

# Monetary and Fiscal Policy Switching with Time-Varying Volatilities\*

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## Abstract

This paper extends the ongoing literature on regime change. The extension allows time variation in disturbance variances of interest rate rules for monetary policy and tax rules for fiscal policy that switch stochastically between two regimes. We achieve superior modelings of monetary and fiscal policy rules with quarterly U.S. data.

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# 1 Introduction

In a series of recent papers, Davig and Leeper (2007, 2011) and Chung *et al.* (2007) resolve an inconsistency in the literature pertaining to the assumption that policy regimes are fixed and analyze the implications of regime switching. In doing so, they investigate how the impacts and analysis of macropolicies can change in environments in which policy regimes evolve according to a Markov process. In particular, they consider interest rate rules for monetary policy and tax rules for fiscal policy that switch stochastically between two regimes. In one regime monetary policy follows the Taylor (1993) principle (raising the nominal interest rate by more than any expected rise in inflation) and taxes rise strongly with increases in the real value of government debt; in another regime monetary policy does not follow the Taylor principle and taxes follow an exogenous stochastic process.

Although Davig and Leeper (2007, 2011) and Chung *et al.* (2007) allow for heteroscedastic errors across regimes, they assume homoscedastic errors within regimes. In this paper, we follow Sims and Zha (2006) who argue that heteroscedastic errors are essential for fitting U.S. time series, and relax the assumption that the variance of the errors in each regime is constant and instead introduce time variation in disturbance variances, as in Cai (1994), Hamilton and Susmel (1994), Gray (1996), and Dueker (1997). We apply Bollerslev's (1986) generalized autoregressive conditional heteroscedasticity (GARCH) model and achieve superior modeling of parametric (monetary and fiscal) policy regime-switching rules in which the unconditional variance is constant but the conditional variance, like the conditional mean, is a random variable depending on current and past information.

The outline of the paper is as follows. In Section 2 we present the regime-switching GARCH model of macro policies with time-varying disturbance variances. In Section 3 we discuss the data and presents the estimation results. The final section briefly concludes the paper.

## 2 Switching-Regime GARCH Macro Policies

Chung *et al.* (2007) and Davig and Leeper (2007, 2011) consider a standard Taylor (1993) monetary policy rule

$$i_t = a_0(s_{m,t}) + a_1(s_{m,t})\pi_t + a_2(s_{m,t})y_t + \varepsilon_{m,t} \quad (1)$$

where  $i_t$  is the nominal interest rate,  $\pi_t$  the inflation rate,  $y_t$  the output gap,  $\varepsilon_{m,t} = \sqrt{h_m(s_{m,t})}\epsilon_t$  and  $\epsilon_t \sim iid N(0, 1)$ .  $s_{m,t}$  indicates the unobservable monetary policy regime, and is assumed to follow a first order, homogenous, two-state Markov chain governed by the transition matrix

$$\mathbf{\Pi}_m = \begin{pmatrix} p_{m,11} & p_{m,12} \\ p_{m,21} & p_{m,22} \end{pmatrix}$$

where  $p_{m,ij} = P[s_{m,t} = j | s_{m,t-1} = i]$ ,  $i, j = 1, 2$ , and  $p_{m,11} = 1 - p_{m,21}$  and  $p_{m,12} = 1 - p_{m,22}$ .

Chung *et al.* (2007) and Davig and Leeper (2007, 2011) also consider the following fiscal policy rule

$$\tau_t = \alpha_0(s_{f,t}) + \alpha_1(s_{f,t})b_{t-1} + \alpha_2(s_{f,t})y_t + \alpha_3(s_{f,t})g_t + \varepsilon_{f,t} \quad (2)$$

where  $\tau_t$  is the lump-sum taxes-to-output ratio,  $b_t$  the debt-to-output ratio,  $g_t$  the government purchases-to-output ratio and  $\varepsilon_{f,t} = \sqrt{h_f(s_{f,t})}\epsilon_t$ .  $s_{f,t}$  is the unobservable fiscal policy regime, assumed to also follow a first order, homogenous, two-state Markov chain. The corresponding transition matrix is

