Money, Velocity, and the Stock Market*

Karl Pinno and Apostolos Serletis†
Department of Economics
University of Calgary
Calgary, Alberta, T2N 1N4,
Canada

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Abstract

This paper provides a study of the relationship between money growth variability, velocity, and the stock market, using recent advances in financial econometrics. We estimate a trivariate VARMA, GARCH-in-Mean, BEKK model to quantify the effects of financial market and money supply instability. We investigate the robustness of the results to different definitions of money using monthly Divisia indices for the United States from the Center for Financial Stability (CFS). Empirical evidence supports significance of financial market and money supply volatility, and we conclude that Friedman's money supply volatility hypothesis is alive and well.

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Keywords: Multivariate GARCH; Variability of money growth; Divisia monetary aggregates.

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†Corresponding author. Phone: (403) 220-4092; Fax: (403) 282-5262; E-mail: Serletis@ucalgary.ca; Web: http://econ.ucalgary.ca/serletis.htm.

1 Introduction

A substantial amount of attention has been focused on the behavior of the velocity of money in the United States, primarily as the result of a relatively sudden and unanticipated decline in the velocity series in 1981. The most powerful element of this statistically unusual decline in velocity is the collapse of the longer-run relationship connecting money to both income and prices. In fact, whether velocity is stable or at least predictable is essential to any empirical interpretation of the monetarist position and especially relevant for some problems of potential importance in the practical conduct of monetary policy. The debate that has arisen mostly concerns the rather abrupt decline in velocity, and quite a few specific hypotheses regarding the determinants of velocity have evolved from this discussion. Among the propositions advanced, those most often cited involve the influence of structural changes in the financial sector, tax cuts, inflation (or expected inflation), changes in government expenditures, changes in energy prices, and money growth along with its variability — see especially, Hall and Noble (1987), Judd and Motley (1984), Tatom (1983), Santoni (1987), and Fisher and Serletis (1989), for more on these and other suggested (and sometimes empirically supported) influences.

It is the variability of money growth and the performance of the stock market that we wish to explore as an influence on velocity in this paper. Regarding the former, there are essentially two avenues of influence, proposed by Friedman (1983, 1984). The first involves the behavior of real income and the second concerns the behavior of the demand for money (accommodated by money supply). Although for the *real income effect*, Friedman's position is not precisely stated, for the *money demand effect*, he argues that increased uncertainty in financial markets, due to the increased volatility of money after the 1979 change of policy-operating techniques in the United States, has produced both an increased demand for money for (essentially) precautionary purposes and, assuming the money supply process accommodates the pressure, a rise in money holdings relative to nominal income. Thus, the authorities willing, the equilibrium stock of money would increase and velocity would decrease, with increases in the volatility of money growth.

Regarding the relationship between velocity and stock prices, as Friedman (1988, pp. 221-222) put it, "the inverse relation between stock prices and monetary velocity (or direct relation between stock prices and the level of real cash balances per unit of income) can be rationalized in three different ways: (1) A rise in stock prices means an increase in nominal wealth and generally, given the wider fluctuation in stock prices than in income, also in the ratio of wealth to income. The higher wealth to income ratio can be expected to be reflected in a higher money to income ratio or a lower velocity. (2) A rise in stock prices reflects an increase in the expected return from risky assets relative to safe assets. Such a change in relative valuation need not be accompanied by a lower degree of risk aversion or a greater risk preference. The resulting increase in risk could be offset by increasing the weight of relatively safe assets in an aggregate portfolio, for example, by reducing the weight of long-term bonds and increasing the weight of short-term fixed-income securities plus money. (3) A rise in stock prices may be taken to imply a rise in the dollar volume of financial transactions, increasing the quantity of money demanded to facilitate transactions. Offsetting these factors is (4) a substitution effect. The higher the real stock price, the more attractive are equities as a component of the portfolio. The relative strength of the inverse effect of items 1-3 and the positive effect of item 4 is an empirical question."

We have three objectives in this paper. Our first objective is to test Friedman's money supply volatility hypothesis using recent advances in the financial econometrics literature. Our second objective is to investigate the relationship between monetary velocity and the stock market. Our third objective is to investigate different measures of monetary velocity to deal with anomalies that arise because of different definitions of money. The money measures employed are monthly Divisia indices (from 1967:1 to 2015:08) for the United States from the Center for Financial Stability (CFS). In this regard, the Center for Financial Stability has initiated a new Divisia monetary aggregates database, maintained within its program Advances in Monetary and Financial Measurement (AMFM). The director of the program is William A. Barnett, the inventor of the Divisia monetary aggregates — see Barnett (1980). The new CFS Divisia monetary aggregates are available at www.centerforfinancialstability.org/amfm.php and are documented in detail in Barnett *et al.* (2013). They represent an improvement over the earlier St. Louis Fed's Monetary Services Indices (MSI). It is our objective in this paper to use the new CFS Divisia monetary aggregates and provide a comparison among the narrower monetary aggregates, M1, M2M, MZM, M2, and ALL, and the broad monetary aggregates, M3, M4+, and M4-. In doing so, we build on Serletis and Shahmoradi (2006) and specify and estimate a trivariate GARCH-in-Mean model, allowing for the possibilities of spillovers in the variance-covariance structure for money growth, velocity growth, and stock market returns.

The paper is organized as follows. Section 2 discusses the data and Section 3 briefly describes the traditional approach to testing such hypotheses and reviews the relevant literature. Section 4 provides a description of the trivariate GARCH-in-Mean model of money growth, velocity growth, and stock returns that we use to test the Friedman hypotheses. In Section 5 we present and discuss the empirical results while in Section 6 we discuss international data and evidence. The final section concludes the paper.

2 The Data

We use monthly United States data, over the period from 1967:1 to 2015:08, on three variables: money, velocity, and stock prices. To measure money, we use the new Center of Financial Stability Divisia monetary aggregates. They are rigorously founded in economic aggregation and index-number theory, and recent research has shown that they predict inflation and the business cycle better than conventional (simple-sum) monetary aggregates. See, for example, Serletis *et al.* (2013), Serletis and Gogas (2014), and Serletis and Rahman (2009, 2015). In addition to using the new CFS Divisia data we also make comparisons among eight CFS Divisia monetary aggregates: the narrower monetary aggregates, M1, M2M, MZM, M2, and ALL, and the broad monetary aggregates, M3, M4+, and M4-.

In calculating velocity we use the Industrial Production Index (IPI) from the Federal Reserve Economic Database (FRED), maintained by the Federal Reserve Bank of St. Louis, and calculate velocity as

$$V = \frac{P \times Y}{M}$$

where P is the Consumer Price Index (all items), Y is the industrial production index (IPI), and M is a monetary aggregate. To provide some perspective on the behavior of the velocity series, we provide graphical representations of the CFS Divisia velocity series at each of the eight levels of monetary aggregation — M1, M2M, MZM, M2, ALL, M3, M4+, and M4- — in Figures 1 to 8, respectively; shaded areas indicate NBER recessions. Finally, we use the S&P 500 index to capture the performance of the stock market in the United States.

In Table 1 we conduct a number of unit root and stationarity tests, which includes the Augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)], the Dickey-Fuller GLS test [see Elliot, Rothenberg, and Stock (1996)], the Phillips-Perron test [see Phillips and Perron (1988)], and the KPSS test for stationarity [see Kwiatkowski *et al.* (1992)], in the first differences of the logarithms of each series. In Table 2, we test the same series for serial correlation, heteroscedasticy, and normality using the Ljung Box Q-statistic [see Ljung and Box (1979)], a test for ARCH effects, and the Jarque-Bera (1980) test, respectively. We find evidence that the stationary series exhibit autocorrelation and are leptokurtic — these properties drive our choice of econometric specification.

3 Money, Velocity, and Stock Prices

As discussed in Serletis and Shahmoradi (2006), the traditional approach to testing Friedman's hypothesis that money growth volatility is causal factor in changes in velocity is based on the use of the Granger-causality method. See, for example, Hall and Noble (1987), Brocato and Smith (1989), Mehra (1989), Fisher and Serletis (1989), and Thornton (1995). In particular, it is assumed that the relevant information is contained in the present and past values of these variables and the following bivariate autoregressive representation is used

$$v_t = \alpha_0 + \sum_{j=1}^{r} \alpha_j v_{t-j} + \sum_{j=1}^{s} \beta_j VOL_{t-j} + \varepsilon_t \tag{1}$$

where $v_t = \Delta \log V_t$, with Δ being the difference operator and V_t the level of money velocity. In equation (1), VOL_t is the level of money growth ($\mu_t = \Delta \log M_t$ where M_t is a money supply series) volatility (calculated as a moving standard deviation of money growth, μ_t) and ε_t is a white noise disturbance. The parameters in the specification are α_0, α_j, and β_j. In this framework, if the null hypothesis of $\beta_j = 0$ for $j = 1, ..., s$ can be rejected, then the conclusion is that the data show causality.

Based on this framework, the literature has produced mixed results regarding the relationship between money growth volatility and monetary velocity. For example, Hall and Noble (1987) and Fisher and Serletis (1989) find a causal relation from money growth variability to velocity. However, Brocato and Smith (1989) and Mehra (1989) show that the Granger-causality result is not robust to some changes of specification and the sample period, while Thornton (1995) concludes that "the Friedman hypothesis would appear to have little general applicability." These contributions, however, are quite outdated by now, since their data incorporate observations before the 1990s. Also, only Fisher and Serletis (1989) search for the relationship using Divisia measures of the money supply (although of an older vintage). It is our objective in this paper to use the new CFS Divisia monetary aggregates to reconsider Friedman's hypothesis that money growth volatility is causal factor in changes in velocity.

We start with reporting on the results for Granger causality from monetary variability and stock prices to monetary velocity. In doing so, we use the following regression equation

$$v_t = \alpha_0 + \sum_{j=1}^{r} \alpha_j v_{t-j} + \sum_{j=1}^{s} \beta_j VOL_{t-j} + \sum_{j=1}^{k} \gamma_j s_{t-j} + \varepsilon_t \qquad (2)$$

where v_t and VOL_t are defined as in equation (1), $s_t = \Delta \log S_t$, with S_t being the S&P 500 index, and ε_t is a white noise disturbance. Again, if the null hypothesis of $\beta_j = 0$ for $j = 1, ..., s$ can be rejected, then the conclusion is that the data show causality from monetary variability to velocity. Similarly, if the null hypothesis of $\gamma_j = 0$ for $j = 1, ..., k$ can be rejected, then the conclusion is that the data show causality from stock prices to monetary velocity.

Before the test statistics can be computed, it is necessary to select a procedure for choosing the lag lengths, r, s, and k in equation (2), since Granger causality tests are often very sensitive to the order of the lags. Instead of assuming that the lag lengths have the same value, as is often done in the literature on causality testing, we use the Akaike Information Criterion (AIC) with a maximum value of 12 for each of r, s, and k in equation (2), and by running 1728 regressions for each trivariate relationship we choose the one that produces the smallest value for the AIC. We present these optimal lag length specifications in the AIC(r, s, k) column of Table 3, and the F-statistics and corresponding p-values in the next two columns. As can be seen, the null hypothesis of $\beta_j = 0$ for $j = 1, ..., s$ is rejected for CFS M1, M2M, MZM, M2 and ALL. That is five of the eight monetary series show causality from monetary variability to velocity. In all eight trivariate estimations, we reject the null hypothesis of $\gamma_j = 0$ for $j = 1, ..., s$ in favor of the alternative that the data show causality from stock prices to monetary velocity.

4 A VARMA, GARCH-in-Mean BEKK Model

In this section, we use a trivariate BEKK framework [see Engle and Kroner (1995) for more details] to model velocity, money growth, and stock prices and volatilities as a system. This model allows for rich dynamics in the variance-covariance structure of the series, making it possible to model interactions in both the values and the conditional variances of the variables. The formulation allows us to model the transmission of volatility from one series to another, and estimate the effects of volatility in any of the three series on each of the other series.

