abundance: Earth Crustal Abundance: Haynes and Lide (2011), Solar System Abundance: Newsome (1995). Source (Nuclear Process): Arnett (1996) 'BB' = Big Bang. Processes involving chemical symbols are "nuclear burning" of that element; '*s*:' *s*-process (slow neutron capture), 'α-f': "Alpha-rich freezeout, explosive" (Arnett 1996, p.524), '*p*': *p*-process, neon and oxygen shell burning, '*r*': *r*-process (rapid neutron capture), '*x*' = "spallation by cosmic ray interactions with the interstellar medium" (Arnett 1996, p.524). "N/A" = not available.