$$\mathbf{\Pi}_f = \begin{pmatrix} p_{f,11} & p_{f,12} \\ p_{f,21} & p_{f,22} \end{pmatrix}$$

where  $p_{f,ij} = P[s_{f,t} = j | s_{f,t-1} = i]$ ,  $i, j = 1, 2$ , and  $p_{f,11} = 1 - p_{f,21}$  and  $p_{f,12} = 1 - p_{f,22}$ . Equation (1) suggests that monetary policy responds to the inflation rate,  $\pi_t$ , and the output gap,  $y_t$ , differently in different regimes, since  $a_1(s_{m,t})$  and  $a_2(s_{m,t})$ , respectively, vary across regimes. Also, equation (2) suggests that fiscal policy responds to the lagged debt-to-output ratio,  $b_{t-1}$ , the output gap,  $y_t$ , and the government purchases-to-output ratio,  $g_t$ , differently in different regimes, since  $\alpha_1(s_{f,t})$ ,  $\alpha_2(s_{f,t})$  and  $\alpha_3(s_{f,t})$ , respectively, vary across regimes.

In this paper, we relax the assumption that the variance of the errors in each regime is constant and instead assume that it is time-varying. In particular, we assume the following conditional variance equation for monetary policy

$$h_{m,t}(s_{m,t}) = b_0(s_{m,t}) + b_1(s_{m,t})\varepsilon_{m,t-1}^2 + b_2(s_{m,t})\bar{h}_{m,t-1} \quad (3)$$

where  $\bar{h}_{m,t-1}$  denotes the regime-independent variance, as in Gray (1996) who uses this approach to remove the path-dependence in the regime-switching GARCH estimation. We also assume the following conditional variance equation for fiscal policy

$$h_{f,t}(s_{f,t}) = \beta_0(s_{f,t}) + \beta_1(s_{f,t})\varepsilon_{f,t-1}^2 + \beta_2(s_{f,t})\bar{h}_{f,t-1} \quad (4)$$

where  $\bar{h}_{f,t-1}$  is the regime-independent variance of fiscal policy.

### 3 Data and Estimation Results

We use quarterly data for the United States over the period from 1949:Q1 to 2015:Q2. However, in estimating the monetary policy rule, equations (1) and (3), we only use data from 1960:Q1 to 2007:Q4, thus excluding the period before the 1960s with different approaches to monetary policy and the zero lower bound period after 2008 during which monetary policy has been passive.

We use the same time series as in Davig and Leeper (2011). In particular, the nominal interest rate,  $i_t$ , is the three-month Treasury bill rate in the secondary market. We define  $\pi_t$  to be the inflation rate over the past four quarters, with the inflation rate calculated as

the log difference in the GDP deflator. The output gap,  $y_t$ , is the log deviation of nominal GDP from potential nominal GDP,  $\tau_t$  is the nominal GDP share of federal tax receipts net of total federal transfer payments,  $b_{t-1}$  is the lagged nominal GDP share of the market value of privately held gross federal debt over the past four quarters, and  $g_t$  is the nominal GDP share of federal government consumption expenditures and gross investment.

The three-month Treasury bill rate and the GDP deflator are obtained from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. The potential nominal GDP series is obtained from the Congressional Budget Office and the privately held gross federal debt series from the Federal Reserve Bank of Dallas. Finally, the nominal GDP, federal tax receipts, total federal transfer payments, government consumption expenditures, and gross investment series are all obtained from the U.S. Bureau of Economic Analysis.

To validate our regime-switching conditional heteroscedasticity models of the policy rules, we first estimate equations (1) and (2) under the assumption of homoscedastic errors and test for ARCH effects in the residuals using Engle’s (1982) Lagrange multiplier test. However, this test cannot be applied to the estimated errors directly, since the errors are regime-dependent. As suggested by Daniel (2008), we follow Gouriéroux *et al.* (1987) and calculate the regime-independent generalized errors which are the expected errors conditional on lagged information. We apply the Engle (1982) ARCH test to the generalized errors and find significant ARCH effects up to 10 lags in both policy rules. We therefore proceed to estimate regime-switching conditional heteroscedasticity models.