We choose a trivariate vector autoregressive moving average VARMA$(\vartheta, 1)$ specification for the mean equation, with ν_t, μ_t, and s_t forming the dependent variables

$$\boldsymbol{z}_t = \phi + \sum_{j=1}^{\vartheta} \boldsymbol{\Gamma}_j \boldsymbol{z}_{t-j} + \boldsymbol{\Psi}\sqrt{\boldsymbol{h}_t} + \boldsymbol{\Theta}\boldsymbol{\epsilon}_{t-1} + \boldsymbol{\epsilon}_t \tag{3}$$

$$\boldsymbol{\epsilon}_t | \, \Omega_{t-1} \sim (\boldsymbol{0}, \boldsymbol{H}_t), \quad \boldsymbol{H}_t = \begin{bmatrix} h_{vvt} & h_{v\mu,t} & h_{vs,t} \\ h_{\mu v,t} & h_{\mu\mu,t} & h_{\mu s,t} \\ h_{sv,t} & h_{s\mu,t} & h_{ss,t} \end{bmatrix}$$

where Ω_{t-1} is the information set available in period $t-1$ and

$$\boldsymbol{z}_t = \begin{bmatrix} v_t \\ \mu_t \\ s_t \end{bmatrix}; \quad \boldsymbol{\epsilon}_t = \begin{bmatrix} \epsilon_{v,t} \\ \epsilon_{\mu,t} \\ \epsilon_{s,t} \end{bmatrix}; \quad \boldsymbol{h}_t = \begin{bmatrix} h_{vv,t} \\ h_{\mu\mu,t} \\ h_{ss,t} \end{bmatrix};$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix}; \quad \boldsymbol{\Psi} = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix}; \quad \boldsymbol{\Theta} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix}.$$

We use the asymmetric version of the BEKK model, introduced by Grier *et al.* (2004), for the variance equation. We choose the BEKK(1,1) specification, which is a multivariate extension of GARCH(1,1). Thus, the variance equation is

$$\boldsymbol{H}_t = \boldsymbol{C}'\boldsymbol{C} + \boldsymbol{B}'\boldsymbol{H}_{t-1}\boldsymbol{B} + \boldsymbol{A}'\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{A} \tag{4}$$

where $\boldsymbol{C}$, $\boldsymbol{B}$, and $\boldsymbol{A}$, are 3×3 matrices with $\boldsymbol{C}$ being a triangular matrix to ensure positive definiteness of $\boldsymbol{H}$. This specification allows past volatilities, $\boldsymbol{H}_{t-1}$, as well as lagged values of

$\boldsymbol{\epsilon\epsilon}'$ to show up in estimating current volatilities of velocity, money growth, and stock returns. Since the $\boldsymbol{H}$ matrix is symmetric, equation (4) produces six unique equations modeling the dynamic variances of velocity, money growth, and stock returns, as well the covariances between them.

4.1 Estimation Matters

The trivariate BEKK model formed by equations (3) and (4) was estimated in Estima RATS using quasi-Maximum Likelihood. We used the BFGS (Broyden, Fletcher, Goldfarb & Shanno) estimation algorithm, which is recommended for GARCH models. Tables 4-11 report the coefficients obtained (with significance levels in parentheses), as well as key diagnostics for the standardized residuals

$$\hat{z}_{jt} = \frac{\epsilon_{jt}}{\sqrt{\hat{h}_{jt}}}$$

for $j = \nu_t$, μ_t, and s_t. The optimal lag structure was determined using the 'varlagselect' command and the AIC criterion in RATS. As part of our model identification process, we attempted to 'over fit' the lag structure (adding one lag), but were unable in each case to get convergence.

Tables 4-11, broadly speaking, are each composed of three panels. In Panel A, we present the parameter estimates for the conditional mean equation. We are most interested in the [1,2] and [1,3] elements ($\hat{\gamma}_{v\mu}$ and $\hat{\gamma}_{vs}$, respectively) of the $\boldsymbol{\Gamma}_j$ matrices, as they show how lagged monetary growth and lagged stock market returns affect changes in velocity. A structure like the one we employ in this paper is designed to capture simultaneous short run effects (in our case each lag is a month). If Friedman's *money demand effect* dominates, in the short run we would expect to find that (at least) the total delay multiplier for $\hat{\gamma}_{v\mu}$ to be negative. Alternatively, since monetary policy is typically countercyclical, in the standard textbook short-run macro model, increases in the money supply lead to increases in aggregate demand and velocity, since the price level is assumed to be fixed. If this effect dominates, in the short run one might expect, a priori, that (at least) the total delay multiplier for $\hat{\gamma}_{v\mu}$ would be positive. Using the previously motivated analysis of Friedman (1988, pp. 221-222), we will interpret negative values of $\hat{\gamma}_{vs}$ parameters (and the total delay multiplier) to provide support that in the short run the 'wealth effect' is dominating (points 1-3). Alternatively, we will interpret positive values of $\hat{\gamma}_{vs}$ parameters (and the total delay multiplier) to provide support that the 'substitution effect' is dominating (point 4). Our main research question however is centered around testing for empirical evidence of Friedman's *money demand effect.*

The $\boldsymbol{\Psi}$ matrix holds the parameter estimates of the conditional standard deviation, $\sqrt{\boldsymbol{h}_t}$, on the dependent variables. For our research questions, we are most interested in elements [1,2] and [1,3] of this matrix ($\hat{\psi}_{v\mu}$ and $\hat{\psi}_{vs}$, respectively), as they provide estimates of the impact of money supply and stock prices conditional volatility on the velocity of money. If $\hat{\psi}_{v\mu}$ is negative and statistically significant it would provide empirical support for the *money demand effect.* If $\hat{\psi}_{vs}$ is negative and statistically significant we might conclude the same regarding the effect of stock market volatility on the demand for money. However, if $\hat{\psi}_{vs}$ is positive and significant and $\hat{\psi}_{v\mu}$ is negative and significant, we cannot rule out the *money demand effect.* It may just be that higher risks come with higher returns (traditional financial theory) *and* that the substitution effects are dominating (point 4); in fact, traditional measures of stock valuations, like P/E ratios, reached historic highs during the sample period. Of secondary interest is $\hat{\psi}_{s\mu}$, the [3,2] element of the $\boldsymbol{\Psi}$ matrix, as it represents the impact of money supply volatility on the broad stock market. Our a priori expectation would be that $\hat{\psi}_{s\mu}$, if significant, would have a negative sign (since more uncertainly around Fed rate policy leads to lower asset returns).

The $\boldsymbol{\Theta}$ matrix provides estimates of the impact of last period's innovations, and the statistical significance of its elements provides an indication of the appropriateness of the moving average structure. We will focus our attention specifically on the [1,2], [1,3], and [3,2] elements ($\hat{\theta}_{v\mu}$, $\hat{\theta}_{vs}$, and $\hat{\theta}_{s\mu}$, respectively) of this matrix. Panel B, is a set of estimates that are calculated using the aforementioned standardized residuals. These diagnostics will tell us whether our lag specification has effectively dealt with autocorrelation. Panel C lists the parameters for the conditional variance equation. The significance of the parameters in the $\boldsymbol{A}'\boldsymbol{A}$ matrix suggest volatility changes with lagged shocks; whereas the significance of the parameters in the $\boldsymbol{B}'\boldsymbol{B}$ matrix tell us that there is also momentum in the system driving the conditional variance. Considered together, if the majority of these parameters are statistically significant, they validate the econometric specification of a GARCH process.

5 Empirical Evidence

Tables 4-11 present the empirical results for each of the eight monetary aggregates. In what follows, we only discuss in detail the results in Table 4 and summarize the major results from panel A of Tables 4-11 in Table 12.

In Table 4, CFS M1 is the monetary aggregate under investigation and 4 lags are used in the trivariate estimation. The parameters for lagged M1 money growth, $\hat{\gamma}_{v\mu}$, (the [1,2] elements of the $\boldsymbol{\Gamma}$ matrices) are significant for all lags; the 1st lag is negative, and subsequent lags are positive. The combined delay multiplier of a 1% increase in the growth rate of the money supply is a $-1.172 + 0.497 + 0.307 + 0.178 = -0.190\%$ decrease in velocity. This result is consistent with Friedman's *money demand effect* hypothesis. For the S&P 500 growth parameter, $\hat{\gamma}_{vs}$, only the 2nd lag (the [1,3] element in the $\boldsymbol{\Gamma}_2$ matrix) shows any significance, suggesting that a 1% increase in the money growth rate leads to a 0.022% increase in velocity. This parameter is statistically significant. The conditional variance parameter of money supply on velocity,$\hat{\psi}_{v\mu}$, is not significant. The volatility parameter of the S&P 500 on velocity, $\hat{\psi}_{vs}$, is equal to 0.073 and is significant at a 10% level. The monthly standard deviation of the growth in the S&P 500, is 4.399% . If we take this as a proxy for the 'average' value of the conditional variance, $\sqrt{h_{ss,t}}$, then on average we expect monthly S&P 500 volatility to increase velocity by $4.399\% \times 0.073 = 0.321\%$ — a value that is economically significant.

Both $\hat{\psi}_{vs}$ and $\hat{\gamma}_{vs}$ are positive suggesting that the substitution effect dominates. The volatility parameter of money supply on S&P 500 growth, $\hat{\psi}_{s\mu}$, is equal to -3.555 and is significant at a 10% level. The monthly standard deviation of the growth of M1 is 0.58%. Following the same logic as above, on average, we expect monthly money supply growth volatility to decrease monthly S&P 500 returns by $0.58\% \times (-3.555) = -2.075\%$. Finally, five of the nine estimates from the $\mathbf{\Theta}$ matrix are significant suggesting that on balance last period's shocks matter for current period growth rates. Of specific interest to us, $\hat{\theta}_{v\mu}$, $\hat{\theta}_{vs}$, and $\hat{\theta}_{s\mu}$ are all significant.

We turn our attention to panel B of Table 4, where we conduct a battery of misspecification tests, using robust versions of the standard test statistics based on the standardized residuals. The Ljung-Box Q-statistic, for testing serial correlation, cannot reject the null of no autocorrelation (at conventional significance levels) for the values and the squared values of the standardized residuals in seven of the 12 cases. In the remaining five cases, two are significant at only a 10% level. While the results are mixed, it appears that our lag specification, has, on balance done away, with autocorrelation. The results in panel C of Table 4 suggest strong evidence of the GARCH specification as there is only one parameter not significant in the $\boldsymbol{A}$ matrix. All other parameter as significant at a 1% level as are all the parameters in the $\boldsymbol{B}$ matrix.

The results in Tables 5-11 can be interpreted in the same fashion as those in Table 4. For the purpose of our analysis, we summarize the major results from panel A of Tables 4-11 in Table 12. We display the effects (%) on velocity of the total delay multipliers (of lagged changes) for changes in money supply and S&P500 ($\sum \hat{\gamma}_{v\mu}$ and $\sum \hat{\gamma}_{vs}$), their volatility parameters$\left(\hat{\psi}_{v\mu} \text{ and } \hat{\psi}_{vs}\right)$, and the expected impact of those volatility parameters $\left(\hat{\psi}_{v\mu} \times \sigma_{\mu} \text{ and } \hat{\psi}_{vs} \times \sigma_{s}\right)$. Seven of the eight delay multipliers for both money supply growth and S&P 500 returns are positive. When both total delay multipliers are positive (for six of the eight series) the money supply multiplier is larger than the S&P 500 multiplier. This is to be expected, because monetary policy inflates assets prices. The volatility parameter of money supply growth on velocity, $\hat{\psi}_{v\mu}$,is significant for six of the eight series; of which 4 are negative, giving direct support to Friedman's money demand effect. The volatility parameter of S&P 500 returns on velocity, $\hat{\psi}_{vs}$, is also significant for six of the eight series; of which two are negative, giving direct support to Friedman's money demand effect. The four series where $\hat{\psi}_{vs} > 0$ coincide with $\sum \hat{\gamma}_{vs} > 0$ — this could be interpreted as Friedman's real income effect being dominated by the substitution effect.

Our model specification has non significant (NS) volatility parameters in the most narrow series M1, and the most broadest M3, M4+, and M4-. It is only in the broadest series M4+ and M4- where $\hat{\psi}_{v\mu} > 0$, a sign that is counter to what one would expect from parameter estimates for Friedman's money demand effect. For each series $\left|\hat{\psi}_{v\mu}\right| > \left|\hat{\psi}_{vs}\right|$ which means, like the lagged impacts, that the volatility from the money supply growth has larger impacts than the volatility of S&P 500 returns. However, because the standard deviation of the S&P 500 returns ranged from being 7.5 to 10.2 times larger than that of the CFS Divisia series, our estimated average impact of stock market volatility on velocity is greater than the estimated average impact of money supply volatility, $\left|\hat{\psi}_{v\mu} \times \sigma_\mu\right| < \left|\hat{\psi}_{vs} \times \sigma_s\right|$. As discussed in a previous section, we found five series of money supply growth to Granger cause velocity, M1, M2M, MZM, M2, and ALL. While $\hat{\psi}_{v\mu}$ is insignificant for M1, with the other four of the series we find the strongest empirical evidence consistent with Friedman's money demand effect. S&P returns are shown to Granger cause all the velocity series. Our VARMA model finds statistical significance in S&P returns volatility on velocity for only six of the eight series. Across the series, considering the estimates of $\mathbf{\Theta}$ and panels B and C together, we have strong support for the use of a VARMA GARCH-in Mean BEKK specification.