Parameters estimates of the regime-switching GARCH models, equations (1) and (3) and (2) and (4), are presented in Tables 1 and 2 for monetary and fiscal policy, respectively; the estimated transition probabilities are also shown in panel C. As shown in panel A of Table 1, monetary policy fluctuates between being active, responding strongly to inflation consistent with the Taylor principle ( $\hat{\alpha}_1 = 1.655$  with a  $p$ -value of 0.000), and being passive, not responding in accordance with the Taylor principle ( $\hat{\alpha}_1 = 0.691$  with a  $p$ -value of 0.000). Moreover, passive monetary policy responds almost twice as strongly to the output gap ( $\hat{\alpha}_2 = 0.203$  in the passive regime while  $\hat{\alpha}_2 = 0.127$  in the active regime), suggesting that the Fed pays less attention to inflation stabilization, consistent with Davig and Leeper (2007, 2011).

Regarding fiscal policy, as shown in panel A of Table 2, it breaks into periods when it is active, responding negatively to debt ( $\hat{\alpha}_1 = -0.004$  with a  $p$ -value of 0.000), and periods when it is passive, responding positively to debt ( $\hat{\alpha}_1 = 0.017$  with a  $p$ -value of 0.000). Also, consistent with Davig and Leeper (2007, 2011), active tax policy reacts strongly to government spending, but by less than one-to-one ( $\hat{\alpha}_3 = 0.901$  with a  $p$ -value of 0.000), while passive tax policy reacts more weakly to government spending ( $\hat{\alpha}_3 = 0.396$  with a  $p$ -value of 0.000). Finally, in both regimes, tax policy reacts positively to the output gap,  $y_t$ ;  $\hat{\alpha}_2 = 0.262$  with a  $p$ -value of 0.000 in the active regime and  $\hat{\alpha}_2 = 0.345$  with a  $p$ -value of 0.000 in the passive regime.

Figures 1 and 2 show the smoothed probabilities of active and passive monetary and fiscal policy, respectively; these probabilities are reported at time  $t$ , conditional on the full sample information. In particular, the smoothed probabilities are highly consistent with Davig and Leeper (2007) and their interpretation of the evidence. For example, our estimates show that fiscal policy was active in the first half of the 1980s when the Reagan Administration’s Economic Recovery Plan of 1981 was implemented. However, the probability of passive tax policy significantly increased by the mid-1980s. The reason is that legislation was passed in 1982 and 1984 to raise revenues in response to the rapidly increasing debt-output ratio. Moreover, as can be seen in Figure 2, fiscal policy has been active in the aftermath of the global financial crisis, consistent with the fact that in response to the global financial crisis, in October 2008, the U.S. Congress passed the Emergency Economic Stabilization Act, establishing a \$700 billion bailout program known as the Troubled Asset Relief Program (TARP). Moreover, in February 2009, the American Recovery and Reinvestment Act was passed, a \$787 billion fiscal stimulus package.

In panel B of Table 1, we find evidence of positive and statistically significant ARCH effects in both monetary policy regimes, but a positive and statistically significant GARCH effect only in the active regime. The most striking finding is the very high ARCH coefficient on  $\varepsilon_{m,t-1}^2$  when monetary policy is passive ( $\hat{b}_1 = 1.068$  with a  $p$ -value of 0.000) relative to when it is active ( $\hat{b}_1 = 0.488$  with a  $p$ -value of 0.000). This suggests that when monetary policy is passive, monetary policy volatility reacts intensely to monetary shocks. In the case of fiscal policy, as can be seen in panel B of Table 2, we find evidence of positive and statistically significant ARCH effects (of similar magnitude) in both regimes, but not evidence of GARCH effects.

## 4 Robustness and Shortcomings

To address the issue of whether our model is over-parameterized, for each of the monetary and fiscal policy rules we turn off regime changes in policy and GARCH errors, but permit the variance of the policy shock to vary across regimes. For example, in the case of monetary policy we estimate the rule

$$i_t = a_0 + a_1\pi_t + a_2y_t + \varepsilon_{m,t}$$

where  $\varepsilon_{m,t} = \sqrt{h(s_{\varepsilon,t})}\epsilon_t$ ,  $\epsilon_t \sim iid N(0, 1)$ , and  $s_{\varepsilon,t}$  indicates the unobservable monetary policy shock regime assumed to follow a first order, homogenous, two-state Markov chain. We take a similar approach regarding the fiscal policy rule.