6 International Data and Evidence

Over the years, the performance of the Divisia monetary aggregates has also been investigated by a large number of studies, more recently by Serletis and Gogas (2014), Hendrickson (2014), Belongia and Ireland (2014, 2015a, 2015b), Serletis and Istiak (2016), and Serletis and Koustas (2016), among others. As noted by Serletis and Istiak (2016, p. 313), "a number of central banks throughout the world also produce Divisia monetary data that could be used for similar investigations. These central banks are: the European Central Bank, the Bank of Israel, the Bank of Japan, the National Bank of Poland, the Bank of England, the International Monetary Fund, and the Federal Reserve Bank of st. Louis. In fact, as an indication of the level of international acceptance of the Divisia approach to monetary aggregation, see the International Monetary Fund's official data document, *Monetary and Financial Statistics: Compilation Guide*, 2008, pp. 183-184." See also the online library at http://www.centerforfinancialstability.org/amfm.php that links to Divisia monetary aggregates data and studies for over 40 countries throughout the world, including Australia, Canada, China, India, Japan, and the United Kingdom, among others.

7 Conclusion

This paper provides a study of the relationship between money growth variability, velocity, and the stock market, using recent advances in financial econometrics. We estimate a trivariate VARMA, GARCH-in-Mean, BEKK model to quantify the effects of financial market and money supply instability. We examine how stock market and money supply growth and volatility impact velocity and investigate the robustness of the results to different definitions of money using monthly Divisia indices for the United States from the Center for Financial Stability (CFS). We find evidence of Friedman's hypothesis that the variability of money growth helps predict velocity. We also find evidence that stock market volatility has positive and significant effects on monetary velocity, consistent with Friedman's substitution effect that the higher the stock prices, the more attractive are equities as a component of the portfolio.

Our results are robust to alternative narrow definitions of the money supply, but there is no support for Friedman's hypothesis with the broader monetary aggregates. The reason for this is that the broader the money supply definition, the more difficult it is to tease out the effects of two factors: the volatility of the money supply and the volatility of the stock market. In this regard, the recent policy of the Federal Reserve of keeping interest rates low for a historically long time (low volatility of rates), in response to the Great Recession, has inflated assets prices. As noted by Narayana Kocherlakota, President of the Minneapolis Federal Reserve, on April 18, 2013: " ... these dramatic changes in asset demand and asset supply are likely to persist over a considerable period of time—possibly the next five to 10 years. If that forecast holds true, it follows that the FOMC will only be able to meet its congressionally mandated objectives over that time frame by taking policy actions that ensure that the real interest rate remains unusually low. I point out that low real interest rates can be expected to be associated with financial market phenomena—like high asset price volatility—that are seen as signifying instability. This is my second main message: For many years to come, the FOMC will only be able to achieve its objectives by following policies that necessarily give rise to signs of financial market instability." Our paper represents a significant contribution to quantifying the effects of increased financial market and money supply instability on monetary velocity.

References

[1] Barnett, W.A. "Economic monetary aggregates: An application of aggregation and index number theory." *Journal of Econometrics* 14 (1980), 11-48.

[2] Barnett, W.A., J. Liu, R.S. Mattson, and J. van den Noort. "The new CFS Divisia monetary aggregates: Design, construction, and data sources." *Open Economies Review* 24 (2013), 101-124.

[3] Belongia, M.T. and P.N. Ireland. "The Barnett critique after three decades: A new Keynesian analysis." *Journal of Econometrics* 183 (2014), 5-21.

[4] Belongia, M.T. and P.N. Ireland. "A 'working' wolution to the question of nominal GDP targeting." *Macroeconomic Dynamics* 9 (2015a), 508–534.

[5] Belongia, M.T. and P.N. Ireland. "Interest rates and money in the measurement of monetary policy." *Journal of Business and Economic Statistics* 33 (2015b), 255-269.

[6] Brocato, J. and K.L. Smith. "Velocity and the variability of monetary growth: Evidence from Granger causality tests." *Journal of Money, Credit, and Banking* 21 (1989), 258-261.

[7] Dickey, D.A., and W.A. Fuller. "Likelihood ratio statistics for autoregressive time series with a unit root." *Econometrica* 49 (1981), 1057-72.

[8] Elliot, G., T.J. Rothenberg, and J.H. Stock. "Efficient tests for an autoregressive unit root." *Econometrica* 64 (1996), 813-836.

[9] Engle, R.F. and K.F. Kroner. "Multivariate simultaneous generalized ARCH." *Econometric Theory* 11 (1995), 122-150.

[10] Fisher, D. and A. Serletis. "Velocity and the growth of money in the United States, 1970-1985." *Journal of Macroeconomics* 3 (1989), 323-332.

[11] Friedman, M. "Monetary variability: The United States and Japan." *Journal of Money, Credit, and Banking* 15 (1983), 339-343.

[12] Friedman, M. "Lessons fron the 1979-1982 monetary policy experiment." *American Economic Review* Papers and Proceedings 74 (1984), 397-400.

[13] Friedman, M. "Money and the stock market." *Journal of Political Economy* 96 (1988), 221-245.

[14] Grier, K.B., Ó.T. Henry, N. Olekalns, and K. Shields. "The asymmetric effects of uncertainty on inflation and output growth." *Journal of Applied Econometrics* 19 (2004), 551-565.

[15] Hall, T.E., and N.R. Noble. "Velocity and the variability of money growth: Evidence from Granger causality tests." *Journal of Money, Credit, and Banking* 19 (1987), 112-116.

[16] Hendrickson, J.R. "Redundancy or mismeasurement? A reappraisal of money." *Macroeconomic Dynamics* 18 (2014), 1437-1465.

[17] Jarque, C.M. and A.K. Bera. "Efficient tests for normality, homoscedasticity, and serial independence of regression residuals." *Economics Letters* 6 (1980), 255-259.

[18] Judd, J.P. and B. Motley. "The great velocity decline of 1982-83: A comparative analysis of M1 and M2." Federal Reserve Bank of San Francisco *Economic Review* (1984), 56-74.

[19] Kocherlakota, N. *https://www.minneapolisfed.org/news-and-events/presidents-speeches/low-real-interest-rates-executive-summary* (2013).

[20] Kroner, K.F. and V.K. Ng. "Modeling asymmetric comovements of asset returns." *The Review of Financial Studies* 11 (1998), 817-844.

[21] Kwiatkowski, D., P.C.B. Phillips, P. Schmidt, and Y. Shin. "Testing the null hypothesis of stationarity against the alternative of a unit root." *Journal of Econometrics* 54 (1992), 159-178.

[22] Ljung, T. and G. Box. "On a measure of lack of fit in time series models." *Biometrica* 66 (1979), 66-72.

[23] Mehra, Y.P. "Velocity and the variability of money growth: Evidence from Granger causality tests." *Journal of Money, Credit, and Banking* 21 (1989), 262-266.

[24] Phillips, P.C.B., and P. Perron. "Testing for a unit root in time series regression." *Biometrica* 75 (1988), 335-346.

[25] Santoni, G.J. "Changes in wealth and the velocity of money." Federal Reserve Bank of St. Louis *Review* 67 (1987), 16-26.

[26] Serletis, A. and P. Gogas. "Divisia monetary aggregates, the great ratios, and classical money demand functions." *Journal of Money, Credit and Banking* 46 (2014), 229-241.

[27] Serletis, A. and K. Istiak. "Are the responses of the U.S. economy asymmetric to positive and negative money supply shocks?" *Open Economies Review* 27 (2016), 303-316.

[28] Serletis, A. and Z. Koustas. “Monetary neutrality.” *Macroeconomic Dynamics* (2016, forthcoming).

[29] Serletis, A. and S. Rahman. “The output effects of money growth uncertainty: Evidence from a multivariate GARCH-in-Mean VAR.” *Open Economies Review* 20 (2009), 607-630.

[30] Serletis, A. and S. Rahman. “On the output effects of monetary variability.” *Open Economies Review* 26 (2015), 225-236.

[31] Serletis, A. and A. Shahmoradi. “Velocity and the variability of money growth: Evidence from a VARMA, GARCH-M model.” *Macroeconomic Dynamics* 10 (2006), 652-666.

[32] Serletis, A., K. Istiak, and P. Gogas “Interest rates, leverage, and money.” *Open Economies Review* 24 (2013), 51-78.

[33] Tatom, J.A. “Was the 1982 velocity decline unusual?” Federal Reserve Bank of St. Louis *Review* 67 (1983), 5-15.

[34] Thornton, J. “Friedman’s money supply volatility hypothesis: Some international evidence.” *Journal of Money, Credit, and Banking* 27 (1995), 288-292.

Figure 1. CFS Divisia M1 velocity and its growth rate

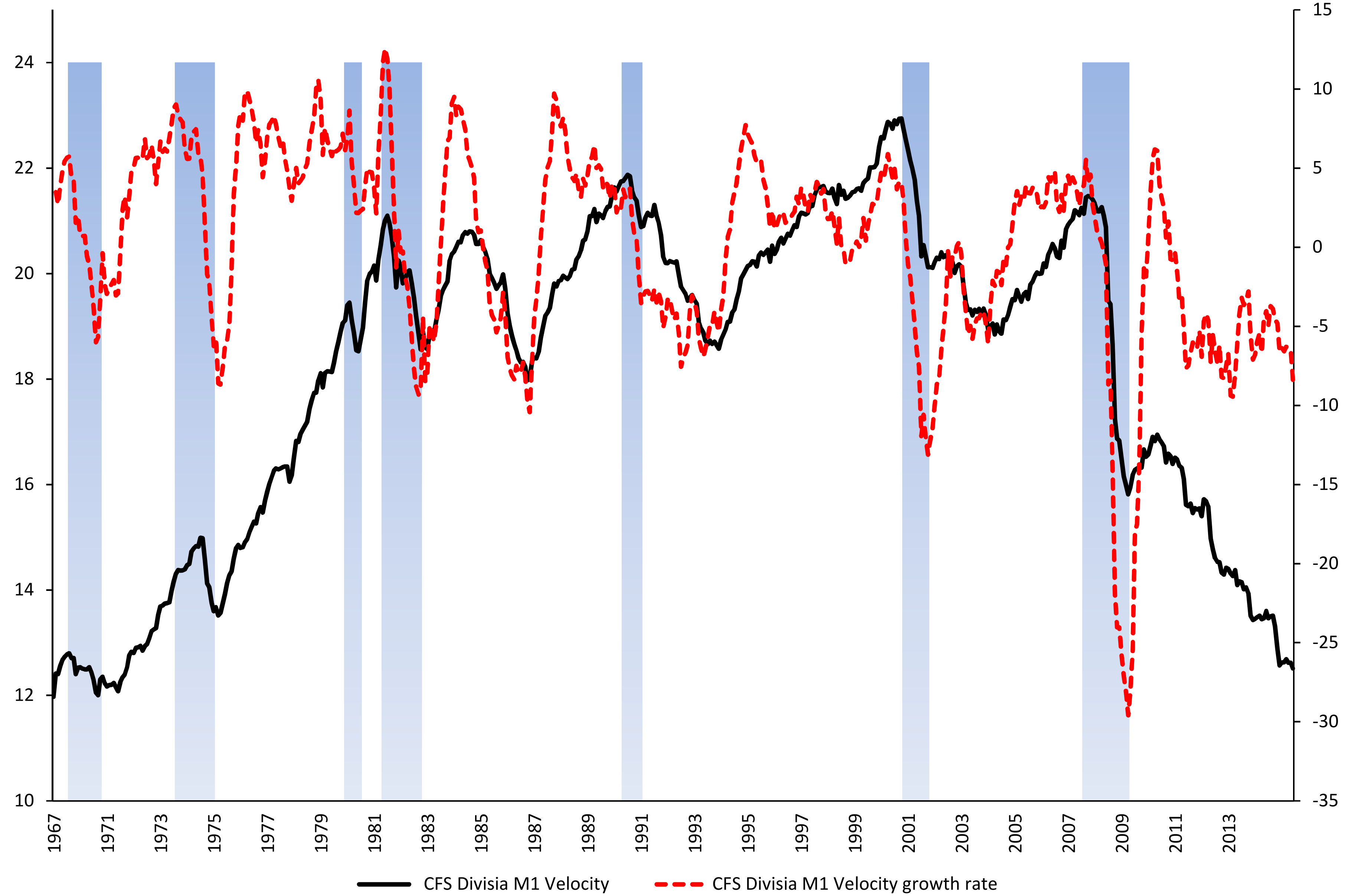

Figure 2. CFS Divisia M2M velocity and its growth rate

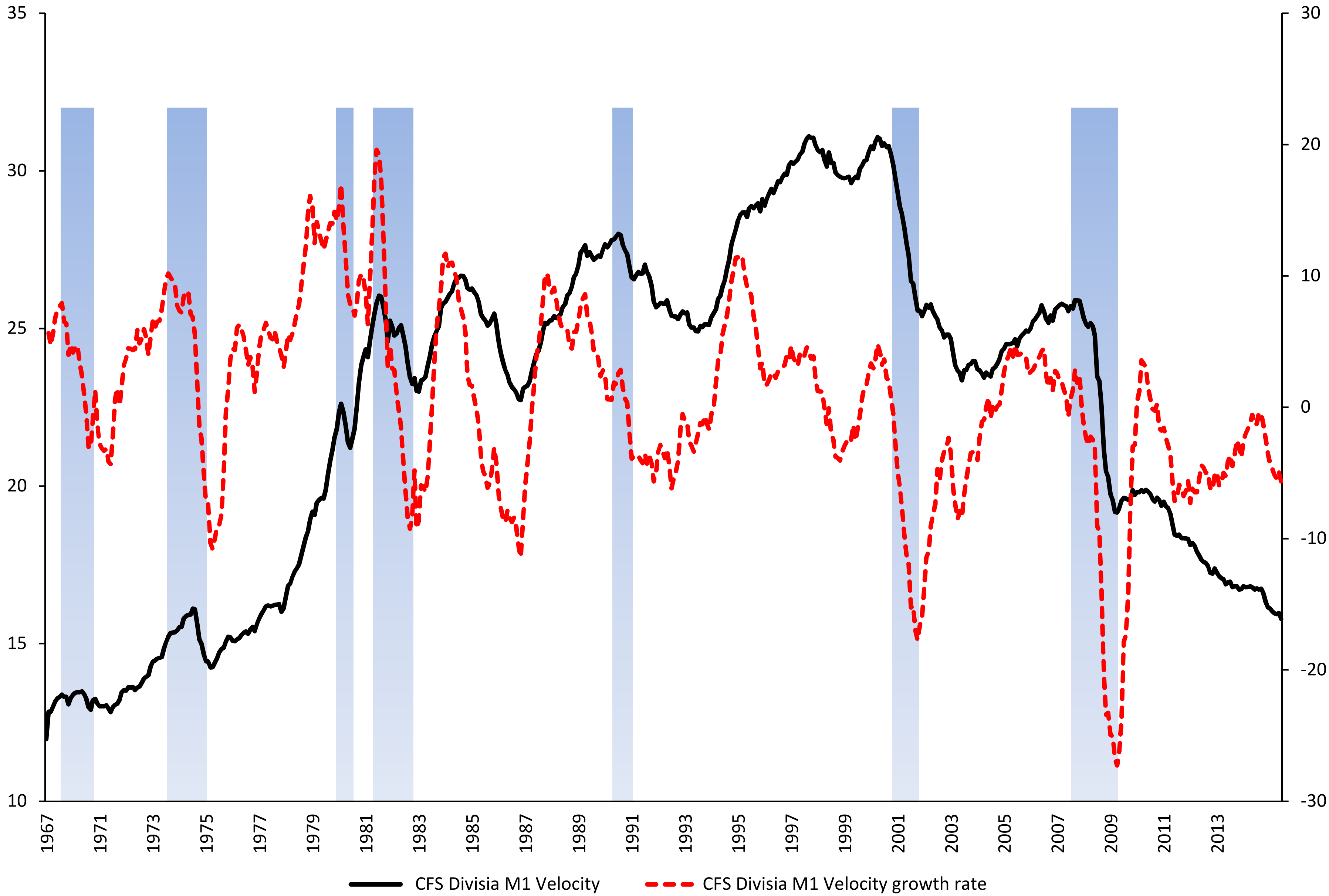

Figure 3. CFS Divisia MZM velocity and its growth rate

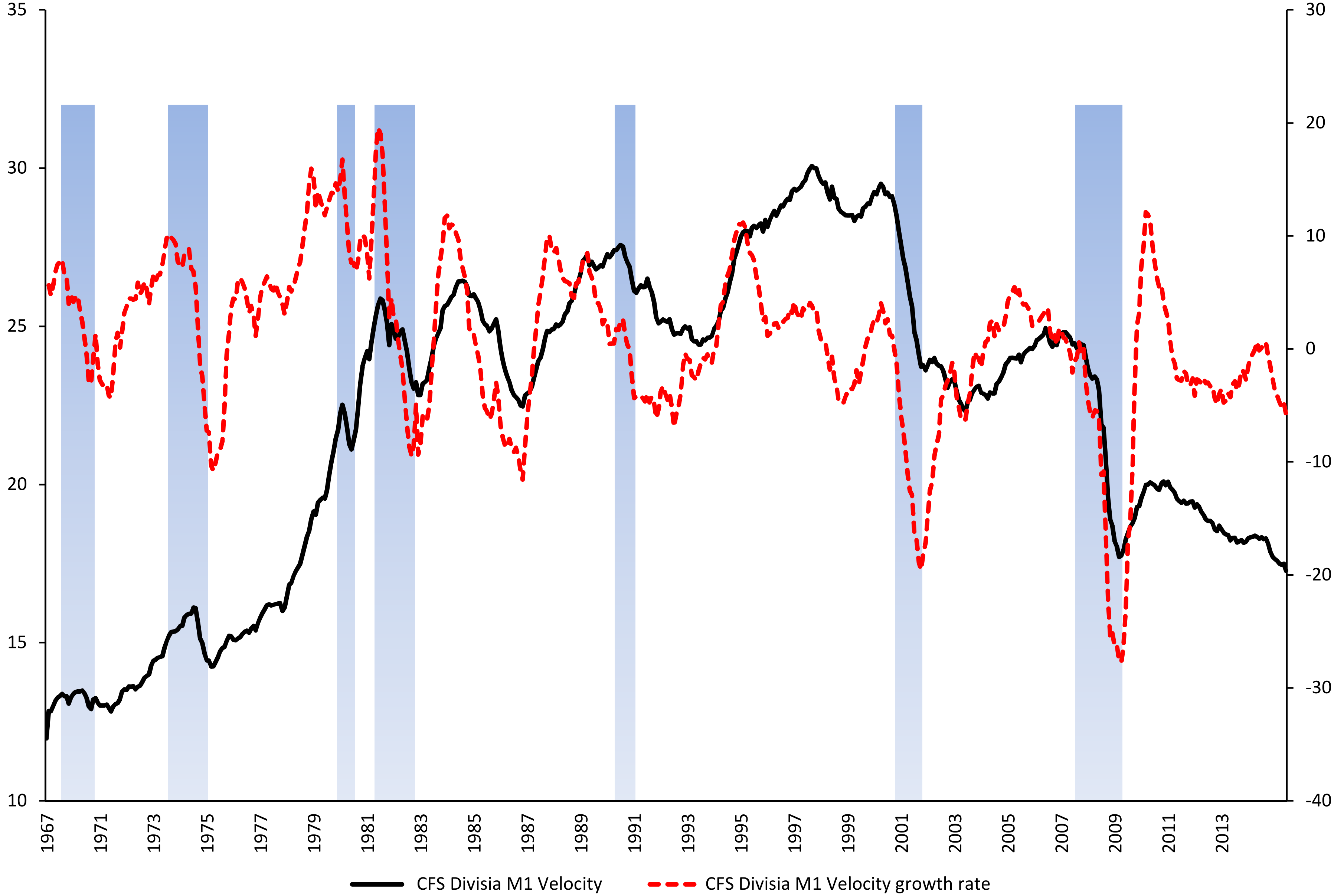

Figure 4. CFS Divisia M2 velocity and its growth rate

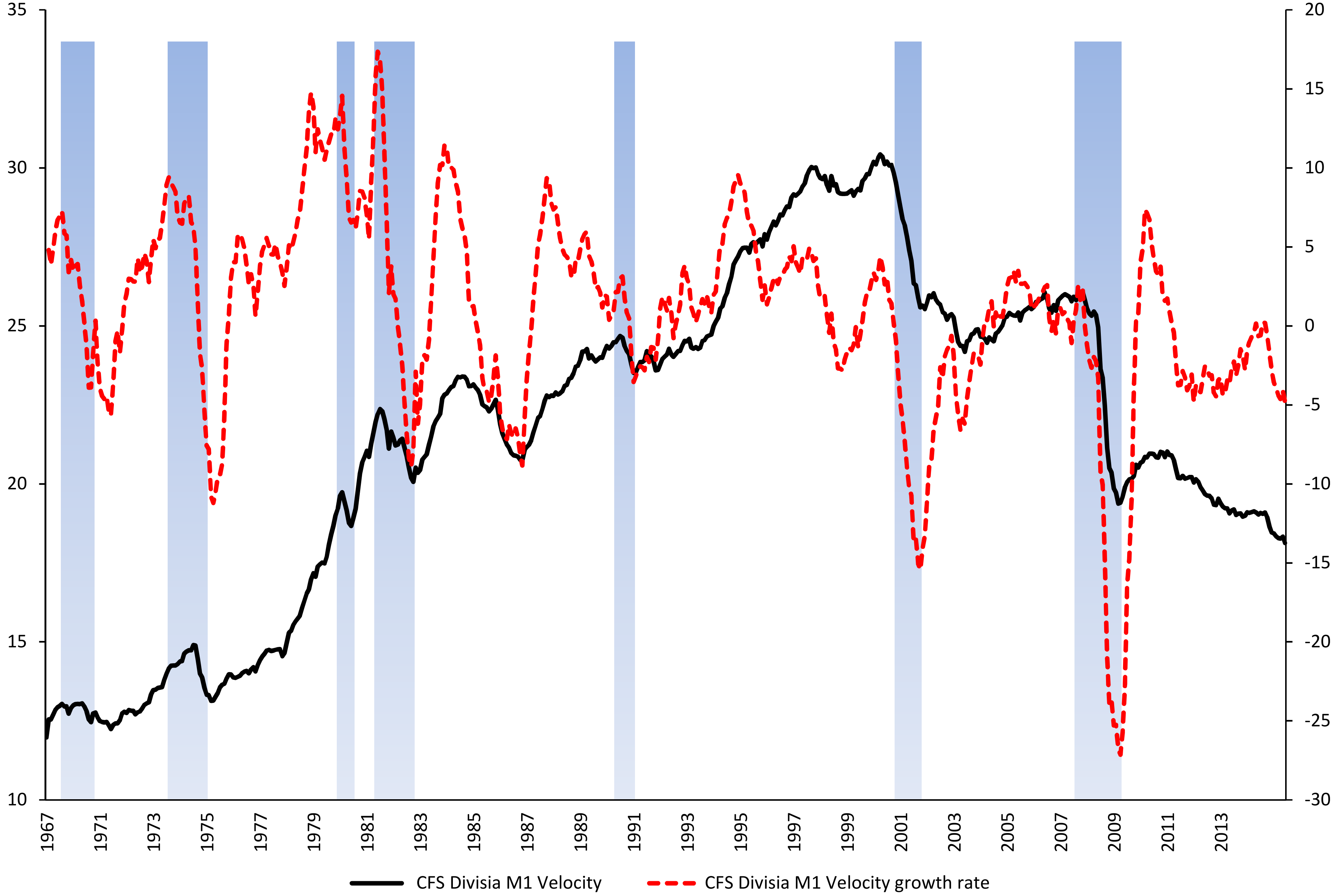

Figure 5. CFS Divisia ALL velocity and its growth rate

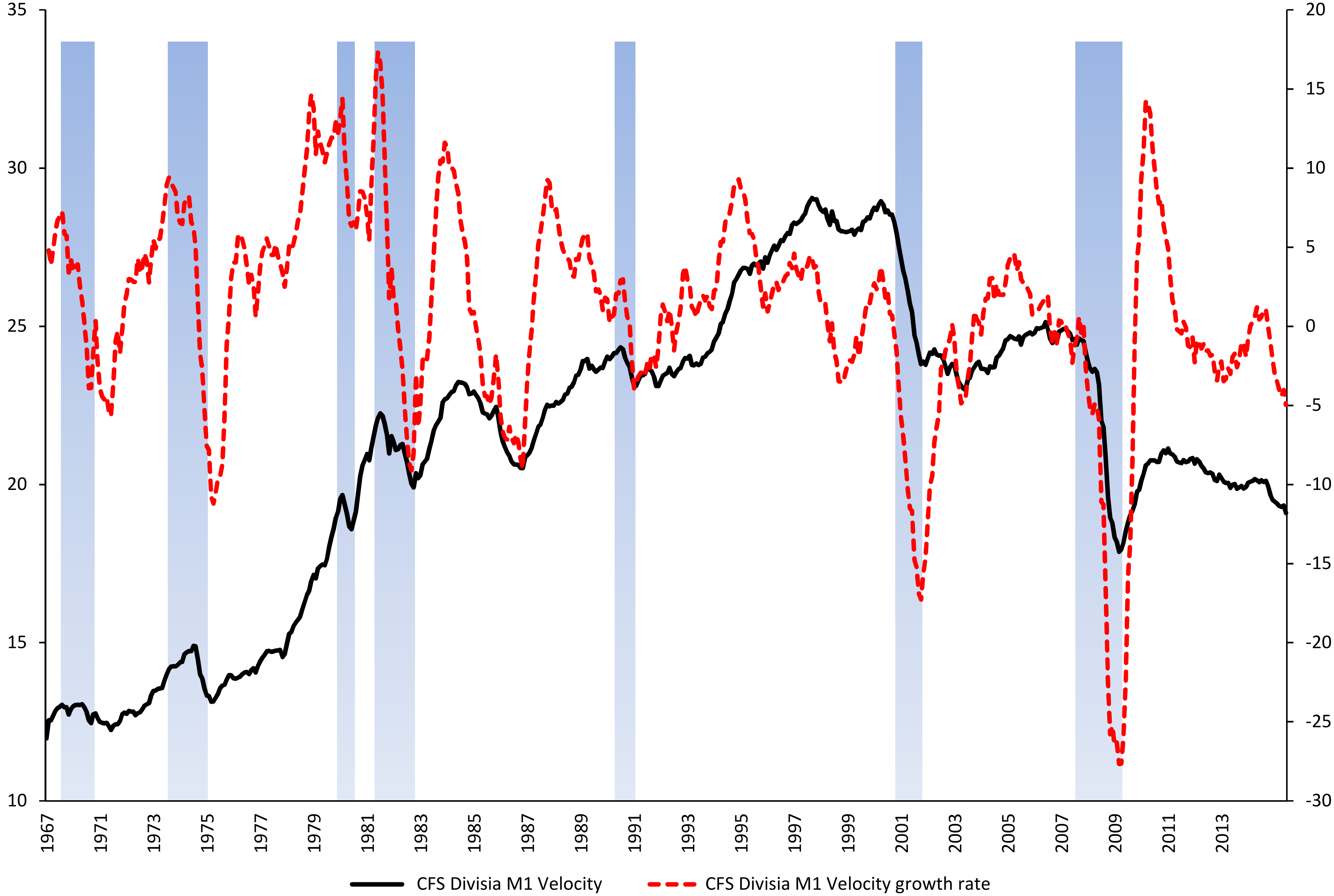

Figure 6. CFS Divisia M3 velocity and its growth rate

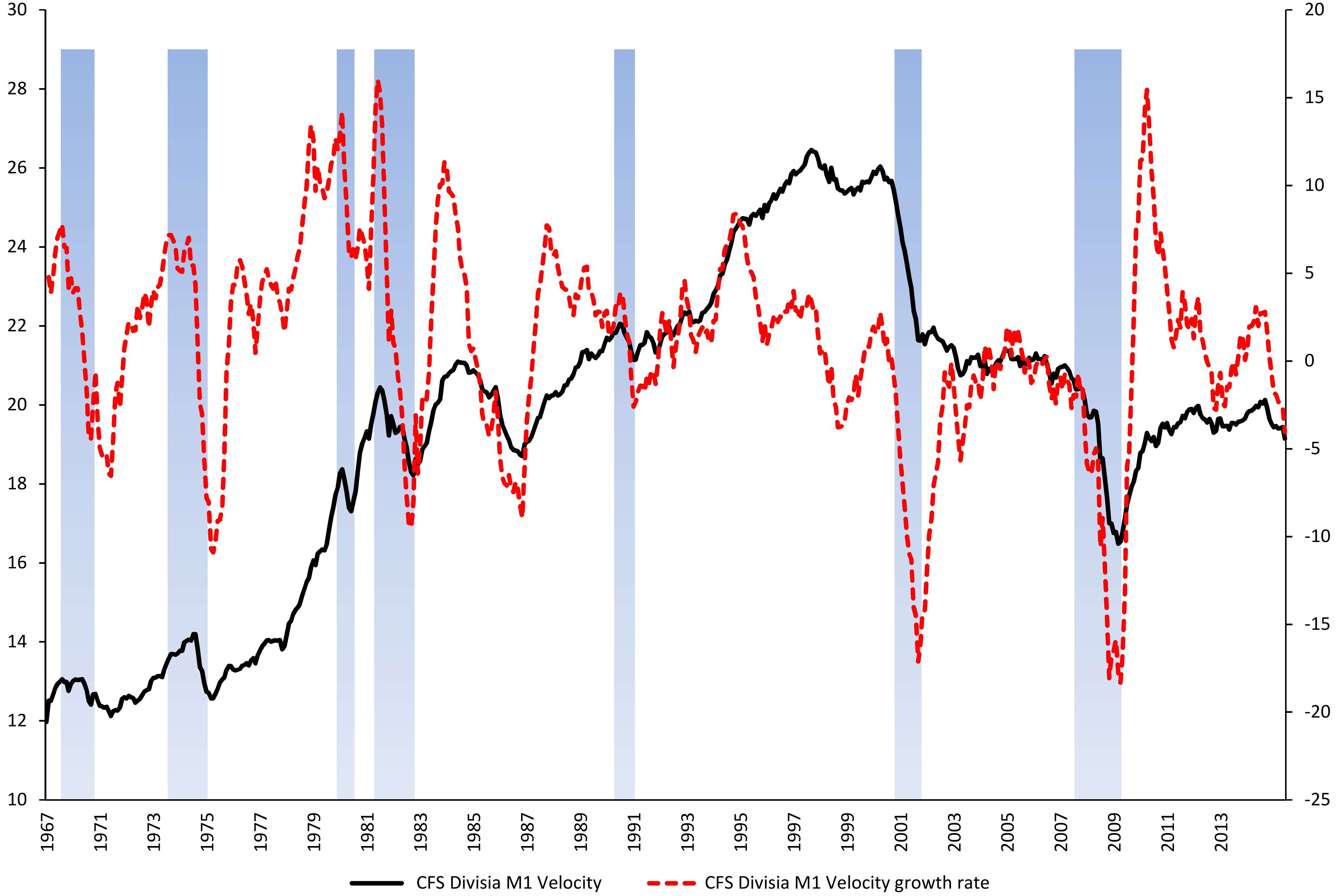

Figure 7. CFS Divisia M4+ velocity and its growth rate

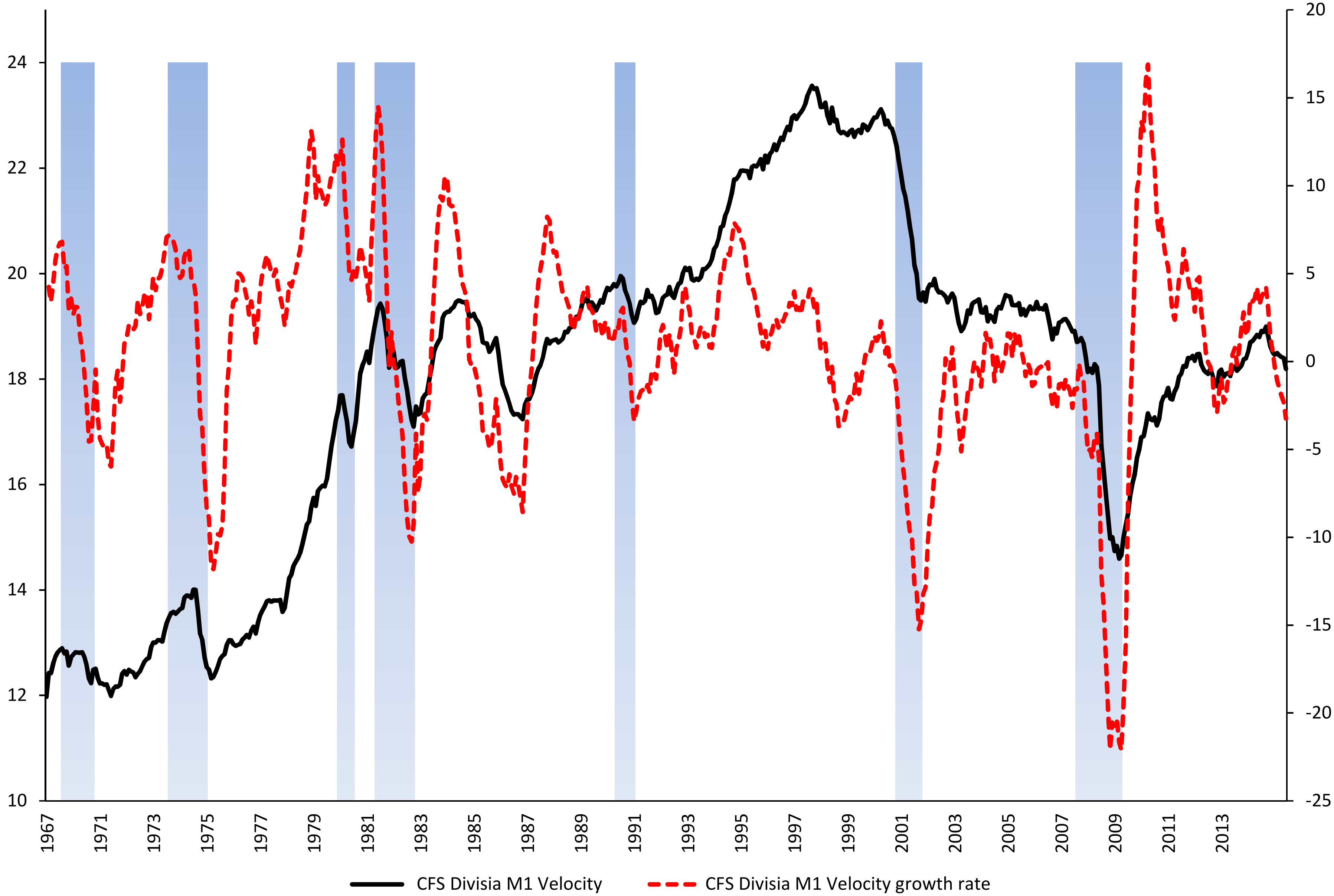

Figure 8. CFS Divisia M4- velocity and its growth rate

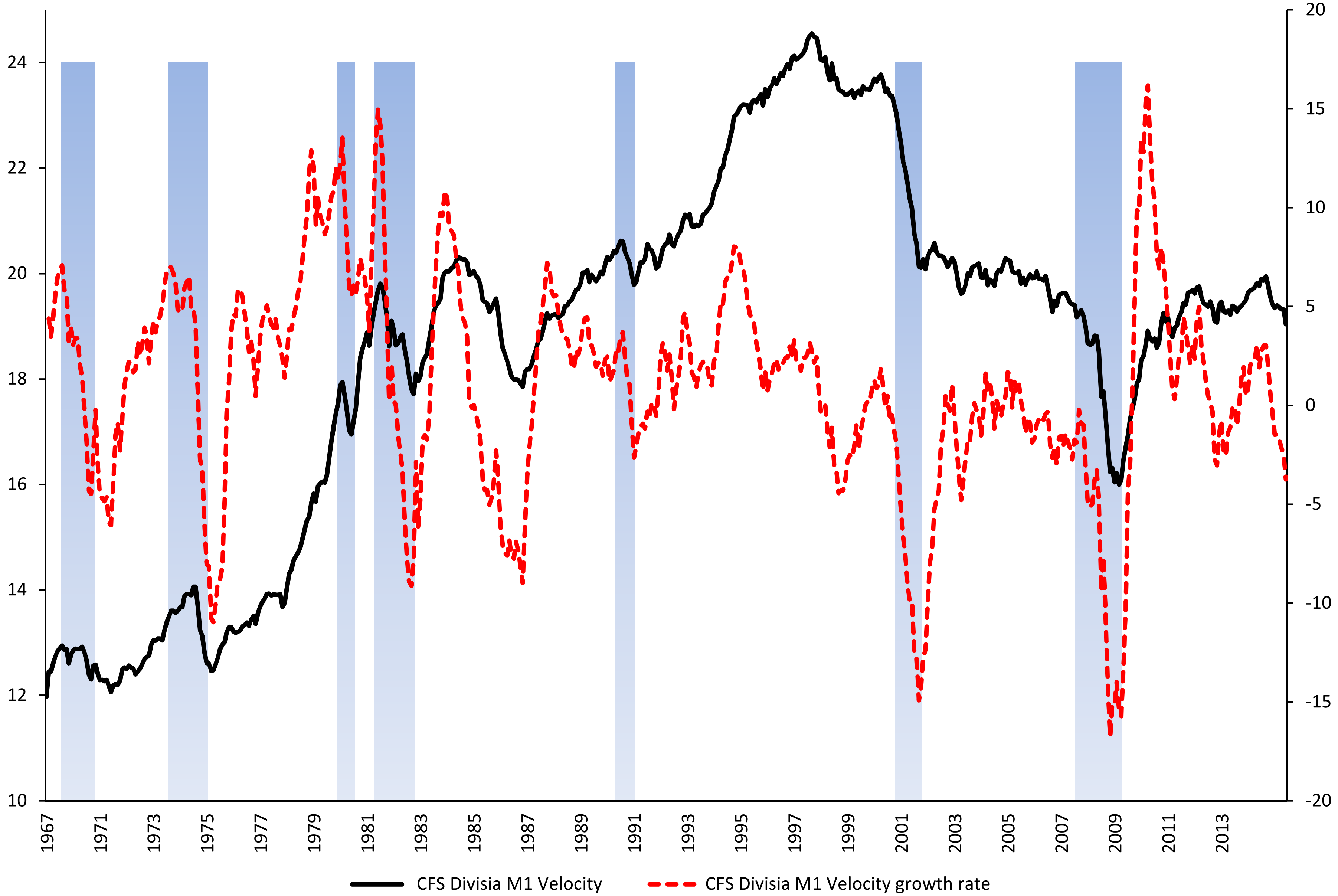

Table 1. Unit Root and stationarity tests

	Series	Unit root tests			KPSS tests	
		DF-GLS	ADF	$Z(t_{\hat{\alpha}})$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
Divisia M1	v_t	-2.306^{*}	-8.736^{**}	17.862^{**}	1.659^{**}	0.092
	μ_t	-3.640^{**}	-8.713^{**}	-20.222^{**}	0.208	0.125
Divisia M2M	v_t	-2.411^{*}	-8.012^{**}	-14.442^{**}	2.178^{**}	0.109
	μ_t	-3.833^{**}	-7.247^{**}	-12.893^{**}	0.569^{*}	0.089
Divisia MZM	v_t	-2.491^{*}	-8.003^{**}	-13.974^{**}	1.627^{**}	0.079
	μ_t	-4.410^{**}	-7.373^{**}	-12.043^{**}	0.222	0.061
Divisia M2	v_t	-2.194^{*}	-8.114^{**}	-15.237^{**}	1.154^{**}	0.107
	μ_t	-3.397^{**}	-7.747^{**}	-14.185^{**}	0.212	0.188^{*}
Divisia ALL	v_t	-2.270^{*}	-8.047^{**}	-14.652^{**}	0.847^{**}	0.077
	μ_t	-4.019^{**}	-7.413^{**}	-12.629^{**}	0.114	0.105
Divisia M3	v_t	-1.932	-9.247^{**}	-16.729^{**}	20.84^{**}	0.116
	μ_t	-2.637^{**}	-7.092^{**}	-15.925^{**}	0.487^{*}	$0.157*$
Divisia M4	v_t	-2.070^{*}	-8.852^{**}	-16.428^{**}	0.544^{*}	0.094
	μ_t	-2.716^{**}	-7.398^{**}	-16.748^{**}	0.920^{**}	0.144
Divisia M4-	v_t	-1.885	-9.409^{**}	-17.341^{**}	0.655^{*}	0.102
	μ_t	-2.648^{**}	-6.824^{**}	-16.260^{**}	0.813^{**}	0.154^{*}
S&P 500	s_t	-2.695^{**}	-23.01^{**}	-23.055^{**}	0.098	0.088

Note: ** indicates a 1% significance level, * indicates a 5% significance level.

Table 2. Tests for serial correlation, arch, and normality

	Series	$Q(12)$	$Q(24)$	$Q^2(12)$	$Q^2(24)$	ARCH(12)	J-B
Divisia M1	v_t	181.443**	197.513**	124.844**	126.805**	133.071**	2212.66**
	μ_t	66.272**	103.046**	56.348**	74.140**	45.606**	2519.89**
Divisia M2M	v_t	405.847**	421.647**	217.674**	223.485**	155.522**	483.133**
	μ_t	442.585**	497.656**	150.720**	161.868**	138.051**	483.011**
Divisia MZM	v_t	410.476**	422.906**	270.501**	278.830**	166.852**	483.07**
	μ_t	454.708**	497.988**	159.889**	177.453**	120.98**	445.838**
Divisia M2	v_t	306.310**	318.064**	226.097**	229.919**	170.477**	825.488**
	μ_t	297.940**	341.882**	106.206**	121.325**	94.423**	478.857**
Divisia ALL	v_t	339.225**	357.878**	273.696**	281.160**	176.036**	755.235**
	μ_t	408.659**	444.767**	122.349**	137.473**	89.678**	380.661**
Divisia M3	v_t	198.419**	218.358**	183.171**	202.488**	86.553**	189.618**
	μ_t	326.587**	387.595**	138.738**	152.110**	115.15**	111.452**
Divisia M4	v_t	218.437**	238.305**	145.423**	157.764**	131.698**	603.831**