This allows discontinuous regime switching in residuals, instead of GARCH, to capture changes in the shock variance that are very abrupt and discontinuous. We use the Bayes factor to conduct model comparison, since the unrestricted and restricted models are not

nested. In particular, the Bayes factor (BF) is

$$\text{BF} = \frac{\Pr(\text{Data}|\text{Unrestricted model})}{\Pr(\text{Data}|\text{Restricted model})}.$$

According to Kass and Raftery (1995), the logarithm of the Bayes factor could be approximated using Schwarz's Bayesian information criterion (BIC), as follows

$$2 \ln \text{BF} = -(\text{BIC of Unrestricted model} - \text{BIC of Restricted model}),$$

and if  $2 \ln \text{BF} > 10$  then the superiority of the unrestricted model is decisive. In our case, with the monetary policy rule,  $2 \ln \text{BF} = 209.376$ , and with the fiscal policy rule,  $2 \ln \text{BF} = 169.214$ , suggesting that the regime-switching GARCH model of monetary and fiscal policy is superior compared to its restricted version.

We would like to note that our simple two-equation model of monetary and fiscal policy rules may suffer from endogeneity and misspecification. In particular, the independent variables at time  $t$ , such as output, inflation, and government spending, in regression equations (1) and (2) are endogenously determined. Estimating the policy rules as part of a large system of equations, as in Sims and Zha (2006), or solving rational expectations models, as in Farmer *et al.* (2009, 2010), will address this problem. However, the focus of our paper is the econometric modeling of time-varying volatility, following existing empirical works by Davig and Leeper (2007, 2011), and we ignore these issues. Our results may not stand when the endogeneity and misspecification issues are taken care of, but we leave this as an area for potentially productive future research.

## 5 Conclusion

This paper extends the ongoing literature on regime change initiated by Lucas (1976), building on Sims and Zha (2006), Davig and Leeper (2007, 2011), and Chung *et al.* (2007). The extension allows time variation in disturbance variances of interest rate rules for monetary policy and tax rules for fiscal policy that switch stochastically between two regimes. We achieve superior modeling of monetary and fiscal policy rules with quarterly U.S. data.

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Table 1. Switching-Regime GARCH Model of Monetary Policy

Coefficient	Monetary policy	
	Active	Passive
A. Conditional mean equation		
$a_0$	0.030 (0.000)	0.025 (0.000)
$a_1$	1.655 (0.000)	0.691 (0.000)
$a_2$	0.127 (0.000)	0.203 (0.000)
B. Conditional variance equation		
$b_0$	0.000 (0.001)	0.000 (0.006)
$b_1$	0.488 (0.000)	1.068 (0.000)
$b_2$	0.064 (0.013)	0.000
C. Transition probabilities		
$P(s_{m,t} = \text{active}   s_{m,t-1} = \text{passive})$	0.016 (0.000)	
$P(s_{m,t} = \text{active}   s_{m,t-1} = \text{active})$	0.935 (0.000)	

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*Notes:* Sample period, quarterly data, 1960:Q1-2007:Q4.  
A coefficient of 0.000 indicates that the nonnegativity constraint is binding.  
Numbers in parentheses are  $p$ -values.