	μ_t	281.248**	322.458	118.493**	128.218**	131.698**	181.592**
Divisia M4-	v_t	161.233**	182.657	215.552**	244.881**	96.946**	128.083**
	μ_t	374.722**	449.863**	93.588**	104.469**	123.994**	161.354**
S&P 500	s_t	11.734	20.706	38.453**	46.634**	31.890**	205.263**

Note: ** indicates a 1% significance level, * indicates a 5% significance level.

Table 3

TESTS OF CAUSALITY FROM MONETARY VARIABILTY AND STOCK PRICES TO MONETARY VELOCITY

$$v_t = \alpha_0 + \sum_{j=1}^{r} \alpha_j \nu_{t-j} + \sum_{j=1}^{s} \beta_j VOL_{t-j} + \sum_{j=1}^{k} \gamma_j s_{t-j} + \varepsilon_t$$

	Causality from VOL_t to v_t			Causality from s_t to v_t	
Aggregate	AIC(r, s, k)	F-statistic	p-value	F-statistic	p-value
Divisia M1	$(3, 7, 2)$	2.869	0.006	11.769	0.000
Divisia M2M	$(3, 6, 3)$	4.023	0.001	9.844	0.000
Divisia MZM	$(12, 6, 3)$	2.840	0.010	10.936	0.000
Divisia M2	$(3, 5, 3)$	3.635	0.003	10.500	0.000
Divisia ALL	$(3, 5, 3)$	3.302	0.006	11.850	0.000
Divisia M4+	$(8, 1, 3)$	2.125	0.145	9.549	0.000
Divisia M4-	$(12, 1, 3)$	1.149	0.284	8.871	0.000
Divisia M3	$(12, 1, 3)$	1.020	0.313	9.998	0.000

Table 4. Trivariate Varma, Garch-in-Mean, BEKK Model Parameter Estimates With CFS Divisia M1 Money, Velocity, And Stock Prices

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} 0.005\ (0.029) \\ 0.003\ (0.020) \\ 0.065\ (0.001) \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} -0.407\ (0.016) & -1.172\ (0.053) & 0.011\ (0.726) \\ -0.071\ (0.403) & 0.153\ (0.629) & 0.028\ (0.076) \\ -0.537\ (0.708) & -15.407\ (0.005) & -0.622\ (0.030) \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} 0.223\ (0.001) & 0.497\ (0.020) & 0.022\ (0.024) \\ 0.034\ (0.300) & 0.146\ (0.159) & -0.001\ (0.754) \\ 0.037\ (0.947) & 3.741\ (0.037) & -0.163\ (0.031) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} 0.273\ (0.000) & 0.307\ (0.002) & 0.011\ (0.274) \\ -0.049\ (0.033) & 0.096\ (0.110) & 0.006\ (0.198) \\ -0.796\ (0.074) & 0.746\ (0.402) & -0.139\ (0.074) \end{bmatrix}; \boldsymbol{\Gamma}_4 = \begin{bmatrix} 0.128\ (0.082) & 0.178\ (0.034) & 0.011\ (0.113) \\ 0.034\ (0.256) & 0.032\ (0.329) & -0.004\ (0.100) \\ -1.349\ (0.018) & 0.606\ (0.528) & 0.021\ (0.773) \end{bmatrix};$$

$$\boldsymbol{\Psi} = \begin{bmatrix} -0.752\ (0.008) & -0.065\ (0.829) & 0.073\ (0.064) \\ 0.193\ (0.020) & -0.091\ (0.500) & -0.043\ (0.022) \\ 2.208\ (0.280) & -3.555\ (0.060) & 0.076\ (0.770) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} 0.538\ (0.002) & 1.150\ (0.059) & 0.004\ (0.891) \\ 0.115\ (0.172) & 0.166\ (0.604) & -0.035\ (0.033) \\ -0.230\ (0.872) & 14.367\ (0.008) & 0.584\ (0.040) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{υ_t}	−0.045	1.001	15.835 (0.199)	21.051 (0.050)	39.408 (0.025)	32.434 (0.117)
z_{μ_t}	0.018	0.965	19.862 (0.070)	20.654 (0.056)	50.538 (0.001)	29.355 (0.207)
z_{s_t}	−0.073	0.951	14.545 (0.267)	12.006 (0.445)	20.866 (0.467)	19.701 (0.714)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.001\ (0.084) & 0.000\ (0.022) & -0.014\ (0.001) \\ & 0.000\ (0.995) & 0.000\ (0.994) \\ & & 0.000\ (0.997) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.542\ (0.000) & 0.093\ (0.001) & 4.716\ (0.000) \\ -0.396\ (0.000) & 0.983\ (0.000) & 4.399\ (0.000) \\ -0.103\ (0.000) & -0.003\ (0.515) & 0.395\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.284\ (0.000) & -0.060\ (0.001) & -1.123\ (0.000) \\ -0.141\ (0.255) & 0.377\ (0.000) & -1.584\ (0.001) \\ 0.065\ (0.000) & -0.028\ (0.000) & -0.318\ (0.000) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

Table 5. Trivariate Varma, Garch-in-Mean, BEKK Model Parameter Estimates With CFS Divisia M2M Money, Velocity, And Stock Prices

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} -0.003\ (0.000) \\ -0.005\ (0.000) \\ -0.023\ (0.012) \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} -3.620\ (0.000) & 3.521\ (0.000) & -0.197\ (0.000) \\ -6.631\ (0.000) & 6.227\ (0.000) & -0.376\ (0.000) \\ -34.202\ (0.000) & 22.779\ (0.000) & -2.079\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} 1.118\ (0.000) & -2.176\ (0.000) & 0.033\ (0.145) \\ 1.582\ (0.000) & -3.721\ (0.000) & 0.031\ (0.435) \\ 7.908\ (0.000) & -16.528\ (0.000) & 0.134\ (0.463) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} 0.930\ (0.000) & 0.658\ (0.000) & 0.066\ (0.003) \\ 1.105\ (0.000) & 0.751\ (0.000) & 0.087\ (0.013) \\ 5.572\ (0.000) & 3.029\ (0.000) & 0.506\ (0.001) \end{bmatrix}; \boldsymbol{\Gamma}_4 = \begin{bmatrix} 1.178\ (0.000) & 0.650\ (0.000) & 0.068\ (0.000) \\ 1.854\ (0.000) & 1.068\ (0.000) & 0.128\ (0.000) \\ 8.639\ (0.000) & 5.528\ (0.000) & 0.644\ (0.000) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_5 = \begin{bmatrix} 0.190\ (0.185) & 0.081\ (0.441) & -0.027\ (0.281) \\ 0.334\ (0.163) & 0.191\ (0.272) & -0.038\ (0.356) \\ 1.768\ (0.133) & 1.333\ (0.144) & -0.118\ (0.541) \end{bmatrix};$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.106\ (0.151) & -0.169\ (0.042) & -0.091\ (0.000) \\ 0.192\ (0.005) & -0.372\ (0.000) & -0.074\ (0.006) \\ 0.998\ (0.296) & -3.266\ (0.007) & 0.194\ (0.373) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} 3.848\ (0.000) & -3.694\ (0.000) & 0.204\ (0.000) \\ 6.572\ (0.000) & -5.752\ (0.000) & 0.375\ (0.000) \\ 33.711\ (0.000) & -22.136\ (0.000) & 2.028\ (0.000) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{υ_t}	−0.092	0.981	20.371 (0.060)	14.632 (0.262)	49.578 (0.002)	26.368 (0.335)
z_{μ_t}	0.041	0.954	29.756 (0.003)	14.379 (0.277)	49.315 (0.002)	25.994(0.354)
z_{s_t}	−0.077	0.970	11.043 (0.525)	13.509 (0.333)	18.725 (0.767)	24.215 (0.449)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.004\ (0.000) & 0.000\ (0.775) & 0.000\ (0.988) \\ & 0.002\ (0.000) & 0.002\ (0.729) \\ & & 0.016\ (0.001) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.683\ (0.000) & -0.024\ (0.548) & 0.972\ (0.083) \\ 0.144\ (0.298) & 0.443\ (0.000) & -5.603(0.000) \\ -\ 0.062\ (0.000) & 0.031\ (0.000) & 0.691\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.567\ (0.000) & -0.159\ (0.000) & -0.571\ (0.024) \\ 0.470\ (0.000) & 0.173\ (0.000) & 0.573\ (0.294) \\ 0.038\ (0.000) & -0.056\ (0.000) & -0.123\ (0.008) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