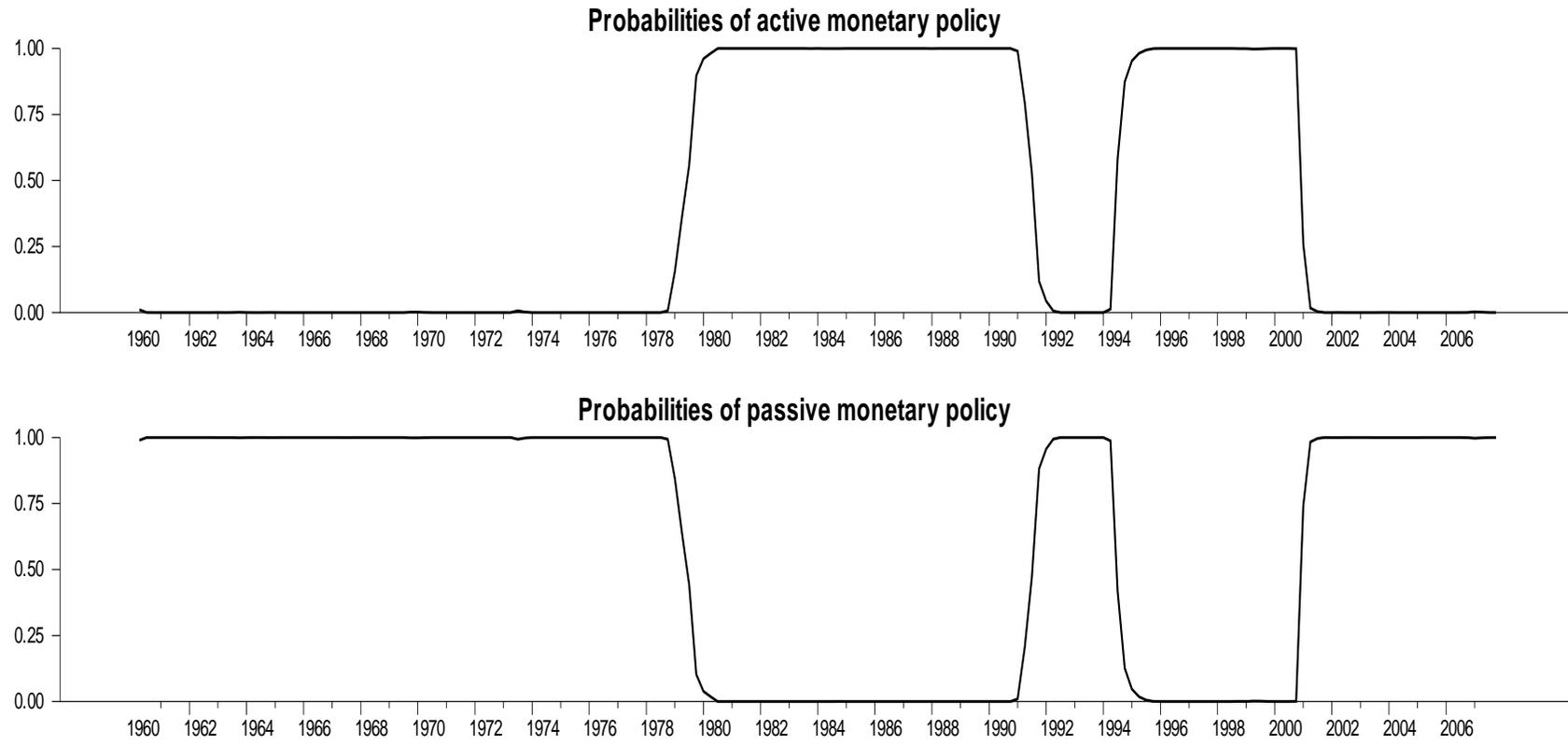
Table 2. Switching-Regime GARCH Model of Fiscal Policy

Coefficient	Fiscal policy	
	Active	Passive
A. Conditional mean equation		
$\alpha_0$	0.001 (0.000)	0.025 (0.000)
$\alpha_1$	-0.004 (0.000)	0.017 (0.000)
$\alpha_2$	0.262 (0.000)	0.345 (0.000)
$\alpha_3$	0.901 (0.000)	0.396 (0.000)
B. Conditional variance equation		
$\beta_0$	0.000 (0.013)	0.000 (0.002)
$\beta_1$	0.774 (0.000)	0.756 (0.000)
$\beta_2$	0.090 (0.328)	0.064 (0.296)
C. Transition probabilities		
$P(s_{f,t} = \text{active}   s_{f,t-1} = \text{passive})$	0.054 (0.001)	
$P(s_{f,t} = \text{active}   s_{f,t-1} = \text{active})$	0.923 (0.000)	

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*Notes:* Sample period, quarterly data, 1949:Q1-2015:Q2.  
Numbers in parentheses are  $p$ -values.

# Figure 1. Monetary Regime Probabilities



**Figure 2. Fiscal Regime Probabilities**