Table 6. Trivariate Varma, Garch-in-Mean, BEKK Model Parameter Estimates With CFS Divisia MZM Money, Velocity, And Stock Prices

A. Conditional mean equation

$$\boldsymbol{\phi}=\begin{bmatrix} -0.014\ (0.018) \\ 0.000\ (0.661) \\ 0.130\ (0.002) \end{bmatrix};\ \boldsymbol{\Gamma}_1=\begin{bmatrix} -0.364\ (0.024) & 0.682\ (0.471) & 0.064\ (0.004) \\ -0.174\ (0.018) & 1.303\ (0.000) & -0.009\ (0.318) \\ -4.161\ (0.001) & -13.496\ (0.016) & -0.492\ (0.001) \end{bmatrix};\ \boldsymbol{\Gamma}_2=\begin{bmatrix} 0.376\ (0.000) & -0.418\ (0.430) & 0.043\ (0.000) \\ 0.001\ (0.969) & -0.543\ (0.000) & 0.003\ (0.548) \\ 0.178\ (0.734) & 5.475\ (0.105) & -0.055\ (0.483) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3=\begin{bmatrix} 0.370\ (0.000) & 0.511\ (0.000) & 0.031\ (0.003) \\ -0.006\ (0.832) & 0.111\ (0.087) & 0.004\ (0.316) \\ -0.370\ (0.477) & -0.292\ (0.755) & 0.062\ (0.430) \end{bmatrix};\ \boldsymbol{\Gamma}_4=\begin{bmatrix} 0.297\ (0.000) & 0.075\ (0.485) & 0.007\ (0.379) \\ 0.094\ (0.001) & 0.032\ (0.602) & 0.007\ (0.066) \\ -0.140\ (0.814) & 1.374\ (0.026) & 0.059\ (0.307) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_5=\begin{bmatrix} -0.028\ (0.550) & -0.180\ (0.054) & 0.000\ (0.972) \\ 0.020\ (0.311) & 0.033\ (0.508) & 0.001\ (0.721) \\ 0.104\ (0.744) & -0.301\ (0.645) & 0.140\ (0.005) \end{bmatrix};$$

$$\boldsymbol{\Psi}=\begin{bmatrix} 0.108\ (0.615) & -2.401\ (0.000) & 0.455\ (0.001) \\ 0.021\ (0.511) & -0.020\ (0.893) & -0.003\ (0.908) \\ -2.727\ (0.014) & 13.562\ (0.000) & -2.762\ (.003) \end{bmatrix};\ \boldsymbol{\Theta}=\begin{bmatrix} 0.642\ (0.000) & -0.946\ (0.324) & -0.047\ (0.030) \\ 0.131\ (0.086) & -0.775\ (0.000) & 0.005\ (0.619) \\ 3.571\ (0.003) & 13.582\ (0.017) & 0.459\ (0.002) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{υ_t}	−0.041	1.012	17.929 (0.118)	28.321 (0.005)	45.965 (0.005)	41.465 (0.015)
z_{μ_t}	0.000	0.989	29.507 (0.003)	15.864 (0.198)	59.076 (0.000)	20.499 (0.668)
z_{s_t}	−0.007	0.987	9.763 (0.637)	17.556 (0.130)	20.295 (0.680)	33.000 (0.104)

C. Conditional variance-covariance structure

$$\boldsymbol{C}=\begin{bmatrix} 0.005\ (0.000) & -0.001\ (0.067) & -0.003\ (0.451) \\ & 0.000\ (0.335) & 0.010\ (0.084) \\ & & 0.003\ (0.831) \end{bmatrix};\ \boldsymbol{B}=\begin{bmatrix} 0.315\ (0.001) & 0.170\ (0.000) & 1.116\ (0.013) \\ -0.561\ (0.000) & 0.980\ (0.000) & -1.373\ (0.002) \\ -0.007\ (0.683) & 0.006\ (0.232) & 0.897\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A}=\begin{bmatrix} 0.567\ (0.00) & -0.138\ (0.000) & -0.360 (0.135) \\ 0.315\ (0.002) & 0.168\ (0.000) & 0.947\ (0.002) \\ -0.012\ (0.270) & -0.026\ (0.000) & -0.231\ (0.000) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

Table 7. Trivariate Varma, Garch-in-Mean, BEKK Model Parameter Estimates With CFS Divisia M2 Money, Velocity, And Stock Prices

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} -0.018\ (0.002) \\ -0.002\ (0.377) \\ -0.099\ (0.044) \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} -1.277\ (0.016) & 3.214\ (0.059) & 0.008\ (0.684) \\ -0.290\ (0.294) & 1.708\ (0.000) & -0.011\ (0.282) \\ -16.864\ (0.008) & 19.300\ (0.159) & -0.282\ (0.179) \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} 0.418\ (0.002) & -1.810\ (0.063) & 0.303\ (0.064) \\ 0.029\ (0.601) & -0.700\ (0.009) & 0.000\ (0.953) \\ 2.876\ (0.041) & -13.978\ (0.084) & 0.078\ (0.565) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} 0.489\ (0.000) & 0.609\ (0.004) & 0.054\ (0.006) \\ 0.010\ (0.859) & 0.112\ (0.164) & 0.006\ (0.498) \\ 2.304\ (0.081) & 3.721\ (0.050) & 3.393\ (0.037) \end{bmatrix}; \boldsymbol{\Gamma}_4 = \begin{bmatrix} 0.592\ (0.001) & 0.225\ (0.201) & 0.036\ (0.032) \\ 0.126\ (0.153) & 0.000\ (0.998) & 0.007\ (0.220) \\ 4.124\ (0.030) & 2.815\ (0.111) & 0.382\ (0.021) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_5 = \begin{bmatrix} 0.133\ (0.108) & 0.093\ (0.618) & 0.011\ (0.377) \\ 0.034\ (0.259) & 0.062\ (0.218) & 0.005\ (0.239) \\ 1.488\ (0.058) & 0.478\ (0.761) & 0.132\ (0.234) \end{bmatrix};$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.188\ (0.066) & -1.532\ (0.000) & 0.313\ (0.000) \\ 0.041\ (0.201) & -0.276\ (0.081) & 0.042\ (0.153) \\ -1.173\ (0.294) & -7.0028\ (0.009) & 2.330\ (0.000) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} 1.438\ (.006) & -3.423\ (0.046) & -0.001\ (0.941) \\ 0.263\ (0.345) & -1.229\ (0.004) & 0.012\ (0.230) \\ 16.612\ (0.009) & -19.497\ (0.153) & 0.285\ (0.170) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{υ_t}	−0.112	0.986	21.632 (0.042)	14.183 (0.289)	52.643 (0.001)	27.492 (0.282)
z_{μ_t}	0.043	0.966	27.394 (0.007)	13.314 (0.347)	50.882 (0.001)	27.003 (0.304)
z_{s_t}	−0.053	0.951	18.457 (0.103)	15.653 (0.208)	33.788 (0.089)	23.819 (0.472)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.004\ (0.000) & -0.001\ (0.000) & -0.011\ (0.003) \\ & 0.001\ (0.000) & 0.008\ (0.067) \\ & & 0.007\ (0.255) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.655\ (0.000) & 0.057\ (0.057) & 0.405\ (0.320) \\ -0.044\ (0.763) & 0.470\ (0.000) & -6.479\ (0.000) \\ 0.003\ (0.762) & -0.009\ (0.064) & 0.766\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.564\ (0.000) & -0.188\ (0.000) & -0.901\ (0.000) \\ 0.358\ (0.000) & 0.223\ (0.000) & 1.302\ (0.000) \\ 0.010\ (0.370) & -0.045\ (0.000) & -0.288\ (0.000) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

Table 8. Trivariate Varma, Garch-in-Mean, BEKK Model Parameter Estimates With CFS Divisia ALL Money, Velocity, And Stock Prices

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} -0.011\ (0.010) \\ 0.005\ (0.005) \\ 0.096\ (0.001) \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} -0.454\ (0.059) & -0.103\ (0.841) & 0.055\ (0.003) \\ 0.037\ (0.708) & -0.029\ (0.858) & -0.010\ (0.342) \\ -5.741\ (0.003) & -9.516\ (0.004) & -0.601\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} 0.289\ (0.000) & -0.032\ (0.900) & 0.039\ (0.000) \\ -0.065\ (0.024) & 0.261\ (0.001) & -0.010\ (0.008) \\ 0.360\ (0.538) & 2.387\ (0.191) & 0.005\ (0.939) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} 0.305\ (0.000) & 0.532\ (0.000) & 0.031\ (0.002) \\ -0.085\ (0.000) & -0.046\ (0.369) & -0.006\ (0.109) \\ -0.032\ (0.941) & 0.296\ (0.758) & 0.104\ (0.143) \end{bmatrix}; \boldsymbol{\Gamma}_4 = \begin{bmatrix} 0.295\ (0.000) & 0.304\ (0.014) & 0.009\ (0.318) \\ -0.048\ (0.047) & -0.083\ (0.111) & 0.002\ (0.585) \\ 0.488\ (0.323) & 1.643\ (0.057) & 0.062\ (0.296) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_5 = \begin{bmatrix} 0.054\ (0.385) & -0.059\ (0.544) & 0.009\ (0.287) \\ 0.008\ (0.717) & 0.056\ (0.165) & 0.004\ (0.171) \\ 0.381\ (0.411) & -0.185\ (0.797) & 0.122\ (0.296) \end{bmatrix}; \boldsymbol{\Gamma}_6 = \begin{bmatrix} 0.024\ (0.579) & -0.150\ (0.109) & -0.001\ (0.866) \\ 0.021\ (0.236) & 0.167\ (0.000) & 0.007\ (0.029) \\ 0.228\ (0.452) & 0.361\ (0.655) & 0.034\ (0.589) \end{bmatrix}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} -0.148\ (0.597) & -1.444\ (0.040) & 0.362\ (0.001) \\ 0.297\ (0.012) & -0.693\ (0.039) & -0.045\ (0.301) \\ -3.485\ (0.020) & 10.962\ (0.013) & -1.642\ (0.020) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} 0.690\ (0.003) & -0.224\ (0.662) & -0.035\ (0.046) \\ -0.064\ (0.506) & 0.595\ (0.000) & 0.001\ (0.904) \\ 5.027\ (0.010) & 9.304\ (0.005) & 0.604\ (0.000) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{υ_t}	−0.069	1.006	18.090 (0.113)	24.758 (0.016)	49.011 (0.002)	37.363 (0.040)
z_{μ_t}	0.030	0.978	24.856 (0.016)	10.549 (0.568)	52.083 (0.001)	15.441 (0.907)
z_{s_t}	−0.033	0.991	7.868 (0.795)	11.409 (0.494)	22.792 (0.532)	26.435 (0.332)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.005\ (0.000) & 0.000\ (0.158) & -0.007\ (0.160) \\ & 0.000\ (0.646) & 0.015\ (0.004) \\ & & 0.001\ (0.978) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.369\ (0.000) & 0.149\ (0.000) & 1.139\ (0.058) \\ -0.577\ (0.000) & 0.961\ (0.000) & -2.092\ (0.004) \\ 0.000\ (0.987) & 0.002\ (0.692) & 0.834\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.523\ (0.000) & -0.114\ (0.000) & -0.083\ (0.779) \\ 0.275\ (0.013) & 0.228\ (0.000) & 1.553\ (0.011) \\ -0.006\ (0.615) & -0.023\ (0.000) & -0.236\ (0.000) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

Table 9. Trivariate Varma, Garch-in-Mean, BEKK Model Parameter Estimates With CFS Divisia M3 Money, Velocity, And Stock Prices

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} -0.002\ (0.731) \\ 0.003\ (0.148) \\ 0.013\ (0.533) \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} -0.517\ (0.040) & -0.559\ (0.303) & 0.015\ (0.524) \\ 0.304\ (0.195) & 1.104\ (0.000) & -0.030\ (0.248) \\ -0.990\ (0.503) & -6.217\ (0.045) & -0.719\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} 0.302\ (0.000) & 0.368\ (0.096) & 0.029\ (0.001) \\ -0.129\ (0.015) & -0.294\ (0.001) & -0.001\ (0.729) \\ 0.302\ (0.484) & 1.450\ (0.244) & -0.008\ (0.881) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} 0.275\ (0.000) & 0.312\ (0.002) & 0.028\ (0.010) \\ -0.053\ (0.194) & -0.024\ (0.730) & -0.008\ (0.297) \\ -0.117\ (0.735) & -0.381\ (0.561) & 0.046\ (0.520) \end{bmatrix}; \boldsymbol{\Gamma}_4 = \begin{bmatrix} 0.028\ (0.000) & 0.273\ (0.012) & 0.014\ (0.084) \\ -0.049\ (0.268) & -0.110\ (0.183) & -0.002\ (0.622) \\ -0.567\ (0.116) & 0.201\ (0.781) & 0.034\ (0.517) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_5 = \begin{bmatrix} -0.023\ (0.663) & -0.043\ (0.645) & 0.004\ (0.622) \\ -0.010\ (0.755) & 0.111\ (0.065) & 0.002\ (0.670) \\ -0.395\ (0.214) & -0.099\ (0.861) & 0.082\ (0.096) \end{bmatrix}; \boldsymbol{\Gamma}_6 = \begin{bmatrix} 0.042\ (0.402) & -0.125\ (0.183) & -0.001\ (0.881) \\ 0.046\ (0.053) & 0.047\ (0.359) & 0.003\ (0.464) \\ 0.361\ (0.187) & 0.706\ (0.231) & 0.057\ (0.147) \end{bmatrix}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.309\ (0.401) & 0.138\ (0.797) & -0.032\ (0.793) \\ 0.166\ (0.140) & -0.163\ (0.355) & -0.080\ (0.195) \\ -0.679\ (0.579) & -5.159\ (0.045) & 0.959\ (0.066) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} 0.733\ (0.004) & 0.596\ (0.282) & -0.014\ (0.555) \\ -0.302\ (0.205) & -0.804\ (0.000) & 0.034\ (0.192) \\ 0.669\ (0.642) & 5.956\ (0.057) & 0.762\ (0.000) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{υ_t}	−0.037	0.982	17.538 (0.131)	19.819 (0.071)	44.828 (0.006)	30.843 (0.158)
z_{μ_t}	0.024	0.961	11.810 (0.461)	6.800 (0.871)	30.294 (0.175)	10.002 (0.995)
z_{s_t}	−0.004	0.987	3.222 (0.994)	18.080 (0.113)	11.779 (0.982)	27.030 (0.303)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.005\ (0.000) & 0.000\ (0.885) & 0.007\ (0.081) \\ & 0.000\ (0.138) & 0.034\ (0.000) \\ & & 0.000\ (0.998) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.398\ (0.001) & 0.028\ (0.376) & -1.851\ (0.036) \\ -0.675\ (0.000) & 0.909\ (0.000) & -2.505\ (0.005) \\ -0.009\ (0.767) & 0.007\ (0.120) & 0.375\ (0.003) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.335\ (0.000) & -0.069\ (0.000) & -0.241\ (0.415) \\ 0.383\ (0.001) & 0.360\ (0.000) & 3.328\ (0.000) \\ 0.048\ (0.000) & 0.001\ (0.676) & -0.130\ (0.006) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

Table 10. Trivariate Varma, Garch-in-Mean, BEKK Model Parameter Estimates With CFS Divisia M4+ WITH TREASURY Money, Velocity, And Stock Prices

A. Conditional mean equation

$$\boldsymbol{\phi}=\begin{bmatrix} -0.001\ (0.797) \\ 0.002\ (0.405) \\ 0.057\ (0.003) \end{bmatrix};\ \boldsymbol{\Gamma}_1=\begin{bmatrix} -0.820\ (0.000) & 0.039\ (0.954) & 0.138\ (0.000) \\ 0.289\ (0.078) & 0.458\ (0.103) & -0.016\ (0.621) \\ -5.085\ (0.000) & -4.827\ (0.059) & -0.055\ (0.737) \end{bmatrix};\ \boldsymbol{\Gamma}_2=\begin{bmatrix} 0.394\ (0.000) & 0.166\ (0.510) & 0.036\ (0.000) \\ -0.098\ (0.080) & -0.067\ (0.544) & 0.000\ (0.925) \\ 0.725\ (0.100) & -0.142\ (0.897) & 0.051\ (0.307) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3=\begin{bmatrix} 0.244\ (0.000) & 0.312\ (0.010) & 0.027\ (0.026) \\ -0.065\ (0.004) & -0.036\ (0.535) & -0.007\ (0.237) \\ -0.026\ (0.924) & -\ 0.153\ (0.752) & 0.132\ (0.012) \end{bmatrix};\ \boldsymbol{\Gamma}_4=\begin{bmatrix} 0.267\ (0.000) & 0.383\ (0.001) & -0.003\ (0.736) \\ -0.054\ (0.054) & -0.118\ (0.42) & 0.004\ (0.207) \\ 0.269\ (0.389) & 0.894\ (0.114) & 0.022\ (0.625) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_5=\begin{bmatrix} 0.030\ (0.593) & 0.049\ (0.645) & 0.001\ (0.878) \\ -0.007\ (0.786) & 0.083\ (0.077) & 0.002\ (0.567) \\ 0.322\ (0.257) & -0.671\ (0.155) & 0.049\ (0.253) \end{bmatrix};$$

$$\boldsymbol{\Psi}=\begin{bmatrix} -0.980\ (0.093) & 2.167\ (0.002) & -0.081\ (0.312) \\ 0.089\ (0.611) & -1.005\ (0.000) & 0.098\ (0.037) \\ -6.078\ (0.017) & -0.775\ (0.663) & 0.511\ (0.272) \end{bmatrix};\ \boldsymbol{\Theta}=\begin{bmatrix} 1.051\ (0.000) & -0.073\ (0.918) & -0.125\ (0.001) \\ -0.286\ (0.084) & -0.137\ (0.627) & 0.014\ (0.660) \\ 4.407\ (0.001) & 3.443\ (0.189) & 0.061\ (0.719) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{υ_t}	−0.012	0.986	17.493 (0.132)	39.895 (0.000)	33.578 (0.093)	59.809 (0.000)
z_{μ_t}	0.014	0.973	15.051 (0.239)	7.948 (0.789)	27.453 (0.284)	59.809 (0.000)
z_{s_t}	0.032	0.979	6.422 (0.893)	6.724 (0.875)	13.195 (0.963)	14.517 (0.934)

C. Conditional variance-covariance structure

$$\boldsymbol{C}=\begin{bmatrix} 0.004\ (0.000) & 0.000\ (0.046) & -0.007\ (0.015) \\ & 0.000\ (0.010) & -0.006\ (0.002) \\ & & 0.000\ (0.999) \end{bmatrix};\ \boldsymbol{B}=\begin{bmatrix} 0.709\ (0.000) & -0.045\ (0.091) & 0.513\ (0.196) \\ -0.324\ (0.001) & 0.845\ (0.000) & 0.901\ (0.129) \\ 0.017\ (0.011) & 0.003\ (0.317) & 0.921\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A}=\begin{bmatrix} 0.159\ (0.001) & 0.065\ (0.000) & 0.977\ (0.000) \\ 0.434\ (0.000) & -0.320\ (0.000) & 2.447\ (0.000) \\ -0.056\ (0.000) & 0.014\ (0.019) & 0.219\ (0.000) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

TABLE 11. TRIVARIATE VARMA, GARCH-IN-MEAN, BEKK MODEL PARAMETER ESTIMATES WITH CFS DIVISIA M4- WITH NO TREASURY MONEY, VELOCITY, AND STOCK PRICES

A. Conditional mean equation

$$\boldsymbol{\phi} = \begin{bmatrix} 0.002\ (0.143) \\ 0.000\ (0.550) \\ -0.013\ (0.231) \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} -0.157\ (0.504) & 0.431\ (0.227) & -0.018\ (0.651) \\ 0.451\ (0.005) & 1.092\ (0.000) & 0.023\ (0.184) \\ -4.647\ (0.151) & 0.702\ (0.672) & 0.694\ (0.008) \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} 0.152\ (0.004) & -0.081\ (0.604) & 0.025\ (0.003) \\ -0.096\ (0.004) & -0.185\ (0.024) & -0.006\ (0.258) \\ 0.571\ (0.249) & -1.165\ (0.298) & 0.085\ (0.255) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} 0.283\ (0.000) & 0.289\ (0.004) & 0.013\ (0.157) \\ -0.111\ (0.007) & -0.088(0.210) & -0.003\ (0.521) \\ 0.435\ (0.515) & 1.153\ (0.142) & 0.141\ (0.119) \end{bmatrix}; \boldsymbol{\Gamma}_4 = \begin{bmatrix} 0.173\ (0.017) & 0.252\ (0.018) & 0.003\ (0.660) \\ -0.042\ (0.342) & -0.209\ (0.007) & -0.005\ (0.229) \\ 1.389\ (0.077) & 1.176\ (0.321) & -0.001\ (0.993) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_5 = \begin{bmatrix} -0.043\ (0.349) & -0.030\ (0.731) & 0.009\ (0.206) \\ -0.013\ (0.719) & 0.157\ (0.014) & 0.000\ (0.999) \\ 0.330\ (0.554) & 0.335\ (0.719) & 0.154\ (0.005) \end{bmatrix}; \boldsymbol{\Gamma}_6 = \begin{bmatrix} 0.017\ (0.637) & -0.238\ (0.021) & 0.006\ (0.402) \\ 0.007\ (0.769) & 0.040\ (0.492) & -0.006\ (0.216) \\ -0.196\ (0.504) & -0.766\ (0.421) & -0.072\ (0.285) \end{bmatrix}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.061\ (0.800) & 0.765\ (0.034) & -0.197\ (0.005) \\ 0.212\ (0.033) & -0.351\ (0.019) & 0.000\ (0.994) \\ -1.977\ (0.134) & 3.046\ (0.161) & 0.282\ (0.646) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} 0.249\ (0.298) & -\ 0.563\ (0.121) & 0.037\ (0.376) \\ -0.428\ (0.008) & -0.797\ (0.000) & -0.024\ (0.158) \\ 4.587\ (0.163) & -0.465\ (0.772) & -0.749\ (0.04) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(12)$	$Q^2(12)$	$Q(24)$	$Q^2(24)$
z_{v_t}	0.012	0.979	25.619 (0.012)	12.235 (0.427)	50.578 (0.001)	27.881 (0.265)
z_{μ_t}	−0.028	0.966	19.414 (0.079)	12.216 (0.429)	37.630 (0.038)	26.156 (0.345)
z_{s_t}	−0.026	0.997	10.405 (0.580)	13.688 (0.321)	17.704 (0.817)	26.597 (0.324)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.003\ (0.001) & 0.000\ (0.644) & -\ 0.029\ (0.000) \\ & 0.000\ (0.100) & 0.000\ (0.100) \\ & & 0.000\ (0.999) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.000\ (0.999) & 0.098\ (0.001) & 2.296\ (0.001) \\ -0.938\ (0.000) & 1.005\ (0.000) & 2.950\ (0.014) \\ -0.113\ (0.000) & -0.009\ (0.255) & -\ 0.551\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.430\ (0.000) & -0.062\ (0.002) & -1.191\ (0.001) \\ 0.327\ (0.003) & 0.363\ (0.000) & 0.920\ (0.152) \\ -0.030\ (0.004) & 0.000\ (0.887) & 0.188\ (0.000) \end{bmatrix}.$$

Note : Sample period, monthly data: January, 1967 to August, 2015. Numbers in parentheses are tail areas of tests.

Table 12. Summary of important parameters from Tables 4-11

	Impact (%) on velocity (v_t)					
	$\sum \hat{\gamma}_{v\mu}$	$\sum \hat{\gamma}_{vs}$	$\hat{\psi}_{v\mu}$	$\hat{\psi}_{v\mu} \times \sigma_\mu$	$\hat{\psi}_{vs}$	$\hat{\psi}_{vs} \times \sigma_s$
Divisia M1	−0.190	0.022	NS	NS	0.073	0.321
Divisia M2M	2.653	−0.03	−0.169	−0.090	−0.091	−0.400
Divisia MZM	0.331	0.138	−2.401	−1.249	0.455	2.002
Divisia M2	2.013	0.393	−1.532	−0.659	0.313	1.377
Divisia ALL	0.836	0.125	−1.444	−0.635	0.362	1.592
Divisia M3	0.953	0.071	NS	NS	NS	NS
Divisia M4+	0.695	0.201	2.167	1.018	NS	NS
Divisia M4-	0.303	0.025	0.765	0.360	−0.197	−0.867